Cosmic antifriction and accelerated expansion

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We explain an accelerated expansion of the present Universe, suggested from observations of supernovae of type Ia at high redshift, by introducing an antifrictional force that is self-consistently exerted on the particles of the cosmic substratum. Cosmic antifriction, which is intimately related to "particle production," is shown to give rise to an effective negative pressure of the cosmic medium. While other explanations for an accelerated expansion (cosmological constant, quintessence) introduce a component of dark energy in addition to "standard" cold dark matter (CDM) we resort to a phenomenological one-component model of CDM with internal self-interactions. We demonstrate how the dynamics of the cold dark matter model with a cosmological constant may be recovered as a special case of cosmic antifriction. We discuss the connection with two-component models and obtain an attractor behavior for the ratio of the energy densities of both components which provides a possible phenomenological solution to the coincidence problem.

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I. INTRODUCTION

There is evidence from supernova type Ia (SN Ia) data for our present Universe to be in a state of accelerated expansion [1–5]. This interpretation, which is indirectly also backed up by recent data from the balloon experiments of Boomerang [6] and Maxima [7], according to which we live in a flat Universe, requires a cosmic medium with sufficiently high negative pressure to violate the strong energy condition (SEC), $\rho+3P>0$. Cosmic matter with negative pressure is now known as "dark energy." The problem why the density of the dark energy is of the order of the matter density just at the present epoch is the "coincidence problem" [10]. An obvious dark energy candidate is a cosmological constant of the order of the current critical density. Another option is a scalar field called "quintessence," either with a suitable potential [8–17] or with a nonstandard kinetic term [18].

Negative pressures also occur in a different context. As was first pointed out by Zel'dovich [19] and Hu [20], quantum processes in the early Universe, such as cosmological particle production, may phenomenologically be equivalent to effective negative bulk pressures. Numerous investigations have subsequently explored this analogy [21–29]. For gaseous matter with specific internal self-interactions negative cosmic bulk pressures can be derived within the framework of relativistic gas dynamics [30–33]. In particular, it turned out that accelerated expansion (inflation) of the Universe can be driven by a self-interacting gas [30–33].

The observational data of the early 1990s left ample room for a presently non-negligible dissipative bulk stress on cosmological scales, as has been pointed out by two of us in Ref. [34]. Hypothetically, this bulk stress was ascribed to internal interactions inside the dark matter, the latter assumed to be the dynamically dominating component of the Universe. According to the state of knowledge at that time our studies were restricted to matter for which the SEC holds. Taking into account the recent development sketched above, it seems natural to investigate a present phase of accelerated expansion from a gas dynamical point of view.

The present work is based on the assumption that the observational evidence for an effective cosmological constant is an indication for the existence of additional interactions within the cosmic medium, which macroscopically manifest themselves as negative pressures. Our strategy is the following: Instead of introducing from the outset a new kind of matter ("quintessence" or "Q matter") with a negative pressure in addition to cold dark matter (CDM), we start with a one-component description of the cosmic medium. Although the corresponding substratum is assumed to consist of non-relativistic particles, it does not represent simple dust since we include interactions within the matter. These interactions turn out to be equivalent to effective one-particle forces, which are self-consistently exerted by the cosmic me-

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dium on each of its individual particles. We show that the cosmological principle restricts these forces to act like "friction" or "antifriction." Generalized equilibrium requirements for the cosmic medium relate those forces to temperature and chemical potential. For a non-relativistic substratum only cosmic antifriction generates a negative fluid bulk pressure. We demonstrate that a suitable amount of cosmic antifriction leads to a SEC violation equivalent to an accelerated expansion of the Universe. On this basis we argue that the hypothesis of cosmic antifriction offers an alternative interpretation of the magnitude-redshift relation for type Ia supernovas.

We present three different phenomenological models of cosmic antifriction which give rise to Hubble diagrams consistent with the SN Ia data. We estimate the redshifts at which the accelerated expansion started. Although SN Ia data alone cannot discriminate between those models, observations of the cosmic microwave radiation anisotropies rule out one of them.

In a subsequent step we demonstrate how the antifriction dynamics may be decomposed into a two-component picture with one component being Q matter, the other one CDM. This procedure reveals that a negative fluid pressure due to antifriction may be dynamically equivalent to the accelerating effect of dominating Q matter. For a specific ansatz for the cosmic force and for a specific decomposition one recovers the cold dark matter model with a cosmological constant (ΛCDM). Moreover, it turns out that the same amount of accelerated expansion of the presently observed Universe is compatible with different splittings of the total energy density into the energy densities of Q matter and CDM. In particular, there are decompositions in which Q matter decays into CDM. For a specific decay rate there exists a stable attractor solution with a fixed ratio of the energy densities of CDM and Q matter, which indicates a possible solution of the coincidence problem.

The paper is organized as follows. Section II recalls some basic relations and features on the possible role of a cosmic bulk pressure. In Sec. III we provide the basic relations of a kinetic theory for self-interacting gases. We show that a certain class of particle number non-preserving interactions may be mapped onto effective one-particle forces. The cosmological principle implies that these forces necessarily describe a friction or an antifriction within the cosmic medium. We determine the strength of the force and realize that only antifriction is compatible with a negative pressure. The back reaction of the antifrictional self-interactions on the cosmological dynamics is considered in Sec. IV for three different antifriction models. The luminosity and angular distances as functions of redshift are studied for all three models. Corresponding two-fluid models are established in Sec. V which provide the basis for a discussion of the coincidence problem. Section VI sums up our conclusions on the possible role of cosmic antifriction. Units have been chosen so that $c=k_B=\hbar=1$.

II. COSMIC BULK PRESSURE

We suppose the Universe to be describable by the stress-energy tensor of an imperfect fluid

$$T^{ab}_{\rm eff}=(\rho+P)u^a u^b+Pg^{ab}, \tag{1}$$

where ρ is the energy density, measured by an observer comoving with the fluid four-velocity u^a normalized according to $u_a u^a=-1$. The effective pressure P splits into two parts,

$$P=p+\Pi, \tag{2}$$

p being the equilibrium pressure with $p\geqslant 0$ for gaseous matter and Π a non-equilibrium part. For a perfect fluid we have $\Pi=0$, i.e., $P\geqslant 0$. For a conventional viscous fluid $\Pi\leqslant 0$ is valid during expansion, e.g., if kinetic energy of the fluid is transferred to internal degrees of freedom. In the first-order Eckart theory one has $\Pi=-3H\zeta$ (see, e.g., [35]), where $H\equiv u^a_{;a}/3=\dot{a}/a$ is the Hubble expansion rate and $\zeta\geqslant 0$ is the coefficient of bulk viscosity. Within the more satisfactory second-order theories Π becomes a dynamical degree of freedom (see, e.g., [36–39]). Both the first- and the second-order theories are valid under the condition $|\Pi|<p$, such that the effective pressure P of a viscous fluid or gas is positive.

Apart from viscosity, particle number non-conserving interactions inside the matter may, as discussed above, lead to an effective bulk pressure. This includes particle production out of the gravitational field. The fact that $\Pi\leqslant 0$ if there is particle production and that $P<0$ is possible under such conditions may be demonstrated as follows. Let the cosmic matter be characterized by the particle flow vector

$$N^i=nu^i, \tag{3}$$

where n is the particle number density. In case the fluid particle number is not preserved, the number density changes according to the balance [23,24]

$$N^a_{;a}=\dot{n}+3Hn=n\Gamma, \tag{4}$$

where $\Gamma=\dot{N}/N$ is the change rate of the number $N\equiv na^3$ of particles in a comoving volume a^3. For $\Gamma>0$ we have particle creation, for $\Gamma<0$ particles are annihilated. Conservation of the effective stress-energy tensor (1) implies

$$\dot{\rho}+3H(\rho+P)=0. \tag{5}$$

With the help of the Gibbs equation

$$Tds=d\frac{\rho}{n}+pd\frac{1}{n}, \tag{6}$$

where s is the entropy per particle and T the temperature, and with the balances (4) and (5) we obtain

$$nT\dot{s}=-3H\Pi-(\rho+p)\Gamma. \tag{7}$$

If the particle number N is conserved, i.e., for $\Gamma=0$, the second law of thermodynamics implies $\Pi\leqslant 0$ in an expanding universe.

If the particle number is not conserved one may define "isentropic" (or "adiabatic") particle production by $\dot{s}=0$, which here means "constant entropy per particle." Under this condition the equilibrium entropy per particle does not change as it does in dissipative processes. Instead, one can associate a viscous pressure to the particle production rate [23,24]:

$$\dot{s}=0 \quad \Rightarrow \quad \Pi=-(\rho+p)\frac{\Gamma}{3H}. \tag{8}$$

The cosmic substratum is not a conventional dissipative fluid but a perfect fluid with varying particle number. Obviously, $\Gamma\geqslant 0$ guarantees $\Pi\leqslant 0$. Substantial particle production is a phenomenon which is reasonably to be expected in the early Universe. It is less clear whether such processes are operative at the present epoch as well. However, given that the nature of CDM is unknown, there seems to be some room for speculations in this direction. As easily seen, $P<0$ in the case of dust ($p=0$) and $P\sim-\rho$ is possible if $\Gamma/(3H)=O(1)$. Compared to typical rates of particle physics this requires an extremely small particle production rate only. Below we shall comment on the origin of negative bulk pressure in more detail.

Referring to matter creation as a relevant cosmological mechanism may remind us of corresponding processes within the steady state model [40]. However, different from the latter, our considerations are fully within Einstein's theory. Moreover, we shall trace the production process to internal interactions within the system.

For a universe of the Friedmann-Lemaître-Robertson-Walker (FLRW) type with scale factor a we have

$$2\frac{\ddot{a}}{a}+\left(\frac{\dot{a}}{a}\right)^2+\frac{k}{a^2}=-8\pi GP \quad (k=1,0,-1) \tag{9}$$

and

$$\frac{k}{a^2H^2}=\Omega-1, \quad \Omega\equiv\frac{\rho}{\rho_c}=\frac{8\pi G}{3H^2}\rho, \tag{10}$$

implying

$$\Pi=-\frac{1}{4\pi G}\left[\left(\frac{1}{2}\Omega-q\right)H^2+4\pi Gp\right], \tag{11}$$

where $q\equiv-\ddot{a}/(aH^2)$ is the deceleration parameter. In the standard big-bang scenario the non-equilibrium pressure Π is ignored and since p cannot become negative it follows $\Omega(t)\leqslant 2q(t)$. But it is obvious that as long as $p(t)$ does not vanish and both pressures are of similar magnitude, $\Omega(t)$ can be either larger or smaller than $2q(t)$. Usually the current value of the hydrostatic pressure is approximated by the state equation of dust $p_0=0$. The recent SN Ia data seem to confirm the existence of an effectively negative pressure of the cosmic medium. Assuming $p_0=0$ in Eq. (11), the current value of the bulk stress can then be expressed as

$$\Pi_0=-\frac{1}{3}\left(1-2\frac{q_0}{\Omega_0}\right)\rho_0, \tag{12}$$

where the current energy density ρ_0 is of the order of the critical energy density $\rho_{c0}=3H_0^2/(8\pi G)$. For $q_0>0$ we get $3|\Pi_0|<\rho_0$, which is in agreement with the strong energy condition. The latter is violated, however, for $q_0<0$. As already mentioned, for "conventional" viscous matter without particle production the non-equilibrium part Π of the pressure is smaller in magnitude than the equilibrium contribution p, so that $P>0$. Here we argue that relation (8) offers the option of understanding the existence of a negative bulk pressure as a manifestation of cosmological "particle production." In the following sections we show how such a kind of negative pressure may emerge as a consequence of specific internal interactions within the cosmic substratum.

III. KINETIC THEORY FOR SELF-INTERACTING GASES

A. Basic relations

The one-particle distribution function $f=f(x,p)$ of a relativistic gas is supposed to obey the Boltzmann equation

$$L[f]\equiv p^i f_{,i}-\Gamma^i_{kl}p^k p^l\frac{\partial f}{\partial p^i}=C[f]+\mathcal{S}(x,p), \tag{13}$$

where $C[f]$ is Boltzmann's collision integral. The term $\mathcal{S}(x,p)$ on the right-hand side takes into account additional interactions which cannot be reduced to elastic, binary interactions. In particular, it describes production or decay processes of particles.

The particle number flow four-vector N^i and the energy momentum tensor T^{ik} are defined in a standard way (see, e.g., [41]) as

$$N^i=\int dP\, p^i f(x,p), \quad T^{ik}=\int dP\, p^i p^k f(x,p). \tag{14}$$

The integrals in the definitions (14) and in the following are integrals over the entire mass shell, characterized by $p^i p_i=-m^2$ and $p^0>0$. The entropy flow vector S^a is given by [41,42]

$$S^a=-\int dP\, p^a[f\ln f-f], \tag{15}$$

where we have restricted ourselves to the case of classical Maxwell-Boltzmann particles.

Using the general relationship [43]

$$\left[\int p^{a_1}\cdots p^{a_n}p^b f\, dP\right]_{;b}=\int p^{a_1}\cdots p^{a_n}L[f]dP \tag{16}$$

and Eq. (13) we find

$$N^a_{;a}=\int dP(C[f]+\mathcal{S}), \quad T^{ak}_{;k}=\int dP\, p^a(C[f]+\mathcal{S}), \tag{17}$$

and

$$S^a_{;a}\equiv\sigma_C+\sigma_{\mathcal{S}}, \tag{18}$$

where

$$\sigma_C = -\int dP\, C[f] \ln f, \tag{19}$$

and

$$\sigma_{\mathcal{S}} = -\int dP\, \mathcal{S} \ln f. \tag{20}$$

Under the condition that with respect to the elastic part of the interactions the gas is at equilibrium, the expression $\ln f$ is a linear combination of the collision invariants 1 and p^a and the collision integral $C[f]$ vanishes. The corresponding equilibrium distribution function becomes (see, e.g., [41])

$$f^0(x,p) = \exp[\alpha + \beta_a p^a], \tag{21}$$

where $\alpha = \alpha(x)$ and $\beta_a(x)$ is timelike. This implies $\sigma_C = 0$, i.e., there is entropy production only due to $\sigma_{\mathcal{S}}$.

With f replaced by f^0 in the definitions (14) and (15), N^a, T^{ab}, and S^a may be split with respect to the unique four-velocity u^a according to

$$N^a = nu^a, \quad T^{ab} = \rho u^a u^b + p h^{ab}, \quad S^a = nsu^a. \tag{22}$$

Note that we have identified here the general fluid quantities n, ρ, and p of the previous sections with those emerging from the Maxwell-Boltzmann gas dynamics. The exact integral expressions for n, ρ, and p may be found, e.g., in Ref. [44]. The entropy per particle s is

$$s = \frac{\rho + p}{nT} - \frac{\mu}{T}. \tag{23}$$

Here we have used the identifications $\beta_i = \beta u_i$, $\beta = T^{-1}$, and $\alpha = \mu/T$ with μ being the chemical potential.

Use of the equilibrium distribution function (21) in the balances (17) yields

$$\dot{n} + 3Hn = n\Gamma \equiv \int dP\, \mathcal{S}^0, \tag{24}$$

and

$$\begin{aligned} u^a[\dot{\rho} + 3H(\rho + p)] + (\rho + p)\dot{u}^a + p_{,b}h^{ab} \\ = -t^a \\ \equiv \int dP\, p^a \mathcal{S}^0, \end{aligned} \tag{25}$$

where $\mathcal{S}^0$ denotes the source term $\mathcal{S}$ for $f = f^0$. We consider the special case that $\mathcal{S}^0$ depends linearly on f^0. For reasons that will become clear shortly, we suppose that the factor of proportionality can be written in terms of a suitable projection of a quantity F^i which will turn out to play the role of an effective one-particle force:

$$\mathcal{S}^0 = -m\beta_i F^i f^0. \tag{26}$$

The constant factor $-m$ has been chosen for later convenience. The expression (26) may be regarded as a special case of the more general structure

$$\mathcal{S} = -mF^i \frac{\partial f}{\partial p^i}. \tag{27}$$

It is straightforward to realize that a "collision" term of this form may be taken to the left-hand side of Boltzmann's equation (13), resulting in

$$p^i f_{,i} - \Gamma^i_{kl} p^k p^l \frac{\partial f}{\partial p^i} + mF^i \frac{\partial f}{\partial p^i} = C[f]. \tag{28}$$

The left-hand side of this equation can be regarded as

$$\frac{df(x,p)}{d\lambda} \equiv \frac{\partial f}{\partial x^i}\frac{dx^i}{d\lambda} + \frac{\partial f}{\partial p^i}\frac{dp^i}{d\lambda}$$

with

$$\frac{dx^i}{d\lambda} = p^i, \quad \frac{Dp^i}{d\lambda} = mF^i. \tag{29}$$

Equations (29) are the equations of motion for gas particles that move under the influence of a force field $F^i = F^i(x,p)$. The quantity λ is a parameter along the particle worldline which for massive particles may be related to the proper time τ by $\lambda = \tau/m$. Consequently, a specific "collisional" interaction, described by a "source" term $\mathcal{S}$, may be mapped onto an effective one-particle force F^i. This demonstrates that there exists a certain freedom to interpret collisional events in terms of forces. (This freedom can also be used in the reverse direction, i.e., to interpret (parts of) forces as collisions [45].) We emphasize that our approach is different from the "canonical" theory of particles in a force field for which the force term $mF^i\,\partial f/\partial p^i$ in Eq. (28) is replaced by $m\partial(F^i f)/\partial p^i$ [45]. While both approaches are consistent with the equations of motion (29), they coincide only for $\partial F^i/\partial p^i = 0$, which holds, e.g., for the Lorentz force. In the cases of interest here we will have $\partial F^i/\partial p^i \neq 0$.

B. Cosmic forces

Since we assume the Universe to be homogeneous and isotropic at large scales we ask for forces that are consistent with the cosmological principle. In such a case the metric of space-time is of the Robertson-Walker form and the energy-momentum tensor is given by Eq. (1). Since the particle four-momenta are normalized according to $p^i p_i = -m^2$, the force F^i has to satisfy the relation $p_i F^i = 0$. The momentum of a comoving particle is $p^i_{(c)} = mu^i$. From its definition comoving particles are force-free and thus $F^i(mu) = 0$. This property follows also from the relation

$$\frac{Du^i}{d\tau} = u^i_{;n}\frac{p^n}{m}, \tag{30}$$

which for $p^i_{(c)} = mu^i$ via $u^i_{;n}u^n \equiv \dot{u}^i \propto F^i$ requires a vanishing force since the cosmological principle implies $\dot{u}^i = 0$.

On a spatial slice Σ_t, normal to u^i, the force field F^i has to be independent of the spatial position, otherwise homogeneity would be violated, thus $F^i = F^i(p;t)$. Since p^i and u^i are generally independent four-vectors, we may decompose according to

$$F^i = Ap^i + Bmu^i, \tag{31}$$

where A and B are arbitrary functions of p^i with dimension 1/time. From $F^i p_i = 0$ we find

$$F^i = \frac{B}{m}(-Ep^i + m^2 u^i), \tag{32}$$

with $E \equiv -p^i u_i$ being the particle energy as measured by a comoving observer. For a comoving particle one has $E = m$ and we consistently recover that $F^i(mu) = 0$ for all B. A particle which exactly moves with the mean macroscopic four-velocity is force free. The temporal and spatial projections of the force field give

$$u_i F^i = \frac{B}{m}(E^2 - m^2), \quad e_i F^i = -\frac{B}{m} E\sqrt{E^2 - m^2}, \tag{33}$$

where

$$e^i \equiv \frac{1}{\sqrt{E^2 - m^2}}(p^i - Eu^i) \tag{34}$$

is the spatial direction of the particle momentum ($e^i u_i = 0, \quad e^i e_i = 1$). Due to spatial isotropy B may not depend on the spatial direction e^i, thus $B = B(E;t)$. The expression (32) is the most general force field consistent with the cosmological principle.

According to the projections (33), the force is acting parallel or anti-parallel to the motion of the particle under consideration, depending on the sign of B. For non-relativistic particles it should be a good approximation to assume that B is independent of E. With $E = m + \varepsilon$, where $\varepsilon = mv^2/2 \ll m$, we find at leading order in the velocity $e_i F^i \simeq -B(m)mv$. This is nothing but Stokes' law of friction. For $B > 0$ the force field may be interpreted as cosmic friction, for $B < 0$ as cosmic antifriction. We conclude that cosmic (anti)friction is the most general force field which is compatible with the cosmological principle.

With the expressions (26) for $\mathcal{S}^0$ and the equivalent force (32) we may calculate the ''source'' terms in the balances (24) and (25). For a B independent of E, the results are

$$\Gamma = -3B \tag{35}$$

and

$$t^a = 3Bu^a(\rho + p). \tag{36}$$

In general, neither the particle number nor the energy momentum are conserved. Consequently, T^{ab} in Eq. (22) is *not* the quantity which will appear on the right-hand side of Einstein's field equations. Equation (35) clarifies that the force strength determines the particle production (decay) rate $\Gamma = \dot{N}/N$. The entropy production density (18) is determined by

$$S^a_{;a} = \sigma_{\mathcal{S}} = -\alpha N^a_{;a} - \beta_a T^{ab}_{;b} = ns\Gamma = -3nsB. \tag{37}$$

A production of particles is characterized by $\Gamma > 0$ and corresponds to $B < 0$, i.e., an antifrictional force, while a decay of particles is equivalent to an effective friction.

C. Generalized equilibrium solutions

To obtain the conditions under which the equilibrium distribution (21) is preserved even under the action of an (anti)frictional force we insert the expression (21) into the Boltzmann equation (28) which yields

$$p^a \alpha_{,a} + \beta_{(a;b)} p^a p^b = -m\beta_i F^i. \tag{38}$$

Since $\beta_i \equiv u_i/T$, it is only the projection $u_i F^i$ of the force which is relevant here. If this projection vanishes, relation (38) reduces to the ''global'' equilibrium condition of standard relativistic kinetic theory, i.e., to $\alpha = \text{const}$ and either to the Killing vector condition $\beta_{(a;b)} = 0$ for $m > 0$, or to the conformal Killing vector condition $\beta_{(a;b)} = \phi(x) g_{ab}$ for $m = 0$. The Friedmann models do not supply a timelike Killing vector, thus there is no equilibrium solution for particles with finite mass. However, in the non-relativistic limit $T \ll m$ a quasi-equilibrium, characterized by $\alpha = m/T + \text{const}$, $\beta_i = u_i/T$, and $T \propto a^{-2}$, exists. In the case of cosmic (anti)friction we find a similar quasi-equilibrium solution for Friedmann models which reads

$$\alpha = \frac{m}{T} + \text{const}, \quad \beta_i = \frac{u_i}{T}, \quad \frac{\dot{T}}{T} = -2\left(\frac{\dot{a}}{a} + B\right). \tag{39}$$

With $3B = -\Gamma = -\dot{N}/N$, according to relation (35), the temperature behavior is

$$T \propto a^{-2} N^{2/3}. \tag{40}$$

For vanishing (anti)friction the particle number is constant and the familiar $T \propto a^{-2}$ dependence for non-relativistic matter is recovered.

The explicit knowledge of the force F^i in terms of Γ now allows us to study the motion of the matter particles explicitly. Contracting the equation of motion (29) with the macroscopic four-velocity results in

$$\frac{D(u_i p^i)}{d\tau} \equiv -\frac{dE}{d\tau} = u_i F^i + \frac{1}{m} u_{i;k} p^i p^k. \tag{41}$$

With Eq. (33) and under the condition of spatial homogeneity we have

$$\frac{dE}{d\tau} = -\frac{B+H}{m}(E^2 - m^2). \tag{42}$$

Since $d\tau = dt(m/E)$ and $dE/dt \equiv \dot{E}$ where $E = m + \varepsilon$ with $\varepsilon \ll m$, we finally obtain for the evolution of the non-relativistic (kinetic) energy the expression

$$\dot{\varepsilon} = -2\left(H - \frac{\Gamma}{3}\right)\varepsilon. \tag{43}$$

Consequently,

$$\varepsilon \propto a^{-2} N^{2/3} \propto T, \tag{44}$$

the equipartition theorem. Since the exponent $\alpha + \beta_a p^a = \alpha - E/T$ of the function (21) in the non-relativistic limit, with $\alpha = m/T + \text{const}$ and $E = m + \varepsilon$, reduces to $\alpha - E/T \rightarrow \text{const} + \varepsilon/T$, relation (44) demonstrates explicitly the invariance of the equilibrium distribution (21). The non-relativistic velocity scales as $v \propto a^{-1} N^{1/3}$.

With Eqs. (25) and (36) the fluid energy balance becomes

$$\dot{\rho} + 3H(\rho + p) = -3B(\rho + p) = \Gamma(\rho + p). \tag{45}$$

Introducing the quantity

$$\Pi \equiv \frac{B}{H}(\rho + p) = -\frac{\Gamma}{3H}(\rho + p), \tag{46}$$

Eq. (45) may be written in the form of the energy balance (5), where Π plays the role of an effective viscous pressure according to the definition (2), thus we have $P = p + \Pi$. The energy balance for T^{ab} with a generally non-vanishing source in Eq. (45) is identical to the conservation law $u_a T^{ak}_{\text{eff};k} = 0$ with the energy momentum tensor (1). It is *this* energy-momentum tensor that appears on the right-hand side of Einstein's field equations [see the discussion following Eq. (36)]. With the reinterpretation (46) of the sources on the right-hand side of the balance (25) in terms of an effective pressure of the medium the latter becomes a "closed" system. In the following we shall restrict ourselves to negative pressures Π, corresponding to the production of particles and to an antifrictional force. We emphasize again that despite the non-vanishing entropy production $S^a_{;a}$ the microscopic particles are always governed by an equilibrium distribution function. In this context $S^a_{;a} > 0$ describes just an enlargement of the phase space of the system but not a dissipative process. Although inter-particle collisions are necessary to establish an initial equilibrium characterized by Eq. (21), this equilibrium may then be maintained even under the influence of the antifrictional force and in the absence of further collisional interactions which might have been frozen out. The force which gives rise to a negative fluid pressure is compatible with an equilibrium distribution of the particles during the expansion. This feature is an essential advantage of the presented approach since it allows us to apply standard gas dynamical concepts to characterize "exotic" matter forms.

IV. COSMOLOGICAL DYNAMICS

Let us assume the cosmic substratum after matter-radiation decoupling to be non-relativistic matter with internal antifriction, characterized by the energy-momentum tensor (1) with $p \ll \rho$ and

$$P \approx \Pi = -\frac{\Gamma}{3H}\rho = -\frac{|B|}{H}\rho. \tag{47}$$

Bearing in mind that $\Gamma = \dot{N}/N$ the energy balance (45) may be integrated to yield

$$\rho = \rho_0 \frac{N}{N_0}\left(\frac{a_0}{a}\right)^3. \tag{48}$$

The index 0 again denotes the present epoch. According to the Friedmann equation for the spatially flat case, $8\pi G\rho = 3H^2$, to which we restrict ourselves from now on, the corresponding Hubble rate is given by

$$H = H_0\left[\frac{N}{N_0}\left(\frac{a_0}{a}\right)^3\right]^{1/2}. \tag{49}$$

For the ratio Π/ρ we obtain

$$\frac{\Pi}{\rho} = \frac{B}{H} = \frac{B}{H_0}\left[\frac{N_0}{N}\left(\frac{a}{a_0}\right)^3\right]^{1/2}. \tag{50}$$

The "particle number" changes as

$$N = N_0 \exp\left[-3\int_t^{t_0} dt|B|\right]. \tag{51}$$

For vanishing antifriction, corresponding to a conserved particle number, we have $N = N_0$.

The ratio $|B|/H$ enters the Hubble law for small redshift z (d_L is the luminosity distance),

$$H_0 d_L = z + \frac{1}{2}(1 - q_0)z^2 + \cdots, \tag{52}$$

via the deceleration parameter q, which for $k = 0$ and non-relativistic matter becomes

$$q = \frac{1}{2} - \frac{3}{2}\frac{|B|}{H}. \tag{53}$$

Generally, the luminosity distance d_L in a spatially flat universe may be written as

$$d_{\rm L} = (1+z)\int_0^z \frac{dz}{H(z)}. \tag{54}$$

As usual [46] this is related to the angular distance $d_{\rm A}$ of an object as

$$d_{\rm A}(z) = (1+z)^{-2} d_{\rm L}(z). \tag{55}$$

Relation (47), the field equations (9) and (10) for $k = 0$ and $p \ll \rho$ may be combined to yield

$$\frac{\Pi}{\rho} = \frac{B}{H} = -1 - \frac{2}{3}\frac{\dot{H}}{H^2}. \tag{56}$$

It is convenient to express the Hubble rate as a function of redshift $z = (a_0/a) - 1$. With

$$\dot{H} = -H' H (1+z),$$

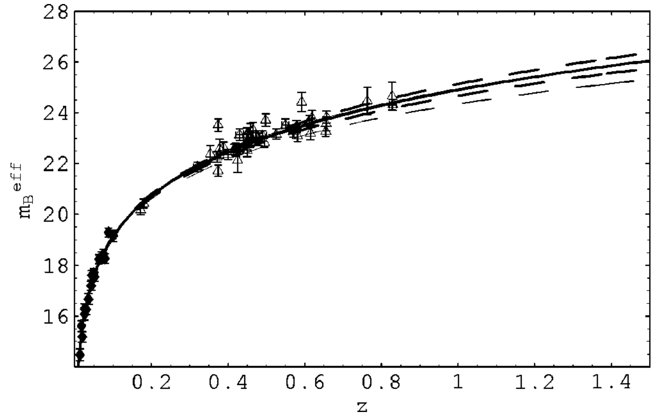

FIG. 1. Hubble diagram for $(\Omega_M,\Omega_\Lambda)=(1,0)$ (thin, dashed line), (0.3,0.7) (thin line) compared to case A with the values $\nu=0.7,0.5,0.3$ (thick lines from top to bottom). The data points are taken from Perlmutter *et al.* [3], the diamonds are the Calán/Tolodo SN1a data, the triangles are those of the SCP.

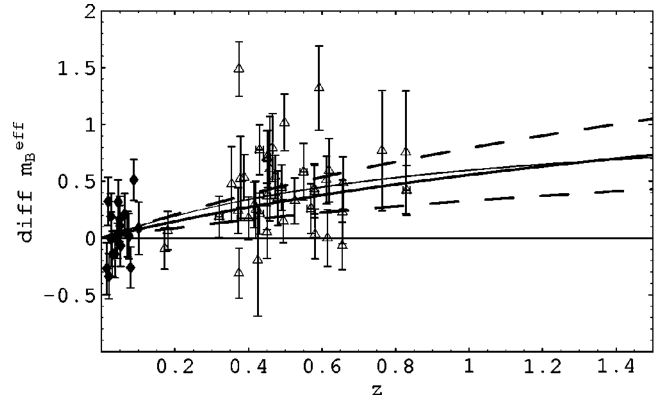

FIG. 2. Differences of the magnitudes with respect to a $(\Omega_M,\Omega_\Lambda)=(1,0)$ universe versus redshift. The plotted models and data are the same as in Fig. 1. Note that other authors often depict the magnitude difference with respect to an empty universe, which has negative spatial curvature and is thus incompatible with inflation.

where $H'\equiv dH/dz$, the resulting equation is

$$\frac{H'}{B+H}=\frac{3}{2(1+z)}. \tag{57}$$

To establish specific models, assumptions about B are necessary. In the following we solve the cosmological dynamics for three different choices of B and compare the results with the Hubble diagrams from recent SN Ia measurements. Although SN Ia data cannot discriminate between our different models, since they provide equally good fits for redshifts that are accessible with SN of type Ia, the additional information from the angular distance to the last scattering surface and cosmic microwave background (CMB) measurements will rule out one of the choices even without any detailed statistical analysis.

A. The case $B\propto -H$

The simplest possible choice is apparently $B=-\nu H$ with $\nu=$const. According to Eq. (47) this ansatz is equivalent to assuming a constant ratio Π/ρ, which should be considered realistic at most piecewise. With this assumption we get a Hubble rate

$$H(z)=H_0(1+z)^{(3/2)(1-\nu)}. \tag{58}$$

Note that the same power-law behavior follows if a perfect fluid with the equation of state $-\nu=w=p/\rho$ is assumed. We recover the matter dominated Universe for $\nu=0$ and the vacuum dominated universe for $\nu=1$. Equation (54) is easily integrated to provide an explicit expression for the luminosity distance ($\nu\neq 1/3$)

$$H_0 d_L=\frac{2}{3\nu-1}[(1+z)^{(1/2)(1+3\nu)}-1-z]. \tag{59}$$

This result coincides with the one by Lima and Alcaniz [47]. The corresponding luminosity distance-redshift relation is shown in Fig. 1 while Fig. 2 shows the difference to the standard cold dark matter (SCDM) model. We find that $\nu=0.5$ gives a good fit to the observations of SN Ia. At first sight one might have expected the best fit for $\nu=0.7$ since it is this value that reproduces an equation of state $P/\rho=P_0/\rho_0=-0.7$. However, as already indicated, a constant ratio P/ρ is realistic only piecewise. The fact that the better fit is $\nu=0.5$ suggests that ν must have been smaller for $z>0$ than for $z=0$. This illustrates the circumstance that the SN Ia observations for higher z do not directly reflect the cosmological equation of state at the present time but, as to be seen from the integral in the expression (54) for d_L, depend on the entire dynamics from redshift z to redshift 0.

The "particle number" changes according to

$$\frac{N}{N_0}=\exp\left(-3\int_0^z\frac{|B|}{H}\frac{dz}{(1+z)}\right)=(1+z)^{-3\nu}. \tag{60}$$

The energy density [cf. Eq. (48)] decays as $\rho\propto(1+z)^{3(1-\nu)}$, the temperature [cf. Eq. (40)] as $T\propto(1+z)^{2(1-\nu)}$. For the "best fit" $\nu=0.5$ there is a decrease in N/N_0 by about 3(5) for a redshift of 1(2). Over a larger interval N might decrease by many orders of magnitude. This again reflects the unrealistic nature of $P/\rho=$const over a large range of z values.

The observations of SN type Ia are restricted to redshifts of order 1. In order to test a specific model of antifriction at redshift $z\gg 1$, additional information is required. A very promising possibility are CMB anisotropy data. Acoustic oscillations in the early Universe give rise to peaks and dips in the band power spectrum of these anisotropies. The typical length scale of the largest acoustic oscillations is given by the sound horizon $R_s=c_s/H$ at the time of photon decoupling, where c_s is the sound velocity. This physical length corresponds to the presently observed angular scale [46] of the first acoustic peak

$$\delta_{\text{1st peak}}\approx\frac{R_s(z_{\text{dec}})}{d_A(z_{\text{dec}})}. \tag{61}$$

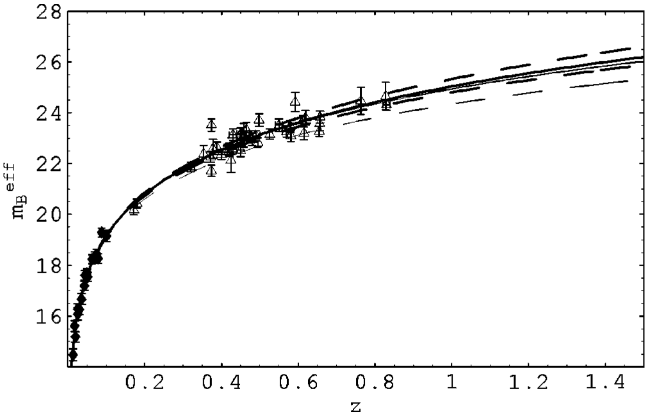

FIG. 3. Same as in Fig. 1 but for case B with $|B_0|/H_0=0.9,0.7,0.5$ (thick lines from top to bottom).

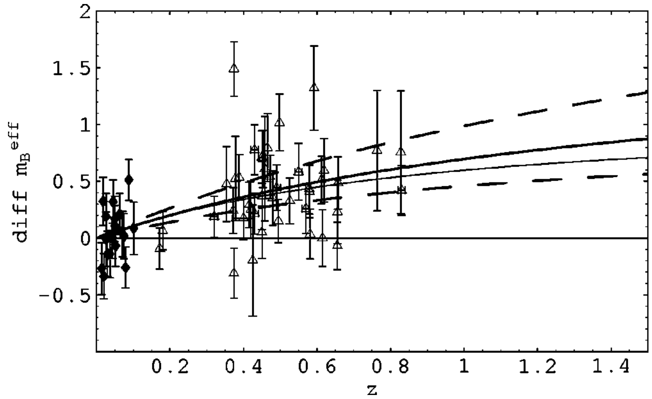

FIG. 4. Differences of the magnitudes versus redshift for the models in Fig. 3.

In fact, the corresponding observational data rule out any constant value for P/ρ. Namely, insertion of Eqs. (58), (55), and (59) leads to ($z_{dec}\gg1$)

$$\delta_{1st\ peak}\approx c_s\frac{3\nu-1}{2}\approx 0.14\ \text{rad}, \qquad (62)$$

which for $\nu=0.5$ and $c_s=\sqrt{1/3}$ exceeds the detected $\delta_{1st\ peak}\simeq 0.9°=0.016$ rad [6,7] by about one order of magnitude. Thus, a constant effective equation of state over a large range in redshift is incompatible with observations. To illustrate this feature we plot in Fig. 7 the angular scale under which the Hubble radius $H^{-1}(z)$ is seen for the "best fit" $\nu=0.5$. At $z=z_{dec}\approx 1100$ the Hubble radius differs from the sound horizon only by a factor $c_s\approx\sqrt{1/3}$. The present model corresponds to the upper curve. In the following we consider two cases for which P/ρ is time varying. Since we are interested in a modification of the cosmological dynamics at late times we focus on two models where this ratio is increasing with time.

B. The case $B=-|B_0|=$const

From Eq. (47) follows that the simplest choice leading to an increasing ratio $|\Pi|/\rho$ is a constant value of B. From Eq. (57) we obtain with $\nu_0\equiv|B_0|/H_0$,

$$H(z)=H_0[(1-\nu_0)(1+z)^{3/2}+\nu_0]. \qquad (63)$$

This solution describes a transition from matter domination to vacuum domination of the Universe. In the past, $z\gg1$, we have $H\propto(1+z)^{2/3}$, a matter dominated universe, while for the future, $z\to-1$, H, and thus ρ are constant. The expansion of the Universe starts to accelerate when $H(z)<3|B|$ [cf. Eq. (53)], which happens for a redshift z_{acc}, given by

$$1+z_{acc}=\left[\frac{2\nu_0}{1-\nu_0}\right]^{2/3}. \qquad (64)$$

In Figs. 3 and 4 we compare the Hubble diagram for this model to recent observations of SN Ia. Here we find a best fit value $\nu_0=0.7$. For this model the Universe starts acceleration at $z_{acc}\approx1.8$. The increase in the particle number may be expressed as

$$\frac{N}{N_0}=\frac{[(1-\nu_0)(1+z)^{3/2}+\nu_0]^2}{(1+z)^3}, \qquad (65)$$

which gives an increase by a factor of about 5 since z_{acc}.

Also for this model the angular scale of the Hubble radius is shown in Fig. 7 for $\nu_0=0.7$ (second curve). Obviously, here the Hubble scale is of the order of the scale of the first acoustic peak, i.e., this model is consistent with the CMB observations. However, a more quantitative statement could only be made on the basis of a Boltzmann code, which is beyond the scope of our present work.

C. The case $B\propto-H^{-1}$

Another choice leading to an increasing ratio $|\Pi|/\rho$ is $|B|\propto H^{-1}$, equivalent to an ansatz

$$\frac{|B|}{H}=\frac{1}{\mu+1}\frac{H_0^2}{H^2}. \qquad (66)$$

The specific choice of the constant factor $1/(\mu+1)$ was made for later convenience. Integration of Eq. (57) with the ansatz (66) yields

$$H=\frac{H_0}{\sqrt{1+\mu}}[\mu(1+z)^3+1]^{1/2}. \qquad (67)$$

For $z\gg1$ we have

$$H\propto(1+z)^{3/2}, \qquad (68)$$

which is characteristic of a matter-dominated universe. For the opposite case $z\to-1$ the Hubble rate approaches the constant value

$$H\to\frac{H_0}{\sqrt{\mu+1}}. \qquad (69)$$

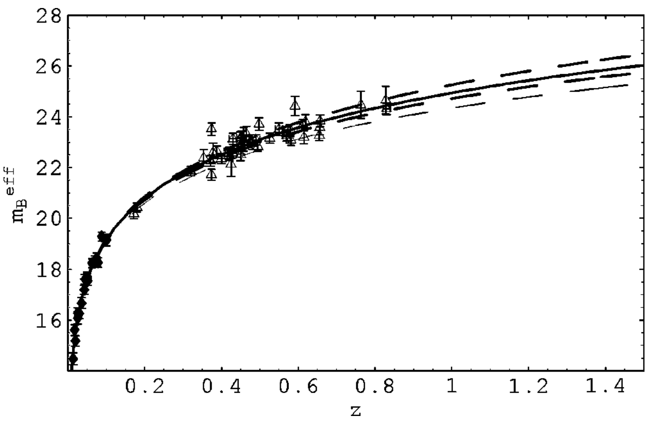

FIG. 5. Same as in Fig. 1 but for case C with $1/(1+\mu)=0.9,0.7,0.5$ (thick lines from top to bottom). Note that this case is identical to ΛCDM, with $\Omega_\Lambda=1/(1+\mu)$.

The Hubble rate (67) implies again a transition from a matter-dominated universe at $z \gg 1$ to a de Sitter universe as $z \rightarrow -1$. Equations (67) and (54) determine the luminosity distance, which is plotted in Figs. 5 and 6. This case in fact includes all ΛCDM models, as can be easily seen by replacing $1/(\mu+1) \rightarrow \Omega_\Lambda$ and $\mu/(\mu+1) \rightarrow \Omega_{\mathrm{CDM}}$. As expected, the best fit model has $1/(1+\mu)=0.7$.

To obtain the redshift at which acceleration starts we write

$$\frac{|B|}{H}=\frac{1}{\mu(1+z)^3+1}. \tag{70}$$

The condition $|B|/H \geqslant 1/3$ for accelerated expansion [cf. Eq. (53)] gives $1+z_{\mathrm{acc}}=(2/\mu)^{1/3}$. The growth of N follows from

$$N=N_0\frac{\mu(1+z)^3+1}{(\mu+1)(1+z)^3}. \tag{71}$$

For the favored value $1/(1+\mu)=0.7$ accelerated expansion starts at $z_{\mathrm{acc}} \approx 0.67$, which is in the expected range $0.5<z_{\mathrm{acc}}<1$ (cf. Ref. [48]), and N has grown since then by a factor of about 3.

Again we show an estimate of the angular scale of the Hubble radius [for $1/(1+\mu)=0.7$] in comparison to the

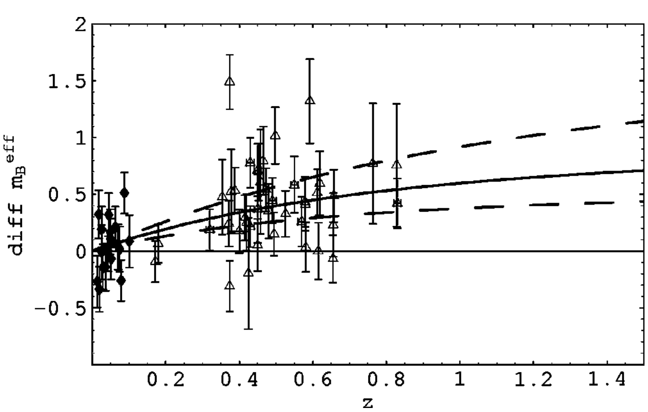

FIG. 6. Differences of the magnitudes versus redshift for the models in Fig. 5.

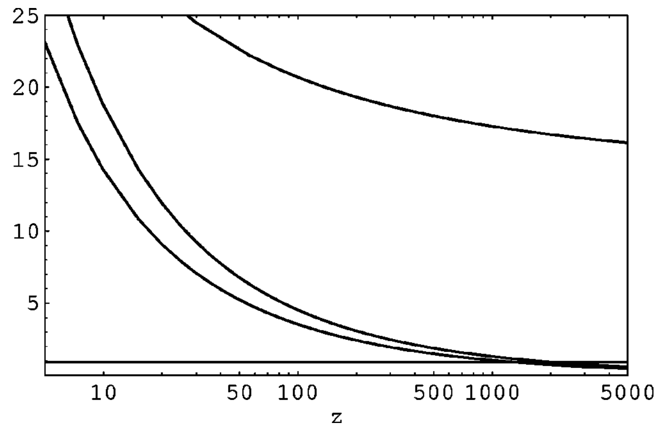

FIG. 7. The angular scale $H^{-1}(z)/d_{\mathrm{A}}(z)$ in degrees under which the Hubble radius H^{-1} is seen for the models of Secs. IV A, IV B, and IV C (from top to bottom). For comparison we indicate the angular scale of the observed first acoustic peak ($\simeq 0.9°$) by the horizontal line.

scale of the first acoustic peak in Fig. 7 (lower curve). This model is consistent with observations as well. The difference to the former model is large enough that it should give rise to a significant difference in the CMB predictions. We do not further elaborate this point here.

V. TWO COMPONENT MODELS AND THE COINCIDENCE PROBLEM

A. Interacting fluids

In this section we investigate in which sense cosmological antifriction may be regarded as effective description for an underlying two-component model. To this purpose we decompose the total energy density (48) into a "conventional" matter part (subscript M) and a "Q matter" part (subscript Q):

$$\rho=\rho_M+\rho_Q, \tag{72}$$

with

$$\rho_M \equiv r\rho_0\left(\frac{a_0}{a}\right)^3 \quad \text{and} \quad \rho_Q \equiv \left[\frac{N}{N_0}-r\right]\rho_0\left(\frac{a_0}{a}\right)^3, \tag{73}$$

where $r=r(a)>0$ with $r<N/N_0$ and $r_0 \equiv r(a_0)<1$. The factor r_0 fixes the ratio of both components at a_0:

$$\frac{\rho_M(a_0)}{\rho_Q(a_0)}=\frac{r_0}{1-r_0}. \tag{74}$$

For $r=$const the Universe evolves as though the cosmic substratum would consist of two non-interacting components: Non-relativistic matter with $p_M=0$ and Q-matter with an equation of state

$$p_Q=\Pi=\frac{B}{H}\rho=-\frac{|B|}{H}\frac{N/N_0}{(N/N_0)-r}\rho_Q \quad (r=\text{const}). \tag{75}$$

For an expanding universe the pressure Π is always negative. For any form of the potential of a quintessence model the corresponding coefficient of antifriction is easily obtained from Eq. (75).

If r is allowed to vary we have

$$\dot{\rho}_M + 3H\rho_M = \frac{\dot{r}}{r}\rho_M . \tag{76}$$

For $\dot{r}>0$ the term on the right-hand side may be regarded as a matter "source." This corresponds to a "sink" in the energy balance of the Q component,

$$\dot{\rho}_Q + 3H\rho_Q = \Gamma(\rho_Q+\rho_M) - \frac{\dot{r}}{r}\rho_M . \tag{77}$$

By introducing the effective pressures

$$P_M = \Pi_M \equiv -\frac{\dot{r}/r}{3H}\rho_M , \tag{78}$$

and

$$P_Q = \Pi - \Pi_M = -\left[\frac{\Gamma}{3H}\frac{N}{N_0} - \frac{\dot{r}}{3H}\right]\frac{\rho_Q}{(N/N_0)-r}, \tag{79}$$

the balances (76) and (77) become

$$\dot{\rho}_M + 3H(\rho_M + P_M) = 0 \tag{80}$$

and

$$\dot{\rho}_Q + 3H(\rho_Q + P_Q) = 0, \tag{81}$$

respectively. The circumstance that for $\dot{r}>0$ *both* components have a negative effective pressure (given that $|\Pi_M| < |\Pi|$) is essential for a discussion of the "coincidence problem" in the present context.

Supernovas Ia observations suggest that $|P|/\rho = |B|/H$ is of the order unity in the present epoch. Within our approach this raises the question: Why is the coefficient of antifriction $-B$ of the order of the Hubble rate just at the present epoch? As demonstrated in Sec. IV A for the case $B \propto -H$, the CMB data rule out that this was the case through the entire evolution of the Universe since decoupling. Thus it seems to be a coincidence that we live in this special era. The decomposition (72) allows us to relate our consideration to the usual discussion of the coincidence problem. It is obvious that all models according to which the Universe is made of a noninteracting mixture of quintessence and CDM correspond to $r=$const. In this special case our approach does not provide any new insight into the coincidence problem. For $\dot{r}>0$, however, it offers a solution which is similar to the one proposed by Chimento *et al.* [49]. The quantity of interest is the ratio ρ_M/ρ_Q which is governed by the equation

$$\left(\frac{\rho_M}{\rho_Q}\right)^{\cdot} = 3H\left[\frac{\rho_M}{\rho_Q}\right]\left[\frac{P_Q}{\rho_Q} - \frac{P_M}{\rho_M}\right]. \tag{82}$$

For $P_M=0$, or equivalently $r=$const, one has $P_Q=\Pi<0$, i.e., the ratio ρ_M/ρ_Q continuously decreases and for large cosmological times one has $\rho_M \ll \rho_Q$. In other words, the matter component becomes dynamically negligible. However, if an exchange between both components is admitted, which amounts to a nonvanishing quantity $P_M=\Pi_M$, there exists a second stationary solution of Eq. (82), namely,

$$\frac{P_Q}{\rho_Q} = \frac{P_M}{\rho_M}. \tag{83}$$

Combining relations (78) and (79) we obtain

$$\frac{P_Q}{\rho_Q} - \frac{P_M}{\rho_M} = \frac{1}{3H}\frac{N/N_0}{(N/N_0)-r}\left[\frac{\dot{r}}{r} - \Gamma\right]. \tag{84}$$

Obviously, the condition (83) is equivalent to

$$\frac{\dot{r}}{r} = \frac{\dot{N}}{N} \quad \Rightarrow \quad r = r_0\frac{N}{N_0}. \tag{85}$$

Via Eq. (73) the stationarity condition (85) for the ratio ρ_M/ρ_Q provides us with

$$\left(\frac{\rho_M}{\rho_Q}\right)_s = \frac{r_0}{1-r_0}, \tag{86}$$

which, according to the split (73), is just the present ratio of both components. Concerning its possible role for the coincidence problem it is interesting to investigate the stability properties of the solution (86). To this purpose we consider deviations from this stationary value:

$$\frac{\rho_M}{\rho_Q} = \left(\frac{\rho_M}{\rho_Q}\right)_s + \delta. \tag{87}$$

Since $(\rho_M/\rho_Q)^{\cdot}_s = 0$, the resulting equation is

$$\dot{\delta} = 3H\left[\left(\frac{\rho_M}{\rho_Q}\right)_s + \delta\right]\left[\frac{P_Q}{\rho_Q} - \frac{P_M}{\rho_M}\right]. \tag{88}$$

By visualizing this dynamics in a $\dot{\delta}-\delta$ diagram, the stationary solution (86) is an attractor solution for

$$\frac{P_Q}{\rho_Q} - \frac{P_M}{\rho_M} < 0 \quad \Rightarrow \quad 0 < \frac{\dot{r}}{r} < \Gamma = \frac{\dot{N}}{N}. \tag{89}$$

The rate $\dot{N}/N$ represents a limit for the rate $\dot{r}/r$ at which energy is transferred from the Q component to the matter. We conclude that a certain decay of vacuumlike Q matter into CDM offers a potentially promising approach to the coincidence problem. The phenomenological concept of a decaying vacuum is widely used in the physics of the early Universe in order to describe the decreasing dynamical role of an effective cosmological "constant" connected with a transition from an initial inflationary period to a subsequent FLRW behavior [50,51]. It is noteworthy that a similar mechanism seems to be relevant also for the transition from matter dominance to "vacuum" dominance in the late Universe. At first sight it might seem counter-intuitive that a decay of the vacuum at the same time leads to an apparent dominance of the latter. However, for the previously discussed model $B \propto -H^{-1}$ (Sec. IV C) we shall confirm explicitly that such kind of transition is indeed consistent with a positive rate $\dot{r}/r > 0$.

At this point we come back to our previous statements [following Eqs. (8) and (46)] on the nature of "particle production" in the presented formalism. According to Eq. (75) the ratio $|B|/H$, equivalent to $\Gamma/3H$, determines the contribution of the vacuum to the total energy density. Cosmic antifriction may be viewed as a vacuum effect which is connected with a nonvanishing particle production rate Γ. For an equation of state $\rho_M = n_M m$ it is obvious that $\dot{r}/r$ is the production rate of matter particles out of the decaying vacuum component Q. It represents a real physical particle production. According to Eq. (89) the rate $\dot{r}/r$ is smaller than $\Gamma \equiv \dot{N}/N$. Only for the "stationary" solution (85) the physical particle production rate coincides with Γ. For $r = \mathrm{const}$ the particle production is entirely connected with the Q component.

In our basic setting the cosmic substratum is entirely made of non-relativistic particles that are governed by an equilibrium distribution function. The splitting (73) has revealed that this "generalized" equilibrium description may be regarded as a two-fluid model of "conventional" and "exotic" matter. The point is that also the exotic matter is described in terms of conventional matter particles, only that the property of being exotic requires a "production" process. If generated at a certain rate, conventional particles in the expanding Universe effectively exhibit vacuum-like properties that manifest themselves macroscopically through a negative pressure.

Our fluid approach formally implies that a particle number is attributed to the vacuum component as well. At first sight the concept of a particle number of the vacuum might appear obscure. In a fluid picture, however, it appears quite naturally (cf. Ref. [51]), as long as the interpretation of the first moment (14) of the distribution function as "particle number flow" is maintained. As in any two-fluid picture, the production rate Γ naturally splits into

$$n\Gamma = n_M \Gamma_M + n_Q \Gamma_Q, \tag{90}$$

implying a split of the total particle number density n of the one-component model into $n = n_M + n_Q$, by which the notions of a particle number density n_Q of Q "particles" and a corresponding change rate Γ_Q are introduced. $\Gamma_M = \dot{r}/r$ is the rate by which the CDM particle number changes and n_M is the CDM particle number density. A non-vanishing Γ does not necessarily imply a non-vanishing Γ_M but may be a feature of the fluid picture of "vacuum matter," characterized by $\Gamma_Q \neq 0$.

B. The case $B \propto -H^{-1}$

In this section we focus on the previously discussed case $B \propto -H^{-1}$ [cf. Eqs. (66)–(71)] for which the energy density is given by

$$\rho = \frac{\rho_0}{\mu+1}[\mu(1+z)^3 + 1]. \tag{91}$$

Performing the splitting (73), it is convenient to replace r by

$$r(z) = \frac{\mu}{\mu+1} f(z), \quad r_0 = \frac{\mu}{\mu+1}, \quad f_0 = 1. \tag{92}$$

The energy density (91) then decomposes into

$$\rho_M = \frac{\mu}{\mu+1}\rho_0(1+z)^3 f(z),$$

$$\rho_Q = \frac{\rho_0}{\mu+1}[1 - [f(z)-1]\mu(1+z)^3]. \tag{93}$$

For the special case $f = 1$ this splitting characterizes a non-interacting mixture of non-relativistic matter and vacuum. This is just the ΛCDM model as already mentioned below Eq. (69). In the general case $f = f(z)$ both components interact. More specifically, there will be a decay of the Q component into matter. The constant μ is the ratio

$$\mu = \frac{\rho_M(z=0)}{\rho_Q(z=0)}. \tag{94}$$

Here it is expedient to emphasize that the basic one-component dynamics (67) and (70) is compatible with any continuous positive-definite but otherwise arbitrary function f. The splitting into two components does not affect the quantity Π/ρ at all. However, different splittings may produce different perturbation spectra. In particular, isocurvature perturbations may occur which should be sensitive to the type of splitting. Corresponding effects are expected to leave an imprint on the CMB anisotropies and are potential tools to discriminate between different choices. Furthermore, the underlying two-component dynamics is relevant for the coincidence problem. The corresponding stationary solution $\rho_M/\rho_Q = \mathrm{const}$ is easily found. The condition (85), together with Eqs. (92) and (71), provides

$$f_s = \frac{1}{\mu+1}[\mu + (1+z)^{-3}], \tag{95}$$

where the subscript s again indicates stationarity according to Eq. (86). The obtained solution (95) for f is expected to describe the splitting (93) for large cosmological times. For the corresponding asymptotic behavior of ρ_M and ρ_Q we obtain

$$(\rho_M)_s = \frac{\mu\rho_0}{(\mu+1)^2}[1 + \mu(1+z)^3] \tag{96}$$

and

$$(\rho_Q)_s = \frac{\rho_0}{(\mu+1)^2}[1 + \mu(1+z)^3], \tag{97}$$

respectively. These asymptotic expressions satisfy

$$\left(\frac{\rho_M}{\rho_Q}\right)_s = \mu = \frac{r_0}{1-r_0} = \text{const.} \tag{98}$$

Both components redshift at the same rate. There is a permanent energy transfer from the Q component to the matter component. Without transfer ρ_M would redshift as a^{-3} while ρ_Q would remain constant. The transfer makes ρ_Q redshift also and, on the other hand, ρ_M to redshift at a lower rate than without transfer. In general, the redshifts differ. For a specific amount of transfer, however, given by the expression (95) for f, the rates just coincide. We conclude that a fixed ratio ρ_M/ρ_Q is compatible with cosmic antifriction. If our Universe follows such an asymptotic solution presently, this should manifest itself by a violation of the CDM particle number. If we are still far from the asymptotic regime, the CDM particle production might be negligible. These considerations of the coincidence problem give rise to a picture according to which an initial cosmological term did not completely vanish during the early stages of the cosmological evolution but still exists and even continues to decay. At large cosmological times not only the cosmological term itself becomes dynamically relevant again but also its decay properties may be essential for the asymptotic state of the Universe (ignoring here a recently discussed scenario with a less bleak eschatological picture [52]).

VI. SUMMARY

We have introduced the concept of cosmic antifriction to discuss the possible origin of an accelerated expansion of the present Universe. Cosmic antifriction relies on a one-component picture of the cosmic substratum which is regarded as a self-interacting gas of non-relativistic particles in (generalized) equilibrium. Together with simple assumptions about the interaction rate it allowed us to establish exactly solvable models of the cosmological dynamics. Cosmic antifriction leads to a negative bulk pressure which may well account for the magnitude-redshift data of type Ia supernovae. The ΛCDM scenario is recovered as a special case of cosmic antifriction. For the models of Secs. IV B and IV C, which are consistent both with the SN Ia observations and with CMB anisotropy data, we find the beginning of the phase of accelerated expansion at redshifts $z_{acc} \approx 1.8$ and $z_{acc} \approx 0.7$, respectively. The one-component dynamics may be split into a two-fluid mixture in which a Q matter component decays into CDM. For a suitable decay rate there exists an attractor solution characterized by a fixed ratio of the energy densities of both components, which indicates a possible solution of the coincidence problem. As for scenarios with a cosmological constant or quintessence, the microphysical evidence for our models remains open. This shortcoming seems presently unavoidable and reflects our basic ignorance concerning the substance our Universe is made of.

ACKNOWLEDGMENTS

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