Quantum entropy of the Kerr black hole arising from gravitational perturbation

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The quantum entropy of the Kerr black hole arising from gravitational perturbation is investigated by using the Null tetrad and 't Hooft's brick-wall model. It is shown that the effect of the graviton's spin on the subleading correction is dependent on the square of the spins and the angular momentum per unit mass of the black hole, and the contribution of the logarithmic term to the entropy will be positive, zero, and negative for different value of a/r_+.

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By comparing black hole physics with thermodynamics and from the discovery of black hole evaporation, Bekenstein and Hawking [1,2] found that black hole entropy is proportional to the area of the event horizon, i.e., $S=A_H/4$. This discovery is one of the most profound ones in black hole physics. However, the issue of the exact statistical origin of the black hole entropy, i.e., what degrees of freedom are counted by the entropy of black holes, has remained a challenging one. Recently, much effort has been concentrated on the problem [3–16]. 't Hooft [3] proposed a "brick-wall" model (BWM) in which the black hole entropy is identified with the statistical-mechanical entropy arising from a thermal bath of quantum fields propagating outside the horizon. The BWM was used in the studies of the statistical-mechanical entropy arising from scalar fields for static black holes [3,6,12] and for stationary axisymmetric black holes [9]. The method also can be applied to calculate quantum entropy due to the electromagnetic field for the Reissner-Nordström black hole [11] and for the general static spherical static black holes [17]. Recently, by using the BWM, we [18] investigated the effects of the spin of photons and Dirac particles on the entropies of the Kerr-Newman black hole.

It has been believed that a black hole can exist in thermal equilibrium with a heat bath possessing a characteristic temperature distribution. The heat bath could cause the change of the space-time geometry by back reaction. From the semiclassical Einstein equations $R_{\mu\nu}-\frac{1}{2}g_{\mu\nu}R=\langle T_{\mu\nu}\rangle$, we know that general metric can be approximately written as $g_{\mu\nu}=g^A_{\mu\nu}+g^B_{\mu\nu}$, where $g^A_{\mu\nu}$ represents the classical background space-time and $g^B_{\mu\nu}$ the disturbance. An interesting open question is how the gravitational perturbation affects the entropy of the black holes. The purpose of this paper is to investigate the question by a concrete example, i.e., study the quantum entropy of the Kerr black hole [19] arising from the gravitational disturbance by using the BWM.

A straightforward way to get perturbation equations for gravitation is to insert the general metric just given into the Einstein field equations and then obtain linear equations for the perturbation. But even in the simplest static and spherically symmetric case, to decouple the perturbation equations involves considerable algebraic complexity. Therefore, in the case where the background metric is stationary, the replacement of spherical symmetry by axial symmetry means that to decouple the equations is no longer possible by this way. Fortunately, there is an alternative approach to the problem, which is provided by the null tetrad formalism. In the following, we first introduce the null tetrad to decouple gravitational perturbation equations, then we seek the total number of modes under proper gauge, and after that we calculate a free energy and the quantum entropy of the Kerr black hole.

To decouple gravitational perturbation equations in space-time of the Kerr black hole in the Boyer-Lindquist coordinates (t,r,θ,φ), we introduce the null tetrad

$$
\begin{aligned}
l^A_\mu&=\frac{1}{\Delta}(\Delta,\ -\Sigma,\ 0,\ -a\Delta\sin^2\theta),\\
n^A_\mu&=\frac{1}{2\Sigma}(\Delta,\ \Sigma,\ 0,\ -a\Delta\sin^2\theta),\\
m^A_\mu&=-\frac{\bar{\rho}}{\sqrt{2}}[ia\sin\theta,\ 0,\ -\Sigma,\ -i(r^2+a^2)\sin\theta],\\
\bar{m}^A_\mu&=-\frac{\rho}{\sqrt{2}}[-ia\sin\theta,\ 0,\ -\Sigma,\ i(r^2+a^2)\sin\theta],
\end{aligned}
\tag{1}
$$

here and hereafter the superscript "A" represents the unperturbed values in the stationary space-time, $\rho=-(r-ia\cos\theta)^{-1}$, $\Sigma=r^2+a^2\cos^2\theta$, $\Delta=(r-r_+)(r-r_-)$, $r_\pm=M\pm\sqrt{M^2-a^2}$, and r_+, M, and a represent the radius of the event horizon, the mass, and the angular momentum per unit mass of the Kerr black hole, respectively. The nonvan-

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ishing unperturbed spin coefficients and components of the Weyl tensor of the Kerr black hole in the null tetrad (1) can then be expressed as [20,21]

$$\rho^A=-\frac{1}{r-ia\cos\theta},\quad \beta^A=-\frac{\bar{\rho}\cot\theta}{2\sqrt{2}},$$

$$\pi^A=\frac{ia\rho^2\sin\theta}{\sqrt{2}},\quad \tau^A=-\frac{ia\rho\bar{\rho}\sin\theta}{\sqrt{2}},$$

$$\mu^A=\frac{\rho^2\bar{\rho}\Delta}{2},\quad \gamma^A=\mu^A+\frac{\rho\bar{\rho}(r-M)}{2},$$

$$\alpha^A=\pi^A-\bar{\beta}^A,\quad \psi_2^A=M\rho^3. \tag{2}$$

When the stationary Kerr black hole is gravitationally perturbed by the incidence of gravitational waves, the quantities which vanish in the stationary state will become quantities of the first order of smallness and can be described by ψ_0, ψ_1, ψ_3, ψ_4, κ, σ, λ, and ν. We know from Ref. [20] that, in a linear perturbation theory, ψ_0 and ψ_4 are gauge invariant quantities while ψ_1 and ψ_3 are not. Consequently, we may choose a gauge (i.e., subject the tetrad basis to an infinitesimal rotation) in which ψ_1 and ψ_3 vanish without affecting ψ_0 and ψ_4. If we choose such a gauge and assume that

$$\psi_0=R_{+2}(r)\Theta_{+2}(\theta)e^{-i(Et-m\varphi)},$$

$$\psi_4=\frac{1}{\bar{\rho}^4}R_{-2}(r)\Theta_{-2}(\theta)e^{-i(Et-m\varphi)}$$

$$(E\ \text{and}\ m\ \text{are constants}) \tag{3}$$

we find that κ, σ, λ, and ν can be written as [20]

$$\kappa=-\frac{\sqrt{2}}{6M}\bar{\rho}^2R_{+2}\left(\mathcal{L}_2-\frac{3ia\sin\theta}{\bar{\rho}}\right)\Theta_{+2},$$

$$\sigma=\frac{1}{6M}\frac{\bar{\rho}^2}{\rho}\Theta_{+2}\Delta\left(\mathcal{D}_2^\dagger-\frac{3}{\bar{\rho}}\right)R_{+2},$$

$$\nu=\frac{\sqrt{2}}{6M}\frac{1}{\rho^2}R_{-2}\left(\mathcal{L}_2^\dagger-\frac{3ia\sin\theta}{\bar{\rho}}\right)\Theta_{-2},$$

$$\lambda=\frac{1}{6M}\frac{2}{\bar{\rho}}\Theta_{-2}\left(\mathcal{D}_0-\frac{3}{\bar{\rho}}\right)R_{-2},$$

and decoupled equations are given by [20]

$$(\Delta\mathcal{D}_1\mathcal{D}_2^\dagger-6iEr)R_{+2}(r)=\lambda_{+2}R_{+2}(r),$$

$$(\Delta\mathcal{D}_{-1}^\dagger\mathcal{D}_0+6iEr)R_{-2}(r)=\lambda_{-2}R_{-2}(r),$$

$$(\mathcal{L}_{-1}^\dagger\mathcal{L}_2+6aE\cos\theta)\Theta_{+2}(\theta)=-\lambda_{+2}\Theta_{+2}(\theta),$$

$$(\mathcal{L}_{-1}\mathcal{L}_2^\dagger-6aE\cos\theta)\Theta_{-2}(\theta)=-\lambda_{-2}\Theta_{-2}(\theta), \tag{4}$$

where λ_{+2} and λ_{-2} are separation constants, and

$$\mathcal{D}_n\equiv\frac{\partial}{\partial r}+\frac{i(r^2+a^2)E-ma}{\Delta}+2n\frac{r-M}{\Delta},$$

$$\mathcal{D}_n^\dagger\equiv\frac{\partial}{\partial r}-\frac{i(r^2+a^2)E-ma}{\Delta}+2n\frac{r-M}{\Delta},$$

$$\mathcal{L}_n\equiv\frac{\partial}{\partial\theta}+aE\sin\theta-\frac{m}{\sin\theta}+n\cot\theta,$$

and

$$\mathcal{L}_n^\dagger\equiv\frac{\partial}{\partial\theta}-aE\sin\theta-\frac{m}{\sin\theta}+n\cot\theta.$$

Equation (4) can be explicitly expressed as

$$\Delta\frac{d^2R_s}{dr^2}+6(r-M)\frac{dR_s}{dr}+\left[2s+4isrE+\frac{K_1^2-2isK_1(r-M)}{\Delta}-\lambda_s^2\right]R_s=0\quad(s=+2),$$

$$\Delta\frac{d^2R_s}{dr^2}-2(r-M)\frac{dR_s}{dr}+\left[+4isrE+\frac{K_1^2-2isK_1(r-M)}{\Delta}-\lambda_s^2\right]R_s=0\quad(s=-2),$$

$$\frac{d^2\Theta_s}{d\theta^2}+\cot\theta\frac{d\Theta_s}{d\theta}+\left[2maE-a^2E^2\sin^2\theta-\frac{m^2}{\sin^2\theta}+2asE\cos\theta+\frac{2sm\cos\theta}{\sin^2\theta}-s-s^2\cot^2\theta+\lambda_s^2\right]\Theta_s=0\quad(s=+2),$$

$$\frac{d^2\Theta_s}{d\theta^2}+\cot\theta\frac{d\Theta_s}{d\theta}+\left[2maE-a^2E^2\sin^2\theta-\frac{m^2}{\sin^2\theta}+2asE\cos\theta+\frac{2sm\cos\theta}{\sin^2\theta}+s-s^2\cot^2\theta+\lambda_s^2\right]\Theta_s=0\quad(s=-2), \tag{5}$$

where $K_1=(r^2+a^2)E-ma$. We now adopt the WKB approximation by writing the mode functions as $R_s(r)=\tilde{R}_s(r)e^{-ik_s[E,m,k_s(\theta),r,\theta]r}$, $\Theta_s(\theta)=\tilde{\Theta}_s(\theta)e^{-ik_s(\theta)\theta}$, and supposing that the amplitudes $\tilde{R}_s(r)$ and $\tilde{\Theta}_s(\theta)$ are slowly varying functions, that is to say,

$$\left|\frac{1}{\tilde{R}_s}\frac{d\tilde{R}_s}{dr}\right| \ll |k_s(E,m,k_s(\theta),r,\theta)|, \left|\frac{1}{\tilde{R}_s}\frac{d^2\tilde{R}_s}{dr^2}\right| \ll |k_s(E,m,k_s(\theta),r,\theta)|^2,$$

$$\left|\frac{1}{\tilde{\Theta}_s}\frac{d\tilde{\Theta}_s}{dr}\right| \ll |k_s(\theta)|, \quad \left|\frac{1}{\tilde{\Theta}_s}\frac{d^2\tilde{\Theta}_s}{dr^2}\right| \ll |k_s(\theta)|^2.$$

Thus, from Eqs. (3) and (5), we know that both $k_{+2}(E,m,k_{+2}(\theta),r,\theta)$ for ψ_0 and $k_{-2}(E,m,k_{-2}(\theta),r,\theta)$ for ψ_4 can be expressed as

$$\begin{aligned} k_s(E,m,k_s(\theta),r,\theta)^2 &= \frac{[(r^2+a^2)E-ma]^2}{\Delta^2} + \frac{1}{\Delta}\left(2maE - a^2E^2\sin^2\theta - \frac{m^2}{\sin^2\theta} - k_s(\theta)^2 + 2saE\cos\theta + \frac{2sm\cos\theta}{\sin^2\theta} + s - s^2\cot^2\theta\right) \\ & (s=+2 \text{ for } \psi_0 \text{ and } s=-2 \text{ for } \psi_4). \end{aligned} \tag{6}$$

The number of modes for each component ψ_i with E, m, and k_θ takes the form $n_s(E,m,k_s(\theta)) = (1/\pi)\int d\theta \int_{r_+ + h}^{L} dr k_s(E,m,k_s(\theta),r,\theta)$. Here, we introduced 't Hooft's brick-wall boundary conditions: the gravitational filed wave functions are cut off outside the horizon, i.e., $\psi_0 = \psi_4 = 0$ at Σ_h which stays at a small distance h from the event horizon r_+. There is also an infrared cutoff $\psi_0 = \psi_4 = 0$ at $r = L$, where the infrared cutoff L is chosen so that the quantum gas is inside the null cylinder, a surface where the co-rotation velocity reaches the velocity of light.

Thermal equilibrium between a quantum gas and a stationary axisymmetric black hole at temperature $1/\beta$ is only possible when the gas is rigidly rotating with the angular velocity equal to the velocity of the black hole horizon Ω_H. Therefore, it is rational to assume that the gravitational field is rotating with angular velocity $\Omega_0 = \Omega_H$ near the event horizon. For such an equilibrium ensemble of states, the free energy is

$$\begin{aligned} \beta F &= \int dm \int dk_s(\theta) \int dn_s(E,m,k_s(\theta)) \ln[1 - e^{-\beta(E-\Omega_0 m)}] \\ &= -\beta \int dm \int dk_s(\theta) \int \frac{n_s[E+\Omega_0 m, m, k_s(\theta)]}{e^{\beta E}-1} dE \\ &= -\beta \int \frac{n(E)}{e^{\beta E}-1} dE, \end{aligned} \tag{7}$$

with

$$\begin{aligned} n(E) &\equiv \sum_s n_s(E) \\ &= \sum_s \int dm \int dk_s(\theta) \int n_s[E+\Omega_0 m, m, k_s(\theta)], \end{aligned} \tag{8}$$

where the function $n(E)$ presents the total number of modes with energy less than E. In order to carry out the calculation for the $n(E)$ we recast Eqs. (6) into the forms

$$\begin{aligned} & k_s(E,m,k_s(\theta),r,\theta) \\ &= \sqrt{\frac{-g_{rr}g_{\varphi\varphi}}{g_{tt}g_{\varphi\varphi}-g_{t\varphi}^2}}\left\{(E-m\Omega)^2 + \left(g_{tt} - \frac{g_{t\varphi}^2}{g_{\varphi\varphi}}\right)\left[\frac{k_s(\theta)^2}{g_{\theta\theta}} + \left(\frac{m}{\sqrt{g_{\varphi\varphi}}} - \frac{s\sqrt{g_{\varphi\varphi}}\cos\theta}{g_{\theta\theta}\sin^2\theta}\right)^2 + \frac{s^2}{g_{\theta\theta}}\left(1 - \frac{g_{\varphi\varphi}}{g_{\theta\theta}\sin^2\theta}\right) \right.\right. \\ & \left.\left. \times \cot^2\theta - \frac{s}{g_{\theta\theta}}(1 - 2aE\cos\theta)\right]\right\}^{1/2} \quad (s = \pm 2), \end{aligned} \tag{9}$$

where the function $\Omega \equiv -g_{t\varphi}/g_{\varphi\varphi}$ and its value on the event horizon is equal to Ω_H. Substituting Eq. (9) into Eq. (8) and carrying out the integrations we find

$$\begin{aligned} n(E) &= \frac{4E^3}{3\pi}\left(\frac{\beta_H}{4\pi}\right)^3 \int d\theta \left[\sqrt{g_{\theta\theta}g_{\varphi\varphi}}\left(\frac{1}{h}\frac{\partial g^{rr}}{\partial r} - \left\{\frac{\partial^2 g^{rr}}{\partial r^2} + \frac{3}{2}\frac{\partial g^{rr}}{\partial r}\frac{\partial \ln f}{\partial r} - \frac{2\pi}{\beta_H\sqrt{f}}\left(\frac{1}{g_{\theta\theta}}\frac{\partial g_{\theta\theta}}{\partial r} + \frac{1}{g_{\varphi\varphi}}\frac{\partial g_{\varphi\varphi}}{\partial r}\right) \right.\right.\right. \\ & \left.\left.\left. - \frac{2g_{\varphi\varphi}}{f}\left[\frac{\partial}{\partial r}\left(\frac{g_{t\varphi}}{g_{\varphi\varphi}}\right)\right]^2\right\}\ln\frac{L}{h}\right)\right]_{r_+} + \frac{2s^2E}{\pi}\left(\frac{\beta_H}{4\pi}\right) \\ & \times \int d\theta \left[\sqrt{g_{\theta\theta}g_{\varphi\varphi}}\left(1 - \frac{g_{\varphi\varphi}}{g_{\theta\theta}\sin^2\theta}\right)\frac{\cot^2\theta}{g_{\theta\theta}}\right]_{r_+} \ln\frac{L}{h}, \end{aligned} \tag{10}$$

where $f \equiv -g_{rr}(g_{tt} - g_{t\varphi^2}/g_{\varphi\varphi})$. With the aid of expression (10), we can work out the free energy defined by Eq. (7). Then, the relation between the entropy and the free energy, $S = \beta^2 \partial F/\partial\beta$, shows that

$$\begin{aligned} S &= \frac{A_H}{24\pi\epsilon^2} - \frac{1}{180}\int d\theta \left\{\sqrt{g_{\theta\theta}g_{\varphi\varphi}}\left[\frac{\partial^2 g^{rr}}{\partial r^2} + \frac{3}{2}\frac{\partial g^{rr}}{\partial r}\frac{\partial \ln f}{\partial r} - \frac{2\pi}{\beta\sqrt{f}}\left(\frac{1}{g_{\theta\theta}}\frac{\partial g_{\theta\theta}}{\partial r} + \frac{1}{g_{\varphi\varphi}}\frac{\partial g_{\varphi\varphi}}{\partial r}\right) - \frac{2g_{\varphi\varphi}}{f}\left(\frac{\partial}{\partial r}\frac{g_{t\varphi}}{g_{\varphi\varphi}}\right)^2\right] \right. \\ & \left. + \frac{s^2}{6}\int d\theta \left[\sqrt{g_{\theta\theta}g_{\varphi\varphi}}\left(1 - \frac{g_{\varphi\varphi}}{g_{\theta\theta}\sin^2\theta}\right)\frac{\cot^2\theta}{g_{\theta\theta}}\right]_{r_+}\right\}_{r_+} \ln\frac{\Lambda}{\epsilon}, \end{aligned} \tag{11}$$

where $\delta^2 = 2\epsilon^2/15$ and $\Lambda^2 = L\epsilon^2/h$ [6] (where $\delta = \int_{r_+}^{r_+ + h}\sqrt{g_{rr}}dr \approx 2\sqrt{h/(\partial g^{rr}/\partial r)_{r_+}}$ is the proper distance from the horizon to Σ_h, ϵ is the ultraviolet cutoff, and Λ is the infrared cutoff [22]), and $A_H = \int d\varphi \int d\theta (\sqrt{g_{\theta\theta} g_{\varphi\varphi}})_{r_+}$ is area of the event horizon. By inserting the metric of the Kerr black hole into Eq. (11), we finally find that the quantum entropy of the Kerr black hole due to the gravitational perturbation is given by

$$S = \frac{A_H}{24\pi\epsilon^2} + \left\{ \frac{2}{45} + \frac{s^2}{6}\left[1 - \frac{r_+^2 + a^2}{a r_+}\arctan\left(\frac{a}{r_+}\right)\right]\right\}\ln\frac{\Lambda}{\epsilon}. \tag{12}$$

Several remarks regarding the main result (12) of the paper are in order: (I) The result is different from quantum entropy of the Kerr black hole caused by the scalar field which coincides with that of the Schwarzschild black hole [16], i.e., $S_{\text{Scalar, Kerr}} = A_H/48\pi\epsilon^2 + \frac{1}{45}\ln(\Lambda/\epsilon)$. The disagreement exists even if $s=0$ in Eq. (12) (by an overall 2 factor). The discrepancy is originated by summing over polarizations. (II) Figure 1 shows that the logarithmic term will increase the entropy in range $a/r_+ = [0, 0.319\,366)$; and decrease the entropy for $a/r_+ = (0.319\,366, 1]$. The term does not affect the entropy when $a/r_+ \approx 0.319\,366$. The reason is that the terms for s^2 decrease the entropy except static case, $a=0$. (III) The subleading logarithmic correction of the quantum entropy depends on the spins of the graviton just in quadratic term s^2. We know from each component of the field that the number of modes for every component field contains both terms of the s and s^2. However, the linear terms of s are eliminated by each other when we sum over all components to get the total number of modes. (IV) The contribution of the spins to the logarithmic term is related to the rotation of the black hole. For the static case, i.e., $a=0$, the terms for s^2 in the results (12) vanishes. It is shown that the spins of the particles affect the logarithmic correction of the entropy in case of the interaction between the spins of the particles and the rotation of the black hole takes place. It should be noted that since the ψ_i cannot be decoupled in the Kerr-Newman space-time [20] the quantum entropy of the Kerr-Newman black hole is an interesting problem and is under consideration.

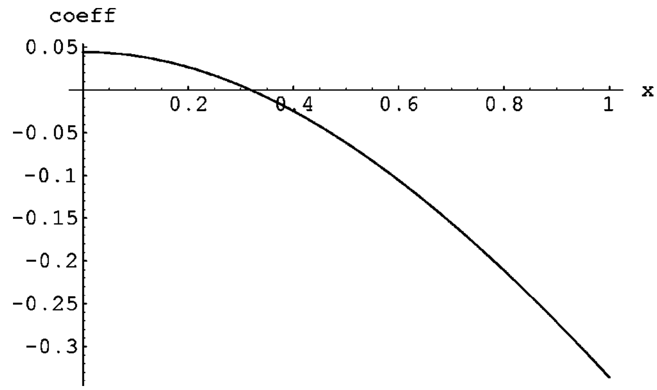

FIG. 1. The coefficient of the logarithm term runs from 0.044 444 4 to $-0.336\,086$ as $x = a/r_+$ from 0 to 1, and is equal to zero at point $x \approx 0.319\,366$. It shows that the logarithmic term increases the entropy in range $a/r_+ = [0, 0.319\,366)$; decreases the entropy for $a/r_+ = (0.319\,366, 1]$; and does not affect the entropy when $a/r_+ \approx 0.319\,366$. The reason is that the terms for s^2 decrease the entropy except static case, $a=0$.

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