Gauge unification in higher dimensions

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A complete 5-dimensional $SU(5)$ unified theory is constructed which, on compactification on the orbifold with two different Z_2's (Z_2 and Z_2'), yields the minimal supersymmetric standard model. The orbifold accomplishes $SU(5)$ gauge symmetry breaking, doublet-triplet splitting, and a vanishing of proton decay from operators of dimension 5. Until 4D supersymmetry is broken, all proton decay from dimension 4 and dimension 5 operators is forced to vanish by an exact $U(1)_R$ symmetry. Quarks and leptons and their Yukawa interactions are located at the Z_2 orbifold fixed points, where $SU(5)$ is unbroken. A new mechanism for introducing $SU(5)$ breaking into the quark and lepton masses is introduced, which originates from the $SU(5)$ violation in the zero-mode structure of bulk multiplets. Even though $SU(5)$ is absent at the Z_2' orbifold fixed point, the brane threshold corrections to gauge coupling unification are argued to be negligibly small, while the logarithmic corrections are small and in a direction which improves the agreement with the experimental measurements of the gauge couplings. Furthermore, the X gauge boson mass is lowered, so that $p \rightarrow e^+ \pi^0$ is expected with a rate within about one order of magnitude of the current limit. Supersymmetry breaking occurs on the Z_2' orbifold fixed point, and is felt directly by the gauge and Higgs sectors, while squarks and sleptons acquire mass via gaugino mediation, solving the supersymmetric flavor problem.

I. INTRODUCTION

The successful prediction of the weak mixing angle is a major achievement of particle theory of recent decades [1]. It suggests that weak-scale supersymmetry with two light Higgs doublets will soon be discovered at colliders, and there should be no exotic states in an energy desert up to $M_U \approx 10^{16}$ GeV. The physics immediately above this unification scale has been viewed in three frameworks: 4 dimensional grand unified field theories [2], higher-dimensional grand unified theories motivated by string theory [3], and string theory [4]. Grand unification in 4D provides a simple and elegant understanding of the quantum numbers of the quarks and leptons in a generation, but these successes are open to doubt because of several issues which require considerable effort to overcome. Chief amongst these are:

How is the grand unified gauge symmetry to be broken?

Why are the weak doublet Higgs bosons split in mass from the colored triplet partners?

Why have we not already observed proton decay induced by dimension 5 operators?

Why should the two Higgs doublets be light when the standard model gauge symmetry allows a large mass term?

In this paper we study higher-dimensional grand unified field theories above M_U, without inputing suggestions from string theory, for example on the number of extra dimensions and the gauge group. We follow a "bottom-up" approach, seeking simple solutions for the above issues. Many of the methods we use, for example for gauge symmetry breaking and doublet-triplet splitting, were introduced in the string-motivated context [3]. Recently, Kawamura has shown how the first two issues above are simply and elegantly solved in the case of an $SU(5)$ unified gauge symmetry [5] using the orbifold $S^1/(Z_2 \times Z_2')$ previously introduced for breaking weak-scale supersymmetry [6]. Orbifolding along one axis, using a parity which is $+$ for the weak direction but $-$ for

the strong direction, automatically breaks $SU(5)$ to the standard model gauge group, and gives zero modes for the Higgs doublets but not for the triplet partners. There is no need for Higgs multiplets at the unified scale with a scalar potential designed to give the correct unified symmetry breaking, and there is no need to arrange couplings in such a way that only the Higgs triplets acquire mass. An important recent observation is that to proceed further with construction of such theories it is necessary to consider how the quarks and leptons transform under the $Z_2 \times Z_2'$ symmetry [7]. However, the brane interactions advocated explicitly break the 5-dimensional $SU(5)$ gauge symmetry. In this paper we require that all interactions preserve this symmetry.

In this paper we construct completely realistic unified theories based on the orbifold $S^1/(Z_2 \times Z_2')$. We show that the answer to the first question is that, from the 5D viewpoint, the unified gauge symmetry is unbroken but takes a restricted form, while from the 4D viewpoint there is no unbroken Georgi-Glashow $SU(5)$. We show that higher dimensional unified theories generically do not have proton decay from dimension 5 operators. In particular the usual dimension 5 proton decay, resulting from the exchange of the colored triplet Higgs multiplet [8], is absent because a $U(1)_R$ symmetry forces a special form for the masses of these states. Furthermore, this $U(1)_R$ symmetry forbids the appearance of dimension 5 operators which violate baryon number in the superpotential. This also allows a fundamental distinction between Higgs and matter fields, and provides an understanding of why the Higgs doublet multiplets are light even though they are vectorial under the standard model: they are protected from a mass term by the same bulk $U(1)_R$ symmetry that forbids proton decay at dimension 5. Thus we see that the four issues of grand unification listed above are automatically solved in higher dimensional theories. The framework for such 5 dimensional theories compactified on a

$S^1/(Z_2 \times Z_2')$ orbifold is given in Sec. II, together with an elucidation of some of their general properties. A complete theory is given with gauge group $SU(5)$ in Sec. III.

In addition to these accomplishments, the complete theory constructed in Sec. III illustrates several new results:

Unified quark and lepton multiplets, and Yukawa couplings on orbifold fixed points, can be made consistent with the orbifolding parity which restricts the unified symmetry.

Even though the unified gauge symmetry is restricted at orbifold fixed points, the threshold corrections to the weak mixing angle, both at the classical and quantum level, are small, preserving the successful prediction for the weak mixing angle.

The orbifold construction for the unified gauge symmetry breaking provides a natural location for gaugino mediated supersymmetry breaking [9]—all the ingredients needed for gaugino mediation are automatically present in the theory.

Even though the theory possesses an $SU(5)$ gauge symmetry which, from the higher dimensional viewpoint is unbroken, the Yukawa couplings, which respect this symmetry, need not have the usual $SU(5)$ relations amongst the massless 4D states.

The first two points are crucial if these theories are to predict $\sin^2\theta_w$ and provide an understanding of the quark and lepton gauge quantum numbers. The last two points allow us to construct completely realistic theories.

II. UNIFIED GAUGE SYMMETRY TRANSFORMATIONS ON ORBIFOLD SPACETIME

In this section we introduce a class of higher dimensional unified field theories, concentrating on the nature of the gauge symmetry transformation. We then discuss several features which are generic to this class. For simplicity we consider a single compact extra dimension, $y(=x_5)$, and assume a fixed radius with size given by the unification scale. We take the unified gauge interactions in 5 dimensions to have gauge group G. The Higgs doublets also propagate in 5 dimensions as components of hypermultiplets. Using 4 dimensional superfield notation, we write the vector multiplet as (V,Σ), where V is a 4D vector multiplet and Σ a chiral adjoint, and the hypermultiplet as (H,H^c), where H and H^c are chiral multiplets with opposite gauge transformations.

The form of the gauge transformations under G can be restricted by compactifying on the orbifold S^1/Z_2 with a parity, P, which acts on the vector representation of G, making some components positive and some components negative: $P=(+,+,\ldots-,-,\ldots)$. The orbifold symmetry on any tensor field ϕ is then defined by $\phi(-y)=P\phi(y)$, where P acts separately on all vector indices of ϕ, and an overall sign choice for the parity of the multiplet may also be included in P. It is understood that there is a relative minus sign between the transformation of $H^c(\Sigma)$ and $H(V)$, as required by the P invariance of the 5D gauge interactions. In certain cases P is a discrete gauge transformation, but it need not be.

A non-supersymmetric theory with $G=SU(5)$ and $P=(-,-,-,+,+)$ was considered by Kawamura [10]; here we discuss a supersymmetric version. The Higgs bosons are taken to lie in a hypermultiplet $(H,H^c)(x,y)$, with H and H^c
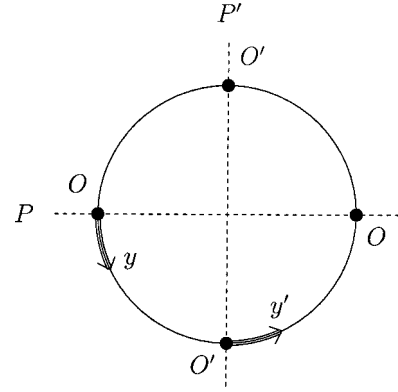

FIG. 1. $S^1/(Z_2 \times Z_2')$ orbifold in the fifth dimension.

chiral multiplets transforming as **5** and $\overline{\mathbf{5}}$ representations. The orbifold projection accomplishes doublet-triplet splitting, in the sense that H has a weak doublet zero mode but not a color triplet zero mode. However, the projections work oppositely for H^c which contains only a color triplet zero mode. Similarly, while the X gauge bosons have negative P and therefore no zero mode, the exotic color triplet, weak doublet states in the chiral adjoint, Σ_X, does have a zero mode. Such exotic light states are generic to orbifolding with a single Z_2 and lead to an incorrect prediction for the weak mixing angle. These exotic states can be removed by introducing two sets of different orbifold parities, giving additional structures to the spacetime which we study in the rest of this paper.

We choose to view this spacetime as illustrated in Fig. 1. There are two reflection symmetries, $y \to -y$ and $y' \to -y'$, each with its own orbifold parity, P and P', acting on the fields. Each reflection introduces special points, O and O', which are fixed points of the transformation. The physical space can be taken to be $0 \le y \le \pi R/2$, and is the usual S'/Z_2 orbifold. Nevertheless, we find it convenient to label the pattern in Fig. 1 as $S'/(Z_2 \times Z_2')$. The components of the vector multiplet can be assembled into four groups, $V_{PP'}$, according to their transformation properties. They have Kaluza-Klein (KK) mode expansions as given in Eqs. (5)–(8), with $\phi \to V$. Before orbifolding the gauge transformations are arbitrary functions of position $\xi=\xi(y)$ for each generator (we suppress the dependence on the usual four dimensions). However, the non-trivial orbifold quantum numbers of the gauge particles imply that the most general set of gauge transformations are restricted to have the form

$$U=\exp[i(\xi_{++}(y)T_{++}+\xi_{+-}(y)T_{+-}$$
$$+\xi_{-+}(y)T_{-+}+\xi_{--}(y)T_{--})]. \qquad (1)$$

The generators of the gauge group T are labeled by the P and P' quantum numbers of the corresponding gauge boson. The gauge functions $\xi_{PP'}(y)$ also have the KK mode expansions similar to those of Eqs. (5)–(8), with $\phi \to \xi$. At an arbitrary point in the bulk all generators of the gauge group are operative. However, at the fixed points O the gauge transformations generated by $T_{-P'}$ vanish, because $\xi_{-P'}(y=0,\pi R)=0$. Thus locally at these points the symmetry

should be thought of as the subgroup H generated by the set of generators $T_{+P'}$.[1] Similarly for the points O'; hence

$$
\begin{array}{cc}
O & O' \\
G \rightarrow H, & G \rightarrow H'.
\end{array}
\tag{2}
$$

Relative to the theory with arbitrary G gauge transformations on a circle, one can view the orbifold procedure as leading to a local explicit breaking of some of these gauge symmetries at the fixed points. In the model of the next section, H' is the standard model gauge group, so that O' is simply not affected by the unified gauge transformations. From the 5D viewpoint, working on the orbifold spacetime, the gauge symmetry simply takes a restricted form—the theory does not involve any gauge symmetry breaking.

Once a KK mode expansion is made, one finds that the 5D gauge transformation corresponds to infinite towers of gauge transformations, which mix up KK modes of different levels, associated with the KK gauge boson modes $A_\mu^{(n)}$. In this 4D picture, ∂_y acts like a symmetry breaking vacuum expectation value and mixes $A_\mu^{(n)}$ with $\partial_\mu A_5^{(n)}$, giving a mass to the $n \neq 0$ modes. Without an orbifold procedure, the theory possesses an infinite number of gauge transformations parametrized by an integer n for each generator. However, after the orbifolding, some of the gauge transformations are projected out due to the non-trivial orbifold quantum numbers of the gauge parameters, and n can no longer take arbitrary values. Therefore, from a 4D viewpoint, the orbifold procedure corresponds to imposing only the restricted sets of gauge transformations on the theory. The unbroken gauge symmetry of the 4D theory is generated by T_{++}, and is the intersection of H and H'.

Our class of theories includes many possibilities for G, P, and P'. We require that the choice of G and P is such that H, the group of local gauge transformations at O, is, or contains, $SU(5)$. To obtain an understanding of the quark and lepton quantum numbers we require that the three generations are described by brane fields at O in representations of H: $3(T_{10} + F_{\bar{5}})$ for $H = SU(5)$. Furthermore, Yukawa couplings of this matter to the bulk Higgs field will be placed at O. Notice that the standard model quarks, leptons, gauge fields and Higgs bosons are transforming under a unified symmetry group, such as $SU(5)$, which from the 5D viewpoint, is *unbroken*. How can this be possible? The answer is that this is not the same 4D $SU(5)$ symmetry that Georgi and Glashow introduced. While it acts in a standard fashion on the quarks and leptons, its action on the gauge and Higgs fields, which live in the bulk is non-standard. For example, consider the gauge transformation induced by the generator T_X, which is in $SU(5)/(SU(3)_C \times SU(2)_L \times U(1)_Y)$, assuming that P' has been chosen such that $T_X = T_{X+-}$. This assignment guarantees that if the Higgs doublets are $h_{2++}(y)$, as they should be to possess zero modes, then the color triplet partners will be $h_{3+-}(y)$, and will not have zero

modes. Making a KK mode expansion, one discovers that the T_X transformation rotates the zero-mode $h_2^{(0)}$ into a combination of massive triplet KK modes $h_3^{(n)}$, and, from the 4D viewpoint, this gauge transformation is broken. It is because $\xi_X = \xi_{X+-}$ has no zero-mode contribution that there is no 4D Georgi-Glashow $SU(5)$ symmetry. A central thesis of this paper is that the grand unified symmetry is an inherently extra-dimensional symmetry—it is not a 4D symmetry which must be spontaneously broken, and it does not contain the 4D Georgi-Glashow $SU(5)$ as a subgroup.

The construction of theories of the type outlined above is not guaranteed. There are certain consistency conditions that must be imposed. All interactions, both bulk and brane, must be invariant under each parity, that is under $y \rightarrow -y$ and $\phi(-y) \rightarrow P\phi(y)$, and similarly for P'. For the bulk gauge interactions this means that the group structure constants must obey $f^{\hat{a}\hat{b}\hat{c}} = f^{\hat{a}bc} = 0$, where $a(\hat{a})$ runs over generators even (odd) under P. Important constraints result because the brane interactions at O, which include the Yukawa couplings for the quarks and leptons, must be symmetrical under both P' and the gauge symmetry H. Constructing such interactions is non-trivial, and will be discussed in detail in the explicit theory of Sec. III.

The parity P' must be chosen so that h_3 and V_X are odd and have no zero modes. This implies that the gauge symmetry H' at the fixed points O' does not contain $SU(5)$. Hence brane interactions at O' violate the Georgi-Glashow $SU(5)$ symmetry. This raises the crucial question as to whether the prediction for the weak mixing angle survives in these theories—is Georgi-Glashow $SU(5)$ necessary for a successful prediction of the weak mixing angle? We find that it is not: the three standard model gauge interactions are unified into a single non-Abelian higher dimensional gauge symmetry, so that a zeroth order relation amongst the three gauge couplings is preserved. Threshold corrections have two origins: brane kinetic terms for gauge fields located at O', and $SU(5)$ splitting of the KK multiplets of the Higgs and gauge towers, which are inherent to our class of theories. In the next section we show that these threshold corrections are under control and small. It is remarkable and non-trivial that, even if the brane gauge kinetic terms violate Georgi-Glashow $SU(5)$ strongly, the resulting tree-level corrections to the weak mixing angle are negligible.

Proton decay from color triplet Higgsino exchange vanishes in our class of theories. These Higgsinos acquire mass via the KK mode expansion of operators of the form $H\partial_y H^c$. The Dirac mass couples the triplet Higgsino to a state in H^c which does not couple to quarks and leptons. It is well known that the dimension 5 proton decay problem can be solved by the form of the triplet Higgsino mass matrix; in higher dimensional unification the structure of the theory requires this dimension 5 proton decay to be absent. The form of the mass matrix is guaranteed by a $U(1)_R$ symmetry of the 5D gauge interactions. As shown in the next section, this can be trivially extended to the brane matter fields so that all proton decay at dimension 5 is absent, and R parity automatically results, giving proton stability also at dimension 4.

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[1] If interactions on O contain Σ fields, there are additional constraints on the form of the interactions coming from $\partial_y \xi_{-P'}(y = 0, \pi R) \neq 0$.

Since matter resides at O, where the gauge transformations include those of $SU(5)$, the Yukawa interactions are necessarily $SU(5)$ symmetric, leading to the unified fermion mass relation $m_s/m_d = m_\mu/m_e$, which conflicts with data by an order of magnitude. Does this exclude our framework? In 4D unified theories, acceptable mass relations follow from using higher dimensional operators involving factors of $SU(5)$ symmetry breaking vacuum expectation values, $\langle \Sigma_{SU(5)} \rangle$. Indeed the Yukawa matrices can be viewed as expansions in $\langle \Sigma_{SU(5)} \rangle / M_{Pl}$ [11], allowing the construction of predictive unified theories of fermion masses [12,13]. This option is not available to us as there are no $SU(5)$ breaking vacuum expectation values. We find that our framework leads to an alternative origin for $SU(5)$ breaking Yukawa interactions, and we are able to introduce a new, highly constrained, class of predictive unified theories of fermion masses. The only origin of $SU(5)$ breaking at O is through the zero-mode structure of bulk fields with non-zero wavefunctions at O. We therefore introduce a new class of bulk matter in a vector representation of G as hypermultiplets $(B, B^c) + (\bar{B}, \bar{B}^c)$. The symmetry quantum numbers of these fields are assigned such that brane interactions at O give mass terms and Yukawa interactions which involve both the brane matter and the bulk matter. For example, when G is $SU(5)$, and B is taken to be a 5-plet, the brane interactions at O include the mass terms $B(\bar{B} + F_{\bar{5}})$ and the Yukawa interactions $T_{10}(\bar{B} + F_{\bar{5}})H_{\bar{5}}$. This leads to realistic masses, as discussed in the next section.

Below the compactification scale our theory is the minimal supersymmetric standard model. While there are a variety of possibilities for supersymmetry breaking, it is readily apparent that our class of theories provides a very natural setting for gaugino mediation. Gaugino mediation [9] requires the standard model gauge fields to propagate in a bulk which contains at least two branes. One brane has quarks and leptons localized to it, while the other is the location of supersymmetry breaking. Clearly, in our class of theories the supersymmetry breaking should reside at O'.

III. AN $SU(5)$ THEORY

A. Orbifold symmetry structure

In this section we construct a complete unified $SU(5)$ theory in 5 dimensions and discuss some of its phenomenological aspects. We begin by reviewing the bulk structure of the model of Ref. [5]. The 5D spacetime is a direct product of 4D Minkowski spacetime M^4 and an extra dimension compactified on the $S^1/(Z_2 \times Z_2')$ orbifold, with coordinates x^μ ($\mu = 0,1,2,3$) and $y(=x^5)$, respectively. The $S^1/(Z_2 \times Z_2')$ orbifold can conveniently be viewed as a circle of radius R divided by two Z_2 transformations; $Z_2: y \rightarrow -y$ and $Z_2': y' \rightarrow -y'$ where $y' = y - \pi R/2$. The physical space is then an interval $y:[0, \pi R/2]$ which has two branes at the two orbifold fixed points at $y = 0$ and $\pi R/2$. (The branes at $y = \pi R$ and $-\pi R/2$ are identified with those at $y = 0$ and $\pi R/2$, respectively.)

Under the $Z_2 \times Z_2'$ symmetry, a generic 5D bulk field $\phi(x^\mu, y)$ has a definite transformation property

TABLE I. The (Z_2, Z_2') transformation properties for the bulk gauge and Higgs multiplets.

(P,P')	4D $N=1$ superfield			Mass
$(+,+)$	V^a,	H_F,	$H_{\bar{F}}$	$2n/R$
$(+,-)$	$V^{\hat{a}}$,	H_C,	$H_{\bar{C}}$	$(2n+1)/R$
$(-,+)$	$\Sigma^{\hat{a}}$,	H_C^c,	$H_{\bar{C}}^c$	$(2n+1)/R$
$(-,-)$	Σ^a,	H_F^c,	$H_{\bar{F}}^c$	$(2n+2)/R$

$$\phi(x^\mu, y) \rightarrow \phi(x^\mu, -y) = P\phi(x^\mu, y), \quad (3)$$

$$\phi(x^\mu, y') \rightarrow \phi(x^\mu, -y') = P'\phi(x^\mu, y'), \quad (4)$$

where the eigenvalues of P and P' must be ± 1. Denoting the field with $(P, P') = (\pm 1, \pm 1)$ by $\phi_{\pm \pm}$, we obtain the following mode expansions [6]:

$$\phi_{++}(x^\mu, y) = \sum_{n=0}^{\infty} \frac{1}{\sqrt{2^{\delta_{n,0}}\pi R}} \phi_{++}^{(2n)}(x^\mu)\cos\frac{2ny}{R}, \quad (5)$$

$$\phi_{+-}(x^\mu, y) = \sum_{n=0}^{\infty} \frac{1}{\sqrt{\pi R}} \phi_{+-}^{(2n+1)}(x^\mu)\cos\frac{(2n+1)y}{R}, \quad (6)$$

$$\phi_{-+}(x^\mu, y) = \sum_{n=0}^{\infty} \frac{1}{\sqrt{\pi R}} \phi_{-+}^{(2n+1)}(x^\mu)\sin\frac{(2n+1)y}{R}, \quad (7)$$

$$\phi_{--}(x^\mu, y) = \sum_{n=0}^{\infty} \frac{1}{\sqrt{\pi R}} \phi_{--}^{(2n+2)}(x^\mu)\sin\frac{(2n+2)y}{R}, \quad (8)$$

where 4D fields $\phi_{++}^{(2n)}$, $\phi_{+-}^{(2n+1)}$, $\phi_{-+}^{(2n+1)}$, and $\phi_{--}^{(2n+2)}$ acquire masses $2n/R$, $(2n+1)/R$, $(2n+1)/R$, and $(2n+2)/R$ upon compactification. Zero modes are contained only in ϕ_{++} fields, so that the matter content of the massless sector is smaller than that of the full 5D multiplet.

In the 5D bulk, we have $SU(5)$ gauge supermultiplets and two Higgs hypermultiplets that transform as **5** and **$\bar{5}$**. The 5D gauge supermultiplet contains a vector boson A_M ($M = 0,1,2,3,5$), two gauginos λ and λ', and a real scalar σ, which is decomposed into a vector supermultiplet $V(A_\mu, \lambda)$ and a chiral multiplet in the adjoint representation $\Sigma[(\sigma + iA_5)/\sqrt{2}, \lambda']$ under $N=1$ supersymmetry in 4D. The hypermultiplet, which consists of two complex scalars ϕ and ϕ^c and two Weyl fermions ψ and ψ^c, forms two 4D $N=1$ chiral multiplets $\Phi(\phi, \psi)$ and $\Phi^c(\phi^c, \psi^c)$ transforming as conjugate representations with each other under the gauge group. Here Φ runs over the two Higgs hypermultiplets, H_5 and $H_{\bar{5}}$. [H_5^c and $H_{\bar{5}}^c$ transform as **$\bar{5}$** and **5** under the $SU(5)$.]

The 5D $SU(5)$ gauge symmetry is "broken" by the orbifold compactification to a 4D $SU(3)_C \times SU(2)_L \times U(1)_Y$ gauge symmetry by choosing $P = (+,+,+,+,+)$ and $P' = (-,-,-,+,+)$ acting on the **5**. Each Z_2 reflection is taken to preserve the same 4D $N=1$ supersymmetry. The

(Z_2, Z_2') charges for all components of the vector and Higgs multiplets are shown in Table I. Here, the indices a and $\hat{a}$ denote the unbroken and broken $SU(5)$ generators, T^a and $T^{\hat{a}}$, respectively. The C and F represent the color triplet and weak doublet components of the Higgs multiplets, respectively: $H_\mathbf{5} \supset \{H_C, H_F\}$, $H_{\bar{\mathbf{5}}} \supset \{H_{\bar{C}}, H_{\bar{F}}\}$, $H_\mathbf{5}^c \supset \{H_C^c, H_F^c\}$, and $H_{\bar{\mathbf{5}}}^c \supset \{H_{\bar{C}}^c, H_{\bar{F}}^c\}$. Since only $(+,+)$ fields have zero modes, the massless sector consists of $N=1$ $SU(3)_C \times SU(2)_L \times U(1)_Y$ vector multiplets $V^{a(0)}$ with two Higgs doublet chiral superfields $H_F^{(0)}$ and $H_{\bar{F}}^{(0)}$. Thus, the doublet-triplet splitting problem is naturally solved in this framework [5]. The higher modes for the vector multiplets $V^{a(2n)}$ $(n>0)$ eat $\Sigma^{a(2n)}$ becoming massive vector multiplets, and similarly for the $V^{\hat{a}(2n+1)}$ and $\Sigma^{\hat{a}(2n+1)}$ $(n \geq 0)$. An interesting point is that the non-zero modes for the Higgs fields have mass terms of the form $H_F^{(2n)} H_F^{c(2n)}$, $H_{\bar{F}}^{(2n)} H_{\bar{F}}^{c(2n)}$, $H_C^{(2n+1)} H_C^{c(2n+1)}$, and $H_{\bar{C}}^{(2n+1)} H_{\bar{C}}^{c(2n+1)}$, which will become important when we consider dimension 5 proton decay later.

How should quarks and leptons be incorporated into this theory? In this paper we concentrate on the case where they are localized on a brane.[2] An important point is that when we introduce brane-localized matter and/or interactions on some orbifold fixed point O, they must preserve only the gauge symmetry with non-vanishing gauge transformations at this point $\xi_O^a \neq 0$. This corresponds to the symmetry which remains unbroken on this fixed point after compactification. Specifically, if we introduce the quark and lepton fields on the branes at $y=(0, \pi R)$ their multiplet structures and interactions must preserve $SU(5)$ symmetry and $N=1$ supersymmetry, while if they are located on the $y=\pm \pi R/2$ branes they only have to preserve $N=1$ supersymmetry and the standard model gauge symmetry. For the $SU(5)$ gauge symmetry to provide an understanding of the quark and lepton quantum numbers, they must reside on the $y=(0, \pi R)$ branes. Therefore, we put three generations of $N=1$ chiral superfields $T_\mathbf{10} \supset \{Q, U, E\}$ and $F_{\bar{\mathbf{5}}} \supset \{D, L\}$ on the $y=(0, \pi R)$ branes.

What are the transformation properties of the quark and lepton superfields under the $Z_2 \times Z_2'$ symmetry? The parities

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[2]An interesting alternative is to put quarks and leptons in the bulk. Then, if we only introduce hypermultiplets $(T_\mathbf{10}, T_\mathbf{10}^c)$ and $(F_{\bar{\mathbf{5}}}, F_{\bar{\mathbf{5}}}^c)$ in the bulk, the low-energy matter content is not that of the minimal supersymmetric standard model. However, if we further introduce $(T_\mathbf{10}', T_\mathbf{10}'^c)$ and $(F_{\bar{\mathbf{5}}}', F_{\bar{\mathbf{5}}}'^c)$ and assign opposite P' parities for $T_\mathbf{10}$ and $T_\mathbf{10}'$ (and for $F_{\bar{\mathbf{5}}}$ and $F_{\bar{\mathbf{5}}}'$), it is possible to recover the correct low-energy matter content. The Yukawa couplings are placed on either $y=(0, \pi R)$ or $y=\pm \pi R/2$ branes. The resulting theory is not "grand unified theory" in the usual sense, since D and L (Q and U, E) come from different (hyper)multiplets. The proton decay from broken gauge boson exchange is absent, and there is no $SU(5)$ relation among the Yukawa couplings. Nevertheless, the theory still keeps the desired features of the "grand unified theory": the quantization of hypercharges and the unification of the three gauge couplings.

P under Z_2 must obviously be plus. Then, the parities P' under Z_2' are determined by the requirement that any operators written on the $y=(0, \pi R)$ branes must transform $SU(5)$ covariantly under Z_2'. This is because if various terms in an $SU(5)$-invariant operator on the $y=0$ brane transform differently under the Z_2', the corresponding terms on the $y=\pi R$ brane are not $SU(5)$ invariant, contradicting the observation made in Eq. (1). In particular, since the kinetic terms for the $T_\mathbf{10}$ and $F_{\bar{\mathbf{5}}}$ fields must transform covariantly under the Z_2', there are only four possibilities for the assignment of the P' quantum numbers: (i) $P'(Q,U,D,L,E)= \pm(+,-,-,+,-)$ or (ii) $P'(Q,U,D,L,E)=\pm(-,+,-,+,+)$.[3] The case $(-,+,-,+,+)$ follows from requiring $(-,-,-,+,+)$ on each **5** index, and the other three possibilities result from overall sign changes on $T_\mathbf{10}$ and/or $F_{\bar{\mathbf{5}}}$.

Once the P' quantum numbers for the brane fields are determined as above, we can work out the transformation properties of the Yukawa coupling $[T_\mathbf{10} T_\mathbf{10} H_\mathbf{5}]_{\theta^2}$ and $[T_\mathbf{10} F_{\bar{\mathbf{5}}} H_{\bar{\mathbf{5}}}]_{\theta^2}$ located on the brane. It turns out that $P'(T_\mathbf{10} T_\mathbf{10} H_\mathbf{5}) = P'(T_\mathbf{10} F_{\bar{\mathbf{5}}} H_{\bar{\mathbf{5}}}) = -$ in the case (i), and $P'(T_\mathbf{10} T_\mathbf{10} H_\mathbf{5}) = -P'(T_\mathbf{10} F_{\bar{\mathbf{5}}} H_{\bar{\mathbf{5}}}) = -$ in the case (ii). Therefore, the $Z_2 \times Z_2'$ invariant Yukawa interactions are

$$
\mathcal{L}_5 = \int d^2\theta \left[\frac{1}{2} \{\delta(y) - \delta(y-\pi R)\} \sqrt{2\pi R}\, y_u T_\mathbf{10} T_\mathbf{10} H_\mathbf{5} \right.
$$
$$
\left. + \frac{1}{2} \{\delta(y) \mp \delta(y-\pi R)\} \sqrt{2\pi R}\, y_d T_\mathbf{10} F_{\bar{\mathbf{5}}} H_{\bar{\mathbf{5}}} \right] + \text{H.c.},
$$
(9)

where $\mp$ takes $-$ and $+$ in the case of (i) and (ii), respectively. (We can easily check that the above expression is invariant under both Z_2:$\{y \to -y$, $T_\mathbf{10} T_\mathbf{10} H_\mathbf{5} \to T_\mathbf{10} T_\mathbf{10} H_\mathbf{5}$, $T_\mathbf{10} F_{\bar{\mathbf{5}}} H_{\bar{\mathbf{5}}} \to T_\mathbf{10} F_{\bar{\mathbf{5}}} H_{\bar{\mathbf{5}}}\}$ and Z_2':$\{y' \to -y'$, $T_\mathbf{10} T_\mathbf{10} H_\mathbf{5} \to -T_\mathbf{10} T_\mathbf{10} H_\mathbf{5}$, $T_\mathbf{10} F_{\bar{\mathbf{5}}} H_{\bar{\mathbf{5}}} \to \mp T_\mathbf{10} F_{\bar{\mathbf{5}}} H_{\bar{\mathbf{5}}}\}$.) After integrating out the extra dimensional coordinate y, we get $SU(5)$-invariant Yukawa couplings in 4D,

$$
\mathcal{L}_4 = \sum_{n=0}^{\infty} \int d^2\theta \left[\sqrt{2} y_u \left(\frac{1}{\sqrt{2^{\delta_{n,0}}}} QUH_F^{(2n)} + QQH_C^{(2n+1)} \right. \right.
$$
$$
\left. + UEH_C^{(2n+1)} \right) + \sqrt{2} y_d \left(\frac{1}{\sqrt{2^{\delta_{n,0}}}} QDH_{\bar{F}}^{(2n)} \right.
$$
$$
\left. \left. + \frac{1}{\sqrt{2^{\delta_{n,0}}}} LEH_{\bar{F}}^{(2n)} + QLH_C^{(2n+1)} + UDH_C^{(2n+1)} \right) \right] + \text{H.c.},
$$
(10)

where the interactions between the zero-mode Higgs doublets and quarks and leptons are precisely those of the minimal supersymmetric $SU(5)$ model. [Note that the $SU(5)$ transformation caused by the broken generators $T^{\hat{a}}$ mixes different levels of KK modes, due to non-trivial profiles of

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[3]This assignment is different from that adopted in Ref. [7].

the broken gauge transformation parameters $\hat{\xi}^{\hat{a}}(y)$ in the extra dimension.]

B. Gauge coupling unification

We now discuss some phenomenological issues of the model, beginning with gauge coupling unification. Since the heavy unified gauge boson masses are given by $1/R$, it is natural to identify $1/R = M_U$, the unification scale, and this is indeed correct to zeroth order approximation. Then, the corrections to this naive identification come from two sources.

First, since the brane interactions only have to preserve symmetries which remain unbroken locally at that fixed point, we can introduce brane kinetic terms for the $SU(3)_C$, $SU(2)_L$, and $U(1)_Y$ gauge fields at $y = \pm \pi R/2$ with three different gauge couplings. [The operators on the $y = \pm \pi R/2$ branes do not have to preserve the full $SU(5)$, since the broken gauge transformation parameters $\hat{\xi}^{\hat{a}}$ vanish on these branes.] However, we can expect that the effect of these $SU(5)$-violating brane kinetic terms is small by making the following observation. Consider the 5D theory where the bulk and brane gauge couplings have almost equal strength. Then, after integrating out y, the zero-mode gauge couplings are dominated by the bulk contributions because of the spread of the wave function of the zero-mode gauge boson. Since the bulk gauge couplings are necessarily $SU(5)$ symmetric, we find that the $SU(5)$-violating effect coming from brane ones is highly suppressed.

To illustrate the above point explicitly, we here considered an extreme case where the 5D theory is truly strongly coupled at the cutoff scale M_* where the field theory is presumably incorporated into some more fundamental theory. Then, the bulk and brane gauge couplings g_5 and g_{4i} defined by

$$\mathcal{L}_5 = \int d^2\theta \left[\frac{1}{g_5^2} \mathcal{W}^\alpha \mathcal{W}_\alpha + \frac{1}{2} \left\{ \delta\left(y - \frac{\pi}{2} R\right) \right. \right.$$
$$\left. \left. + \delta\left(y + \frac{\pi}{2} R\right) \right\} \frac{1}{g_{4i}^2} \mathcal{W}_i^\alpha \mathcal{W}_{i\alpha} \right] + \text{H.c.,} \qquad (11)$$

are estimated to be $1/g_5^2 \sim M_*/(24\pi^3)$ and $1/g_{4i}^2 \sim 1/(16\pi^2)$ using a strong coupling analysis in higher dimensions [14]. Here, g_5 and g_{4i} are the $SU(5)$-invariant and $SU(5)$-violating contributions. Note that the brane contributions vanish for $\mathcal{W}_X^\alpha$, which is odd under Z_2', but are nonzero and different for $\mathcal{W}_i^\alpha$, where i runs over $SU(3)_C$, $SU(2)_L$, and $U(1)_Y$, so that the couplings g_{4i} differ from each other by factors of order unity. On integrating over y, we obtain zero-mode 4D gauge couplings g_{0i} at the unification scale

$$\frac{1}{g_{0i}^2} = \left(\frac{2\pi R}{g_5^2} + \frac{1}{g_{4i}^2} \right) \sim \left(\frac{M_* R}{12\pi^2} + \frac{1}{16\pi^2} \right). \qquad (12)$$

Since we know that $g_{0i} \sim 1$, we must take $M_* R \sim 12\pi^2$ in this strongly coupled case. This shows that the

$SU(5)$-violating contribution from the brane kinetic terms on $y = \pm \pi R/2$ (the second term in the above equations) is suppressed by an amount equivalent to a loop factor $1/(16\pi^2)$ relative to the dominant $SU(5)$-preserving contribution (the first term). Of course, this is a tree-level correction, and the brane contribution is small relative to the bulk term because of the large volume factor $2\pi R M_*$. Note that in the realistic case the theory is not necessarily strongly coupled at the cutoff scale ($M_* R \lesssim 12\pi^2$) as we will see later. Even then, however, it is true that the brane piece is generically suppressed compared with the bulk piece, after integrating out y, due to the volume factor $2\pi R M_*$. For comparable values of the dimensionless couplings $g_5^2 M_*$ and g_{4i}^2, this corresponds, even in the case of $M_* R = 4$, to much less than a 1% correction to the prediction of the weak mixing angle.

The second correction to gauge unification originates from the running of the gauge couplings above the compactification scale due to the KK modes not filling degenerate $SU(5)$ multiplets.[4] From the 4D point of view, the zero-mode gauge couplings g_{0i} at the compactification scale $M_c (= 1/R)$ is given by [15]

$$\frac{1}{g_{0i}^2(M_c)} \simeq \frac{1}{g_0^2(M_*)} - \frac{b}{8\pi^2}(M_* R - 1) + \frac{b_i'}{8\pi^2} \ln(M_* R), \qquad (13)$$

where b and b_i' are constants of $O(1)$. A crucial observation is that the coefficient b is $SU(5)$ symmetric, i.e., b is the same for $SU(3)_C$, $SU(2)_L$, and $U(1)_Y$. This is because the power-law contributions come from renormalizations of 5D kinetic terms, which must be $SU(5)$ symmetric. Since the sum of the first two terms must be $O(1)$ to have $g_{0i}(M_c) \sim 1$, the $SU(5)$-violating contribution from the running (the third term in the right-hand side of the above equation) is suppressed by a loop factor because the running is only logarithmic. Furthermore, the differences $b_i' - b_j'$ are smaller than the corresponding differences of the beta-function coefficients at low energies $[(b_3', b_2', b_1') = (0,2,6)]$, so that we arrive at the following picture. As the three gauge couplings are evolved from the weak scale to higher energies, they approach each other with the usual logarithmic running, and at the compactification scale M_c they take almost equal values but are still slightly different. The further relative evolution above M_c is slower, but finally the three couplings unify at the cutoff scale M_*, where our 5D theory is incorporated into some more fundamental theory. Note that $M_c < M_U < M_*$, where M_U is the usual zeroth order value of the unification scale, $M_U \simeq 2 \times 10^{16}$ GeV. We assume the physics above M_* to be $SU(5)$ symmetric.

To estimate the threshold correction coming from this second source, we consider the one-loop renormalization group equations for the three gauge couplings. Assuming that the couplings take a unified value g_* at M_*, they take the following form:

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[4] We thank D. Smith and N. Weiner for discussions on this issue.

$$\alpha_i^{-1}(m_Z) = \alpha_*^{-1}(M_*) + \frac{1}{2\pi}\left\{\alpha_i \ln\frac{m_{\mathrm{SUSY}}}{m_Z} + \beta_i \ln\frac{M_*}{m_Z}\right.$$
$$\left. + \gamma_i \sum_{n=0}^{N_l} \ln\frac{M_*}{(2n+2)M_c} + \delta_i \sum_{n=0}^{N_l} \ln\frac{M_*}{(2n+1)M_c}\right\},$$
$$(14)$$

where $(\alpha_1,\alpha_2,\alpha_3) = (-5/2, -25/6, -4)$, $(\beta_1,\beta_2,\beta_3) = (33/5, 1, -3)$, $(\gamma_1,\gamma_2,\gamma_3) = (6/5, -2, -6)$, and $(\delta_1,\delta_2,\delta_3) = (-46/5, -6, -2)$. Here, we have assumed a common mass m_{SUSY} for the superparticles for simplicity, and the sum on n includes all KK modes below M_*, so that $(2N_l+2)M_c \leq M_*$.[5] Contributions proportional to γ_i result from the KK modes of the Higgs doublets and of the standard model gauge bosons and their $N=2$ partners, while terms proportional to δ_i result from the KK modes of the Higgs triplets and from the vectors and chiral adjoints of the broken $SU(5)$ generators. In 4D unified theories one typically only considers threshold corrections from a few $SU(5)$-split multiplets with mass close to M_U. In the present theory there are $SU(5)$-split KK multiplets right up to M_*.

Taking a linear combination of the three equations, we obtain

$$(5\alpha_1^{-1} - 3\alpha_2^{-1} - 2\alpha_3^{-1})(m_Z)$$
$$= \frac{1}{2\pi}\left\{8\ln\frac{m_{\mathrm{SUSY}}}{m_Z} + 36\ln\frac{(2N_l+2)M_c}{m_Z}\right.$$
$$\left. - 24\sum_{n=0}^{N_l}\ln\frac{(2n+2)}{(2n+1)}\right\},$$
$$(15)$$

where we have set $M_* = (2N_l+2)M_c$. The corresponding linear combination in the usual 4D minimal supersymmetric $SU(5)$ grand unified theory takes the form

$$(5\alpha_1^{-1} - 3\alpha_2^{-1} - 2\alpha_3^{-1})(m_Z) = \frac{1}{2\pi}\left\{8\ln\frac{m_{\mathrm{SUSY}}}{m_Z} + 36\ln\frac{M_U}{m_Z}\right\},$$
$$(16)$$

where $M_U = (M_\Sigma^2 M_V)^{1/3}$, and M_Σ and M_V are the adjoint Higgs and the broken gauge boson masses, respectively [17]. Therefore, we find the following correspondence between the two theories:

$$\ln\frac{M_c}{m_Z} = \ln\frac{M_U}{m_Z} + \frac{2}{3}\sum_{n=0}^{N_l}\ln\frac{(2n+2)}{(2n+1)} - \ln(2N_l+2), \quad (17)$$

from the gauge running point of view. Since the experimental values of the gauge couplings restrict M_U to the range 1×10^{16} GeV $\lesssim M_U \lesssim 3 \times 10^{16}$ GeV, we can find the range

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TABLE II. $U(1)_R$ charges for chiral superfields.

	Σ	H_5	$H_{\bar{5}}$	H_5^c	$H_{\bar{5}}^c$	T_{10}	$F_{\bar{5}}$	N_1
$U(1)_R$	0	0	0	2	2	1	1	1

of M_c for a given N_l. For instance, if we take $N_l = 4$ ($M_* = 10M_c$), we find that the compactification scale must be in the range

$$3 \times 10^{15} \text{ GeV} \lesssim M_c \lesssim 8 \times 10^{15} \text{ GeV}, \quad (18)$$

which is somewhat lower than the usual 4D unification scale $M_U \simeq 2 \times 10^{16}$ GeV.[6] There is also an independent bound on N_l coming from another linear combination $(5\alpha_1^{-1} - 12\alpha_2^{-1} + 7\alpha_3^{-1})(m_Z)$, but it is rather weak due to the experimental uncertainty of the strong coupling constant $\alpha_3(m_Z)$.

The above estimate is not precise, since only the leading logarithmic contributions are included. A detailed analysis of these corrections to gauge coupling unification will be given in Ref. [18]. Nevertheless, the number given in Eq. (18) is very encouraging in that the dimension 6 proton decay, $p \to e^+\pi^0$, induced by the exchange of an X gauge boson, may be seen in the near future. The present experimental limit from Super-Kamiokande, $\tau_{p\to e^+\pi^0} > 1.6 \times 10^{33}$ yr [19], is translated into $M_c \gtrsim 5 \times 10^{15}$ GeV, remembering that the X gauge boson mass is M_c and the coupling of the X gauge boson to quarks is $\sqrt{2}$ times larger than that of the standard model gauge boson. Thus $N_l \gtrsim 4$, corresponding to a hierarchy $M_*/M_c \gtrsim 10$, could be excluded by an increase in the experimental limit on $\tau_{p\to e^+\pi^0}$ by a factor of 6. In fact, if we require $N_l > 1$, so that there is some energy interval where the theory is described by higher dimensional field theory ($M_*/M_c \gtrsim 4$), we obtain an upper bound on the compactification scale $M_c < 1.4 \times 10^{16}$ GeV, which means that increasing the experimental limit by a factor of 60 covers the entire parameter space of the model.

C. $U(1)_R$ symmetry

Knowing that $1/R$ is somewhat smaller than M_U, one may worry about too fast proton decay caused by an exchange of the colored Higgs multiplet, since they couple to quarks and squarks through the interactions of Eq. (10). However, these dimension 5 proton decay operators are not generated in our theory. The mass terms for the colored Higgs supermultiplets are

$$\mathcal{L}_4 = \sum_{n=0}^{\infty} \int d^2\theta \left(\frac{1}{R}H_C^{(2n+1)}H_C^{c(2n+1)} + \frac{1}{R}H_{\bar{C}}^{(2n+1)}H_{\bar{C}}^{c(2n+1)}\right)$$
$$+ \text{H.c.}, \quad (19)$$

coupling H to H^c. The conjugate fields, $H_C^{c(2n+1)}$ and

$H_{\bar{C}}^{c(2n+1)}$, do not couple directly to the quark and lepton superfields because of the $U(1)_R$ symmetry shown in Table II and discussed below. This symmetry is only broken by small supersymmetry breaking effects, so we predict that there is no proton decay induced by dimension 5 operators.

In order to be complete, however, we must also forbid the brane interactions

$$
\mathcal{L}_5 = \int d^2\theta \Bigg[\frac{1}{2}\{\delta(y) + \delta(y - \pi R)\} H_{\mathbf{5}} H_{\bar{\mathbf{5}}}
$$
$$
+ \frac{1}{2}\{\delta(y) \pm \delta(y - \pi R)\} T_{\mathbf{10}} T_{\mathbf{10}} T_{\mathbf{10}} F_{\bar{\mathbf{5}}} \Bigg] + \text{H.c.}, \quad (20)
$$

where $\pm$ takes $+$ and $-$ in the case of (i) and (ii), respectively. For this purpose, it is useful to notice that the 5D bulk Lagrangian possesses a continuous $U(1)_R$ symmetry. This bulk $U(1)_R$ is a linear combination of the $U(1)$ subgroup of the $SU(2)_R$ automorphism group of $N=2$ supersymmetry algebra and a vector-like non-R $U(1)$ symmetry under which the Higgs fields transform as $H_{\mathbf{5}}(-1), H_{\bar{\mathbf{5}}}(-1)$, $H_{\mathbf{5}}^c(+1), H_{\bar{\mathbf{5}}}^c(+1)$. The point is that we can extend this $U(1)_R$ to the full theory by assigning appropriate charges to the quark and lepton superfields. The full $U(1)_R$ symmetry, with charge assignments given in Table II, allows the Yukawa couplings in Eq. (9). Hence, imposing the matter quantum numbers of Table II, all interactions of Eq. (20) are forbidden by the $U(1)_R$ symmetry. This not only solves the problem of dimension 5 proton decay completely [both from tree-level brane operators in Eq. (20) and from colored Higgsino exchanges], but also naturally explains why the weak Higgs doublets H_F and $H_{\bar{F}}$ are light. Note that this $U(1)_R$ also forbids unwanted brane operators $[F_{\bar{\mathbf{5}}} H_{\mathbf{5}}]_{\theta^2}$ and $[T_{\mathbf{10}} F_{\bar{\mathbf{5}}} F_{\bar{\mathbf{5}}}]_{\theta^2}$, since it contains the usual R-parity as a discrete subgroup.[7]

D. Supersymmetry breaking

An important question is the origin of 4 dimensional $N=1$ supersymmetry breaking. In 4D it is well known that the supersymmetry breaking must be isolated to some degree from the particles and interactions of the minimal supersymmetric standard model. In fact, the argument of Dimopoulos and Georgi shows that this isolation is only necessary for the matter fields [2]. It is therefore quite clear that the ideal location for supersymmetry breaking is the branes at $y = \pm \pi R/2$. Remarkably, one immediately finds that this results in a supersymmetry breaking scheme which is precisely that of the gaugino mediation mechanism [9]. Since the gauginos

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[7]The above $U(1)_R$ allows the brane operator $[H_{\mathbf{5}} H_{\bar{\mathbf{5}}}]_{\theta^2 \bar{\theta}^2}$ on the $y = (0, \pi R)$ branes. Thus, it is possible to produce a μ term (a supersymmetric mass term for the Higgs doublets) of the order of the weak scale by the mechanism of Ref. [20] after the breakdown of $N=1$ supersymmetry, through a constant term in the superpotential needed to cancel a positive cosmological constant arising from the supersymmetry breaking. [The superpotential constant term comes from an explicit or spontaneous breaking of the $U(1)_R$.]

and Higgs fields are in the bulk they have direct couplings to the supersymmetry breaking field S, which we take to be a gauge and $U(1)_R$ singlet, on the $y = \pm \pi R/2$ branes

$$
\mathcal{L}_5 = \frac{1}{2}\{\delta(y - \pi R/2) + \delta(y + \pi R/2)\} \Bigg[\int d^2\theta S \mathcal{W}_i^\alpha \mathcal{W}_{i\alpha}
$$
$$
+ \int d^4\theta (S^\dagger H_F H_{\bar{F}} + S^\dagger S H_F H_{\bar{F}}) + \text{H.c.} \Bigg], \quad (21)
$$

where coefficients of order unity, in units of M_*, are omitted. Note that F-component expectation value for the S field breaks the $U(1)_R$ symmetry to R parity. This generates gaugino masses as well as the μ and μB parameters, while the squarks and sleptons obtain masses through radiative corrections so that the supersymmetric flavor problem is naturally solved. Superparticle phenomenology is more tightly constrained than in a general theory of gaugino mediation. The distance between the supersymmetry breaking and matter branes is known quite precisely, and, furthermore, the proton decay constraint does not allow very large values for M_*/M_c, so that the ratio of $\mu B/\mu$ is not a significant problem. It is also remarkable that, even though we have a unified gauge symmetry with precise gauge coupling unification, there is no unification of the three gaugino masses. They originate from the only location in the theory where the gauge transformations $\xi_X(y)$ are forced to vanish, so that the coefficients of $S\mathcal{W}_i^\alpha \mathcal{W}_{i\alpha}$ in Eq. (21) is different for the $SU(3)_C$, $SU(2)_L$ and $U(1)_Y$ terms.

E. Quark and lepton masses

The $SU(5)$ invariance of Eq. (9) guarantees the successful m_b/m_τ mass relation [21], but is not realistic, since it yields $m_s/m_d = m_\mu/m_e$. As discussed at the end of the previous section, if the unified symmetry is broken by an orbifold compactification, there is a new, constrained mechanism for obtaining $SU(5)$ breaking in fermion mass relations. Here we give a simple specific realization of this mechanism which is sufficient to allow realistic fermion masses. We add bulk hypermultiplets which transform as $\mathbf{5} + \bar{\mathbf{5}}$: $(B, B^c) + (\bar{B}, \bar{B}^c)$. We assume that they have no bulk mass term, for simplicity. We assign both B and $\bar{B}$ a $U(1)_R$ quantum number of $+1$, so that these hypermultiplets should be thought of as matter fields rather than Higgs fields. The $Z_2 \times Z_2'$ quantum numbers of these hypermultiplets can be chosen such that the zero modes are either weak doublets or color triplets; both possibilities lead to realistic theories. The $SU(5)$ invariant brane interactions at $y = 0$ relevant for down-type quark and charged lepton mass matrices are

$$
\mathcal{L}_5 = \int d^2\theta \, \delta(y)(B(\bar{B} + F_{\bar{\mathbf{5}}}) + T_{\mathbf{10}}(\bar{B} + F_{\bar{\mathbf{5}}}) H_{\bar{\mathbf{5}}}), \quad (22)
$$

where coupling parameters are omitted. Corresponding interactions are placed at $y = \pi R$ to maintain the $Z_2 \times Z_2'$ invariance of the theory. The lepton doublets and right-handed down quarks are found to lie partly in $\bar{B}$ and partly in $F_{\bar{\mathbf{5}}}$, but in different combinations. Hence in general the $SU(5)$ relations between the down and charged lepton masses are removed. The analysis is very simple when the mass terms of

Eq. (22) are smaller than M_c. Suppose that the zero modes of B are weak doublets. In this case the lepton doublet lies partly in $\bar{B}$ and partly in $F_{\bar{5}}$, while the down quark lies completely in $F_{\bar{5}}$. Hence $T_{10}F_{\bar{5}}H_{\bar{5}}$ contributes to both down and lepton masses, while $T_{10}\bar{B}H_{\bar{5}}$ contributes only to the lepton masses. One can imagine that, for some flavor symmetry reason, these mixing terms are only important for the 2-2 entry of the Yukawa matrices, so that the Georgi-Jarlskog mass matrices follow [12].

Note that the above mechanism could, in principle, introduce supersymmetric flavor problem, since B and $\bar{B}$ fields have supersymmetry breaking masses by coupling to S on the $y=\pm\pi R/2$ branes and give non-universal squark and slepton masses through the mixing with $F_{\bar{5}}$. However, the amount of induced flavor violation strongly depends on the structure of the Yukawa couplings. Suppose the Yukawa coupling of the $\bar{B}$ field, $[T_{10}\bar{B}H_{\bar{5}}]_{\theta^2}$, is of order one. Then, the mixing between $\bar{B}$ and the second generation $F_{\bar{5}}$ with angle of $O(0.01)$ will be sufficient to break unwanted $SU(5)$ relations on fermion masses. In this case, the flavor violating squark and slepton masses are suppressed compared with the gaugino masses, and the induced flavor violating processes at low energy would be sufficiently small.

Neutrino masses can easily be incorporated into our theory. Introducing three right-handed neutrino superfields N_1's, with $U(1)_R$ charge of $+1$, on the $y=(0,\pi R)$ branes, we can write neutrino Yukawa couplings $[F_{\bar{5}}N_1H_5]_{\theta^2}$ and Majorana mass terms for N_1's on the branes, accommodating the conventional see-saw mechanism [22] to explain the smallness of the neutrino masses. Alternatively, we could introduce N_1's on the $y=\pm\pi R/2$ branes and forbid their mass terms by imposing some symmetry such as $U(1)_{B-L}$. Then, with appropriate couplings to heavy bulk fields of masses $\sim M_*$, exponentially suppressed Yukawa couplings, $\exp(-\pi R M_*/2)[F_{\bar{5}}N_1H_5]_{\theta^2}$, are generated via exchanges of these heavy bulk fields. This provides a mechanism of naturally producing small Dirac neutrino masses in the present framework.

IV. CONCLUSIONS

It is well known that compactifying a higher-dimensional gauge field theory on a compact manifold leads to gauge symmetry breaking. For example, compactification on a circle leads to a mass for all gauge bosons corresponding to non-trivial gauge transformations on the circle. The only gauge bosons which remain massless correspond to zero-mode gauge transformations. Compactifying on an orbifold, with an orbifold symmetry which acts non-trivially on the gauge bosons, constrains the form of the gauge transformations, and reduces the number of their zero modes. This removal of zero-mode gauge transformations therefore decreases the gauge symmetry of the resulting 4D theory. Such a reduction can also be seen because the orbifold symmetry removes some of the zero-mode states that would be necessary to realize the 4D gauge symmetry. Kawamura [5] has shown that a 5D theory with $SU(5)$ gauge symmetry compactified on the orbifold $S^1/(Z_2\times Z_2')$ elegantly reduces the

4D gauge symmetry to $SU(3)_C\times SU(2)_L\times U(1)_Y$, and that this is accompanied by a reduction in the zero-mode Higgs states to only those that are weak doublets.

We have constructed a complete 5-dimensional $SU(5)$ unified field theory compactified on the orbifold $S^1/(Z_2\times Z_2')$. The theory possesses the following features:

Quarks and leptons are introduced at orbifold fixed points which preserve $SU(5)$ invariance, thereby yielding an understanding of their gauge quantum numbers.

Quark and lepton masses arise from $SU(5)$ invariant Yukawa couplings at these fixed points, which can therefore display $SU(5)$ fermion mass relations such as $m_b=m_\tau$. Mixing between heavy bulk matter and brane matter can lead to non-$SU(5)$ symmetric mass relations, as the zero-mode structure of the bulk matter is $SU(5)$ violating.

The mass matrix for the color triplet Higgs particles is determined by compactification, and results in the complete absence of dimension 5 proton decay from the exchange of these states.

Until 4D supersymmetry is broken, the theory possesses an exact $U(1)_R$ symmetry. This forbids all proton decay from dimension 4 and dimension 5 operators.

The orbifold has fixed points where $SU(5)$ is broken. This leads to corrections to gauge coupling unification arising from non-$SU(5)$ symmetric brane gauge kinetic terms and from KK modes of gauge and Higgs multiplets which do not fill degenerate $SU(5)$ multiplets. We have argued that the former are negligibly small. The latter yield small corrections to the weak mixing angle prediction, in a direction which improves the agreement between supersymmetric unification and experiment.

The X gauge boson, which induces $p\to e^+\pi^0$, has a mass equal to the compactification scale, $1/R$, which is smaller than the usual 4D unification scale of 2×10^{16} GeV by a factor between 1.4 and 4. We predict that $p\to e^+\pi^0$ will be discovered by further running of the Super-Kamiokande experiment, or at a next generation megaton proton decay detector.

Supersymmetry breaking occurs on a brane distant from the matter brane, resulting in the generation of gaugino masses and the $\mu/\mu B$ parameters. This breaking is communicated to matter by gaugino mediation, so that there is no supersymmetric flavor problem. While supersymmetry breaking breaks $U(1)_R$, it preserves R parity. Since the full $SU(5)$ gauge transformations do not act on the supersymmetry breaking brane, the gaugino mass parameters do not unify.

See-saw neutrino masses arise on the matter brane.

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