Standard model in the latticized bulk

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We construct the manifestly gauge invariant effective Lagrangian in $3+1$ dimensions describing the standard model in $4+1$ dimensions, following the transverse lattice technique. We incorporate split generation fermions and we explore naturalness for two Higgs configurations: a universal Higgs vacuum expectation value (VEV), common to each transverse brane, and a local Higgs VEV centered on a single brane with discrete exponential attenuation to other branes, emulating the split-generation model. Extra dimensions, with explicit Higgs configurations, do not ameliorate the naturalness problem.

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I. INTRODUCTION

Recently we introduced the low energy effective Lagrangian of an extra-dimensional Yang-Mills gauge theory in which gauge fields, fermions, and scalars propagate in the bulk [1,2]. The idea is to ask how an experimentalist would describe the first few Kaluza-Klein (KK) modes of, e.g., the gluon, seen in the detector in an effective Lagrangian in $3+1$ dimensions. Hidden local symmetry implies a much larger gauge group than $SU(3)_{QCD}$ that is spontaneously broken down to $SU(3)_{QCD}$ at low energies, [3], but how should such a model be wired together to emulate extra dimensions? We find that the solution to this problem is a generalized version of the transverse Wilson lattice of Bardeen, Pearson, and Rabinovici [4], where the key idea is to keep some dimensions in the continuum and treat others on the lattice. This leads to a local gauge invariant $3+1$ effective Lagrangian of the continuum $d+1$ theory with a valid description of it's attendant KK modes in the infrared, all a consequence of universality. The theory is manifestly gauge invariant, renormalizable, and can be viewed as a new class of models within $3+1$ dimensions, with novel hidden internal symmetries, dictated by the embedding constraints into extra dimensions.

For example, QCD in the $4+1$ bulk can be described by a sequence of gauge groups with a common coupling, $\Pi_{i=0}^{N}SU(3)_i$, with N chiral $(3_i,\overline{3}_{i-1})$ Φ_i fields connecting the groups sequentially [1]. This can be viewed as a Wilson action for a transverse lattice in x^5, and is shown explicitly to match a compactified continuum $4+1$ Lagrangian truncated in p^5 momentum space in the axial gauge. Power-law running is driven by excitation of the KK modes with increasing mass scale. The renormalization group is just that of a $3+1$ theory with many thresholds and can be readily treated with conventional threshold decoupling techniques [1]. We find that the transverse lattice is indistinguishable from the naive $4+1$ running up to very large mass scales. We called this an ‘‘aliphatic’’ model, since it corresponds to a compactification between a pair of branes with the field strength $G^a_{\mu 5}=0$ on each brane.[1] The aliphatic model is similar to the orbifold construction and contains no undesirable zero-modes, such as massless Nambu-Goldstone bosons (NGB) associated with zero modes of A^a_5 components of the vector potential.

With periodic boundary conditions the spectrum is changed. The KK modes are doubled. However, in the periodic case a zero mode corresponding to the A^a_5 vector potential component appears in the spectrum. As one element of the present paper, we will explicitly compare and contrast these two different choices of boundary conditions, however we will generally adopt the aliphatic structure throughout to avoid this spurious Nambu-Goldstone boson.

Our approach emphasizes that the transverse lattice is a valid ‘‘completion’’ or renormalizable description of extra dimensions within $3+1$ dimensions. We argued that universality allows us to write down any number of theories that can do this, all yielding the same infrared behavior. The transverse lattice is optimal, in our opinion, and can be extended to any number of dimensions [1]. One can view the transverse lattice as a Higgs boson (for links), or gauged chiral Lagrangian, and any of these descriptions will be equally valid [4]. Another example of a high energy completion is the recent paper [2] which proposes a form of ‘‘technicolor’’ to engineer the effective description of $4+1$ dimensions. Note that by opening the closed moose diagram of Ref. [2] and removing one strong condensate, one obtains the aliphatic case, and removes the unwanted extra Nambu-Goldstone boson.

Our approach can readily be extended to discuss a wide range of issues. One can readily construct a supersymmetric transverse lattice, and one should be able to describe gravitational KK modes in this approach as well [5]. Topological

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[1]The name follows the chemical nomenclature for hydrocarbons; aliphatic means ‘‘in a line.’’

and anomaly questions are also readily addressable, and dynamical issues are also better under control, e.g., for the Nambu–Jona-Lasinio model in extra dimensions the present approach greatly simplifies and better defines that analysis [5]. This is relevant because extra dimensions are intrinsically strongly interacting theories at some high energy scale, M_s, (this can be seen from perturbative unitarity constraints in the $3+1$ theory [1]) and this may play a fundamental role in electroweak symmetry breaking (EWSB) [6]. The present paper is, in some regards, a warm-up exercise to return to the study of dynamical (EWSB) in the transverse lattice formalism.[2]

In the present paper we turn our attention to the full standard model. Our goal presently is not ambitious; rather than constructing a new dynamics for EWSB, we wish to use the usual Higgs mechanism to describe the EWSB in the full standard model, and to understand the immediate ramifications of extra dimensions from the point of view of the latticized effective Lagrangian.

This is a transverse lattice description of a standard model in $4+1$ dimensions in which the gauge fields and fermions and Higgs bosons all live in the bulk [6,7,9]. One simple and immediate result is that the KK-modes γ_n, W_n and Z_n are seen to have a fine structure in their spectrum which follows the mass spectrum of the observed standard model γ, W and Z.

For the description of matter fields, we exploit the fact that chiral fermions can always be engineered with arbitrarily massive vectorlike KK modes (using the method as in Ref. [8]), so we need keep only the chiral zero modes. Indeed, it is an advantage of the $3+1$ formalism that we can do this; in a sense the chiral generations are put in by hand, but they can be localized, or split, arbitrarily throughout the bulk.

Presently we will discuss ''split'' inter-generations [9,10], but not the more esoteric split intra-generations [9]. The latter very interesting case raises anomaly questions that we have not yet explored, but which are under current study [5]. In the present paper we will consider the two cases of (i) a Higgs boson which develops a common vacuum expectation value (VEV) on all branes, and (ii) the split-generation model in which we have a localized VEV and generation hierarchy is explained by the ''distance'' a given generation appears from the localized VEV. This has an elegant formulation in the present mode, and indeed we find in the present approach that the split-generation model is more general than an extra-dimensional scheme and may be viewed as a class of generalized standard models in 3+1 dimensions.

In both Higgs VEV configurations we discuss naturalness. These two cases are extreme limits on the range of possibilities. Unfortunately, it appears that extra dimensions cannot solve the naturalness problem of the electroweak hierarchy with a fundamental Higgs boson.

[2]Our construction is not intended for nonpeturbative computations (at least not in the far infrared).

II. STANDARD MODEL EFFECTIVE LAGRANGIAN

A. Incorporation of QCD; 4+1 boundary conditions versus the 3+1 model structure

We wish to describe the low energy effective Lagrangian of the standard model in $4+1$ dimensions using the transverse lattice. We begin with the QCD content. The spectrum of KK modes is sensitive to the structure of the effective Lagrangian in $3+1$, which in turn depends upon the global boundary conditions of the underlying $4+1$ theory. First we examine the simplest case, the *aliphatic model* corresponding to a linear system with free boundary conditions [1]. Then we examine the *periodic model* in which we link the zeroth and Nth fields together with one extra link-Higgs field. These are distinct global systems with characteristically distinct spectra. Which one occurs depends upon the detailed compactification scheme of nature.

Consider the pure gauge Lagrangian in $3+1$ dimensions

$$\mathcal{L}_{\rm QCD}=-\frac{1}{4}\sum_{i=0}^{N} G^a_{i\mu\nu}G^{i\mu\nu a}+\sum_{i=1}^{N} D_\mu\Phi_i^\dagger D^\mu\Phi_i \qquad (2.1)$$

in which we have $N+1$ gauge groups $SU(3)_i$ and N link-Higgs fields, Φ_i forming $(3_i,\overline{3}_{i-1})$ representations. The covariant derivative is defined as $D_\mu=\partial_\mu+i\tilde{g}_3\Sigma_{i=0}^{N}A^a_{i\mu}T^a_i$. The term $\tilde{g}_3$ is a dimensionless gauge coupling constant that is common to all of the $SU(3)_i$ local gauge symmetries. The physical observed low energy QCD coupling will be $g_3 \propto\widetilde{g_3}/\sqrt{N+1}$. T^a_i are the generators of the ith $SU(3)_i$ gauge symmetry, where a is the color index. Thus, $[T^i,T^j]=0$ for $i\neq j$; T^a_i annihilates a field that is singlet under the $SU(3)_i$; when the covariant derivative acts upon Φ_i we have a commutator of the gauge part with Φ_i, $T^{a\dagger}_i$ acting on the left and T^a_{i-1} acting on the right; the ith field strength is $G^{ai}_{\mu\nu} \propto {\rm tr}T^{ai}[D_\mu,D_\nu]$, etc.

A common renormalizable potential can be constructed for each of the link-Higgs fields

$$V(\Phi_j)=\sum_{j=1}^{N}\{-M^2\,{\rm Tr}(\Phi_j^\dagger\Phi_j)+\lambda_1\,{\rm Tr}(\Phi_j^\dagger\Phi_j)^2 +\lambda_2({\rm Tr}(\Phi_j^\dagger\Phi_j))^2+M'[e^{i\theta}\det(\Phi_j)+{\rm H.c.}]\}. \qquad (2.2)$$

We can always arrange the parameters in the potential such that the diagonal components of each Φ_j develop a vacuum expectation value v, and the Higgs boson and $U(1)$ pseudo NGB (PNGB) are heavy. Hence, we can arrange that each Φ_i becomes effectively a nonlinear-σ model field

$$\Phi_i\rightarrow v\,\exp(i\phi_i^aT_i^a/v). \qquad (2.3)$$

Thus, the Φ_i kinetic terms lead to a mass-squared matrix for the gauge fields

$$\sum_{i=1}^{N}\frac{1}{2}\tilde{g}_3^2v^2(A^a_{(i-1)\mu}-A^a_{i\mu})^2. \qquad (2.4)$$

This mass-squared matrix has the structure of a nearest neighbor coupled oscillator Hamiltonian. It can be written as an $(N+1)\times(N+1)$ matrix sandwiched between the column vector $A=(A^a_{0\mu},A^a_{1\mu},\ldots,A^a_{N\mu})$, and its transpose, as A^TMA, where

$$M=\tfrac{1}{2}\tilde{g}_3^2v^2\begin{pmatrix} 1 & -1 & 0 & \cdots & 0\\ -1 & 2 & -1 & \cdots & 0\\ 0 & -1 & 2 & \cdots & 0\\ & & & \cdots & \\ 0 & 0 & \cdots & -1 & 1\end{pmatrix}. \tag{2.5}$$

We can diagonalize the matrix as follows. The gauge fields A^j_μ can be expressed as real linear combinations of the mass eigenstates $\tilde{A}^n_\mu$ as

$$A^j_\mu=\sum_{n=0}^{N} a_{jn}\tilde{A}^n_\mu. \tag{2.6}$$

The a_{jn} form a normalized eigenvector $(\vec{a}_n)$ associated with the nth $n\neq 0$ eigenvalue and has the following components:

$$a_{jn}=\sqrt{\frac{2}{N+1}}\cos\left(\frac{2j+1}{2}\gamma_n\right),\quad j=0,1,\ldots,N, \tag{2.7}$$

where $\gamma_n=\pi n/(N+1)$ and $\mathbf{a}_0=1/\sqrt{N+1}(1,1,\ldots 1)$. The mass terms take the form

$$\mathcal{L}_{\text{mass}}=\frac{1}{2}\tilde{g}_3^2v^2\sum_{j=1}^{N}(A_{j-1}-A_j)^2 \tag{2.8}$$

$$=2\tilde{g}_3^2v^2\sum_{n=0}^{N}\sin\left(\frac{\gamma_n}{2}\right)^2(\tilde{A}^n)^2. \tag{2.9}$$

Hence the KK tower of masses is

$$M_n=2\tilde{g}_3v\sin\left[\frac{\gamma_n}{2}\right],\quad \gamma_n=\frac{n\pi}{N+1},\quad n=0,1,\ldots,N. \tag{2.10}$$

Thus we see that for small n this system has a geometrical KK tower of masses given by

$$M_n\approx\frac{\tilde{g}_3v\pi n}{(N+1)},\quad n\ll N \tag{2.11}$$

and $n=0$ corresponds to the zero-mode gluon. To match on to the spectrum of the KK modes, we require

$$\frac{\tilde{g}_3v\pi}{(N+1)}=\frac{\pi}{R}, \tag{2.12}$$

where R is defined as the size of the fifth dimension compactified on the line segment with the boundary condition $G_{\mu 5}=0$ (equivalent to an orbifold S_1/Z_2). Hence, the aliphatic system with $SU(3)^{N+1}$ and N Φ_i provides a gauge invariant description of the first n KK modes by generating the same mass spectrum.

The zero mode theory is pure QCD with a massless gluon. The zero-mode trilinear coupling constant is $g_3=\tilde{g}_3/\sqrt{N+1}$ [1]. In a geometric picture, the aliphatic model corresponds to a "transverse lattice" description of a full $4+1$ gauge theory [4], where the $4+1$ theory is compactified between two parallel branes at $x^5=0$ and $x^5=R$ and the boundary conditions on the branes are $G^a_{\mu 5}=-G^a_{5\mu}=0$. These boundary conditions insure that no vector gauge invariant field strength is "observable" on the branes. There is no A^a_5 zero mode (all of the N link-Higgs chiral fields have been eaten to provide longitudinal components to the massive KK mode gluons).

Of course, we can always make a periodic extension of the interval $[0,R]$. This leads to a Lagrangian in which we have $N+1$ branes, hence $N+1$ $SU(3)_i$ as before, but now, $N+1$ linking Φ_i Higgs fields

$$\mathcal{L}=-\frac{1}{4}\sum_{i=0}^{N}G^a_{i\mu\nu}G^{i\mu\nu a}+\sum_{i=0}^{N}D_\mu\Phi_i^\dagger D^\mu\Phi_i. \tag{2.13}$$

We now have the additional Φ_0 which is a $(\overline{3}_0,3_N)$ representation linking the first $SU(3)_0$ gauge group to the last $SU(3)_N$. The resulting gauge field mass-squared term becomes

$$\sum_{i=1}^{N+1}\frac{1}{2}\tilde{g}_3^2v^2(A^a_{(i-1)\mu}-A^a_{i\mu})^2, \tag{2.14}$$

where we identify $A^a_{(N+1)\mu}\equiv A^a_{(0)\mu}$. Thus, the mass-squared matrix is now

$$M^2=\frac{1}{2}\tilde{g}_3^2v^2\begin{pmatrix} 2 & -1 & 0 & \cdots & -1\\ -1 & 2 & -1 & \cdots & 0\\ 0 & -1 & 2 & \cdots & 0\\ & & & \cdots & \\ -1 & 0 & \cdots & -1 & 2\end{pmatrix}. \tag{2.15}$$

The diagonalization is now done with a complex representation (suppressing gauge and Lorentz indices; consider N even)

$$A^j_\mu=\sum_{n=-N/2}^{N/2}a_{jn}\tilde{A}^n_\mu, \tag{2.16}$$

where now

$$a_{jn}=\frac{1}{\sqrt{N+1}}\exp\left(i2\pi\frac{nj}{N+1}\right),\quad j=0,1,\ldots,N. \tag{2.17}$$

Note with this definition A_j is periodic, $A_{(N+1)}=A_0$. Reality of A_i dictates that $\tilde{A}^n=\tilde{A}^{-n*}$. One thus obtains for the mass matrix

$$2\tilde{g}_3^2 v^2 \sum_{n=-N/2}^{N/2} \sin^2\left(\frac{\pi n}{N+1}\right)|\tilde{A}^n|^2. \tag{2.18}$$

The spectrum is now

$$2\tilde{g}_3 v \sin\left(\frac{\pi n}{N+1}\right), \quad n=0,1,2,\ldots,N/2. \tag{2.19}$$

We now require

$$\frac{\tilde{g}_3 v}{(N+1)} = \frac{1}{R}. \tag{2.20}$$

Hence, the periodic system with $SU(3)^{N+1}$ and $N+1$ Φ_i provides a gauge invariant description of the first n doubled KK modes, generating the same mass spectrum as in the aliphatic case up to an overall scale factor of 2. [Note that if N were odd the spectrum would include an additional singlet level with $n=(N+1)/2$.] There remains the zero-mode in the spectrum $n=0$, which is a singlet since the reality condition $\tilde{A}^n = \tilde{A}^{-n*}$ implies that $\tilde{A}^0$ is real. However, every nonzero n corresponds to a degenerate doublet of levels.

The zero-mode theory of periodic boundary conditions contains QCD with a massless gluon and a coupling constant $g_3 = \tilde{g}_3/\sqrt{N+1}$. Now, however, there is an additional component: Since we added one extra link Higgs there is a zero-mode chiral field ϕ_0^a which is not part of the normal low energy spectrum of QCD. This field is a color-octet massless Nambu-Goldstone boson (NGB) mode. It would bind with $q\bar{q}$ and with itself to produce exotic mesons. Most exotic would be a bound state of a gluon and ϕ^a. These exotic states might be heavy, and could decay quickly to normal hadrons, so it is unclear whether they are ruled out. In the case of the electroweak part of the standard model, similar objects would also occur as light Nambu-Goldstone bosons, and are likely problematic.

Since our present goal is to construct a low energy model that is the minimal standard model, we are therefore compelled to use the aliphatic boundary conditions to remove these NGBs. Henceforth, throughout the remainder of the paper we will use the aliphatic constructions with $N+1$ gauge fields and N link-Higgs fields.

B. Incorporating $SU(2)_L \times U(1)_Y$

We now consider the pure gauge Lagrangian in $3+1$ dimensions

$$\mathcal{L}_{ew} = -\frac{1}{4}\sum_{i=0}^{N} F^a_{i\mu\nu}F^{i\mu\nu a} - \frac{1}{4}\sum_{i=0}^{N} F_{i\mu\nu}F^{i\mu\nu} + \sum_{i=1}^{N} D_\mu \Phi_i'^{\dagger} D^\mu \Phi_i' + \sum_{i=1}^{N} D_\mu \phi_i^{\dagger} D^\mu \phi_i. \tag{2.21}$$

Here we have $N+1$ copies of the $SU(2)_L \times U(1)_Y$ electroweak standard model. Thus the gauge group is $\Pi_{i=0}^N SU(2)_{iL} \times U(1)_{iY}$ where $F^a_{i\mu\nu}$ ($F_{i\mu\nu}$) is the $SU(2)_{iL}$ ($U(1)_{iY}$) field strength. The N Φ_i' and ϕ_i are elementary scalars. The Φ_i' carry $SU(2)$ charges $(\frac{1}{2}{}_i, \frac{1}{2}{}^C_{i-1})$, where ^{C} denotes charge conjugation, and the ϕ_i carry weak hypercharges $(Y_i, -Y_{i-1})$. These fields correspond to the links of a transverse Wilson lattice in the fifth dimension, x^5.

Note that we will ultimately specify the ϕ charges to be given by $Y_i = Y = 1/3$ throughout. We must choose ϕ_i to carry less than the smallest common unit of the weak hypercharge of all components of the theory. This serves the purpose of constructing the fermion links, as in mass-mixing operators required for the CKM matrix, out of polynomial operators involving ϕ^p, not allowing fractional powers, p. We cannot strictly use a product link, $\tilde{\Phi} = \Phi'\phi$, which is a slight departure from the pure transverse lattice. In what immediately follows we will write Y as a generic parameter.

We arrange potentials for the Φ_i' and ϕ_i so they each acquire VEVs independent of i. Hence, we can again arrange that each field becomes effectively a nonlinear-σ model

$$\Phi_i' \to v_2 \exp(i\phi_i^a \tau^a/2v_2), \quad \phi_i \to \frac{v_1}{\sqrt{2}}\exp(i\phi_i/v_1). \tag{2.22}$$

Thus, the Φ_i' and ϕ_i kinetic terms lead to a mass-squared matrix for the $SU(2)$ and $U(1)$ gauge fields

$$\sum_{i=1}^{N} \frac{1}{2}\tilde{g}_2^2 v_2^2 (A^a_{(i-1)\mu} - A^a_{i\mu})^2 + \sum_{i=1}^{N} \frac{1}{2}\tilde{g}_1^2 v_1^2 Y^2 (A_{(i-1)\mu} - A_{i\mu})^2. \tag{2.23}$$

The gauge fields A^j_μ can again be expressed as linear combinations of the mass eigenstates $\tilde{A}^n_\mu$ as

$$A^j_\mu = \sum_{n=0}^{N} a_{jn}\tilde{A}^n_\mu \tag{2.24}$$

with (in the aliphatic case)

$$a_{jn} = \sqrt{\frac{2}{N+1}}\cos\left(\frac{2j+1}{2}\gamma_n\right), \quad j=0,1,\ldots,N, \tag{2.25}$$

where $\gamma_n = \pi n/(N+1)$. The mass eigenvalues are

$$M_n^{(2)} = 2\tilde{g}_2 v_2 \sin\left[\frac{\gamma_n}{2}\right], \quad M_n^{(1)} = 2\tilde{g}_1 v_1 Y \sin\left[\frac{\gamma_n}{2}\right],$$

$$\gamma_n = \frac{n\pi}{N+1}, \quad n=0,1,\ldots,N. \tag{2.26}$$

Thus we see that for small n this system has a KK tower of masses given by

$$M_n^{(2)} \approx \frac{\tilde{g}_2 v_2 \pi n}{(N+1)}, \quad M_n^{(1)} \approx \frac{\tilde{g}_1 v_1 Y \pi n}{(N+1)}, \quad n \ll N \tag{2.27}$$

and $n=0$ again corresponds to the zero-mode gauge fields.

To match onto the spectrum of the KK modes, we require

$$\frac{\tilde{g}_2 v_2}{(N+1)} = \frac{\tilde{g}_1 v_1 Y}{(N+1)} = \frac{1}{R}. \tag{2.28}$$

The KK modes should have common values owing to geometry. Thus we require, for matching,

$$\frac{v_2}{v_1} = \frac{\tilde{g}_1 Y}{\tilde{g}_2} = Y \tan\theta_W. \tag{2.29}$$

This corresponds to an aliphatic system with $\mathrm{SU}(2)_L^{N+1} \times \mathrm{U}(1)^{N+1}$ and N Φ_i' and ϕ_i providing a gauge invariant description of the first n KK modes.

The zero modes of this pure gauge theory are described by the effective Lagrangian in $3+1$ dimensions

$$\mathcal{L}_{\text{gauge}} = -\frac{1}{4} F^a_{\mu\nu} F^{\mu\nu a} - \frac{1}{4} F_{\mu\nu} F^{\mu\nu}, \tag{2.30}$$

where $F^a_{\mu\nu}$ $(F_{\mu\nu})$ is the $\mathrm{SU}(2)_L$ $(\mathrm{U}(1)_Y)$ field strength. The physical $\mathrm{SU}(2)_L$ $(\mathrm{U}(1)_Y)$ gauge coupling constant is $g_2 \equiv \tilde{g}_2/\sqrt{N+1}$, $(g_1 \equiv \tilde{g}_1/\sqrt{N+1})$ a consequence of using the expansion of Eq. (2.24). The fact that the physical coupling constants are suppressed by $\sim 1/\sqrt{N}$ is just the classical volume suppression of the coupling in the $(4+1)$-dimensional theory.

III. INCORPORATING ELECTROWEAK HIGGS FIELDS

We now introduce $N+1$ Higgs fields, H_i each transforming as $(\frac{1}{2}{}_i)$ under $\mathrm{SU}(2)_i$ [and singlet under $\mathrm{SU}(2)_j$ $j \neq i$], and with weak hypercharges $Y_i = 1$ (and $Y_j = 0$ $j \neq i$). The Lagrangian for the Higgs fields is

$$\mathcal{L}_{\text{Higgs}} = \sum_{i=0}^{N} (D_\mu H_i)^\dagger (D^\mu H_i) - M_0^2 |H_{i+1} - (\Phi'_{i+1} \phi^3_{i+1}/v_1^3 v_2) H_i|^2 - V(H_i), \tag{3.1}$$

where we identify $H_{N+1} = 0$ in the aliphatic case. Here we have chosen $Y = 1/3$, and thus the ϕ^3 link appears. Note that the second term is a latticized covariant derivative in the x^5 direction. Purely from the point of view of the $3+1$ theory it is advantageous to rewrite Eq. (3.1) as

$$\begin{aligned}\mathcal{L}_{\text{Higgs}} = \sum_{i=0}^{N} [&(D_\mu H_i)^\dagger (D^\mu H_i) - 2M_0^2 |H_i|^2 \\ &+ \lambda' (H_{i+1} (\Phi'_{i+1} \phi^3_{i+1}) H_i^\dagger + \text{H.c.}) - V(H_i)] \\ &+ M_0^2 |H_0|^2 + M_0^2 |H_N|^2.\end{aligned} \tag{3.2}$$

The last terms take care of the difference between H_0, H_N and H_i in the aliphatic case. Note that $\lambda' = M_0^2/v_1^3 v_2$. The theory now appears as a conventional $3+1$ multi-Higgs model with a system of mass terms and higher dimension interactions with the link Higgs 2 models.

First we ignore the Higgs potentials, and we gauge away the chiral field components, so $\Phi_i' = v_2$ and $\phi_i = v_1$. We thus have in Eq. (3.1) the nearest neighbor mass terms

$$\mathcal{L}_{\text{Higgs}} = -\sum_{i=1}^{N} M_0^2 |H_{i-1} - H_i|^2 \tag{3.3}$$

which leads to the spectroscopy

$$M_n^2 = 4 M_0^2 \sin^2\left[\frac{\gamma_n}{2}\right], \quad n = 0, 1, \ldots, N. \tag{3.4}$$

Matching onto the spectrum of the KK modes requires

$$\frac{M_0}{(N+1)} = \frac{1}{R}. \tag{3.5}$$

The eigenfields are given by

$$H^j = \sum_{n=0}^{N} a_{jn} \tilde{H}^n \tag{3.6}$$

with the a_{jn} as in Eq. (2.7).

We now incorporate the Higgs potentials. We consider presently a universal Higgs potential common to each brane i (we will consider a nonuniversal configuration in the subsequent section)

$$V(H_i) = -\tilde{m}^2 H_i^\dagger H_i + \frac{\tilde{\lambda}}{2} (H_i^\dagger H_i)^2. \tag{3.7}$$

The presence of the Higgs potential adds a common mass term $-\tilde{m}^2 \Sigma H_i^\dagger H_i$ to each of the H_i in the Lagrangian. This modifies the eigenvalues

$$M_n^2 = 4 M_0^2 \sin^2\left[\frac{\gamma_n}{2}\right] - \tilde{m}^2, \quad n = 0, 1, \ldots, N. \tag{3.8}$$

We see that $-\tilde{m}^2$ is the mass for the zero mode. Hence the zero-mode Lagrangian corresponds to the standard model with a tachyonic Higgs of negative mass-squared $-\tilde{m}^2$.

Let us go to mass eigenbasis and truncate on the zero mode. Hence the zero-mode Higgs potential is

$$V(\tilde{H}_0) = -\tilde{m}^2 \tilde{H}_0^\dagger \tilde{H}_0 + \frac{\tilde{\lambda}}{2(N+1)} (\tilde{H}_0^\dagger \tilde{H}_0)^2. \tag{3.9}$$

Notice the large suppression factor of the quartic interaction term, a consequence of the normalization of the zero-mode component of the Higgs field. This may be interpreted as the volume suppression of the quartic coupling constant in the extra-dimensional theory. Thus, we define the low energy physical quartic coupling as $\lambda = \tilde{\lambda}/(N+1)$. The VEV of the zero-mode Higgs, $\langle \tilde{H}_0 \rangle = (v_0, 0)^T$, thus becomes $v_0^2 = \tilde{m}^2/\lambda = (N+1)\tilde{m}^2/\tilde{\lambda}$. Substituting the zero-mode Higgs field with VEV, the zero-mode Higgs boson kinetic term becomes

$$\mathcal{L}_{\rm Higgs}=\sum_{j=0}^{N}(D_\mu H_j)^\dagger(D^\mu H_j)$$
$$\rightarrow\frac{1}{(N+1)}\sum_{j=0}^{N}\left|\left(i\tilde{g}_2A^a_{j,\mu}\frac{\tau^a}{2}+i\tilde{g}_1A_{j,\mu}\frac{Y}{2}\right)\begin{pmatrix}v_0\\0\end{pmatrix}\right|^2, \tag{3.10}$$

where the $1/(N+1)$ comes from the zero-mode normalization. We can absorb it into renormalized physical couplings, g_1 and g_2

$$\mathcal{L}_{\rm Higgs}\rightarrow\sum_{j=0}^{N}\left|\left(ig_2A^a_{j,\mu}\frac{\tau^a}{2}+ig_1A_{j,\mu}\frac{Y}{2}\right)\begin{pmatrix}v_0\\0\end{pmatrix}\right|^2. \tag{3.11}$$

These terms may be rewritten in term of W, Z, and γ fields on each brane

$$\mathcal{L}_{\rm Higgs}=\sum_{j=0}^{N}M_W^2W^+_{j\mu}W^{j\mu-}+\frac{1}{2}M_Z^2Z_{j\mu}Z^{j\mu}. \tag{3.12}$$

The W_i and Z_i fields are combined with the Nambu-Goldstone bosons π^a. The combined fields are defined as

$$W^\pm_{j\mu}=(A^1_{j,\mu}\pm iA^2_{j,\mu})/\sqrt{2},$$
$$\gamma_{j,\mu}=\sin\theta A^3_{j,\mu}+\cos\theta A_{j,\mu},$$
$$Z_{j,\mu}=\cos\theta A^3_{j,\mu}-\sin\theta A_{j,\mu}$$
$$=\frac{(\tilde{g}_2A^3_{j,\mu}-\tilde{g}_1A_{j,\mu})}{\sqrt{\tilde{g}_1^2+\tilde{g}_2^2}}, \tag{3.13}$$

where $\gamma_{j,\mu}$ is a photon field, while Z_j ($W_{j,\mu}$) is a Z-boson (W-boson) mode.

The masses M_W and M_Z are universal to all the SU(2) $\times$U(1)'s, i.e., to all branes, and they are just the masses of the W and Z measured in the low energy theory

$$M_W^2=\frac{g_2^2v_0^2}{2}, \tag{3.14}$$

$$M_Z^2=\frac{(g_2^2+g_1^2)v_0^2}{2}, \tag{3.15}$$

where g_1, g_2, and v_0 are measured at low energies.

Combining these expressions with the full KK mass formula, we find that the W, Z, and γ KK towers are given by

$$M_\gamma^{n\,2}=4M_0^2\sin^2\frac{\gamma_n}{2}, \tag{3.16}$$

$$M_W^{n2}=M_W^2+4M_0^2\sin^2\frac{\gamma_n}{2}, \tag{3.17}$$

$$M_Z^{n2}=M_Z^2+4M_0^2\sin^2\frac{\gamma_n}{2}. \tag{3.18}$$

Each of the KK mode levels thus has a fine structure determined by the electroweak symmetry breaking.

IV. INCORPORATING FERMIONS

A. Chiral fermions

In $4+1$ dimensions free fermions are vectorlike. Chiral fermion zero modes can be readily engineered. For example, one can use domain wall kinks in a background field which couples to the fermion like a mass term. This can trap a chiral zero mode on the kink [8]. The magnitude of the kink field away from the domain wall can be arbitrarily large, so the vectorlike fermion masses can be made arbitrarily large, and are not directly related to the compactification scale. This means that we need to be concerned at present *only* with the chiral zero modes. That is, from the point of view of our $3+1$ effective Lagrangian approach, if we are only interested in the fermionic zero modes then we can simply incorporate the chiral fermions by hand.

Consider one complete generation of left-handed quarks and leptons, l_L, q_L which are doublets under the specific $\mathrm{SU}(2)_{jL}$ and carrying weak hypercharges $Y_l=-1$, $Y_q=2/3$ under the $\mathrm{U}(1)_{jY}$; the quarks carry color under $\mathrm{SU}(3)_j$; the fermions are sterile under all other gauge groups $i\neq j$. Likewise, we have right-handed SU(2) singlets, l_R, q_{uR}, and q_{dR} carrying weak hypercharges under the $\mathrm{U}(1)_{jY}$. Additional generations can be incorporated with additional fields.

The chiral fermions of a given generation can be placed at a unique brane, distinct from the others. One could go further and split members within a single generation. In a sense this latter approach would emulate the split-fermion construction of Arkani-Hamed and Schmaltz [9]. It leads us into interesting issues involving anomalies, and Wess-Zumino terms in the present formulation which we prefer to address elsewhere. We will emulate more closely the split family model [10], as we will presently consider a complete anomaly free generation on any given brane.

Let us designate the branes which receive the generations by $j=(j_1,j_2,j_3)$ thus the full fermionic Lagrangian becomes

$$\mathcal{L}_{\rm fermion}=\sum_j(\overline{l}_{j,L}\not{D}_jl_{j,L}+\overline{q}_{j,L}\not{D}_jq_{j,L}+\overline{l}_{j,R}\not{D}_jl_{j,R}$$
$$+\overline{q}_{j,uR}\not{D}_jq_{j,uR}+\overline{q}_{j,dR}\not{D}_jq_{j,dR}), \tag{4.1}$$

where $\not{D}_j=\gamma^\mu(\partial_\mu-i\tilde{g}_2A^a_{j,\mu}\tau^a/2-i\tilde{g}_1A_{j,\mu}Y/2)$, and the sum extends over $j=(j_1,j_2,j_3)$. The couplings to the zero–mode gauge boson of e.g., the quarks, are therefore

$$\mathcal{L}_0=\sum_j(\overline{q}_{j,L}\tilde{\not{D}}\,q_{j,L}+\overline{q}_{j,uR}\tilde{\not{D}}\,q_{j,uR}+\overline{q}_{j,dR}\tilde{\not{D}}\,q_{j,dR}), \tag{4.2}$$

where $\tilde{\not{D}}=\gamma^\mu(\partial_\mu-ig_2\tilde{A}^a_{0,\mu}\tau^a/2-ig_1\tilde{A}_{0,\mu}Y_\psi/2)$, in which g_1 and g_2 are the physical gauge coupling constants.

In the preceding discussion we considered a universal Higgs field in the bulk. This translated into $N+1$ Higgs fields, H_i each transforming as $(\frac{1}{2}{}_i)$ (and singlet under $j\neq i$, and with weak hypercharges $Y_i=1$ and $Y_j=0$ $j\neq i$. This led to the zero-mode gauge fields feeling a Higgs VEV of order $m_H^2/\lambda \sim (N+1)m_H^2/\tilde{\lambda}$, which is the conventional standard model result where λ is the physical (renormalized) low energy quartic coupling. Hence, one requires a tiny and unnaturally small Higgs boson mass, m_H, to generate the electroweak symmetry breaking scale. The power law running of the coupling $\tilde{\lambda}$ brings $\tilde{\lambda}$ at the fundamental high energy scale (M_s) down to a low scale $\lambda=\tilde{\lambda}/(N+1)$. To match on to the measured EW theory, one requires the mass squared in the Higgs potential $m_H^2 \lesssim v_0^2$ which may be viewed as the present electroweak radiative bound, whence $\lambda \lesssim 1$. If one saturates perturbative unitarity and assumes $\tilde{\lambda}\sim 16\pi^2$ at M_s, then the KK tower is bounded by $N\lesssim 16\pi^2$.

We would have expected that the natural scale for the Higgs mass is of order the fundamental scale of the theory, M_s. Can we modify the approach to introducing the Higgs in such a way that the light Higgs boson becomes natural? For example, can we engineer a Higgs mass of order M_s^2/N by judicious choice of the structure of the model?

One possibility is to assume that the Higgs potential is nonuniversal, i.e., takes different values of it's parameters for different values of j. The simplest idea is to assume that a single Higgs on the kth brane has a large negative mass-squared $\sim -m_H^2$ and the Higgs gets a VEV on that brane only. This helps considerably, but does not alleviate the naturalness problem. If $\langle H_k\rangle \sim v$ then we get a gauge mass term $\tilde{g}^2(A_k)^2v^2$ where k is unsummed. However $A_k=A_0/\sqrt{N}+\dots$ so again the zero-mode mass term becomes $\tilde{g}^2(A_0)^2v^2/N \sim g^2A_0^2v^2$. This requires that $v=v_0$, which implies that on the kth brane the Higgs mass is given by $v_0^2=m_H^2/\tilde{\lambda}$. Note that now there is no large $(N+1)$ prefactor. Using perturbative unitarity for $\tilde{\lambda}\lesssim 16\pi^2$, we have an upper limit on $m_H\sim 1$ TeV (the Lee-Quigg Thacker bound [11]). Thus, this localization of the Higgs allows us to raise the scale of the Higgs boson somewhat. However, given that we typically want $N \gg 1$ we require $m_H \ll M_s$, so again we have an unnatural situation. These are the two extreme limits of a zero-momentum VEV and a localized (all momentum) VEV.

Despite the fact that the fundamental Higgs field is unnatural in these schemes, it is interesting to examine a latticized version of the split-generation model. Thus we consider a model in which there is a strongly localized Higgs VEV [10]. We assign the Higgs VEV v_0 only to the 0th brane, then the zero-mode gauge fields acquire masses of order $\tilde{g}^2v_0^2/N \sim g_2^2v_0^2$.

The Higgs VEV exponentially attenuates away from the localization point and fermions that are at various distances from the localized VEV will receive different values. We assume the same structure as in Eq. (3.1) where now the Higgs potentials have an i-dependent mass term:

$$V(H_i)=M_i^2H_i^\dagger H_i+\frac{\lambda_i}{2}(H_i^\dagger H_i)^2. \tag{4.3}$$

For concreteness as an explicit example we choose

$$M_{i=0}^2=-\kappa M^2, \quad M_{i\neq 0}^2=+M^2, \quad \lambda_{i\neq 0}=0, \tag{4.4}$$

where κ is a phenomenological parameter. The full Higgs-only potential can be written

$$V_{\text{Higgs}}=-\tilde{M}^2H_0^\dagger H_0+\frac{\lambda}{2}(H_0^\dagger H_0)^2+\sum_{i=1}^{N}\Lambda^2H_i^\dagger H_i - \sum_{i=0}^{N}(M_0^2H_{i+1}^\dagger H_i+\text{H.c.}), \tag{4.5}$$

where we identify $H_{N+1}=0$ ($H_{N+1}=H_0$) in the aliphatic (periodic) case and thus

$$\tilde{M}^2=\kappa M^2-M_0^2, \quad \Lambda^2=M^2+2M_0^2. \tag{4.6}$$

The equation of motion of the H_i is, thus,

$$\Lambda^2H_i=M_0^2H_{i+1}+M_0^2H_{i-1} \quad (i\geq 1) \tag{4.7}$$

which has the solution $H_{i+1}=\epsilon H_i$ where

$$\epsilon=\frac{\Lambda^2-\sqrt{\Lambda^4-4M_0^4}}{2M_0^2}. \tag{4.8}$$

If we substitute the solution back into the action of Eq. (4.8) we see that we obtain

$$V_{\text{Higgs}}=-\tilde{M}^2H_0^\dagger H_0+\frac{\lambda}{2}(H_0^\dagger H_0)^2-M_0^2H_0^\dagger H_1 \tag{4.9}$$

and we can thus minimize the potential on the zeroth brane as

$$\langle\tilde{H}_0\rangle=\begin{pmatrix}v_0\\0\end{pmatrix}, \tag{4.10}$$

where $v_0^2=M_H^2/\lambda$, $H_1=\epsilon H_0$, and $M_H^2=\kappa M^2-M_0^2+\epsilon M_0^2$.

We can substitute the full dynamical Higgs field into this expression

$$\tilde{H}_n=\begin{pmatrix}v_{0n}+h_n/\sqrt{2}\\0\end{pmatrix} \tag{4.11}$$

and we have

$$v_{0n}=\epsilon^n v_0, \quad h_n=\epsilon^n h_0. \tag{4.12}$$

Now, we substitute into the kinetic terms of Eq. (3.2) to obtain the dynamical Higgs field kinetic term

$$\sum_{n=0}^{N}(D_\mu H_i)^\dagger(D^\mu H_i)\rightarrow\frac{1}{2}\left[1+\sum_{n=1}^{N}\epsilon^{2n}\right](\partial h)^2. \tag{4.13}$$

We see that the dynamical Higgs field has a wave-function renomalization constant

$$Z = \left[1 + \sum_{n=1}^{N} \epsilon^{2n}\right] = \frac{1-\epsilon^{2N+2}}{1-\epsilon^2}. \qquad (4.14)$$

Thus, the physical mass of the Higgs field becomes

$$m_H^2 = 2M_H^2/Z. \qquad (4.15)$$

The Higgs is strongly localized in the limit $M_0^2/\Lambda^2 \to 0$. In this limit $\epsilon \to 0$ and the only Higgs field receiving the VEV is effectively H_0. Then the zero-mode gauge masses are given by $\propto \tilde{g}^2 v_0^2/(N+1) \sim g^2 v_0^2$ and we see that v_0 is indeed the electroweak VEV. Since $v_0^2 \sim M_H^2/\lambda$ we see that $M_H \lesssim 1$ TeV, by perturbative unitarity, $\lambda \lesssim 16\pi^2$. We furthermore see that the physical Higgs is heavy, as $m_H^2 \sim 2M_H^2/Z \sim$ TeV. In this case, $\epsilon \sim M_0^2/\Lambda^2 \ll 1$ implies that $M^2 \gg M_0^2$. The most natural way to generate the EW scale M_H^2 is thus to tune a cancellation between κM^2 and $-M_0^2$ and use small ϵ to account for the hierarchy between M_0^2 and the EW scale.

On the other hand, we can delocalize the Higgs with $\epsilon \to 1-\eta$ and $\eta \ll 1$. Then we see that $Z \to (N+1)$. Now the zero-mode gauge masses are given by $\propto \tilde{g}^2 v_0^2 \sim (N+1) g^2 v_0^2$ and we see that $\sqrt{N+1}\, v_0$ is the electroweak VEV. This recovers the universal Higgs configuration described in Sec. III.

B. Localization and the split-generation model

Restoring the link-Higgs fields for gauge covariance, the nearest neighbor interactions generate a profile for the Higgs field of the form $H_j = \Pi_{i=0}^{j} (\epsilon \Phi_i' \phi_i'^3 / v_2 v_1^3) H_0$, which is the discretized version of the exponential attenuation in x^5 away from the source $H(x^5) \sim \exp(-M|x^5|)H(0)$.

For diagonal masses we consider only the fermions placed on a given brane. If there is a complete family of fermions on the jth brane, it is charged under $SU(3)_j \times SU(2)_j \times U(1)_j$ only. We postulate a coupling to the Higgs field H_j as

$$\mathcal{L}_{\text{Yukawa}} = y_{lj} \overline{l}_{j,L} H_j^c l_{j,uR} + y_{uj} \overline{q}_{j,L} H_j q_{j,uR} + y_{dj} \overline{q}_{j,L} H_j^c q_{j,dR} + \text{H.c.} \qquad (4.16)$$

(H^c is the charge-conjugated Higgs field). These fermions thus acquire masses as $\langle H_j \rangle$ becomes nonzero

$$\to \mathcal{L}_{\text{mass}} = y_{lj} v_0 \epsilon^j \overline{l}_j l_j + y_{uj} v_0 \epsilon^j \overline{u}_j u_j + y_{dj} v_0 \epsilon^j \overline{d}_j d_j. \qquad (4.17)$$

If we place the three fermion generations on different branes $j_1 \neq j_2 \neq j_3$, the diagonal hierarchy between the families is generated through the suppression factors ϵ^{j_i} [10].

The off-diagonal terms in the mass matrix must be generated to give a nontrivial CKM matrix. We specialize to quarks. This mixing now arises through higher dimensional operators corresponding to the overlap of the wave functions of the chiral zero-mode fermions localized on different branes

$$\mathcal{L}_{\text{mixed}} = y_{u,il} \overline{q}_{j_i,L} H_{j_i} \left(\Pi_{l=j_i+1}^{j_l} \frac{\overline{\phi}_l^4}{M_f^4} \right) q_{j_l,uR} + y_{d,il} \overline{q}_{j_i,L} H_{j_i}^c \left(\Pi_{l=j_i+1}^{j_l} \frac{\phi_l^2}{M_f^2} \right) q_{j_l,dR}. \qquad (4.18)$$

We emphasize that the mass scale M_f is new, and is related to the masses of the decoupled vectorlike fermions. The above expression effectly mimics the overlapping of fermion wave functions in the setup of split fermions [9,10]. The suppressed off-diagonal mass terms are therefore

$$\mathcal{L}_{\text{mixed}} = y_{u,il} v_0 (\epsilon')^{4|j_l - j_i|} \epsilon^{j_i} \overline{u}_{j_i,L} u_{j_l,R} + y_{d,il} v_0 (\epsilon')^{2|j_l - j_i|} \epsilon^{j_i} \overline{d}_{j_i,L} d_{j_l,R} + \text{H.c.}, \qquad (4.19)$$

where $\epsilon' = v/M_f$. In this manner a model of the CKM matrix can be generated.

We will not presently address the effective Lagrangian and the phenomenology of the split generations in detail at present, in particular the problematic coupling to the KK modes. As a consequence of splitting, this is nonuniversal and flavor-changing neutral current effects occur [12]. One can live with these by raising the compactification mass scale. Of course, at the end of the day we may view this as a $(3+1)$-dimensional model in which there are many mixing interactions and higher dimension operators giving the hierarchy. Perhaps we can discover new GIM symmetries to suppress such effects.

V. DISCUSSION AND CONCLUSION

In conclusion, we have given a description of the standard model in the bulk as a pure $(3+1)$-dimensional effective theory. One can in principle discard the notion of an extra dimension and view this as an extension of the standard model within $3+1$ dimensions with extra discrete symmetries. The connection to extra dimensions is made through the transverse lattice, and this may be viewed as a manifestly gauge invariant low energy effective theory for an extra-dimensional standard model. Softening the link-Higgs fields to dynamical Higgs fields leaves a renormalizable effective Lagrangian (modulo certain higher dimension operators that are involved in fermion mass and mixing angle physics).

The larger gauge invariance needed to describe KK modes in $3+1$ may be viewed as a consequence of hidden local symmetries required to make renormalizable theories of spin-1 objects [3]. Alternatively, this is the expanding local gauge invariance in the bulk that appears as an extra dimension opens up.

In treating the ϕ weak hypercharge link-Higgs fields we have, strictly speaking, departed somewhat from the pure transverse lattice. In the chiral phase we could have used fractional powers of a ϕ link with $Y=1$ to propagate quarks, but we chose the present decomposition to maintain a polynomial effective Lagrangian.

We do not, alas, gain insights into the problem of naturalness of the Higgs mass and electroweak hierarchy. Many issues remain, however, to be addressed in the context of the general transverse lattice approach to describing extra dimensions [5]. For example, how does a dynamical electroweak symmetry breaking scheme emerge in this description [6]? One thing we see immediately in this approach is the emergence of an embedding of QCD as in SU(3) $\rightarrow$SU(3)$\times$SU(3), etc. This is reminiscent of the structure of Topcolor [13], and suggests that class of extra-dimensional models in which the electroweak symmetry is broken dynamically [6].

We view the transverse lattice approach as providing powerful new insights into the construction of new extensions beyond the standard model within 3+1 model building. Many future applications to SUSY, gravity, topology, strong dynamics, and grand unification are foreseeable.

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