Model for decoherence of entangled beauty

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In the context of the entangled $B^0\bar{B}^0$ state produced at the $\Upsilon(4S)$ resonance, we consider a modification of the usual quantum-mechanical time evolution with a dissipative term, which contains only one parameter denoted by λ and respects complete positivity. In this way a decoherence effect is introduced in the time evolution of the two-particle $B^0\bar{B}^0$ state, which becomes stronger with increasing distance between the two particles. While our model of time evolution has decoherence for the two-particle system, we assume that, after the decay of one of the two B mesons, the resulting one-particle state obeys the purely quantum-mechanical time evolution. From the data on dilepton events we derive an upper bound on λ. We also show how λ is related to the so-called "decoherence parameter" ζ, which parametrizes decoherence in neutral flavored meson-antimeson systems.

I. INTRODUCTION

There is increasing interest in recent years in using particle physics phenomena for the study of possible deviations from quantum mechanics (QM). Efforts have concentrated on two types of phenomena: oscillations, such as K^0-$\bar{K}^0$ [1] and neutrino oscillations [2], and quantum entanglement, where particularly suitable systems are the entangled $K^0\bar{K}^0$ and $B^0\bar{B}^0$ states [3] which are produced in e^+e^- collisions at the resonances Φ and $\Upsilon(4S)$, respectively. These states become macroscopically extended objects before they decay. Thus in both types of phenomena macroscopic distances are involved. Furthermore, entangled systems are—due to Einstein-Podolsky-Rosen- (EPR-)Bell correlations [4]—important objects to clearly test QM against local realistic theories. Whereas entangled systems like $K^0\bar{K}^0$ have a rather long-standing and venerable position in the QM literature [5], the physics of neutrino oscillations is a rather recent testing ground for QM; this development has been boosted, in particular, by the now well-established atmospheric neutrino anomaly, but also solar neutrinos and neutrinos in the early universe are discussed in this context.

In this paper we concentrate on possible decoherence effects which might arise due to some fundamental modification of QM or due to the interaction of the system with its "environment," whatever this may be. In the latter case, the idea of the influence of quantum gravity [6,7]—quantum fluctuations in the space-time structure at the Planck mass scale—is especially attractive nowadays. Possible effects of the environment have been investigated intensively in the $K^0\bar{K}^0$ system in Refs. [1] and [8–14]. But also other models of decoherence, like those found in Refs. [15–18], may serve as a working hypothesis. Our model, which we will propose in this paper, has some remote similarity with the models mentioned here, but ours will be tailored to the situation of two particles moving apart in their center of mass system.

In the past, a measure of decoherence for entangled systems has been introduced on pure phenomenological grounds in order to determine quantitatively deviations from pure QM. This simple procedure of multiplying the quantum-mechanical interference term by $1-\zeta$ [19], where ζ is called decoherence parameter, is a basis-dependent concept and works very well as a measure for interpolating continuously between pure QM ($\zeta=0$) and total decoherence ($\zeta=1$). The latter case corresponds to spontaneous factorization, also called Furry's hypothesis [20]. By investigating certain observables, the authors of Refs. [21–25] could show that the entangled $K^0\bar{K}^0$ and $B^0\bar{B}^0$ systems are far from total decoherence, at least when ζ is introduced in relation to the basis of mass eigenstates, so that local realistic theories are highly unlikely. In other words, the presence of the interference term is well established in agreement with QM (see also Ref. [26]), which means that there is quantum interference of massive particles over macroscopic distances.

In this paper we want to present a model of dissipation for entangled systems of two particles. In contrast to the prevailing concept in the literature, where dissipation is introduced at the one-particle level and transferred to the two-particle level through the tensor product structure of the Hilbert space of states (see, e.g., Ref. [12]), we assume the usual quantum-mechanical time evolution for the one-particle states. Thereby we have in mind that entangled two-particle systems become decoherent when they move apart over macroscopic distances, whereas for a one-particle system QM is not modified. Our dissipative term in the two-particle time evolution obeys the condition of complete positivity [27]. For reasons given below, we consider the entangled $B^0\bar{B}^0$ system with negative C parity. By using the experimental value of the ratio R of the number of like-sign dilepton events over opposite-sign dilepton events, we can derive a bound on the strength of the dissipative term. Considering the time-integrated dilepton event rates, our model reproduces precisely the corresponding calculations with the phenomenological decoherence parameter ζ associated with the B_H-B_L basis. As a result, we obtain a remarkably simple formula which relates the dissipative strength to the decoherence parameter ζ. In the context of the observable R, we also compare our model of two-particle decoherence with the case where the analogous dissipative term is introduced already at the one-particle level.

II. MODEL

Before considering the $B^0\bar{B}^0$ system, let us first discuss our model of decoherence in a two-dimensional Hilbert space of states $\mathcal{H}=\mathbf{C}^2$. We allow for a non-Hermitian Hamiltonian H, in order to include the possibility of incorporating particle decay in the Weisskopf-Wigner approximation [28]. We denote the normalized energy eigenstates by $|e_j\rangle$ ($j=1,2$) and have therefore

$$H|e_j\rangle = \lambda_j|e_j\rangle \text{ with } \lambda_j = m_j - \frac{i}{2}\Gamma_j, \quad (2.1)$$

where m_j and Γ_j are real and the latter quantities are positive in addition. Furthermore, we make the crucial assumption that

$$\langle e_1|e_2\rangle = 0 \quad (2.2)$$

despite the non-Hermiticity of H. Including decoherence, the time evolution of the density matrix ρ has the form

$$\frac{d\rho}{dt} = -iH\rho + i\rho H^\dagger - D[\rho]. \quad (2.3)$$

Our model of decoherence consists in assuming that

$$D[\rho] = \lambda(P_1\rho P_2 + P_2\rho P_1), \text{ where } P_j = |e_j\rangle\langle e_j| \quad (2.4)$$

and λ is a positive constant. Such a term is also employed, for instance, in the context of neutrinos in the early universe (see, e.g., Ref. [29]). It can readily be checked that the decoherence term in Eq. (2.4) is of the Lindblad type [27]:

$$D[\rho] = \frac{1}{2}\left(\sum_j A_j^\dagger A_j\rho + \rho\sum_j A_j^\dagger A_j\right) - \sum_j A_j\rho A_j^\dagger, \quad (2.5)$$

if we make the identification $A_j = \sqrt{\lambda}P_j$. Thus the term (2.4) generates a completely positive map; moreover, since $P_j^\dagger = P_j$ and $[P_j,H]=0$, the decoherence term would increase the "von Neumann entropy" and conserve energy in the case of a Hermitian Hamiltonian (see Ref. [30] and references therein). However, what is more important in our discussion is the fact that with the choice (2.4) the equations for the components of ρ decouple. Indeed, with

$$\rho = \sum_{j,k=1}^2 \rho_{jk}|e_j\rangle\langle e_k|, \quad (2.6)$$

where $\rho_{jk} = \rho_{kj}^*$, and with the time evolution (2.3), we obtain

$$\rho_{11}(t) = \rho_{11}(0)\exp(-\Gamma_1 t),$$

$$\rho_{22}(t) = \rho_{22}(0)\exp(-\Gamma_2 t), \quad (2.7)$$

$$\rho_{12}(t) = \rho_{12}(0)\exp\{-[i(m_1-m_2)$$
$$+ (\Gamma_1+\Gamma_2)/2 + \lambda]t\}.$$

Let us for a moment dwell on the motivation for our model of decoherence. To this end we start with the more general setting $A_j = \sqrt{\lambda_j}P_j$ ($j=1,2$ and $\lambda_j>0$) with $P_j = |p_j\rangle\langle p_j|$, where the normalized vectors $|p_j\rangle$ are linearly independent and in general different from the eigenvectors of the Hamiltonian. Note that we also allow for $\langle p_1|p_2\rangle \neq 0$. In any case, one can use formula (2.5) to obtain a completely positive dissipative term in the time evolution (2.3), but in general one will not obtain the form of $D[\rho]$ given by Eq. (2.4). For the time being we want to assume that $H^\dagger = H \neq 0$ holds and that H is nondegenerate in order to avoid trivial considerations. Now we have two possibilities: 1. The system $\{|p_1\rangle,|p_2\rangle\}$ is the system of eigenvectors of H, i.e., it is equivalent to the orthonormal system $\{|e_1\rangle,|e_2\rangle\}$; 2. $\{|p_1\rangle, |p_2\rangle\}$ is not equivalent to the system of eigenvectors of H. In the first case one can show that the form (2.4) of $D[\rho]$ with $\lambda = (\lambda_1+\lambda_2)/2$ is obtained and that

$$\text{case } 1 \Leftrightarrow [H,P_j]=0 \text{ for } j=1,2:$$

$$\lim_{t\to\infty}\rho(t) = P_1\rho(0)P_1 + P_2\rho(0)P_2 \quad (2.8)$$

holds. Furthermore, density matrices P_j, or linear combinations thereof, are constant solutions of the time evolution equation. In the second case, at least one of the vectors $|p_1\rangle$, $|p_2\rangle$ is not an eigenvector of H and one can prove that

$$\text{case } 2 \Leftrightarrow \exists j=1 \text{ or } 2 \text{ with } [H,P_j] \neq 0: \lim_{t\to\infty}\rho(t) = \frac{1}{2}\mathbf{1}, \quad (2.9)$$

independent of $\rho(0)$. We will see in the following—when we apply our model to the $B^0\bar{B}^0$ system—that the first case is closer to our physical intuition (see also last paragraph of this section). The two cases have been described in Refs. [31] and [32] in the context of one-particle decoherence in neutrino oscillations. Note that case 1 is used in Ref. [2] (see also Ref. [30]), whereas case 2 is considered, e.g., in Ref. [33] in the same context. If we allow for $H^\dagger \neq H$, the picture we have developed here gets blurred because then there is a competition between particle decay, i.e., $\lim_{t\to\infty}\text{Tr}\,\rho(t)=0$, and the effect of decoherence. We nevertheless stick to the first case. Note that identifying the orthonormal system given by P_j ($j=1,2$) with the system of eigenvectors of H is not only motivated by the considerations above but also by simplicity; as we have seen in Eq. (2.7) we have decoupled time evolutions as a consequence. We want to stress, however, that in the case of CP violation, which is particularly important for the $K^0\bar{K}^0$ system, it might be useful to allow for small deviations from $\langle e_1|e_2\rangle = 0$, and thus for small deviations of the eigenvectors of the Hamiltonian from the orthonormal system $\{|p_1\rangle,|p_2\rangle\}$.

Returning from general considerations, we now apply our model of decoherence to the case of the two-particle $B^0\bar{B}^0$ state, generated by the decay of the $\Upsilon(4S)$ resonance (for the formalism used in the $B^0\bar{B}^0$ system see, e.g., Ref. [34]). We conceive t as the eigentime of B^0 and $\bar{B}^0$ and make the identification

$$|e_1\rangle = |B_H \otimes B_L\rangle \quad \text{and} \quad |e_2\rangle = |B_L \otimes B_H\rangle, \quad (2.10)$$

where the heavy and the light neutral B states are defined via

$$|B_H\rangle = p|B^0\rangle + q|\bar{B}^0\rangle \quad \text{and} \quad |B_L\rangle = p|B^0\rangle - q|\bar{B}^0\rangle, \quad (2.11)$$

which have eigenvalues

$$\lambda_H = m_H - \frac{i}{2}\Gamma_H \quad \text{and} \quad \lambda_L = m_L - \frac{i}{2}\Gamma_L, \quad (2.12)$$

respectively, of the effective one-particle Hamiltonian H_1. For the two-particle system, we transfer, as usual, the one-particle Hamiltonian to the tensor product of the one-particle Hilbert spaces by using $H = H_1 \otimes \mathbf{1} + \mathbf{1} \otimes H_1$. We imagine that the first factor in the tensor product corresponds to particles moving to the left, whereas the second factor in the tensor product corresponds to right-moving particles. We assume CP conservation in B^0-$\bar{B}^0$ mixing, which is a good approximation [34,35] and corresponds to $|p/q| = 1$. In this case we have $\langle B_H | B_L \rangle = 0$ and therefore $\langle e_1 | e_2 \rangle = 0$. In the following we will set $p = q = 1/\sqrt{2}$. At the $\Upsilon(4S)$ resonance, at $t = 0$, the entangled state

$$|\psi\rangle = \frac{1}{\sqrt{2}}(|e_1\rangle - |e_2\rangle) \quad (2.13)$$

is produced, which is equivalent to the density matrix

$$\rho(0) = \frac{1}{2}(|e_1\rangle\langle e_1| + |e_2\rangle\langle e_2| - |e_1\rangle\langle e_2| - |e_2\rangle\langle e_1|). \quad (2.14)$$

With the time evolution (2.7), the initial condition (2.14) and taking into account that in the case of the vectors (2.10) we have $\lambda_1 = \lambda_2 = m_H + m_L - i\Gamma$ with $\Gamma \equiv (\Gamma_H + \Gamma_L)/2$, we obtain the time evolution

$$\rho(t) = \frac{1}{2}e^{-2\Gamma t}\{|e_1\rangle\langle e_1| + |e_2\rangle\langle e_2|$$

$$- e^{-\lambda t}(|e_1\rangle\langle e_2| + |e_2\rangle\langle e_1|)\}. \quad (2.15)$$

Note that the factor $\exp(-\lambda t)$ in the density matrix (2.15) introduces decoherence as a consequence of the D term in the time evolution (2.3). In other words, for $t > 0$ and $\lambda > 0$, the density matrix (2.15) does not correspond to a pure state anymore.

Having chosen the energy eigenstates (2.10) for the construction of the projectors P_j, our model complies with case 1 (2.8). In this case, we would have no decoherence if $\rho(0) = P_1$ or P_2, though such initial conditions might be unrealistic. This agrees with our intention because in these cases we have no entanglement over macroscopic distances and no reason for modifying the quantum-mechanical time evolution. Note that using the projector states (2.10) confines our Hilbert space of states to a two-dimensional one. Using projector states which are nontrivial orthogonal linear combinations of the states (2.10) would lead to case 2 (2.9), still

with a two-dimensional Hilbert space. However, using projector states which are *not* linear combinations of the states (2.10), like, e.g., $|B^0 \otimes \bar{B}^0\rangle$, $|\bar{B}^0 \otimes B^0\rangle$, entails not only a time evolution into the full four-dimensional Hilbert space of states, including $|B_H \otimes B_H\rangle$ and $|B_L \otimes B_L\rangle$, but also opens up the possibility for more involved schemes of decoherence than given by our simple model.

III. MEASUREMENT

In order to obtain information on the parameter λ, which modifies the time evolution in the $B^0\bar{B}^0$ system, we adopt the following philosophy. We start at $t = 0$ with the density matrix (2.14) for a $B^0\bar{B}^0$ state with negative C parity. This two-particle density matrix follows the time evolution (2.15) and undergoes thereby some decoherence. We imagine a measurement of the B quantum number of the left-moving particle at time t_l and of the right-moving particle at time t_r. For times $\min(t_l, t_r) < t < \max(t_l, t_r)$ we have a one-particle state which we assume to evolve exactly according to QM, with the time evolution given by H_1.

In a mathematical language, we do the following. Assuming for definiteness $t_l < t_r$, at $t = t_l$ we calculate the trace

$$\text{Tr}_l\{(|n\rangle\langle n| \otimes \mathbf{1})\rho(t_l)\} \equiv \rho_r(t_l; t = t_l), \quad (3.1)$$

where Tr_l means the trace evaluated only in the space of the left-moving particles; $\rho(t_l)$ is given by Eq. (2.15), evaluated at $t = t_l$; moreover, we have defined $|1\rangle = |B^0\rangle$ and $|2\rangle = |\bar{B}^0\rangle$, and $n = 1,2$. Consequently, $\rho_r(t_l; t = t_l)$ is a one-particle density matrix for the right-moving ones. For $t > t_l$, it is denoted by $\rho_r(t_l; t)$ and follows the one-particle time evolution. At $t = t_r$, where we measure the B quantum number of the right-moving particles, we finally have ($n' = 1,2$)

$$N(n, t_l; n', t_r) = \text{Tr}\{|n'\rangle\langle n'|\rho_r(t_l; t_r)\}. \quad (3.2)$$

Using all the above formalism and allowing also for $t_l > t_r$, we arrive at

$$N(n, t_l; n', t_r)$$

$$= \frac{1}{2}e^{-\Gamma(t_l + t_r)}\{|\langle n|B_H\rangle|^2|\langle n'|B_L\rangle|^2 e^{-\Delta\Gamma(t_l - t_r)/2}$$

$$+ |\langle n|B_L\rangle|^2|\langle n'|B_H\rangle|^2 e^{\Delta\Gamma(t_l - t_r)/2}$$

$$- e^{-\lambda\min(t_l, t_r)}(\langle n|B_H\rangle\langle n|B_L\rangle^*\langle n'|B_L\rangle$$

$$\times \langle n'|B_H\rangle^* e^{-i\Delta m(t_l - t_r)} + \langle n|B_L\rangle\langle n|B_H\rangle^*\langle n'|B_H\rangle$$

$$\times \langle n'|B_L\rangle^* e^{i\Delta m(t_l - t_r)})\}. \quad (3.3)$$

In this equation we have used the notation $\Delta\Gamma = \Gamma_H - \Gamma_L$ and $\Delta m = m_H - m_L$. For the sake of clarity, we have retained the scalar products $\langle n|B_{H,L}\rangle$ and $\langle n'|B_{H,L}\rangle$. According to our assumption of CP conservation in B^0-$\bar{B}^0$ mixing, we will replace them by their values $\pm 1/\sqrt{2}$. It is easy to check that for $\lambda = 0$ one obtains the usual expressions found in the literature. Note that for $t_l = t_r$ and $n = n'$ we have

$$N(n,t_l;n,t_l)=\frac{1}{4}e^{-2\Gamma t_l}(1-e^{-\lambda t_l}), \qquad (3.4)$$

which is different from zero, in contrast to the standard quantum-mechanical case.

IV. DILEPTONIC DECAYS

In practice, measurement of the B quantum number of neutral mesons in the entangled $B^0\bar{B}^0$ state proceeds via flavor tagging when the mesons decay. Assuming the validity of the $\Delta B = \Delta Q$ rule, in inclusive semileptonic decays l^+ tags B^0 and l^- tags $\bar{B}^0$ ($l=e$ or μ). In the following we will concentrate on dilepton events [36]. Denoting the inclusive semileptonic decay rate by Γ_l, the numbers of dilepton events from the decay of $|\psi\rangle$ (2.13) are then given by the integrals

$$N_{++}=\Gamma_l^2\int_0^\infty dt_l\int_0^\infty dt_r\,N(1,t_l;1,t_r),$$

$$N_{--}=\Gamma_l^2\int_0^\infty dt_l\int_0^\infty dt_r\,N(2,t_l;2,t_r),$$

$$N_{+-}=N_{-+}=\Gamma_l^2\int_0^\infty dt_l\int_0^\infty dt_r\,N(1,t_l;2,t_r).$$
$$(4.1)$$

Defining $x=\Delta m/\Gamma$ and $y=\Delta\Gamma/2\Gamma$ and calculating these integrals leads to the result

$$N_{++}=N_{--}=\frac{\Gamma_l^2}{4\Gamma^2}\left\{\frac{1}{1-y^2}-\frac{1}{1+x^2}[1-\zeta(\Lambda)]\right\},$$
$$(4.2)$$

$$N_{+-}=N_{-+}=\frac{\Gamma_l^2}{4\Gamma^2}\left\{\frac{1}{1-y^2}+\frac{1}{1+x^2}[1-\zeta(\Lambda)]\right\},$$
$$(4.3)$$

where the function $\zeta(\Lambda)$ is given by the simple expression

$$\zeta(\Lambda)=\frac{\Lambda}{2+\Lambda}\quad\text{with}\quad\Lambda=\frac{\lambda}{\Gamma}. \qquad (4.4)$$

It is interesting to note that x does not enter into ζ.

Equations (4.2) and (4.3) reproduce the results of Refs. [22] and [23], where the "decoherence parameter" ζ [19] is introduced phenomenologically in the observable

$$R=\frac{N_{++}+N_{--}}{N_{+-}+N_{-+}}, \qquad (4.5)$$

by multiplying the interference terms in N_{++}, etc., with $(1-\zeta)$. In the model presented here, ζ is expressed by the parameter λ [see Eq. (4.4)], the strength of the dissipative term in the modified time evolution (2.3). It has been discussed in the literature that the above phenomenological procedure of introducing a parameter ζ depends on the basis chosen in B^0-$\bar{B}^0$ space [20,22–24,37]. In the present model it is the ζ associated with the B_H-B_L basis.

Let us perform a numerical estimate of ζ and Λ along the lines presented in Ref. [21]. To this end we use R (4.5), which has been measured by the ARGUS [38] and CLEO

[39] Collaborations. Combining both measurements, we obtain the value $R_{\text{exp}}=0.189\pm0.044$ [21]. As far as x is concerned, we use the value $x_{\text{exp}}=0.740\pm0.031$ obtained by combining the data from all CERN e^+e^- collider LEP experiments [35]. With the approximation $y=0$ in Eqs. (4.2) and (4.3) [35,40] and using the law of propagation of errors, from R_{exp} and x_{exp} we derive the following numerical estimates:

$$\zeta=-0.06\pm0.10\quad\text{and}\quad\Lambda=-0.11\pm0.18. \qquad (4.6)$$

The Belle Collaboration has published data on the correlated semileptonic decay rate as a function of the difference t_l-t_r [41]. Of course, these data could also be used to put a limit on λ, if we integrate $N(n,t_l;n',t_r)$, Eq. (3.3), over t_l+t_r. However, we do not have enough information to perform such a fit.

Let us now compare our model, where decoherence is implemented at the two-particle level, with the case where we have the analogous time evolution (2.3) at the one-particle level [12,13]. We use the same structure of the D term as given by Eq. (2.4), but now with $|e_1\rangle=|B_H\rangle$ and $|e_2\rangle=|B_L\rangle$, instead of Eq. (2.10). We denote the strength of the dissipative term by ξ, in order to distinguish it from λ in the case of two-particle decoherence. Evidently, we have the same time evolution (2.7) at the one-particle level, with λ replaced by ξ. Following the steps to derive Eq. (3.3), we obtain the same formula, except that $\exp[-\lambda\min(t_l,t_r)]$ is replaced by $\exp[-\xi(t_l+t_r)]$. Eventually, we arrive at N_{++} and N_{+-} given by Eqs. (4.2) and (4.3), respectively, where ζ is now given by[1]

$$\zeta(\Xi,x)=\frac{(1+\Xi)^2-1}{(1+\Xi)^2+x^2}\quad\text{with}\quad\Xi=\frac{\xi}{\Gamma}. \qquad (4.7)$$

Thus the two models of decoherence cannot be distinguished on the basis of the time-integrated dilepton event rates, but only on the basis the time-dependent event rates. A numerical estimate analogous to the one performed for Λ leads to the result $\Xi=-0.04\pm0.07$.

V. SUMMARY

In this paper we have considered a model of decoherence applicable in the center of mass system of two particles. Our model reflects the idea that, when the two particles move apart and eventually become macroscopically separated, some "forces" might be operative which de-entangle the quantum-mechanical state as a function of the distance. The dissipative term which we have added to the quantum-mechanical time evolution could be an effective term originating in some modification of QM; it could as well be based on some effective quantum-mechanical description of an interaction of the two-particle system with an unknown envi-

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[1]Note that the result for R with ζ given by Eq. (4.7) agrees with the result for R in first order in Ξ of Ref. [12], if the general Lindblad term in this paper is specialized to energy conservation.

ronment. Our dissipative term respects complete positivity, which—we believe—is a useful physical guiding principle for modifications of the quantum-mechanical time evolution. In compliance with our idea, we assume that, after one of the particles has decayed, the other one follows the quantum-mechanical time evolution. We have applied our model in the case of the entangled $B^0\bar{B}^0$ state with negative C parity, where we have used the data on the B lifetime, the B_H-B_L mass difference measured by observing the time evolution of single neutral B mesons, and the ratio of like-sign over opposite-sign dilepton event rates for the purpose of estimating the strength λ of the dissipative term. In the case of time-integrated dilepton events, our simple model leads to a result which is also obtained by the phenomenological introduction of a "decoherence parameter" ζ in the quantum-mechanical interference terms of the quantities N_{++}, N_{+-}, etc. In the dissipative term $D[\rho]$ we employ the states $|B_H \otimes B_L\rangle$ and $|B_L \otimes B_H\rangle$; this, eventually, modifies the interference terms of N_{++}, N_{+-}, etc., with the ζ associated with the B_H-B_L basis [22,23]. Note that we have neglected CP violation in B^0-$\bar{B}^0$ mixing, which is a good approximation in this system. Transferring our model of decoherence to the $K^0\bar{K}^0$ system is not straightforward, because it requires us to take CP violation and the nonorthogonality of the K_S and K_L states into account. Work on this is in progress.

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