# Inclusive dileptonic rare *B* decays with an extra generation of vectorlike quarks

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We investigate the leading effects of extending the standard model of electroweak interactions by an extra isosinglet up- and down-type quark pair on various distributions and total branching ratio of the inclusive  $B \rightarrow X_s l^+ l^-$  ( $l = e, \mu$ ) rare *B* decays. The presence of the extra vectorlike down quark *D* results in the nonunitarity of the extended quark mixing matrix *V*, which in turn leads to  $b \rightarrow s$  flavor changing neutral currents at the tree level proportional to  $(V^{\dagger}V)_{sb}$ . On the other hand, the effective penguin and box vertex functions are sensitive to the mass of the extra isosinglet up quark  $m_U$ . The experimental upper bound on  $BR(B \rightarrow X_s \mu^+ \mu^-)$  is used to constrain the parameters of the model. It is shown that the shapes of the differential branching ratio and forward-backward asymmetry distribution are very sensitive to the value of the model parameters. We also calculate the *CP* aymmetry distribution of the dileptonic decay in the vectorlike quark model. It is shown that, for a typical choice of the model parameters, asymmetries up to around 10% can be achieved for certain values of the dilepton invariant mass.

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of the model parameters. However, it is shown that the

### I. INTRODUCTION

Rare *B* decays, which proceed via loop effects, are important venues for observing the signals of new physics beyond the standard model (SM). Heavy exotic particles, which are not accessible to experiments at present or near future accelerators, can occur in quantum loops as virtual particles, and therefore, be detected indirectly. *B* factories, CLEO III, and other dedicated *B* experiments are expected to observe new rare *B* decay channels and to improve the precision of those which have already been measured. As a result, by using experimental data on radiative rare B decays, one should be able to constrain the parameters of new physics better than before.

One simple extension of the SM is obtained by adding an extra generation of isosinglet quarks [1]. There are various motivations for this model: down-type vectorlike quarks are predicted by certain grand unified theories such as  $E_6$  in four dimensions, which is inspired by the superstring  $E_8 \times E_8$  model in ten dimensions. Also, it was shown that heavy quarks should be vectorlike if the vacuum is to be stable perturbatively [2]. The presence of these isosinglet quarks leads to nonunitarity of the quark mixing matrix and, consequently, results in nonvanishing flavor changing neutral currents (FCNC's) at tree level. On the other hand, the existence of the extra heavy quarks necessitates the modification of the Wilson coefficients of the operators entering the effective Hamiltonian for rare decays of *K* and *B* mesons [3].

In this paper, we investigate the effects of adding an extra isosinglet pair of quarks, U and D, with charges + 2/3 and - 1/3, respectively, to the SM on the inclusive rare dileptonic B decays  $B \rightarrow X_s l^+ l^-$ . Using the experimental upper bound of the branching ratio, one can constrain the acceptable range shapes of the differential branching ratio and forwardbackward asymmetry distribution is still quite sensitive to these parameters in their restricted domain. We then proceed to calculate the direct *CP* asymmetry distribution associated with  $B \rightarrow X_s l^+ l^-$ , assuming that the imaginary parts of the  $c\bar{c}$  continuum and resonance contributions are the only sources of the strong phase. Our results indicate that the size of the *CP* asymmetry, which is vanishingly small in the SM, and very sensitive to the model parameters in the vectorlike quark model (VQM), can reach up to 10% for certain dilepton invariant mass values.

## II. VECTORLIKE QUARK MODEL (VQM)

In this model, the gauge structure of the SM remains intact except for an additional pair of isosinglet quarks, which we denote them by U and D, as mentioned before. The difference between these new quarks and ordinary quarks of the three SM generations is that, unlike the latter ones, both leftand right-handed components of the former quarks are  $SU(2)_L$  singlets. Therefore, the Dirac mass terms of vectorlike quarks, i.e.,

$$m_U(\bar{U}_L U_R + \bar{U}_R U_L) + m_D(\bar{D}_L D_R + \bar{D}_R D_L), \qquad (1)$$

are invariant under electroweak gauge symmetry. However, the masses of the ordinary quarks arise from their gauge invariant Yukawa couplings to an isodoublet scalar Higgs field  $\phi$  as follows:

$$-f_{d}^{ij}\bar{\psi}_{L}^{i}d_{R}^{j}\phi - f_{u}^{ij}\bar{\psi}_{L}^{i}u_{R}^{j}\tilde{\phi} + \text{H.c.}, \qquad (2)$$

where i, j = 1, 2, 3 covers the three generations of the regular quarks, and the doublet of fermions  $\psi_L^i$  is defined as

$$\psi_L^i = \begin{pmatrix} u^i \\ d^i \end{pmatrix}_L. \tag{3}$$

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At the same time, additional  $SU(2)_L$  invariant Yukawa couplings between vectorlike and ordinary quarks, in the form

$$-f_d^{i4}\bar{\psi}_L^i D_R \phi - f_u^{i4}\bar{\psi}_L^i U_R \tilde{\phi} + \text{H.c.}, \qquad (4)$$

leads to mixing among four up- and down-type quarks of the same charge. As a result, after spontaneous electroweak symmetry breaking due to  $\langle \phi \rangle = v \neq 0$ , we obtain the mass terms

$$\bar{d}_{L}^{\alpha}M_{d}^{\alpha\beta}d_{R}^{\beta} + \bar{u}_{L}^{\alpha}M_{u}^{\alpha\beta}u_{R}^{\beta} + \text{H.c.}, \qquad (5)$$

where  $M_d$  and  $M_u$  are  $4 \times 4$  mass matrices and  $\alpha, \beta = 1 \dots 4$  cover ordinary and vectorlike quarks. In general, the mass matrices are not diagonal, and unitary transformations from weak to mass eigenstates are necessary to achieve diagonalization. Denoting the mass eigenstates by  $u'_{L,R}$  and  $d'_{L,R}$ , we have

$$u_{L,R}^{\alpha} = A_{L,R}^{u}{}^{\alpha\beta} u_{L,R}^{\prime}{}^{\beta}, \quad d_{L,R}^{\alpha} = A_{L,R}^{d}{}^{\alpha\beta} d_{L,R}^{\prime}{}^{\beta}, \tag{6}$$

where the unitary transformation matrices  $A_{L,R}^{u,d}$  are chosen such that  $A_L^{d\dagger}M_dA_R^d$  and  $A_L^{u\dagger}M_uA_R^u$  are diagonal. The interesting property of the VQM is that transformations (6) lead to intergenerational mixing among quarks not only in the charged current sector but also in the neutral current interactions. This is due to the fact that the extra isosinglet quarks carry zero weak isospin and thus, are not involved in  $SU(2)_L$ interactions as weak eigenstates. For example, the charge current interaction term

$$J_{CC}^{W}{}^{\mu} = \sum_{i=1}^{3} I \frac{g}{\sqrt{2}} \overline{u}_{L}^{i} \gamma^{\mu} d_{L}^{i} W_{\mu}^{+} + \text{H.c.}, \qquad (7)$$

transforms to

$$J_{CC}^{W}{}^{\mu} = \sum_{\alpha,\beta=1}^{4} I \frac{g}{\sqrt{2}} \bar{u}_{L}^{\prime \alpha} V^{\alpha\beta} \gamma^{\mu} d_{L}^{\prime \beta} W_{\mu}^{+} + \text{H.c.}, \qquad (8)$$

where

$$V^{\alpha\beta} = \sum_{i=1}^{3} (A_L^{u\dagger})^{\alpha i} (A_L^d)^{i\beta}, \qquad (9)$$

when expressed in terms of mass eigenstates via Eq. (6). *V* is the  $4 \times 4$  generalization of the Cabibbo-Kobayashi-Maskawa (CKM) [4] quark mixing matrix. The fact that the fourth generation is isosinglet, i.e., *i* = 1,2,3, leads to a nonunitarity of the mixing matrix *V* as demonstrated in the following:

$$(V^{\dagger}V)^{\alpha\beta} = \sum_{\delta=1}^{4} V^{\delta\alpha} V^{\delta\beta}$$
$$= \sum_{\delta=1}^{4} \sum_{i,j=1}^{3} \left[ (A_{L}^{ui\delta})^{*} A_{L}^{di\alpha} \right]^{*} (A_{L}^{uj\delta})^{*} A_{L}^{dj\beta}$$

$$= \sum_{i,j=1}^{3} (A_{L}^{di\alpha})^{*} A_{L}^{dj\beta} \sum_{\delta=1}^{4} (A_{L}^{uj\delta})^{*} A_{L}^{ui\delta}$$
$$= \sum_{i=1}^{3} (A_{L}^{di\alpha})^{*} A_{L}^{di\beta} = \delta^{\alpha\beta} - (A_{L}^{d4\alpha})^{*} A_{L}^{d4\beta}.$$
(10)

In obtaining the last line in Eq. (10), the unitarity of the transformation matrix  $A_L^u$  has been utilized. In the same way, one can show

$$(VV^{\dagger})^{\alpha\beta} = \delta^{\alpha\beta} - (A_L^{u4\alpha})^* A_L^{u4\beta}. \tag{11}$$

Equations (10) and (11), together with the unitarity of  $A_L^{u,d}$ , indicate that the quark mixing matrix *V* cannot be unitary. This property of the VQM has interesting consequences in the neutral current sector where nonvanishing tree level FC-NC's, proportional to the deviation of the quark mixing matrix [Eq. (9)] from unitarity, are generated. To demonstrate this explicitly, let us examine the neutral current which is coupled to  $Z_{\mu}$  boson

$$J_{NC}^{Z}{}^{\mu} = I \frac{g}{\cos \theta_{w}} \left( I_{w}^{q} \sum_{i=1}^{3} \bar{q}_{L}^{i} \gamma^{\mu} q_{L}^{i} - Q_{q} \sin^{2} \theta_{w} \right)$$
$$\times \sum_{\delta=1}^{4} \left( \bar{q}_{L}^{\delta} \gamma^{\mu} q_{L}^{\delta} + \bar{q}_{R}^{\delta} \gamma^{\mu} q_{R}^{\delta} \right), \qquad (12)$$

where  $Q_q$  is the electric charge of the quark q. The first term in Eq. (12) is proportional to  $I_w^q$ , the third component of the isospin, which has a value + 1/2 or - 1/2 for the up- or down-type quark, respectively. Consequently, the isosinglet quarks, which have zero isospin, *do not* contribute to this term. As a result, under transformations (6), the neutral current [Eq. (12)] can be expressed in terms of the mass eigenstates as follows:

$$J_{NC}^{Z}{}^{\mu} = I \frac{g}{\cos \theta_{w}} \left( I_{w}^{q} \sum_{i=1}^{3} \sum_{\alpha,\beta=1}^{4} \bar{q}'_{L}^{\alpha} \gamma^{\mu} q_{L}^{\prime \beta} (A_{L}^{q\dagger})^{\alpha i} A_{L}^{q i \beta} - Q_{q} \sin^{2} \theta_{w} \sum_{\delta=1}^{4} \sum_{\alpha,\beta=1}^{4} \bar{q}_{L}^{\prime \alpha} \gamma^{\mu} q_{L}^{\prime \beta} (A_{L}^{q\dagger})^{\alpha \delta} A_{L}^{q \delta \beta} + \bar{q}_{R}^{\prime \alpha} \gamma^{\mu} q_{R}^{\prime \beta} (A_{R}^{q\dagger})^{\alpha \delta} A_{R}^{q \delta \beta} \right)$$
$$= I \frac{g}{\cos \theta_{w}} \sum_{\alpha,\beta=1}^{4} (I_{w}^{q} U^{\alpha \beta} \bar{q}_{L}^{\prime \alpha} \gamma^{\mu} q_{L}^{\prime \beta} - Q_{q} \sin^{2} \theta_{w} \delta^{\alpha \beta} \bar{q}^{\prime \alpha} \gamma^{\mu} q^{\prime \beta}), \qquad (13)$$

where

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We observe that the nonunitarity of the mixing matrix V in the VQM leads to tree level FCNC's in the Z sector. In fact, this property is valid for the neutral currents involving the scalar partner of Z and the Higgs bozon as well. On the other hand, the same nonunitarity parameters  $U^{\alpha\beta}(\alpha \neq \beta)$  of Eqs. (13) and (14) appear at the one-loop level FCNC's as multiplicative factors for terms which are independent of the internal quark mass. These terms are absent in the SM, where CKM quark mixing matrix is unitary.

One can use the existing experimental data to constrain various elements of U. In this work, we are concerned with  $U^{sb}$ , which appears in rare FCNC  $b \rightarrow s$  transitions. In particular, we investigate the shift in the differential branching ratio and total branching ratio, forward-backward and CP asymmetries of the inclusive dileptonic B decays due to the presence of the extra vectorlike quarks. The experimental upper bound on the branching ratio of the  $B \rightarrow X_s \mu^+ \mu^-$  decay is used to constrain  $U^{sb}$  and other VQM parameters, as explained in Sec. III.

## III. RARE B DECAYS $B \rightarrow X_s l^+ l^-$ IN THE VOM

The low-energy effective Lagrangian for  $B \rightarrow X_s l^+ l^-$  can be written as

FIG. 1. The Feynmann diagrams which contribute to B  $\rightarrow X_s l^+ l^-$  in the VQM. (a) The tree level contribution. (b) The box diagram. (c) The penguin diagrams mediated by charged particles. (d) The penguin diagrams mediated by neutral particles. Z and  $\gamma$  coupling to the internal quarks in (c) and (d) are proportional to  $U^{\beta\alpha}$  and  $\delta^{\beta\alpha}$ , respectively.



where

$$L_{\mu} = \gamma_{\mu}(1 - \gamma_5), \quad R_{\mu} = \gamma_{\mu}(1 + \gamma_5),$$
 (16)

$$T_{\mu} = i\sigma_{\mu\nu}(1+\gamma_5)q^{\nu}/q^2.$$
(17)

 $m_b$  is the mass of the b quark and q is the total momentum of the  $l^+l^-$  pair. A and B receive long-distance (LD) contributions from dynamical quark loops and intermediate resonances as well as short-distance (SD) contributions at the tree and one loop level. C is the coefficient of the SD magnetic moment operator. In the VQM, the tree level FCNC diagram of Fig. 1(a) leads to the results

$$A_{1(a)}^{SD} = U^{sb}(-1+2\sin^2\theta_W), \quad B_{1(a)}^{SD} = U^{sb}(2\sin^2\theta_W),$$
(18)

where  $\sin^2 \theta_W \approx 0.23$  ( $\theta_W$  is the weak angle). On the other hand, the box diagram [Fig. 1(b)] contributes to  $A^{SD}$  only, as it is purely  $(V-A)\otimes(V-A)$ :

$$A_{1(b)}^{SD} = -\frac{\alpha}{\pi \sin^2 \theta_W} \sum_{\beta=1}^{4} V_{\beta s}^* V_{\beta b} B_0(x_{\beta}).$$
(19)

The effective vertex function<sup>1</sup>  $B_0(x_\beta)$  (subscript "0" indicates that QCD corrections are not included), where  $x_\beta = m_\beta^2/M_W^2$  with  $m_\beta$  being the mass of the virtual quark in the loop, has the following expression [6]:

$$B_0(x) = \frac{1}{4} \left[ 1 + \frac{x}{1-x} + \frac{x \ln x}{(1-x)^2} \right].$$
 (20)

In the SM with the CKM quark mixing picture, due to unitarity, the constant term in the effective vertex functions like  $B_0(x)$ , when summed over all three generations, adds up to zero, and therefore, is usually omitted. However, in the VQM at one loop order, such constant terms make contributions proportional to the nonunitarity parameter  $U^{sb}$ , and thus their significance depends on the specific process which is under investigation. For example, in the radiative  $B \rightarrow X_s \gamma$  decay process, where a tree level FCNC like Fig. 1(a) is absent, the above mentioned nonunitarity contributions are quite crucial [7]. For the dileptonic  $B \rightarrow X_s l^+ l^-$  process, however, the dominant nonunitarity  $U^{sb}$  effect appears at the tree level [Eq. (18)], compared to which the above mentioned contributions are suppressed by a factor  $\alpha/\pi$ , and thus, can be safely ignored at the leading order.

Photon- and Z-penguin diagrams [Figs. 1(c) and 1(d)] also contribute to A and B. Figure 1(c) consists of the usual SM diagrams due to W and its unphysical Higgs partner exchange, that now include an extra virtual up-type quark insertion in the loop as well. On the other hand, Fig. 1(d) illustrates penguin diagrams via Z, its unphysical scalar Higgs partner and physical Higgs exchange, which are peculiar to the VQM. Because of the property  $\sum_{\delta=1}^{4} U^{\alpha\delta} U^{\delta\beta}$  $= U^{\alpha\beta}$ , these latter diagrams can all be shown to be proportional to  $U^{sb}$ , and therefore, are subleading as compared to the tree level FCNC contribution. Consequently, one has

$$A_{1(c)}^{SD} = -\frac{\alpha}{\pi} \sum_{\beta=1}^{4} V_{\beta s}^{*} V_{\beta b} \left[ \frac{1}{4} D_{0}(x_{\beta}) + \left( 1 - \frac{1}{2 \sin^{2} \theta_{W}} \right) C_{0}(x_{\beta}) \right],$$
$$B_{1(c)}^{SD} = -\frac{\alpha}{\pi} \sum_{\beta=1}^{4} V_{\beta s}^{*} V_{\beta b} \left[ \frac{1}{4} D_{0}(x_{\beta}) + C_{0}(x_{\beta}) \right],$$
(21)

where the effective photon and *Z* vertex functions are defined as:

$$D_0(x) = -\frac{4}{9}\ln x + \frac{-19x^3 + 25x^2}{36(x-1)^3} + \frac{x^2(5x^2 - 2x - 6)}{18(x-1)^4}\ln x,$$
(22)

$$C_0(x) = \frac{x}{8} \left[ \frac{x-6}{x-1} + \frac{3x+2}{(x-1)^2} \ln x \right].$$

At this point, an explanation is in order. The Z-penguin diagram in the VQM has a divergent term that is absent in the SM due to the unitarity of the quark mixing matrix which is no longer applicable here. However, this divergence is removed by renormalizing the tree level FCNC which exists in the VQM Lagrangian. Therefore, the effective vertex function  $C_0$  is renormalization scheme dependent and the expression given in Eq. (22) is in fact, in a modified minimal subtraction scheme. However, due to the reason pointed out after Eq. (20), this scheme dependence is not relevant for the dileptonic decay process at hand, where, compared to the tree level, the contribution of a one-loop-level constant term is subleading.

Equations (18), (19), and (21) determine the SD part of the coefficients A and B:

$$A^{SD} = A^{SD}_{1(a)} + A^{SD}_{1(b)} + A^{SD}_{1(c)},$$
  

$$B^{SD} = B^{SD}_{1(a)} + B^{SD}_{1(c)}.$$
(23)

Coefficient C in Eq. (15) receives SD contributions only, and is given as

$$C = \frac{\alpha}{\pi} \sum_{\beta=1}^{4} V_{\beta s}^* V_{\beta b} \frac{1}{4} D_0'(x_\beta), \qquad (24)$$

where the effective magnetic moment vertex function  $D_0'$  is evaluated to be

$$D_0'(x) = -\frac{8x^3 + 5x^2 - 7x}{12(1-x)^3} + \frac{x^2(2-3x)}{2(1-x)^4} \ln x.$$
 (25)

The LD contributions enter A and B coefficients through charm-quark loop ( $c\bar{c}$  continuum) and the intermediate resonances  $\psi$  and  $\psi'$ :

$$A^{LD} = B^{LD} = \frac{\alpha}{2\pi} V_{cs}^* V_{cb} [3C_1 + C_2] (\tau^{\text{cont}} + \tau^{\text{res}}).$$
(26)

 $C_1$  and  $C_2$  are the Wilson coefficients of the current-current operators  $O_1$  and  $O_2$ , respectively, which are defined as follows:

$$O_1 = \overline{s} L^{\mu} b \overline{c} L_{\mu} c, \quad O_2 = \overline{c} L^{\mu} b \overline{s} L_{\mu} c. \tag{27}$$

To avoid the scale dependence issue, which is associated with the QCD corrected Wilson coefficients, we take the combination  $3C_1+C_2$  in Eq. (26) as a phenomenological parameter whose magnitude  $|3C_1+C_2|=0.72$  can be determined from the data on the semi-inclusive  $B \rightarrow X_s \psi$  [8]. The  $c\bar{c}$  continuum contribution is obtained from joining the *c* and  $\bar{c}$  legs of the four Fermi operators in Eq. (27) [9],

$$\tau^{\text{cont}} = -g\left(\frac{m_c}{m_b}, z\right),\tag{28}$$

<sup>&</sup>lt;sup>1</sup>We use the notation which is used in Ref. [5] for the effective box and penguin vertices.

where  $z = q^2/m_b^2$  (q the total momentum of the dileptons) and

$$g(y,z) = \begin{cases} \frac{4}{9} \ln y^2 - \frac{8}{27} - \frac{16y^2}{9z} + \frac{2}{9} \sqrt{1 - \frac{4y^2}{z}} \left(2 + \frac{4y^2}{z}\right) \left(\ln \left|\frac{1 + \sqrt{1 - \frac{4y^2}{z}}}{1 - \sqrt{1 - \frac{4y^2}{z}}}\right| + i, \pi\right), \quad z \ge 4y^2 \\ \frac{4}{9} \ln y^2 - \frac{8}{27} - \frac{16y^2}{9z} + \frac{4}{9} \sqrt{\frac{4y^2}{z} - 1} \left(2 + \frac{4y^2}{z}\right) \arctan \frac{1}{\sqrt{\frac{4y^2}{z} - 1}}, \quad z \le 4y^2. \end{cases}$$
(29)

On the other hand, the resonance contributions from  $\psi$  and  $\psi'$  can be incorporated by using a Breit-Wigner form for the resonance propagator [10]:

$$\tau^{\rm res} = \frac{16\pi^2}{9} \left( \frac{f_{\psi}^2(q^2)/m_{\psi}^2}{m_{\psi}^2 - q^2 - im_{\psi}\Gamma_{\psi}} + (\psi \to \psi') \right).$$
(30)

 $f_{\psi(\psi')}$  is the decay constant of the vector meson, which is defined as

$$\langle 0|\bar{c}\gamma_{\mu}c|\psi(\psi')\rangle = f_{\psi(\psi')}\epsilon_{\mu}, \qquad (31)$$

where  $\epsilon_{\mu}$  is the polarization vector.  $m_{\psi(\psi')}$  and  $\Gamma_{\psi(\psi')}$  are the mass and the total decay width of  $\psi(\psi')$ , respectively. In fact, the total branching ratio of  $B \rightarrow X_s l^+ l^-$  is dominated by these two resonances. However, by using some appropriate cuts around the resonances in the differential branching ratio, one can obtain information about the contributing SD physics without any significant interference from the LD sector [11]. The fact that there are various observable distributions associated with the dileptonic rare *B* decays makes these processes excellent venues for examining relevant SD operators within the SM and beyond.

## **IV. RESULTS AND DISCUSSION**

By inserting the coefficients  $A = A^{SD} + A^{LD}$ ,  $B = B^{SD} + B^{LD}$  and *C*, which are obtained from Eqs. (23), (24), and (26), into the effective Lagrangian [Eq. (15)], one can calculate various observables of the decay mode  $B \rightarrow X_s l^+ l^-$ . The differential decay rate for this process, approximated as the free quark decay  $b \rightarrow s l^+ l^-$ , can be written as

$$\frac{1}{BR(B \to X_c e \,\overline{\nu}_e)} \frac{dBR(B \to X_s l^+ l^-)}{dz}$$
$$= \frac{2(1-z)^2}{f(m_c/m_b)|V_{cb}|^2} \left( (|A|^2 + |B|^2)(1+2z) + 2|C|^2(1+2/z) + 6\Re[(A+B)^*C]), \quad (32) \right)$$

where

$$f(x) = 1 - 8x^2 + 8x^6 - x^8 - 24x^4 \ln x.$$
(33)

By normalizing to the semileptonic rate in Eq. (32), the strong dependence on the *b*-quark mass is canceled out.

The VQM parameters appearing in Eq. (32) are the following: The U-quark mass  $m_U$ , the nonunitarity parameter  $U^{sb} = |U^{sb}|e^{i\theta}$ , where  $\theta$  is a weak phase; and  $V_{4s}^*V_{4b} = U^{sb}$  $-(V_{CKM}^+V_{CKM})^{sb}$ .  $V_{CKM}$ , which is our notation for the 3  $\times$ 3 submatrix of the matrix V, consists of the elements representing mixing among three ordinary generations of quarks. Therefore,  $(V_{CKM}^{\dagger}V_{CKM})^{sb}$ , which is zero in the SM, gives a measure of the deviation from unitarity of the threegeneration CKM mixing matrix in the VQM context.

Using the experimental upper bound  $BR(B \rightarrow X_s \mu^+ \mu^-) \le 5.8 \times 10^{-5}$  [12], and assuming the dominance of the tree level contribution [Fig. 1(a)], one can extract the following rough constraint on the magnitude of  $U^{sb}$ :

$$|U^{sb}| \lesssim 10^{-3}.\tag{34}$$

In this work, by taking the above upper limit as a guide, we include all leading contributing factors to the dileptonic rare *B* decay process in obtaining constraints on the VQM model parameters. As we pointed out above, due to the tree level  $U^{sb}$  contribution to the above decay channels, the terms proportional to  $V_{4s}^*V_{4b}$ , which appear at the one-loop order, are significant only if  $V_{4s}^*V_{4b} \approx -(V_{CKM}^{\dagger}V_{CKM})^{sb} \gg U^{sb}$ . Therefore, having this condition in mind, we parametrize our results in terms of  $(V_{CKM}^{\dagger}V_{CKM})^{sb}/|V_{cb}| = \epsilon e^{i\phi}$  instead of  $V_{4s}^*V_{4b}$ , where  $\epsilon = |(V_{CKM}^{\dagger}V_{CKM})^{sb}|/|V_{cb}|$  and  $\phi$  is another weak phase of the model.

As for the numerical values of the ordinary CKM matrix elements, we use  $|V_{cs}| \approx 0.97$ ,  $|V_{cb}| \approx 0.04$  and  $|V_{ts}|/|V_{cb}| \approx 1.1$  [13], which are extracted from various experimental measurements and are not affected by the presence of the new physics. We take  $V_{us}^*V_{ub} \approx 0$  and assume  $V_{cs}^*V_{cb}$  to be real, as is the case, to a good accuracy, in the "standard"



FIG. 2. The differential branching ration vs dilepton invariant mass for various values of the VQM parameters is compared to the SM prediction. Results with  $|U^{sb}| = 10^{-4}$  and  $|U^{sb}| = 10^{-3}$ , for two different values of the *U* quark mass, (i)  $m_U = 200$  GeV and (ii)  $m_U = 400$  GeV, are shown by solid and dashed lines, respectively.

parametrization of the CKM matrix. As a result,  $V_{ts}^*V_{tb}$ , which is not known experimentally, can be expressed in terms of the VQM parameters as

$$\frac{V_{ts}^* V_{tb}}{|V_{cb}|} \approx \epsilon e^{i\phi} - |V_{cs}|. \tag{35}$$

In Fig. 2, the differential branching ratio [Eq. (32)] for some values of the VQM parameters is compared to the SM prediction. To illustrate the effect of various interfering factors, we use  $\epsilon = 0.3$  and all constructive or destructive contribution possibilities of the extra beyond the SM terms.<sup>2</sup> We observe that away from the resonances, where SD operators are dominant, the shift from the SM expectation, depending on the parameter values, can be quite significant. For example, if  $\theta$  and  $\phi$  are both zero, Fig. 2(a) indicates a significant sensitivity of the differential branching ratio to both the non-unitarity parameter  $U^{sb}$  and the vectorlike up quark mass  $m_U$ . Figures 2(b), 2(c), and 2(d) illustrate a reduced sensitivity to these parameters when the new terms generated by the additional quarks enter the effective Lagrangian with a nonvanishing phase. At the same time, the variation of the differential branching ratio with the invariant mass of the dileptons is appreciably larger than that of the pure SM graph, and follows the above-mentioned trend with respect to the relative phase of the extra contributions.

To constrain the model parameters by using the experimental results on  $BR(B \rightarrow X_s \mu^+ \mu^-)$  reported in Ref. [12], we calculate the total branching ratio by integrating Eq. (23) over all the available range of the dileptonic invariant mass but excluding the resonances  $\psi$  and  $\psi'$  with a  $\delta = \pm 0.1$  GeV cut. Our results are depicted in Fig. 3 in the form of acceptable regions in the  $|U^{sb}|$  versus  $m_U$  plane for various choices of the relative sign of the extra contributions. As expected,

<sup>&</sup>lt;sup>2</sup>In view of Eq. (34), this is more or less the minimum requirement to satisfy the condition  $-(V_{\text{CKM}}^{\dagger}V_{\text{CKM}})^{sb} \ge U^{sb}$ .



FIG. 3. The acceptable region (shaded area) of the  $|U^{sb}|$  vs  $m_U$  plane for various values of the VQM parameters obtained by using the experimental upper bound on  $BR(B \rightarrow X_s \mu^+ \mu^-)$ .

the most stringent constraint is obtained if the relative phases  $\theta$  and  $\phi$  both vanish [Fig. 3(a)]. In this case, absolute upper bounds  $|U^{sb}| \leq 1.5 \times 10^{-3}$  and  $m_U \leq 750$  GeV can be inferred for these model parameters. These limits become stronger if a larger value for  $\epsilon$  is used. Figures 3(b) and 3(c) show that, if the extra tree and penguin contributions have positive relative signs, the experimental bound leads to a lower limit for  $m_U$  for larger values of the nonunitarity parameter  $U^{sb}$ . Less strong upper bounds on these parameters result when  $\theta$  and  $\phi$  are both nonzero [Fig. 3(d)].

In addition to the differential and total branching ratios, there are other physical observables associated with the dileptonic rare B decays which can provide crucial information on the contributing amplitudes and their relative phases. Among them are the forward-backward and the CP asym-

metry distributions, which we investigate in the presence of the extra vectorlike quarks. The forward-backward asymmetry distribution is defined as

$$A_{FB}(z) = \frac{\int_{0}^{1} dw d^{2}BR/dw dz - \int_{-1}^{0} dw d^{2}BR/dw dz}{\int_{0}^{1} dw d^{2}BR/dw dz + \int_{-1}^{0} dw d^{2}BR/dw dz},$$
(36)

where  $w = \cos \omega$ , with  $\omega$  being the angle between the momentum of the ingoing *B* meson (or the outgoing *s* quark) and that of  $l^+$  in the center of mass frame of the dileptons. Using the effective Lagrangian [Eq. (15)], a simple form for this asymmetry in the  $m_s = 0$  limit is obtained [14]:

$$A_{FB}(z) = \frac{3}{2} \frac{(|A|^2 - |B|^2)z + 2\Re[(A+B)^*C]}{(|A|^2 + |B|^2)(1+2z) + 2|C|^2(1+2/z) + 6\Re[(A+B)^*C]}.$$
(37)



FIG. 4. The forward-backward asymmetry distribution of  $B \rightarrow X_s \mu^+ \mu^-$  for various values of the VQM parameters is compared to the SM prediction. Results with  $|U^{sb}| = 10^{-4}$  and  $|U^{sb}| = 10^{-3}$ , for two different values of the *U* quark mass, (i)  $m_U = 200$  GeV and (ii)  $m_U = 400$  GeV, are shown by solid and dashed lines, respectively.

Figure 4 illustrates our results for the forward-backward asymmetry distribution of the decay  $B \rightarrow X_s \mu^+ \mu^-$  in the VQM as compared to the SM. From Figs. 4(a) and 4(b), we observe that, when the tree level FCNC's contribute constructively, even though the sign of the asymmetry remains the same as in the SM, its shape, away from the resonances, can be significantly different. On the other hand, as shown in Figs. 4(c) and 4(d), for the destructive contribution of the nonunitarity induced tree level term and large enough values of  $|U^{sb}|$  or  $m_U$ , the sign of  $A_{FB}$  can be opposite to what is predicted by the SM. One important observable in this decay channel is the point of zero asymmetry in the forwardbackward asymmetry distribution, which occurs somewhere below the resonance  $\psi$ . Our investigation reveals that, as far as the VQM is concered, the position of this point is quite stable and is not shifted very much from its SM value for various choices of the model parameters.

The *CP* asymmetry distribution is defined as

$$A_{CP}(z) = \frac{\frac{dBR}{dz}(B \to X_s l^+ l^-) - \frac{dBR}{dz}(\bar{B} \to \bar{X}_s l^+ l^-)}{\frac{dBR}{dz}(B \to X_s l^+ l^-) + \frac{dBR}{dz}(\bar{B} \to \bar{X}_s l^+ l^-)},$$
(38)

where the decay rate for the charge conjugate process  $\overline{B} \rightarrow \overline{X}_s l^+ l^-$  is obtained from the rate for  $B \rightarrow X_s l^+ l^-$  by reversing the sign of the weak phases  $\theta$  and  $\phi$ . Since in the VQM, the effective Lagrangian [Eq. (15)] contains terms with different *CP*-odd weak phases, as well as, LD continuum and resonance contributions which are sources of perturbative and nonperturbative *CP*-even strong phases, the direct *CP* asymmetry [Eq. (38)], unlike the SM, is expected to be nonzero. The size of this asymmetry, which is zero for invariant dilepton masses below the  $2m_c$  threshold due to the vanishing strong phase, depends on the interplay



FIG. 5. The *CP* asymmetry distribution of  $B \rightarrow X_s \mu^+ \mu^-$  for various values of the VQM parameters. Results with  $|U^{sb}| = 10^{-4}$  and  $|U^{sb}| = 10^{-3}$ , for two different values of the *U* quark mass, (i)  $m_U = 200$  GeV and (ii)  $m_U = 400$  GeV, are shown by solid and dashed lines, respectively.

of the various contributing terms. Our results for certain choices of the model parameters are depicted in Fig. 5. From Figs. 5(a) and 5(b), we observe that when the weak phase  $\phi$ is large, smaller values of the nonunitarity parameter lead to significantly larger CP asymmetries sensitive to the U quark mass  $m_U$ . In fact, for a negative relative sign of the tree level contribution ( $\theta = \pi$ ) and  $m_U = 400$  GeV, as shown in Fig. 5(b), asymmetries of the order of 10% can be achieved. On the other hand, for smaller values of  $\phi$ , larger  $|U^{sb}|$  values can generate asymmetries of the order of a few percent if  $m_U \ge 400$  GeV [Fig. 5(c)]. Finally, we find that for a purely real deviation of  $V_{\rm CKM}$  from unitarity, i.e.,  $\phi = 0$  or  $\pi$ , the CP asymmetries away from the resonances are smaller. Figure 5(d) shows a particular instance of this situation where off-resonance-peak CP asymmetries are less than 2%, at most.

In conclusion, we have investigated various observables

of the dileptonic rare *B* decay  $B \rightarrow X_s l^+ l^-$  in the presence of an extra generation of vectorlike quarks. The measurement of this decay channel in the near future should provide more stringent constraints on the model parameters. We showed that the shape of the differential branching ratio and forwardbackward asymmetry distribution can be quite distinct from the SM predictions. *CP* asymmetry in this decay mode, which is near zero in the SM, shows significant sensitivity to the VQM parameters. Asymmetries of up to 10% can be achieved for a typical choice of the parameters which are used in our investigation.

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