Remark on neutrino masses and oscillations in an $SU(3)_L \times U(1)_N$ model with the radiative mechanism

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We have discussed how neutrino masses and oscillations are radiatively generated in an $SU(3)_L \times U(1)_N$ gauge model with a symmetry based on L_e-L_μ-L_τ ($\equiv L'$). The model is characterized by lepton triplets $\psi^i = (\nu^i, l^{-i}, E^{-i})$, where E^{-i} are negatively charged heavy leptons, an $SU(3)_L$ triplet Higgs scalar ξ, and a singlet Higgs scalar k^{++}. These Higgs scalars can be interpreted as a Zee and Zee-Babu scalar for radiative mechanisms. We have demonstrated that the mass hierarchy of $\Delta m^2_{\text{atm}} \gg \Delta m^2_{\odot}$ arises as a consequence of the dynamical hierarchy between L'-conserving one-loop effects and L'-violating two-loop effects, where L' is broken softly in the Higgs potential, and that our model is relevant to yield a quasivacuum solution for the solar neutrino problem.

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There is definitive evidence for neutrino oscillations from atmospheric and solar neutrino observations. For the atmospheric neutrino oscillations, the recent SuperKamiokande (SK) data indicate that the observed deficit of ν_μ is due to the $\nu_\mu \leftrightarrow \nu_\tau$ oscillation [1,2] while for the solar neutrino oscillations [3], the SK, Homestake [4], SAGE [5], GALLEX [6] and GNO [7] data indicate the $\nu_e \leftrightarrow \nu_\mu, \nu_\tau$ oscillation. The existence of these neutrino oscillations implies the neutrinos are massive particles [8]. The mass squared differences for atmospheric oscillations Δm^2_{atm} are measured as $\Delta m^2_{\text{atm}} \sim 3 \times 10^{-3}$ eV^2 [9]. On the other hand, there are some solutions to explain the observed solar neutrino oscillation data as: (1) $\Delta m^2_{\odot} \sim 10^{-5}$ eV^2 for the large mixing angle solution, (2) $\Delta m^2_{\odot} \sim 10^{-6}$ eV^2 for the small mixing angle solution, (3) $\Delta m^2_{\odot} \sim 10^{-7}$ eV^2 for the low mass, low probability solution, (4) $\Delta m^2_{\odot} \sim 10^{-10}$ eV^2 for the vacuum oscillation (VO) solution, and recently proposed (5) $\Delta m^2_{\odot} \sim 10^{-9}$ eV^2 for the quasi-VO (QVO) solution [10]. To sum up, we have $\Delta m^2_{\text{atm}} \sim 10^{-3}$ eV^2 and $\Delta m^2_{\odot} \lesssim 10^{-5}$ eV^2, indicating the hierarchy of $\Delta m^2_{\text{atm}} \gg \Delta m^2_{\odot}$ exists. In the theoretical view, this mass hierarchy suggests that the neutrino mass matrix has bimaximal structure [11,12].

Recently, radiative mechanisms to generate tiny neutrino masses and oscillations in $SU(3)_L \times U(1)_N$ gauge models [13–15] with the L' symmetry have been extensively studied [16–18]. Here $L' \equiv L_e$-L_μ-L_τ is a new lepton number and the conservation of this quantum number is one of the possibilities of the origin of the bimaximal structure [19,20]. Three $SU(3)_L \times U(1)_N$ gauge models are used to accommodate such radiative mechanisms. Each of the $SU(3)_L \times U(1)_N$ models can be distinguished by the lepton triplets ψ^i ($i=1,2,3$) in the models: (a) $\psi^i = (\nu^i, l^i, \omega^{0i})$ model [16], (b) $\psi^i = (\nu^i, l^i, \kappa^{+i})$ model[1] [17], and (c) $\psi^i = (\nu^i, l^i, E^{-i})$ model [18], where ω^{0i}, κ^{+i} and E^{-i} are denoted by electrically neutral heavy leptons, positively charged heavy leptons, and negatively charged heavy leptons, respectively. In model (a) and model (b), the atmospheric neutrino oscillations are generated by a one-loop radiative mechanism with L'-conserving interactions [21], and the solar neutrino oscillations are induced from a two-loop radiative mechanism with L'-violating interactions [22]. Consequently, the mass hierarchy of $\Delta m^2_{\text{atm}} \gg \Delta m^2_{\odot}$ is explained as a result of the smallness of the two-loop effects compared with one-loop effects [23,24]. On the other hand, in model (c), there is no one-loop interaction and both of the atmospheric and solar neutrino oscillations come from two-loop radiative effects [22]. The mass hierarchy of $\Delta m^2_{\text{atm}} \gg \Delta m^2_{\odot}$ is related to the dynamical hierarchy of the L'-conserving and L'-violating two-loop interaction effects.

In this paper, we show that it is possible to construct the other $SU(3)_L \times U(1)_N$ model with lepton triplets $\psi^i = (\nu^i, l^i, E^{-i})$ [25]. The model has similar particle content to model (c); however, one-loop interactions also exist and neutrino masses are induced by the L'-conserving one-loop radiative mechanism as well as the L'-violating two-loop radiative mechanism.

The particle content in our $SU(3)_L \times U(1)_N$ gauge model is summarized as follows:

$$\psi_L^{i=1,2,3} = (\nu^i, l^i, E^i)_L^T : (\mathbf{3}, -2/3),$$

$$l_R^{1,2,3} : (\mathbf{1}, -1), \quad E_R^{1,2,3} : (\mathbf{1}, -1), \tag{1}$$

in the lepton sector, where we have denoted E^{-i} by E^i,

$$Q_L^1 = (u^1, d^1, d'^1)_L^T : (\mathbf{3}, 0),$$

$$Q_L^{i=2,3} = (d^i, -u^i, u'^i)_L^T : (\mathbf{3^*}, 1/3),$$

$$u_R^{1,2,3} : (\mathbf{1}, 2/3), \quad d_R^{1,2,3} : (\mathbf{1}, -1/3),$$

$$u'^{2,3}_R : (\mathbf{1}, 2/3), \quad d'^1_R : (\mathbf{1}, -1/3), \tag{2}$$

in the quark sector, and

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[1]The model with $\psi^i = (\nu^i . l^i, l^{+i})$ has been lately examined to yield tiny neutrino masses and observed neutrino oscillations [15].

TABLE I. L and L' quantum number.

Fields	η,ρ,χ,ξ	ψ_L^1,l_R^1,E_R^1	$\psi_L^{2,3},l_R^{2,3},E_R^{2,3}$	k^{++}
L	0	1	1	-2
L'	0	1	-1	-2

$$\eta=(\eta^0,\eta^-,\overline{\eta^-})^T:(\mathbf{3},-2/3),\quad \rho=(\rho^+,\rho^0,\overline{\rho^0})^T:(\mathbf{3},1/3),$$
$$\chi=(\chi^+,\overline{\chi^0},\chi^0)^T:(\mathbf{3},1/3),\quad \xi=(\xi^{++},\overline{\xi^+},\xi^+)^T:(\mathbf{3},4/3),$$
$$k^{++}:(\mathbf{1},2), \tag{3}$$

in the Higgs sector, where the quantum numbers are specified in parentheses by $(SU(3)_L,U(1)_N)$. Let $N/2$ be the $U(1)_N$ quantum number, then the hypercharge (Y) and electric charge (Q_e) are given by $Y=\lambda^8/\sqrt{3}+N$ and $Q_e=(\lambda^3+Y)/2$, respectively, where λ^a is the $SU(3)$ generator with $\mathrm{Tr}(\lambda^a\lambda^b)=2\delta^{ab}$ $(a,b=1,\ldots,8)$. Three Higgs triplets η,ρ and χ are the minimal set to generate masses of quarks and leptons in $SU(3)\times U(1)_N$ models. An additional Higgs triplet ξ is introduced as a triplet version of the Zee scalar to realize the one-loop radiative mechanism [26] and an additional Higgs singlet k^{++} is introduced to realize the two-loop radiative mechanism [27].

Here, we introduce two constraints to obtain the relevant Yukawa interactions. The first is the $L'\equiv L_e$-L_μ-L_τ conservation imposed on our interactions to reproduce the observed atmospheric neutrino oscillations as mentioned. The L' assignment is shown in Table I. The second is the discrete symmetry based on Z_4 to suppress unwanted flavor-changing-neutral current (FCNC) interactions in the quark sector and the lepton sector. In the quark sector, there are quarks with the same charge, thus, quark mass terms can be generated by ρ and χ between Q_L^1 and down-type quarks and by $\rho^\dagger$ and $\chi^\dagger$ between $Q_L^{2,3}$ and up-type quarks. FCNC is induced from these interactions [28]. Also, in the lepton sector, l^i and E^i $(i=1,2,3)$ has the same charge and the similar FCNC problem can occur. To avoid such interactions, Yukawa interactions must be constrained such that a quark (lepton) flavor gains a mass from only one Higgs scalar [29]. These situations can be realized by introducing the following Z_4 symmetry into the model: $\psi_L^{1,2,3}\to i\psi_L^{1,2,3}$, $l_R^{1,2,3}\to l_R^{1,2,3}$, $E_R^{1,2,3}\to -iE_R^{1,2,3}$, $Q_L^1\to iQ_L^1$, $Q_L^{2,3}\to -iQ_L^{2,3}$, $u_R^{1,2,3}\to u_R^{1,2,3}$, $d_R^{1,2,3}\to d_R^{1,2,3}$, $u_R'^{2,3}\to iu_R'^{2,3}$, $d_R'^1\to -id_R'^1$, $\eta\to i\eta$, $\rho\to i\rho$, $\chi\to -\chi$, $\xi\to -\xi$, and $k^{++}\to ik^{++}$.

With these constraints, the Yukawa interactions are given by

$$-\mathcal{L}_Y=\epsilon^{\alpha\beta\gamma}\sum_{i=2,3}f_{[1i]}\overline{(\psi_{\alpha L}^1)^c}\psi_{\beta L}^i\xi_\gamma+\sum_{i=1,2,3}\overline{\psi_L^i}(f_l^i\rho l_R^i+f_E^i\chi E_R^i)+\sum_{i,j=2,3}f_k^{ij}\overline{(l_R^i)^c}E_R^jk^{++}+\overline{Q_L^1}(\eta U_R^1+\rho D_R^1+\chi D_R'^1)+\sum_{i=2,3}\overline{Q_L^i}(\eta^*D_R^i+\rho^*U_R^i+\chi^*U_R'^i)+(\mathrm{H.c.}), \tag{4}$$

where f's are Yukawa couplings with the relation $f_{[ij]}=-f_{[ji]}$ demanded by the Fermi statics, and right-handed quarks are denoted by $U_R^i=\Sigma_{j=1}^3f_{uj}^iu_R^j$, $D_R^i=\Sigma_{j=1}^3f_{dj}^id_R^j$, $U_R'^i=f_{u'2,3}^iu_R'^{2,3}$, and $D_R'^1=f_{d'1}^1d_R'^1$. For simplicity, we have assumed diagonal mass terms for the leptons. Note that there is no term which can induce FCNC interactions such as $\overline{Q_L^1}\chi D_R^1$, $\overline{Q_L^1}\rho D_R'^1$, $\overline{Q_L^{2,3}}\chi^*U_R^{2,3}$, $\overline{Q_L^{2,3}}\rho^*U_R'^{2,3}$, $\overline{\psi_L^i}\chi l_R^i$, and $\overline{\psi_L^i}\rho E_R^i$.

The Higgs interactions are given by self-Hermitian terms of $\phi_\alpha\phi_\beta^\dagger$ ($\phi=\rho,\ \eta,\ \chi,\ \xi,\ k^{++}$), and two types of non-self-Hermitian Higgs potentials:

$$V_0=\lambda_0\epsilon^{\alpha\beta\gamma}\eta_\alpha\rho_\beta\chi_\gamma+\lambda_1(\eta^\dagger\rho)(\xi^\dagger\chi)+\lambda_2(\eta^\dagger\chi)(\xi^\dagger\rho)+(\mathrm{H.c.}),$$
$$V_b=\mu_b\xi^\dagger\eta k^{++}+(\mathrm{H.c.}), \tag{5}$$

where $\lambda_{0,1,2}$ stands for L'-conserving coupling constants and μ_b denotes the L'-violating mass scale. The interaction of the $\eta\rho\chi$ type in Eq. (5) is a guarantee of the orthogonal choice of vacuum expectation values for three Higgs scalars, η, ρ, and χ as $\langle 0|\eta|0\rangle=(v_\eta,0,0)^T$, $\langle 0|\rho|0\rangle=(0,v_\rho,0)^T$, and $\langle 0|\chi|0\rangle=(0,0,v_\chi)^T$, respectively.

We note that there are two main differences between model (c) discussed in Ref. [18] and the model in this paper (current model). The first is the absence of a $SU(3)_L$ singlet Higgs scalar k'^{++} in the current model. The model (c) has two Zee-Babu type Higgs scalars called k^{++} and k'^{++}, which are needed to realize L'-conserving and L'-violating two-loop interactions. However, in the current model, only one Higgs k^{++} is introduced and no additional singlet Higgs is needed because L'-conserving one-loop effects will serve as the L'-violating two-loop effects in model (c). The second is the different implementation of the discrete symmetry into the models. The discrete symmetry based on Z_2 is required in model (c) to avoid the FCNC interactions and to prevent the realization of the one-loop effects. Meanwhile, in the current model, the discrete symmetry based on Z_4 is introduced and the one-loop effects are allowed.

Now, let us demonstrate how radiative corrections induce neutrino masses in our model. The Yukawa interaction denoted by $\mathcal{L}_Y$ and L'-conserving Higgs potential V_0 work together to generate one-loop interactions as shown in Fig. 1, also $\mathcal{L}_Y$ and L'-violating Higgs potential V_b yield two-loop interactions as shown in Fig. 2. From the one-loop diagrams, we obtained the following Majorana neutrino masses:

$$m_{1i}^{(1)}=f_{[1i]}\left[\lambda_1\frac{m_{l^i}^2F(m_{l^i}^2,m_{\xi^+}^2,m_{\rho^+}^2)-m_e^2F(m_e^2,m_{\xi^+}^2,m_{\rho^+}^2)}{v_\rho^2}+\lambda_2\frac{m_{E^i}^2F(m_{E^i}^2,m_{\overline{\xi^+}}^2,m_{\chi^+}^2)-m_{E^1}^2F(m_{E^1}^2,m_{\overline{\xi^+}}^2,m_{\chi^+}^2)}{v_\chi^2}\right]\times v_\eta v_\rho v_\chi, \tag{6}$$

where

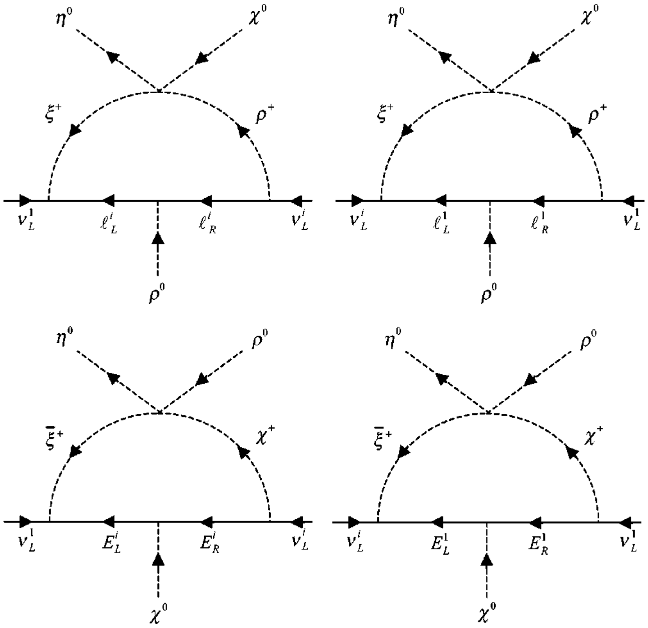

FIG. 1. L'-conserving one-loop diagrams.

$$F(x,y,z)=\frac{1}{16\pi^2}\left[\frac{x\ln x}{(x-y)(x-z)}+\frac{y\ln y}{(y-x)(y-z)}+\frac{z\ln z}{(z-y)(z-x)}\right], \tag{7}$$

and, from the two-loop diagrams, we obtain

$$m_{11}^{(2)}=-2\sum_{i,j=2,3}\lambda_2 f_{[1i]}f_{[1j]}f_k^{ij}\mu_b m_{l^i}m_{E^j}v_\rho v_\chi I^{(2)} \tag{8}$$

with

$$I^{(2)}=\frac{G(m_{l^i}^2,m_{\xi^+}^2)[G(m_{E^j}^2,m_{\bar{\eta}^-}^2)-G(m_{E^j}^2,m_{\bar{\xi}^+}^2)]}{m_k^2(m_{\bar{\eta}^-}^2-m_{\bar{\xi}^+}^2)},$$

$$G(m_a^2,m_b^2)=\frac{1}{16\pi^2}\frac{m_a^2\ln(m_a^2/m_k^2)-m_b^2\ln(m_b^2/m_k^2)}{m_a^2-m_b^2}, \tag{9}$$

where the relation of $m_k \gg$ (masses of other particle) has been used. The outline of the derivation of the two-loop integral, Eq.(9), is shown in the Appendix of Ref. [16].

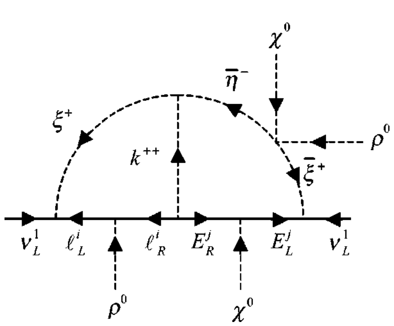

FIG. 2. L'-violating two-loop diagrams.

The neutrino mass matrix is composed of these Majorana masses as

$$M_\nu=\begin{pmatrix} m_{11}^{(2)} & m_{12}^{(1)} & m_{13}^{(1)} \\ m_{12}^{(1)} & 0 & 0 \\ m_{13}^{(1)} & 0 & 0 \end{pmatrix}, \tag{10}$$

from which we find the following relations for the neutrino oscillations in our $SU(3)_L\times U(1)_N$ model

$$\Delta m_{\rm atm}^2=m_{12}^{(1)2}+m_{13}^{(1)2}(\equiv m_\nu^2), \quad \Delta m_\odot^2=2m_\nu|m_{11}^{(2)}|. \tag{11}$$

The bimaximal structure of M_ν is realized by requiring that $|m_{12}^{(1)}|\sim|m_{13}^{(1)}|$, thereby, leading to $m_{E^2}\sim m_{E^3}$ or $m_{E^2,E^3}\ll m_{E^1}$ because the charged lepton contributions are to be neglected in our case. We assume that $m_{E^2}\sim m_{E^3}$ in our analysis.

In order to see that our result, Eq. (11), really regenerates the observed neutrino oscillations, we make the following assumptions on relevant free parameters in the same way as those in Ref. [17]: (1) $v_\eta=v_w/20$, $v_\rho=v_w$ and $v_\chi=10v_w$, where $v_w=(2\sqrt{2}G_F)^{-1/2}=174$ GeV, (2) $m_\xi\sim m_\rho=v_w$, $m_{\chi,k}=10v_w$, $m_{E^2,E^3}=ev_\chi$ to enhance the bimaximal mixing and $m_{E^1}=0.9m_{E^2,E^3}$ to contribute to $\Delta m_{\rm atm}^2$, where e stands for the electromagnetic coupling, (3) $f_{[1i]}\sim 10^{-7}$, $\lambda_1=\lambda_2=f_k^{ij}=1$ and $\mu_b=ev_\chi$, where $f_{[1i]}$ is determined by $\Delta m_{\rm atm}^2=m_{12}^{(1)2}+m_{13}^{(1)2}=3.0\times10^{-3}$ eV^2. From numerical calculations of Eq. (11), we find $f_{[1i]}=0.93\times10^{-7}$ eV^2, which reproduce $\Delta m_{\rm atm}^2=3.0\times10^{-3}$ eV^2 and $\Delta m_\odot^2=0.91\times10^{-9}$ eV^2. As a result, the mixing angle ϑ for atmospheric neutrinos defined by $\cos\vartheta=m_{12}^{(1)}/m_\nu$ is computed to yield $\sin^2 2\vartheta=0.93$, where the charged lepton contributions to $\Delta m_{\rm atm}^2$ give the deviation form $\sin^2 2\vartheta=1$ for the bimaximal mixing case. The estimated $\Delta m_\odot^2$ lies in the allowed region of the QVO solution to the solar neutrino problem.

Summarizing our discussion, we have constructed an $SU(3)_L\times U(1)_N$ gauge model with lepton triplets $\psi^i=(\nu^i,l^{-i},E^{-i})$, where E^{-i} are negatively charged heavy leptons. This model has a triplet version of the Zee scalar ξ and a singlet as the Zee-Babu scalar k^{++}. Owing to the existence of these scalars, our $SU(3)_L\times U(1)_N$ model is capable of generating tiny neutrino masses by the radiative mechanism. The atmospheric neutrino oscillation is related to L'-conserving one-loop interactions, while the solar neutrino oscillation is related to L'-violating two-loop interactions, where $L'\equiv L_e-L_\mu-L_\tau$. As a result, the bimaximal structure of the neutrino mass matrix is enhanced by the approximate degeneracy between masses of heavy leptons of E^2 and E^3. The observed mass hierarchy of $\Delta m_{\rm atm}^2\gg\Delta m_\odot^2$ is explained by the difference between one-loop and two-loop effects. From our numerical estimate, our model reproduces the observed neutrino oscillation data $\Delta m_{\rm atm}^2=3.0\times10^{-3}$ eV^2 with $\sin^2\vartheta=0.93$ and $\Delta m_\odot^2=0.91\times10^{-9}$ eV^2. Our model is, thus, relevant to yield quasivacuum solution for the solar neutrino problem.

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