$B_s \to K$ form factor in the whole kinematically accessible range

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A systematic analysis is presented of the $B_s \to K$ form factor $f(q^2)$ in the whole range of momentum transfer q^2, which would be useful in analyzing the future data on $B_s \to K$ decays and extracting $|V_{ub}|$. With a modified QCD light cone sum rule (LCSR) approach, in which the contributions cancel out from the twist 3 wave functions of the K meson, we investigate in detail the behavior of $f(q^2)$ at small and intermediate q^2 and the nonperturbative quantity $f_{B*}g_{B*B_sK}$ (f_{B*} is the decay constant of B^* meson and g_{B*B_sK} the B^*B_sK strong coupling), whose numerical result is used to the study q^2 dependence of $f(q^2)$ at large q^2 in the single pole approximation. Based on these findings, a form factor model from the best fit is formulated, which applies to the calculation on $f(q^2)$ in the whole kinematically accessible range. Also, a comparison is made with the standard LCSR predictions.

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A study on heavy-to-light exclusive processes plays a complementary role in the determination of the fundamental parameters of the standard model (SM) and in the development of QCD theory. At present, an important task in the SM is to extract the Cabibbo-Kobayashi-Maskawa (CKM) parameter $|V_{ub}|$. Recently, a new QCD factorization formula [1] has been proposed for nonleptonic B decays and has been applied to discuss the phenomenology of $B \to \pi\pi$, πK, and πD. This approach, however, is not adequate to extract $|V_{ub}|$ from the relevant data, for the effects of the long distance QCD are anyway difficult to control in exclusive nonleptonic processes. Semileptonic B decays into a light meson, induced by the $b \to u$ transition, are regarded as the most promising processes suitable for such a purpose. It is the QCD light-cone sum rule (LCSR) approach [2] that can deal effectively with this type of decay channel. For a detailed description of this method, see Ref. [3]. However, a problem to be solved is how to control the pollution by higher twist, especially twist 3 wave functions, which are not well understood and whose influence on the sum rules is considerable in most cases. In Refs. [4,5], an improved LCSR approach has been worked out, to eliminate twist 3 wave functions and enhance the reliability of sum rule calculations, and has been applied to reexamine heavy-to-light form factors in the region of momentum transfer $0 \leqslant q^2 \leqslant m_b^2 - 2m_b\Lambda_{\rm QCD}$, where the operator product expansion (OPE) goes effectively in powers of small light-cone distance x^2.

Most previous works [3,4,6–8] are devoted to discussing $B \to \pi, \rho$ semileptonic transitions within the context of LCSR, with the aim to extract $|V_{ub}|$. A study on $B_s \to K$ semileptonic processes is equally important. As compared with the case $B \to \pi, \rho$, however, the $B_s \to K$ form factors are more difficult to evaluate, for SU(3) breaking corrections to the twist 3 wave functions of K meson have not been investigated completely in the literature. Explicitly, this problem can be avoided in our approach [4,5]. On the other hand, to calculate the semileptonic widths one must find another way to estimate the form factors at the large momentum transfer $m_b^2 - 2m_b\Lambda_{\rm QCD} \leqslant q^2 \leqslant (m_{B_s} - m_K)^2$. In this paper, we investigate the $B_s \to K$ form factor $f(q^2)$ in the whole range of the momentum transfer with the improved LCSR and a pole model.

We start with the definition of the $B_s \to K$ form factors $f(q^2)$ and $\tilde{f}(q^2)$: $\langle K(p)|\bar{u}\gamma_\mu b|B(p+q)\rangle = 2f(q^2)p_\mu + \tilde{f}(q^2)q_\mu$. For $B_s \to K l \tilde{\nu}_l$ transitions, as $l = e, \mu$ we can neglect the contributions from $\tilde{f}(q^2)$ due to the smallness of $m_{e,\mu}$ and therefore only the form factor $f(q^2)$ is relevant. It can precisely be represented as

$$f(q^2) = \frac{f_{B*}g_{B*B_sK}}{2m_{B*}(1 - q^2/m_{B*}^2)} + \int_{\sigma_0}^{\infty} \frac{\rho(\sigma)d\sigma}{1 - q^2/\sigma} = F_G(q^2) + F_H(q^2), \quad (1)$$

with f_{B*} being the decay constant of B^* meson, g_{B*B_sK} the strong coupling defined by $\langle B^*(q,e)K(p)|B_s(p+q)\rangle = -g_{B*B_sK}(p\cdot e)$, m_{B*} the B^* meson mass, and $\rho(\sigma)$ a spectral function with the threshold σ_0. Obviously, $F_G(q^2)$ stands for the contribution from the ground state B^* meson, which describes the principal behavior of $f(q^2)$ around $q^2 = q^2_{\rm max}$, and $F_H(q^2)$ parametrizes the higher state effects in the B^* channel. As we have known, the form factor $f(q^2)$ may be estimated for the small and intermediate momentum transfers by means of LCSR, and also the nonperturbative parameter $f_{B*}g_{B*B_sK}$ is accessible within the same framework. Accordingly, modeling the higher state contributions by a certain assumption and then fitting Eq. (1) to its LCSR result $f_{\rm LC}(q^2)$ in the region accessible to the light-cone OPE, we might derive the form factor $f(q^2)$ in the total kinematical range to a better accuracy. For this purpose, we follow

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the procedure in Refs. [4,5] and consider a chiral current correlator used for a LCSR sum rule calculation on $f_{\rm LC}(q^2)$ and $f_{B*}g_{B*B_sK}$,

$$\Pi_\mu(p,q)=i\int d^4x\, e^{iqx}\langle K(p)|T\{\overline{u}(x)\gamma_\mu(1+\gamma_5)b(x),$$

$$\overline{b}(0)i(1+\gamma_5)s(0)\}|0\rangle$$

$$=F(q^2,(p+q)^2)p_\mu+\widetilde{F}(q^2,(p+q)^2)q_\mu. \tag{2}$$

Inserting complete sets of the relevant intermediate states $|B^H\rangle$ in Eq. (2) and using the definitions $\langle 0|\overline{s}i\gamma_5 b|B_s\rangle=[m_{B_s}^2/(m_b+m_s)]f_{B_s}$ and $\langle 0|\overline{u}\gamma_\mu b|B^*\rangle=m_{B*}f_{B*}e_\mu$, we have the two hadronic representations of the invariant function $F(q^2,(p+q)^2)$,

$$F_1^H(q^2,(p+q)^2)=\frac{2f_{\rm LC}(q^2)m_{B_s}^2f_{B_s}}{(m_b+m_s)[m_{B_s}^2-(p+q)^2]}+\int_{s_0}^{\infty}\frac{\rho_1^H(s)}{s-(p+q)^2}ds, \tag{3}$$

$$F_2^H(q^2,(p+q)^2)=\frac{m_{B_s}^2m_{B*}f_{B_s}f_{B*}g_{B*B_sK}}{(m_b+m_s)(m_{B*}^2-q^2)[m_{B_s}^2-(p+q)^2]}+\int\int\frac{\rho_2^H(s_1,s_2)\Theta(s_1-s_0')\Theta(s_2-s_0)}{(s_1-q^2)[s_2-(p+q)^2]}ds_1\,ds_2. \tag{4}$$

Several definite interpretations for Eqs. (3) and (4) are in order. The two dispersion integrals include, in addition to the contributions of the resonances carrying the same quantum numbers as the corresponding ground states in the pole terms, the effects due to the relevant orbit-excited B mesons. Taking it into account that these orbit-excited states are far from the lowest B_s and B^* mesons, and the lowest two of them are slightly below the first excited B_s and B^* mesons in mass, their contributions can effectively absorbed into a dispersion integral so that thresholds s_0 and s_0' should correspond to the squared masses of the lowest 0^+ B_s and 1^+ B mesons, respectively. On the other hand, the vector current $\overline{u}\gamma_\mu b$ and axial-vector current $\overline{u}\gamma_\mu\gamma_5 b$ couple also to 0^+ and 0^- B mesons, respectively, which should be considered in Eq. (4). The invariant function, however, does not receive such a contribution as we have checked. Therefore it is safe to separate the hadronic expression $F_2^H(q^2,(p+q)^2)$ into a pole term and a dispersion integral.

The remaining task is to calculate the correlator in QCD theory in order to obtain the desired sum rules. To this end, we work in the large spacelike momentum regions: $(p+q)^2\ll 0$ for the $f_{\rm LC}(q^2)$ case and $q^2\ll 0$, $(p+q)^2\ll 0$ for the $f_{B_s}f_{B*}g_{B*B_sK}$ case, so that the light-cone OPE can be used for the correlator under consideration. Making the Borel improvements on the yielded theoretical expression $F^{\rm QCD}(q^2,(p+q)^2)\to\overline{F}_1{}^{\rm QCD}(q^2,M^2)$, $F^{\rm QCD}(q^2,(p+q)^2)\to\overline{F}_2{}^{\rm QCD}(M_1^2,M_2^2)$, and then matching them onto the individual Borel improved hadronic forms via the use of the quark-hadron duality ansatz, the final sum rules for $f_{\rm LC}(q^2)$ and g_{B*B_sK}, to twist-4 accuracy, read, respectively,

$$\begin{aligned}f(q^2)=&\frac{m_b(m_b+m_s)}{m_{B_s}^2f_{B_s}}f_Ke^{m_{B_s}^2/M^2}\Bigg\{\int_\Delta^1\frac{du}{u}e^{-[m_b^2-(q^2-um_K^2)(1-u)]/uM^2}\Bigg[\varphi_K(u)-\frac{4m_b^2}{u^2M^4}g_1(u)+\frac{2}{uM^2}\int_0^u g_2(v)\\&\times dv\left(1+\frac{m_b^2+q^2}{uM^2}\right)\Bigg]+\int_0^1d\alpha\int D\alpha_i\frac{\Theta(\beta-\Delta)}{\beta^2M^2}e^{-[m_b^2-(q^2-\beta m_K^2)(1-\beta)]/\beta M^2}[2\varphi_\perp(\alpha_i)+2\widetilde{\varphi}_\perp(\alpha_i)-\varphi_\parallel(\alpha_i)\\&-\widetilde{\varphi}_\parallel(\alpha_i)]-4m_b^2e^{-s_0/M^2}\Bigg[\frac{1}{(m_b^2-q^2)^2}\left(1+\frac{s_0-q^2}{M^2}\right)g_1(\Delta)-\frac{1}{(s_0-q^2)(m_b^2-q^2)}\frac{dg_1(\Delta)}{du}\Bigg]\\&-2e^{-s_0/M^2}\Bigg[\frac{m_b^2+q^2}{(s_0-q^2)(m_b^2-q^2)}g_2(\Delta)-\frac{1}{(m_b^2-q^2)}\left(1+\frac{m_b^2+q^2}{m_b^2-q^2}\left(1+\frac{s_0-q^2}{M^2}\right)\right)\int_0^\Delta g_2(v)dv\Bigg]\Bigg\},\end{aligned} \tag{5}$$

$$\begin{aligned}f_{B_s}f_{B*}g_{B*B_sK}=&\frac{2m_b(m_b+m_s)f_K}{m_{B_s}^2m_{B*}}e^{(m_{B_s}^2+m_{B*}^2)/2\overline{M}^2}\Bigg\{\overline{M}^2[e^{-(m_b^2+m_K^2/4)/\overline{M}^2}-e^{-s_0/\overline{M}^2}]\varphi_K(1/2)+e^{-(m_b^2+m_K^2/4)/\overline{M}^2}\\&\times\Bigg[g_2(1/2)-\frac{4m_b^2}{\overline{M}^2}\left(g_1(1/2)-\int_0^{1/2}g_2(v)dv\right)+\int_0^{1/2}d\alpha_1\int_{1/2-\alpha_1}^{1-\alpha_1}\frac{d\alpha_3}{\alpha_3}[2\varphi_\perp(\alpha_i)+2\widetilde{\varphi}_\perp(\alpha_i)\\&-\varphi_\parallel(\alpha_i)-\widetilde{\varphi}_\parallel(\alpha_i)]\Bigg]\Bigg\},\end{aligned} \tag{6}$$

where $\varphi_K(u)$ is the twist-2 wave function, while the others have twist 4, and we have used the definitions $\beta=\alpha_1+\alpha\alpha_3$, $\Delta=(m_b^2-q^2)/(s_0-q^2-m_K^2)$, $\overline{M}^2=M_1^2M_2^2/(M_1^2+M_2^2)$, and $D\alpha_i=d\alpha_1 d\alpha_2 d\alpha_3\delta(1-\alpha_1-\alpha_2-\alpha_3)$. It should be understood that in the derivation of Eq. (6) $M_1=M_2$ has been taken due to the fact that B_s and B^* mesons are nearly degenerate in mass, which renders the continuum subtraction reduced to a simple replacement $e^{-(m_b^2+m_K^2/4)/\overline{M}^2}\to e^{-(m_b^2+m_K^2/4)/\overline{M}^2}-e^{-s_0/\overline{M}^2}$ for the leading twist 2 term.

At this point, we put once again an emphasis on that different from the existing LCSR calculations, the twist-3 wave functions make precisely a vanishing contribution to the correlator we choose. This is essentially important for enhancing precision of the LCSR calculation.

We turn now to the numerical discussions on the sum rules. B channel parameters entering the sum rules are the b quark mass m_b, B meson masses m_{B_s} and m_{B^*}, decay constants f_{B_s} and f_{B^*}, and the threshold parameter s_0. We take $m_{B_s}=5.369$ GeV, $m_{B^*}=5.325$ GeV, and $m_b=4.8$ GeV. As for the decay constants f_{B_s} and f_{B^*}, we have to reanalyze them in the two-point QCD sum rule approaches [4,5] with chiral current correlators, to keep a consistency with the sum rules in question. The results are found to be $f_{B_s}=0.142$ GeV and $f_{B^*}=0.132$ GeV, as the threshold parameter $s_0=34$ GeV^2 corresponding to the mean value of squared masses of the lowest 0^+ B_s and 1^+B mesons. For the decay constant of K meson and mass of s quark, we use $f_K=0.16$ GeV and $m_s=0.15$ GeV. The important point is to specify the set of the light cone wave functions of the K meson. Unlike the case of the π meson, SU(3) breaking effects need considering for the distribution amplitudes of K meson. In the work, we use the model presented in Ref. [9] for the leading twist wave function, which is based on an expansion over orthogonal Gegenbauer polynomials with coefficients determined by means of QCD sum rules. The explicit expression is

$$\varphi_K(u)=6u(1-u)\{1+1.8[(2u-1)^2-\tfrac{1}{5}] -0.5(2u-1)[1+1.2[(2u-1)^2-\tfrac{3}{7}]]\}, \quad (7)$$

at the scale $\mu_b=\sqrt{m_{B_s}^2-m_b^2}$, measuring the mean virtuality of the b quark. For the twist 4 wave functions, we neglect the SU(3) breaking effects and utilize the same forms as those of the π meson investigated in Ref. [10].

Having fixed the input parameters, one must look for a reliable range of the Borel parameters M^2 and $\overline{M}^2$, which can be determined by the standard procedure. The fiducial intervals are found to be $8\leqslant M^2\leqslant 17$ GeV^2, depending slightly on q^2, for $q^2=0-17$ GeV^2 and $5\leqslant\overline{M}^2\leqslant 10$ GeV^2. In the two "windows," the twist 4 wave functions contribute less than 9 and 7%, and the continuum states at the levels lower than 25 and 22%, respectively. The sum rule results for $f_{LC}(q^2)$ show a weak dependence on M^2 up to $q^2=17$ GeV^2, varying between ±3 and ±5% relative to their central values. For the product $f_{B_s}f_{B^*}g_{B^*B_sK}$, the resulting sum rule is $f_{B_s}f_{B^*}g_{B^*B_sK}=0.55\,\text{GeV}^2$, the uncertainty due to $\overline{M}^2$ being ±4%. Taking its central value, we get $g_{B^*B_sK}=29$. To evaluate better the B^* pole contribution in Eq. (1), however, we would give a direct sum rule result for $f_{B^*}g_{B^*B_sK}$, which can be obtained utilizing the analytic form instead of the numerical result for the two-point sum rule for f_{B_s} in Eq. (6). The result is $f_{B^*}g_{B^*B_sK}=3.57-4.19$ GeV, depending on the Borel parameters. The sum rule prediction $f_{LC}(q^2)$, together with that from the B^* pole approximation, is illustrated in Fig. 1. It is explicitly demonstrated that a perfect match between them appears at $q^2\approx 15-20\,\text{GeV}^2$.

The influence on the sum rules should be investigated in detail from several important sources of uncertainty: the twist 2 distribution amplitude $\varphi_K(u)$, b quark mass m_b, decay constants f_{B_s} and f_{B^*}, and the threshold parameter s_0. Concerning the light cone wave function $\varphi_K(u)$, there are some determinations other than that in Eq. (7) in the literature. To investigate the sensitivity of the sum rules to the choice of the nonasymptotic coefficients in $\varphi_K(u)$, we consider the two models suggested in Refs. [7] and [8], and confront the resulting sum rules with ours. If adopting $\varphi_K(u)$ in Ref. [7], the resulting changes amount to $-8--9$% for the $f_{LC}(q^2)$ case and to ±5% for the $f_{B^*}g_{B^*B_sK}$ case. Almost the same situation exists for that used in Ref. [8]. Therefore the uncertainties caused by $\varphi_K(u)$ may be estimated at a considerably small level. As for the B channel parameters m_b, f_{B_s}, f_{B^*}, and s_0, considering a correlated variation in the individually allowed ranges would give sufficient information on the uncertainties induced by them. This can be done in such a way where letting m_b vary from 4.7 to 4.9 GeV, we observe the behavior of $f_{LC}(q^2)$ and $f_{B^*}g_{B^*B_sK}$ by requiring that the relevant decay constants take only the best fitting values. We find that such an effect amounts to 6 and 5%, respectively. At present, the total uncertainties in

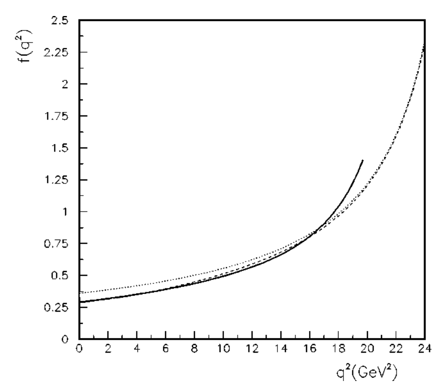

FIG. 1. The $B_s\to K$ form factor $f(q^2)$ in the total kinematical range. The solid line denotes the LCSR result $f_{LC}(q^2)$, which is reliable for $0\leqslant q^2\leqslant 17\,\text{GeV}^2$. The dotted line expresses the B^* pole prediction suitable for large q^2. The best fit of Eq. (8) to $f_{LC}(q^2)$ is illustrated by the dashed line. It should be understood that the plotted curves correspond to the central values of all the relevant parameters.

$f_{\rm LC}(q^2)$ and $f_{B^*}g_{B^*B_sK}$ can respectively be estimated to be 20% and 18%, by adding up linearly all the considered errors.

It is important and interesting to make a comparison of our sum rule results and those from the standard LCSR based on the correlator of vector and pseudoscalar currents, which are easy to obtain using the twist 3 wave functions suggested in Ref. [11], leaving the twist 4 distribution amplitudes unchanged and making a corresponding replacement of the other relevant input parameters in Eqs. (79) and (44) of the second part of Ref. [6]. We observe that the standard approach gives the same matching range as in our case and the resulting deviations from our predictions turn out to be between -10% and -15%, depending on q^2, in the total kinematically accessible region. This denotes that both approaches are essentially compatible with each other within the available errors.

With the yielded findings we would give a specific parametrization for $f(q^2)$ applicable to the whole kinematical region, which is helpful for the future practical application. Assuming the higher state contribution in Eq. (1) to obey $F_H(q^2)=a/(1-bq^2/m_{B^*}^2-cq^4/m_{B^*}^4)$, we have a model for the form factor $f(q^2)$,

$$f(q^2)=\frac{f_{B^*}g_{B^*B_sK}}{2m_{B^*}(1-q^2/m_{B^*}^2)}+\frac{a}{1-bq^2/m_{B^*}^2-cq^4/m_{B^*}^4}. \tag{8}$$

The parameter a can easily be fixed at -0.07, using the central values of $f_{\rm LC}(0)$ and $f_{B^*}g_{B^*B_sK}$. In the region $q^2=0-18$ GeV^2, the best fit of Eq. (8) to $f_{\rm LC}(q^2)$ yields $b=1.11$ and $c=-8.33$. The resulting q^2 dependence of $f(q^2)$ is demonstrated in Fig. 1 too, for a comparison. It turns out that the fitting results reproduce precisely the LCSR prediction up to $q^2=18\,\text{GeV}^2$ and support considerably the single pole description of the $B_s\to K$ form factor $f(q^2)$ at large q^2.

Also, it is worthwhile to look roughly into SU(3) breaking effects in heavy to light decays by considering the ratio of the derived $B_s\to K$ form factor over the corresponding $B\to\pi$ one. The $B\to\pi$ form factor has already been obtained for small and intermediate q^2 in the improved LCSR approach in Ref. [4]. Using all the same methods as in the present case, we can understand its behavior at large q^2 and further get a parametrization holding for the total kinematical range. For the common kinematical region to the two processes, the resulting ratios, a comparable result 1.05–1.15 with that from the standard approach, favor a small SU(3) breaking effect.

We have given a detailed discussion on the $B_s\to K$ form factor $f(q^2)$ in the whole kinematical region. To avoid the contamination with the twist 3 wave functions, in which SU(3) breaking corrections have not been analyzed systematically, an improved LCSR approach with some kind of chiral current correlator has been applied to estimate the form factor $f_{\rm LC}(q^2)$ at small and intermediate q^2. The nonperturbative quantity $f_{B^*}g_{B^*B_sK}$, an important input in the B^* pole model for $f(q^2)$, has also been calculated within the same framework and the sum rule result has been adopted to study the behavior of $f(q^2)$ at large q^2. We find that the resulting $f_{\rm LC}(q^2)$ matches quite well with the estimate from the B^* pole model at $q^2=15-20$ GeV^2. A comparison shows that our predictions are in basic agreement with those from the standard LCSR. Based on our findings, a model for $f(q^2)$ has been worked out, which is applicable to the total kinematically accessible region. The results presented here would be used as analyzing the future data on $B_s\to K$ decays and extracting $|V_{ub}|$. A future lattice calculation of the $B_s\to K$ form factors, which is available for large q^2, will provide a direct test of our predictions. The same approach applies also to discussions of other heavy to light processes.

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