Dilatonic domain walls and curved intersecting branes

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Curved, intersecting brane configurations satisfying the type IIA supergravity equations of motion are found. In eleven dimensions, the models are interpreted in terms of orthogonally intersecting M5-branes, where the world-volumes are curved due to the effects of one or more massless scalar fields. Duality symmetries are employed to generate further type II and heterotic solutions. Some cosmological implications are discussed.

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Intersecting brane configurations in supergravity theories have played a fundamental role in uncovering the conjectured duality symmetries that relate the five perturbative string theories. (For a review, see, e.g., Refs. [1,2].) In this paper, we find a wide class of (nonsupersymmetric) intersecting, curved branes within the context of the $D=10$, $N=2$ and $D=11$, $N=1$ supergravity theories. These solutions provide the seeds for generating further configurations through duality transformations. Moreover, since the world-volumes of the branes are curved, they admit a cosmological interpretation and can therefore provide a framework for addressing questions arising in the recently proposed braneworld scenario [3–6]. Intersecting branes with curved world-volumes were recently derived from string dualities in Ref. [7].

The sector of a D-dimensional supergravity action that leads to a solitonic (magnetically charged) p-brane is given by[1]

$$S=\int d^Dx\sqrt{|g|}\left[R-\frac{1}{2}(\nabla\Phi)^2-\frac{1}{2q!}e^{\alpha\Phi}F^2_{(q)}\right], \quad (1)$$

where R is the Ricci curvature scalar of the spacetime with metric, g_{AB}, $g\equiv\det g_{AB}$, and the constant, α, parametrizes the coupling between the form field and the D-dimensional dilaton, Φ.

A domain wall in D dimensions may be viewed as a solitonic $(D-2)$-brane supported by a 0-form field strength, i.e., a cosmological constant, $F^2_{(0)}=\Lambda^2$. The metric and dilaton field are given by

$$ds^2_D=H^m f_{\mu\nu}dx^\mu dx^\nu+H^n dy^2 \quad (2)$$

$$e^\Phi=H^{-2\alpha/\Delta}, \quad (3)$$

where

$$m\equiv\frac{4}{\Delta(D-2)},\quad n\equiv\frac{4(D-1)}{\Delta(D-2)},$$

$$\Delta\equiv\alpha^2-\frac{2(D-1)}{D-2} \quad (4)$$

are constants [8]. The harmonic function is given by $H=1+m|y|$, where $m^2=\Delta\Lambda^2/4$ and y is the coordinate on the transverse dimension. The mass parameter, m, should be viewed as a piecewise constant of integration arising from a dual reformulation of action (1) in terms of a D-form field strength. The world-volume metric, $f_{\mu\nu}$, is the metric on a $(D-1)$-dimensional Ricci-flat spacetime [9].

In the present analysis, we consider the more general metric ansatz [10–12]:

$$ds^2_D=H^m f_{\mu\nu}dx^\mu dx^\nu+e^{2B}H^n dy^2, \quad (5)$$

where $\{m,n,H(y)\}$ are defined as above and the function $B=B(x)$ depends only on the world-volume coordinates. It can then be verified by direct substitution that the field equations derived by varying the action (1) are solved if the metric and dilaton field are given by [10–12]

$$ds^2_D=H^m e^{-(2/Q(D-3))\varphi}\tilde{f}_{\mu\nu}dx^\mu dx^\nu+H^n e^{(2/Q)\varphi}dy^2 \quad (6)$$

and

$$e^\Phi=e^{-(2/\alpha Q)\varphi}H^{-2\alpha/\Delta}, \quad (7)$$

respectively, where

$$\varphi\equiv QB,\quad Q\equiv\sqrt{2}\left[1+\frac{2}{\alpha^2}+\frac{1}{D-3}\right]^{1/2}, \quad (8)$$

$$\tilde{f}_{\mu\nu}=\Theta^2 f_{\mu\nu},\quad \Theta^2\equiv e^{(2/Q(D-3))\varphi} \quad (9)$$

and $\{\tilde{f}_{\mu\nu},\varphi\}$ satisfy the $(D-1)$-dimensional Einstein field equations for a massless, minimally coupled scalar field:

$${}^{(D-1)}\tilde{R}_{\mu\nu}=\frac{1}{2}\tilde{\nabla}_\mu\varphi\tilde{\nabla}_\nu\varphi,\quad \tilde{\nabla}^2\varphi=0. \quad (10)$$

A tilde denotes quantities calculated with the conformally transformed world-volume metric, $\tilde{f}_{\mu\nu}$. Since the dependence of the fields on the transverse coordinate is *identical* to that of the Ricci-flat solution (2), (3), Eqs. (6), (7) represent

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[1]In this paper, spacetime has the signature $(-,+,\ldots,+)$. Variables in ten and eleven dimensions are represented with the accents ˆ and ˇ, respectively. Upper case, Latin indices take values in the range $A=(0,1,\ldots,D-1)$. Lower case Greek (Latin) indices correspond to world-volume (transverse space) coordinates. A totally antisymmetric p form is defined by $A_{(p)}=(1/p!)A_{A_1\ldots A_p}dx^{A_1}\wedge\ldots\wedge dx^{A_p}$ and has a field strength given by the exterior derivative $F_{(p+1)}=dA_{(p)}$. The Chern-Simons terms arising in the bosonic sectors of the theories are trivial for the configurations we consider.

a curved dilatonic domain wall, where the curvature of the world-volume is induced by the nontrivial variation of the modulus field, φ. This field arises when the transverse space depends directly on the world-volume coordinates in an appropriate way.

We now develop curved intersecting branes in the type IIA supergravity theory. The Neveu-Schwarz–Neveu-Schwarz (NS-NS) sector of the massless theory consists of the graviton, $\hat{g}_{AB}$, the dilaton, $\hat{\Phi}$, and the two-form potential, $\hat{A}_{(2)}$. The Ramond-Ramond (RR) sector is comprised of one-form and three-form potentials, $\hat{A}_{(1)}$ and $\hat{A}_{(3)}$ [13]. The theory admits a 5-brane supported by the NS-NS two-form (NS5-brane) and a D4- and D6-brane supported by the RR three-form and one-forms, respectively. The massless type IIA theory may be derived from the Kaluza-Klein reduction of eleven-dimensional supergravity (M theory) on a circle, S^1. The bosonic sector of this latter theory is given by Eq. (1), where $(D,\alpha,q)=(11,0,4)$. The ten- and eleven-dimensional metrics are related by $d\check{s}^2=e^{-\hat{\Phi}/6}d\hat{s}^2+e^{4\hat{\Phi}/3}(dz_{11}+\hat{A}_{(1)B}dx^B)^2$, where z_{11} denotes the coordinate parametrizing the circle and the conformal factors are chosen such that the ten-dimensional spacetime is the Einstein-frame metric [14]. The corresponding field strengths are related by $\check{F}_{(4)}=\hat{F}_{(4)}+\hat{F}_{(3)}\wedge(dz_{11}+\hat{A}_{(1)})$.

We consider compactifications of type IIA supergravity to six dimensions that are associated with the "wrapping" of a solitonic p-brane. A brane is said to be wrapped when the internal components of an antisymmetric tensor field are placed onto a manifold, X, such that the form-field has a nontrivial flux on that space. The nature of the wrapping is determined by the Betti numbers, b_m, of X, corresponding to the number of independent m-cycles in the internal manifold, or equivalently, to the dimensionality of the cohomology class, $H^m(X)$ (the set of all harmonic m-forms that are closed but not exact). In general, wrapping a p-brane around a given m-cycle in X leads to a solitonic $(p-m)$-brane. The simplest compactifying space is the n-dimensional torus, T^n, with Betti numbers $b_m=n!/[m!(n-m)!]$. The four-torus therefore admits six harmonic two-forms, $dy^a\wedge dy^b$ $(a,b=1,2,3,4)$, and this implies that there exist three self-dual two-forms and three anti–self-dual two-forms on this space:

$$\begin{aligned} J_{\pm}^{(1)}&=dy^1\wedge dy^4\pm dy^2\wedge dy^3\\ J_{\pm}^{(2)}&=dy^2\wedge dy^4\pm dy^3\wedge dy^1\\ J_{\pm}^{(3)}&=dy^3\wedge dy^4\pm dy^1\wedge dy^2. \end{aligned}\tag{11}$$

Thus, we may wrap the NS-NS two-form potential around the corresponding two-cycles by invoking the ansatz $\hat{A}_{(2)}=b(x)J_{\pm}^{(i)}$, where $b=b(x)$ is a scalar function that is constant over the internal manifold. When the other form fields are trivial, the truncated type IIA action is given by Eq. (1), where $(D,\alpha,q)=(10,-1,3)$. Compactification of this action on T^4 leads to ten moduli fields arising from the SL(4,R)$\times R$ toroidal symmetry. For simplicity, we consider only the dynamics of the breathing mode, defined by $\sigma\equiv-\ln[\int_X\sqrt{g}]$, where g is the determinant of the metric on T^4. This is equivalent to choosing the metric ansatz

$$d\hat{s}^2=e^{\sigma/2}ds_6^2+e^{-\sigma/2}ds_4^2(X),\tag{12}$$

where the conformal factor in front of the six-dimensional line element ensures that the standard Einstein-Hilbert action is recovered in the lower dimensions. Assuming the ten-dimensional dilaton and the breathing mode to be constant on T^4 and, furthermore, that the two fields are related by $\hat{\Phi}=-\sigma$, then implies that the reduced six-dimensional action can be written as

$$S=\int d^6x\sqrt{|g|}\left[R-\frac{1}{2}(\nabla x)^2-\frac{1}{2}e^{-\sqrt{2}x}(\nabla b)^2\right],\tag{13}$$

where $x\equiv\sqrt{2}\hat{\Phi}$.

Since the axion field, b, arises only through a total derivative, a generalized Scherk-Schwarz dimensional reduction to five dimensions may now be performed [15]. Compactifying on a circle such that $ds_6^2=e^{-2\hat{\Phi}/3}ds_5^2+e^{2\hat{\Phi}}dy_6^2$, and allowing the axion to have a linear dependence on the compactifying coordinate, $b=\Lambda y_6$, results in a five-dimensional action of the form (1), where $(D,\alpha,q)=(5,\sqrt{14/3},0)$ and $\Phi\equiv-\sqrt{14/3}\hat{\Phi}$.

Thus, the theory admits a curved domain wall (three-brane), where $\Delta=2$. The five-dimensional line element is

$$ds_5^2=H^{2/3}e^{-\varphi/Q}\tilde{f}_{\mu\nu}dx^\mu dx^\nu+H^{8/3}e^{2\varphi/Q}dy_5^2,\tag{14}$$

where $\{\tilde{f}_{\mu\nu},\varphi\}$ satisfy the four-dimensional Einstein equations (10) of general relativity, $Q\equiv\sqrt{27/7}$ and $H=1+m|y_5|$. The corresponding six-dimensional metric is given by

$$ds_6^2=e^{-(9/7Q)\varphi}\tilde{f}_{\mu\nu}dx^\mu dx^\nu+H^2(e^{(12/7Q)\varphi}dy_5^2+e^{(6/7Q)\varphi}dy_6^2)\tag{15}$$

and after oxidizing the solution back to ten dimensions by employing Eq. (12), we find that

$$\begin{aligned} d\hat{s}^2&=H^{-1/2}e^{-(3/2Q)\varphi}\tilde{f}_{\mu\nu}dx^\mu dx^\nu\\ &\quad+H^{3/2}[e^{(3/2Q)\varphi}dy_5^2+e^{(9/14Q)\varphi}dy_6^2]+H^{1/2}e^{(3/14Q)\varphi}ds_4^2,\\ e^{\hat{\Phi}}&=He^{(3/7Q)\varphi},\quad \hat{A}_{(2)}=\Lambda y_6J_{(2)}. \end{aligned}\tag{16}$$

Finally, since Eq. (16) was derived within the context of the massless type IIA theory, a further oxidation to eleven dimensions can be made. We find that

$$\begin{aligned} d\check{s}^2&=H^{-2/3}e^{-(11/7Q)\varphi}\tilde{f}_{\mu\nu}dx^\mu dx^\nu\\ &\quad+H^{4/3}[e^{(10/7Q)\varphi}dy_5^2+e^{(4/7Q)\varphi}(dy_6^2+dz_{11}^2)]\\ &\quad+H^{1/3}e^{(1/7Q)\varphi}ds_4^2,\\ \check{F}_{(4)}&=\Lambda dy^6\wedge J_{(2)}\wedge dz^{11}. \end{aligned}\tag{17}$$

Equation (17) may be interpreted in terms of M5-branes. The metric for two M5-branes orthogonally intersecting over a three-brane can be written as [16–18]

$$ds^2=(H_1H_2)^{2/3}[(H_1H_2)^{-1}f_{\mu\nu}dx^\mu dx^\nu+H_2^{-1}(dy_1^2+dy_2^2)$$
$$+H_1^{-1}(dy_3^2+dy_4^2)+(dy_5^2+dy_6^2+dz_{11}^2)], \qquad (18)$$

where H_i are harmonic functions over (y_5,y_6,z_{11}). When $H_2=1$, this metric represents a M5-brane with world-volume coordinates (x^μ,y_3,y_4) and transverse space spanned by (y_1,y_2,y_5,y_6,z_{11}). The brane is delocalized over the (y_1,y_2) directions. Similarly, when $H_1=1$, the solution represents an M5-brane transverse to (y_3,y_4,y_5,y_6,z_{11}) and smeared over (y_3,y_4). In general, the metric (18) interpolates between these two limits. The transverse dependence of Eq. (17) is recovered when $H_1=H_2$ and this latter solution may therefore be interpreted as two curved M5-branes orthogonally intersecting on a curved three-brane. Since the harmonic function depends only on y_5, the M5-branes are delocalized over the remaining transverse dimensions. A similar analysis follows for the interpretation of Eq. (16) as the orthogonal intersection of two curved NS5-branes on a three-brane.

We now consider a compactification to six dimensions involving the RR one-form potential of the type IIA theory. The coupling of this field to the ten-dimensional dilaton is given by $\alpha=3/2$. Since this field arises only through an exterior derivative, we may consider a generalized Scherk-Schwarz compactification on a four-dimensional manifold, X, where the closed, harmonic two-form field strength is identified with the cohomology class, $H^2(X)$, of X. Such a wrapping of the RR one-form around the four-torus is achieved through the ansatz $\hat{F}_{(2)}=\Lambda J$, where J is a harmonic two-form on T^4 and Λ is an arbitrary constant [19]. For example, if $J=J_+^{(3)}$, the one-form is given by $\hat{A}_{(1)}=\Lambda(y^1dy^2+y^3dy^4)$. Compactifying with the metric ansatz (12), and equating the ten-dimensional dilaton with the breathing mode, $\hat{\Phi}=\sigma$, then implies that the six-dimensional action is given by Eq. (1), where $(D,\alpha,q)=(6,3/\sqrt{2},0)$ and $\Phi=\sqrt{2}\hat{\Phi}$. Thus, it follows from Eq. (4) that $\Delta=2$ and Eqs. (6)–(10) result in a domain wall solution:

$$ds_6^2=H^{1/2}e^{-(2/3Q)\varphi}\tilde{f}_{\mu\nu}dx^\mu dx^\nu+H^{5/2}e^{(2/Q)\varphi}dy_6^2$$
$$e^\Phi=e^{-(\sqrt{8}/3Q)\varphi}H^{-3/\sqrt{2}}, \qquad (19)$$

where $\{\tilde{f}_{\mu\nu},\varphi\}$ solve the five-dimensional Einstein equations (10), $H=1+m|y_6|$ and $Q=4\sqrt{2}/3$. Oxidizing the solution back to ten dimensions then implies that

$$d\hat{s}^2=H^{-1/4}e^{-\varphi/Q}\tilde{f}_{\mu\nu}dx^\mu dx^\nu+H^{7/4}e^{(5/3Q)\varphi}dy_6^2$$
$$+H^{3/4}e^{(1/3Q)\varphi}ds_4^2, \qquad (20)$$

where $e^{\hat{\Phi}}=e^{-(2/3Q)\varphi}H^{-3/2}$.

The metric for two orthogonally intersecting D6-branes on a D4-brane is [17,19]

$$ds^2=(H_1H_2)^{7/8}[(H_1H_2)^{-1}f_{\mu\nu}dx^\mu dx^\nu+dy_6^2$$
$$+H_2^{-1}(dy_1^2+dy_2^2)+H_1^{-1}(dy_3^2+dy_4^2)], \qquad (21)$$

where $H_i=1+m_i|y_6|$. For example, when $H_1=1$, the line element is that of a D6-brane with world-volume and transverse coordinates (x^μ,y_1,y_2) and (y_6,y_3,y_4). The two D6-branes are smoothed over the (y_3,y_4) and (y_1,y_2) directions, respectively, and when $H_1=H_2$, the transverse dependence of the metric coefficients reduces to that given in Eq. (20). This solution therefore represents the orthogonal intersection of two curved D6-branes on a curved four-brane, where the harmonic functions are identified.

To summarize thus far, we have found curved intersecting brane solutions by wrapping type IIA form fields around homology cycles of the four-torus. Further intersecting branes may now be generated from Eq. (20) by employing the duality symmetries of string theory. We first consider the T-duality that maps the type IIA theory onto the type IIB theory, and vice versa. We assume that all fields are independent of one of the world-volume coordinates (x^5) and express the world-volume metric as $ds_5^2=e^{-\chi/\sqrt{3}}\tilde{f}^{(4)}_{\mu\nu}dx^\mu dx^\nu+e^{2\chi/\sqrt{3}}dx_5^2$, where the normalization is chosen such that the four-dimensional metric, $\tilde{f}^{(4)}_{\mu\nu}$, is the Einstein-frame metric and $\{\chi,\varphi\}$ represent two, massless, minimally coupled scalar fields in four dimensions.

Conformally transforming Eq. (20) to the ten-dimensional, type IIA string-frame and performing a T-duality in the x^5 direction results in a type IIB solution representing the intersection of two curved D5-branes supported by the magnetic charge of the RR two-form potential, $B_{\mu\nu}$. The fields are related by $2\Phi_B=2\Phi_A-\ln G_{55}^{(A)}$, $G_{55}^{(B)}=1/G_{55}^{(A)}$ and $B_{5\mu}=-A_\mu$, where G denotes string-frame metrics [20]. Applying an S-duality [21] on the resulting type IIB solution then interchanges the RR and NS-NS two-form potentials and reverses the sign of the dilaton, thus leading to a configuration consisting of two intersecting NS5-branes with a transverse dependence given by Eq. (16). At this level of truncation, such a solution also satisfies the field equations of the type IIA theory and it may therefore be oxidized to eleven dimensions. This results in a further solution of two orthogonally intersecting M5-branes:

$$d\check{s}^2=H^{-2/3}e^{-(4/3Q)\varphi-(2/3\sqrt{3})\chi}\tilde{f}^{(4)}_{\mu\nu}dx^\mu dx^\nu$$
$$+H^{4/3}[e^{(4/3Q)\varphi}(e^{-(5/3\sqrt{3})\chi}dx_5^2+e^{(1/3\sqrt{3})\chi}dy_6^2)$$
$$+e^{(4/3\sqrt{3})\chi}dz_{11}^2]+H^{1/3}e^{(1/3\sqrt{3})\chi}ds_4^2. \qquad (22)$$

Equation (22) is more general than Eq. (17) since the world-volume is curved by two scalar fields. Either of these may be consistently set to zero.

The wrappings around T^4 that we have considered thus far admit a direct generalization to the compact, Ricci-flat, K3 manifold. This is Kummer's quartic surface in CP^3 and admits 22 harmonic two-forms $(b_2=22)$. (For a review of the properties of K3 surfaces, see, e.g., Ref. [22].) When discussing compactifications on K3, it is convenient to view it as an orbifold approximation to T^4, K3$\approx T^4/Z_2$. Such a compactification was considered in detail in Ref. [23]. Six of the harmonic two-forms on K3 correspond to the harmonic two-forms on the four-torus. Thus, wrapping the NS-NS two-form potential of the type IIA theory around one of these two-cycles of the K3 manifold results, after oxidation to eleven dimensions, in the intersecting brane (17), where the internal metric, ds_4^2, is now the metric on K3. The intersecting D6 brane (20) may also be generalized to the K3 case in a similar fashion.

The compactification of the type IIA theory on K3 is important in view of the conjectured strong/weak coupling S duality between this theory and the heterotic theory compactified on T^4 [21,24]. This duality implies that a curved heterotic brane may be derived, for example, from Eq. (16). The relevant transformation rules between the massless fields of the two theories have been summarized in Refs. [23,25] for the case where the heterotic gauge group is broken to $U(1)^{16}$. We consider the transformations relevant to the compactification leading to the type II truncated action (13). The six-dimensional string-frame metrics are related by $G^{\rm II}_{\mu\nu}=\Theta^2 G^{\rm het}_{\mu\nu}$, where $\Theta^2\equiv e^{-2\psi_{\rm het}}$, and the six-dimensional dilatons are given by $\psi_{\rm II}=-\psi_{\rm het}$. In the above type II compactification, we have only considered the breathing mode of the K3 manifold and this is equivalent to assuming that all four radii of the orbifold T^4/Z_2 are equal, i.e., $\hat{G}^{\rm II}_{aa}=\hat{G}^{\rm II}_{bb}$ $(a,b=1,2,3,4)$. This places a restriction on the toric radii in the corresponding compactification of the heterotic theory. Specifically, in the ten-dimensional string frame, three of the internal dimensions are static, $\hat{G}^{\rm het}_{ii}=1$, and the radius of the fourth is given by $\hat{G}^{\rm het}_{44}=(\hat{G}^{\rm II}_{aa})^2$. Finally, the scalar axion field, b, arising from the wrapping of the NS-NS two-form potential around the K3 two-cycle, is related to one of the sixteen U(1) potentials, $\hat{A}^{\rm het}_\mu$, such that $b=A_4/\sqrt{2}$, where the scalar field, A_4, arises from compactification of the U(1) gauge field on the circle parametrized by y^4, i.e., $\hat{A}^{\rm het}=A_4 dy^4$.

These type II/heterotic correspondences may therefore be employed to derive the curved heterotic brane that is S-dual to the type IIA, six-dimensional metric (15). Oxidizing the resulting solution to ten dimensions then yields the heterotic solution

$$\begin{aligned} d\hat{s}^2_{\rm het}=&H^{-1/4}[e^{-(39/28Q)\varphi}\tilde{f}^{(4)}_{\mu\nu}dx^\mu dx^\nu \\ &+e^{-(3/28Q)\varphi}(dy_1^2+dy_2^2+dy_3^2)] \\ &+H^{7/4}e^{(3/4Q)\varphi}(dy_4^2+e^{(6/7Q)\varphi}dy_5^2+dy_6^2) \\ e^{\hat{\Phi}_{\rm het}}=&H^{1/2}e^{(3/14Q)\varphi}, \end{aligned} \tag{23}$$

where $Q=\sqrt{27/7}$, $\hat{F}^{\rm het}_{(2)}=\sqrt{2}\Lambda dy^6\wedge dy^4$ and the metric is expressed in the Einstein frame. Equation (23) represents a curved six-brane, where three of the transverse dimensions of the type IIA solution (16) have become world-volume dimensions in the heterotic solution. The S duality between the SO(32) heterotic and type I theories may also be invoked to derive the corresponding type I brane [24,26]. The vacuum limit ($\varphi=0$) of Eq. (23) is the heterotic/type I six-brane found in Ref. [27] that is T-dual to the bound state of an anti–five-brane and a Kaluza-Klein monopole. Such a solution arises as a special case in the domain-wall/quantum-field–theory correspondence [28,27].

The six-brane (23) is also relevant to the recently introduced compactification ansatz referred to as "braneworld Kaluza-Klein reduction" [29]. In this scheme, the world-volume of a codimension one-brane arising as a solution of a gauged supergravity theory is determined by an ungauged supergravity theory with half the supersymmetry. For example, the massive type IIA supergravity theory of Romans [30] admits a D8-brane, where the curvature of the world-volume is determined by a solution to nine-dimensional, ungauged $N=1$ supergravity. Under appropriate conditions, this latter theory may be derived by a Kaluza-Klein compactification on a circle of the (truncated) ten-dimensional type I theory. It follows, therefore, that the dimensional reduction of the six-brane (23) along a world-volume coordinate results in a five-brane of $D=9$, $N=1$ supergravity. Consequently, following the prescription outlined in Ref. [29], such a brane may be embedded within the D8 solution of the massive type IIA theory. The resulting configuration corresponds to the intersection of an NS5-brane and a D6-brane with a D8-brane.

More general solutions to those presented may be found by noting that the scalar fields $\{x,b\}$ in action (13) parametrize the SL(2,R)/U(1) coset. The action is therefore invariant under a global SL(2,R) symmetry transformation, where the complex scalar field, $\kappa\equiv\lambda b+ie^{\lambda x}$ $(\lambda\equiv 1/\sqrt{2})$, undergoes a fractional linear transformation $\bar{\kappa}=(A\kappa+B)/(C\kappa+D)$ for $AD-BC=1$, and the Einstein-frame metric transforms as a singlet. Given a solution (b,x) to the field equations derived from Eq. (13), the SL(2,R) transformation may be employed to generate a class of solutions where both fields have a nontrivial dependence on the transverse and world-volume coordinates. Moreover, a generalized Scherk-Schwarz compactification of action (13) may also be performed, where the dependence of the fields on the compactifying coordinates is determined by a local SL(2,R) transformation, thereby extending the linear ansatz we invoked for the axion field [31].

A related six-dimensional SL(2,R)/U(1) model is derivable by compactifying eight-dimensional, vacuum Einstein gravity on a nondynamical two-torus, $ds_8^2=ds_6^2+e^{-\Phi}dy_6^2+e^{\Phi}(dy_5+\sigma dy_6)^2$; the $\{\Phi,\sigma\}$ fields parametrize the coset manifold, $ds^2=d\Phi^2+e^{2\Phi}d\sigma^2$, and therefore support a 3-brane after Scherk-Schwarz compactification to five dimensions. This is interesting because a mapping between $D=8$ vacuum Einstein gravity with two commuting spacelike isometries and $D=11$ supergravity was recently established by means of a nonlocal classical duality [32]. Thus, a given solution to one theory acts as a seed for generating new solutions in the other, and vice versa. Indeed, such a correspondence has been employed to generate intersecting M5-branes [32]. The results of the present work imply that analogous curved models may also be found by this procedure. It is also worth remarking that branes intersecting at angles can be derived by applying successive T-S-T duality transformations on orthogonally intersecting configurations [2,33]. It would be interesting to explore such a procedure to derive tilted curved branes.

Finally, we conclude by discussing some of the cosmological implications of the solutions we have derived. Considerable interest has been generated recently by the proposal that our observable, four-dimensional universe corresponds to a domain wall or *p*-brane embedded in a higher-dimensional space [3–6]. A natural generalization of the simplest braneworld scenario is to view our universe as the intersection of two or more higher-dimensional branes [34]. The solutions we have found can be interpreted cosmologically when the scalar field in Eq. (10) is time dependent. Since the world-volume, $\tilde{f}_{\mu\nu}$, is arbitrary, a wide class of spatially anisotropic and inhomogeneous cosmologies may be considered that generalize the standard Friedmann-Robertson-Walker (FRW) models. This is important since deviations from spatial isotropy are expected to have been significant in the very early universe.

To be specific, curved braneworlds may be found directly once a solution to Eq. (10) has been given. Equation (10) represents Einstein's equations sourced by a massless, minimally coupled scalar field and solutions to this latter theory are known [35–37]. In particular, homogeneous and inhomogeneous models containing one or more massless scalar fields were recently reviewed [37] within the context of string-inspired models such as the pre-big bang inflationary cosmology [38]. In this latter scenario, inflation can be interpreted in the Einstein frame as the collapse of a scalar field dominated universe, where the dynamics is determined by Eq. (10) [39]. Thus, our solutions provide a framework for considering pre-big bang inflation in a braneworld setting.

One of the simplest cosmological models is represented by the spatially homogeneous and anisotropic Bianchi type I metric. When the world-volume has this form, it can be shown that the eleven-dimensional metrics that we have derived correspond to *vacuum* solutions of Einstein gravity in the limit where the harmonic function $H=1$. Since the spatial hypersurfaces are Ricci flat, these metrics represent higher-dimensional generalizations of the four-dimensional Kasner solution [40] and it is known that for these models, inflation is possible over a wide region of parameter space [37]. The accelerated expansion of a subset of the spatial dimensions is driven by the collapse of the remaining dimensions [41]. It would be interesting to investigate inflation of this type within the intersecting braneworld context, although a detailed analysis is beyond the scope of the present paper.

An important question in the braneworld scenario is whether gravity can be localized on the domain wall I [42]. In the models considered above, $\Delta=2$. However, in this case, gravity can arise on the world-volume of the brane if the extra coordinate is compact. For example, if the coordinate is restricted to the interval, S^1/Z_2, the domain wall may be located on the orbifold fixed points, as in the Hořava-Witten theory [4,5]. [This model corresponds to $\alpha=-2$ in Eq. (1).]

A significant consequence of viewing our observable universe as a codimension one brane embedded in a five-dimensional "bulk" space is that the effective four-dimensional gravitational field equations include extra terms. These tidal effects are parametrized by the Weyl tensor of the higher-dimensional metric and do not depend specifically on the energy-momentum of matter that is confined to the brane [43]. Hence, the geometry of the bulk can significantly influence the lower-dimensional brane dynamics and in general this implies that the cosmological expansion of the brane cannot be determined unless the form of the higher-dimensional metric is known. In this paper, we have found exact bulk solutions to the type II string theory and M theory field equations and these solutions therefore provide a class of models where the cosmological dynamics of the brane can be determined.

Recently, Feinstein, Kunze, and Vázquez-Mozo considered a related class of five-dimensional domain wall models supported by an exponential potential of the form given in Eq. (1) [12]. These authors included a matter source confined to the brane with a Lagrangian coupled to the scalar field via a Liouville term. In this case, a self-tuning mechanism arises between the matter and the brane tension that causes the effective cosmological constant on the brane to vanish. In principle, a similar analysis may be performed for the intersecting brane configurations derived above by introducing an appropriate matter source.

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[1] R. Argurio, hep-th/9807171.

[2] J.P. Gauntlett, hep-th/9705011, and references therein.

[3] K. Akama, hep-th/0001113; V.A. Rubakov and M.E. Shaposhnikov, Phys. Lett. **159B**, 22 (1985); N. Arkani-Hamed, S. Dimopoulos, and G. Dvali, Phys. Lett. B **429**, 263 (1998); **436**, 257 (1998); M. Gogberashvili, Europhys. Lett. **49**, 396 (2000).

[4] P. Hořava and E. Witten, Nucl. Phys. **B460**, 506 (1996); **B475**, 94 (1996).

[5] A. Lukas, B.A. Ovrut, K.S. Stelle, and D. Waldram, Phys. Rev. D **59**, 086001 (1999); A. Lukas, B.A. Ovrut, and D. Waldram, *ibid.* **60**, 086001 (1999).

[6] L. Randall and R. Sundrum, Phys. Rev. Lett. **83**, 3370 (1999); **83**, 4690 (1999).

[7] G. Papadopoulos, J.G. Russo, and A.A. Tseytlin, Class. Quantum Grav. **17**, 1713 (2000).

[8] H. Lü, C.N. Pope, E. Sezgin, and K.S. Stelle, Nucl. Phys. **B456**, 669 (1995); M. Cvetic, Phys. Rev. Lett. **71**, 815 (1993).

[9] D. Brecher and M.J. Perry, Nucl. Phys. **B566**, 151 (2000).

[10] J.E. Lidsey, Class. Quantum Grav. **17**, L39 (2000); Phys. Rev. D **62**, 083515 (2000).

[11] H. Ochiai and K. Sato, Phys. Lett. B **503**, 404 (2001).

[12] A. Feinstein, K.E. Kunze, and M.A. Vázquez-Mozo,

hep-th/0105182.
[13] M. Huq and M.A. Namazie, Class. Quantum Grav. **2**, 293 (1985); **2**, 597(E) (1985); F. Giani and M. Pernici, Phys. Rev. D **30**, 325 (1984); I.C. Campbell and P.C. West, Nucl. Phys. **B243**, 112 (1984).
[14] H. Lü and C.N. Pope, Nucl. Phys. **B465**, 127 (1996).
[15] J. Scherk and J.H. Schwarz, Nucl. Phys. **B153**, 61 (1979); E. Bergshoeff, M. de Roo, M.B. Green, G. Papadopoulos, and P.K. Townsend, *ibid.* **B470**, 113 (1996); P.M. Cowdall, H. Lü, C.N. Pope, K.S. Stelle, and P.K. Townsend, *ibid.* **B486**, 49 (1997).
[16] G. Papadopoulos and P.K. Townsend, Phys. Lett. B **380**, 273 (1996).
[17] A.A. Tseytlin, Nucl. Phys. **B475**, 149 (1996).
[18] J.P. Gauntlett, D.A. Kastor, and J. Traschen, Nucl. Phys. **B478**, 544 (1996).
[19] I.V. Lavrinenko, H. Lü, and C.N. Pope, Nucl. Phys. **B492**, 278 (1997).
[20] E. Bergshoeff, C. Hull, and T. Ortín, Nucl. Phys. **B451**, 547 (1995).
[21] C. Hull and P.K. Townsend, Nucl. Phys. **B438**, 109 (1995).
[22] P.S. Aspinwall, hep-th/9611137.
[23] H. Lü, C.N. Pope, and K.S. Stelle, Nucl. Phys. **B548**, 87 (1999).
[24] E. Witten, Nucl. Phys. **B443**, 85 (1995).
[25] M. Frau, A. Liccardo, and R. Musto, Nucl. Phys. **B602**, 39 (2001).
[26] C. Hull, Phys. Lett. B **357**, 545 (1995); J. Polchinski and E. Witten, Nucl. Phys. **B460**, 525 (1996).
[27] K. Behrndt, E. Bergshoeff, R. Halbersma, and J. van der Schaar, Class. Quantum Grav. **6**, 3517 (1999).
[28] H. Itzhaki, J.M. Maldacena, J. Sonnenschein, and S. Yankielowicz, Phys. Lett. B **432**, 298 (1998); H.J. Boonstra, K. Skenderis, and P.K. Townsend, J. High Energy Phys. **01**, 003 (1999).
[29] H. Lü and C.N. Pope, Nucl. Phys. **B598**, 992 (2001); M. Cvetič, H. Lü, and C.N. Pope, hep-th/0009183.
[30] L.J. Romans, Phys. Lett. **169B**, 374 (1986).
[31] I.V. Lavrinenko, H. Lü, and C.N. Pope, Class. Quantum Grav. **15**, 2239 (1998); P. Meessen and T. Ortin, Nucl. Phys. **B541**, 195 (1999).
[32] C. Chen, D.V. Gal'tsov, and S.A. Sharakin, Gravitation Cosmol. **5**, 45 (1999); Phys. Lett. B **475**, 269 (2000).
[33] K. Behrndt and M. Cvetič, Phys. Rev. D **56**, 1188 (1997).
[34] N. Arkani-Hamed, S. Dimopoulos, G. Dvali, and N. Kaloper, Phys. Rev. Lett. **84**, 586 (2000); J. High Energy Phys. **12**, 010 (2000); S. Nam, *ibid.* **04**, 002 (2000); N. Kaloper, Phys. Lett. B **474**, 269 (2000); J. Cline, C. Grojean, and G. Servant, *ibid.* **472**, 302 (2000); T. Torma, hep-th/0004021.
[35] D. Kramer, H. Stephani, M.A.H. MacCallum, and E. Herlt, *Exact Solutions of Einstein's Equations* (Cambridge University Press, Cambridge, England, 1980); A. Krasinski, *Inhomogeneous Cosmological Models* (Cambridge University Press, Cambridge, England, 1997).
[36] R. Lazkoz, Phys. Rev. D **60**, 104008 (1999).
[37] J.E. Lidsey, D. Wands, and E.J. Copeland, Phys. Rep. **337**, 343 (2000).
[38] G. Veneziano, Phys. Lett. B **265**, 287 (1991); M. Gasperini and G. Veneziano, Astropart. Phys. **1**, 317 (1993).
[39] A. Buonanno, K.A. Meissner, C. Ungarelli, and G. Veneziano, Phys. Rev. D **57**, 2543 (1998); A. Buonanno, T. Damour, and G. Veneziano, Nucl. Phys. **B543**, 275 (1999); R. Madden, J. High Energy Phys. **01**, 023 (2001).
[40] E. Kasner, Trans. Am. Math. Soc. **27**, 155 (1925).
[41] J.J. Levin and K. Freese, Phys. Rev. D **47**, 4282 (1993); J.J. Levin, Phys. Lett. B **343**, 69 (1995).
[42] M. Cvetič, H. Lü, and C.N. Pope, Phys. Rev. D **63**, 086004 (2001); D. Youm, Nucl. Phys. **B576**, 106 (2000); **B589**, 315 (2000); **B596**, 289 (2001).
[43] T. Shiromizu, K. Maeda, and M. Sasaki, Phys. Rev. D **62**, 024012 (2000); K. Maeda and D. Wands, *ibid.* **62**, 124009 (2000); A. Mennim and R.A. Battye, Class. Quantum Grav. **18**, 2171 (2001).