CP asymmetry of $B\to X_s l^+ l^-$ in the low invariant mass region

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I analyze the CP asymmetry of $B\to X_s l^+ l^-$ based on a model-independent analysis, which includes 12 independent four-Fermi operators. The CP asymmetry is suppressed in the standard model; however, if some new physics makes it much larger, the present or the next generation of B factories may catch the CP violation in this decay mode. In this paper, we study the correlation of the asymmetry and the branching ratio, and then we find only a type of interaction which can enlarge the asymmetry. Therefore, in comparison with experiments, we have the possibility that we can constrain models beyond the standard model.

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I. INTRODUCTION

The inclusive rare B decay $B\to X_s l^+ l^-$ has already been studied by many researchers. It is attractive to investigate this process experimentally or theoretically. This decay mode is experimentally clean as well as $B\to X_s\gamma$, especially in the low invariant mass region, and, when we can use a parton model to study this process theoretically, it is a semileptonic decay. In the standard model (SM), a flavor changing neutral current (FCNC) process appears only through one or more loops. Since $B\to X_s l^+ l^-$ is also a FCNC, new physics can clarify itself to measure this decay. The extended models beyond the SM, such as the minimal supersymmetrized model (MSSM) and the two Higgs doublets model (2HDM), predict some deviation form the SM [1–14]. The SM prediction shows that, for $l=e$ or μ, this mode will be found at the KEKB and the SLAC e^+e^- storage ring PEP-II B factories in the near future. Therefore, in order to search new physics, the study of this process is one of the most interesting topics. In this paper, the final leptons will be muons or electrons throughout.

The CP-violating asymmetry of this decay is also a subject that many physicists investigate. This observable is very sensitive to the complex phase of the Cabibbo-Kobayashi-Maskawa (CKM) matrix elements, so that we have the possibility to find effects beyond the SM. The SM predicts that the CP asymmetry is suppressed, about 10^{-3} or smaller [15,16]. If some non-SM interactions enlarge for the asymmetry to get sizable, we can know the existence beyond SM. This observable has been calculated in MSSM and 2HDM [10–14]. In these models, as well as the SM, the distribution is a function of fewer Wilson coefficients than the full operator basis. In our previous work, we analyzed the branching ratio and the forward-backward (FB) asymmetry, which is an observable corresponding to the size of parity violation in the decay $B\to X_s l^+ l^-$, with a most general model-independent method [17,18]. Generally, the matrix element for the decay $b\to s l^+ l^-$ includes all types of local and $bs\gamma$-induced four-Fermi operators. That is,

$$
\begin{aligned}
\mathcal{M}=\frac{G_F\alpha}{\sqrt{2}\pi}V_{ts}^*V_{tb}\Bigg[& C_{SL}\bar{s}i\sigma_{\mu\nu}\frac{q^\nu}{q^2}(m_s L)b\,\bar{l}\gamma^\mu l \\
&+C_{BR}\bar{s}i\sigma_{\mu\nu}\frac{q^\nu}{q^2}(m_b R)b\,\bar{l}\gamma^\mu l+C_{LL}\bar{s}_L\gamma_\mu b_L\bar{l}_L\gamma^\mu l_L \\
&+C_{LR}\bar{s}_L\gamma_\mu b_L\bar{l}_R\gamma^\mu l_R+C_{RL}\bar{s}_R\gamma_\mu b_R\bar{l}_L\gamma^\mu l_L \\
&+C_{RR}\bar{s}_R\gamma_\mu b_R\bar{l}_R\gamma^\mu l_R+C_{LRLR}\bar{s}_L b_R\bar{l}_L l_R \\
&+C_{RLLR}\bar{s}_R b_L\bar{l}_L l_R+C_{LRRL}\bar{s}_L b_R\bar{l}_R l_L+C_{RLRL}\bar{s}_R b_L\bar{l}_R l_L \\
&+C_T\bar{s}\sigma_{\mu\nu}b\,\bar{l}\sigma^{\mu\nu}l+iC_{TE}\bar{s}\sigma_{\mu\nu}b\,\bar{l}\sigma_{\alpha\beta}l\epsilon^{\mu\nu\alpha\beta}\Bigg],
\end{aligned}
\tag{1}
$$

where C_{XX}'s are the coefficients of the four-Fermi interactions. Among them, there are two $bs\gamma$ induced four-Fermi interactions denoted by C_{SL} and C_{BR}, which correspond to $-2C_7^{\rm eff}$ in the SM, and which are constrained by the experimental data of $b\to s\gamma$. There are four vector-type interactions denoted by C_{LL}, C_{LR}, C_{RL}, and C_{RR}. Two of them (C_{LL}, C_{LR}) are already present in the SM as the combinations of (C_9-C_{10}, C_9+C_{10}). Therefore, they are regarded as the sum of the contributions from the SM and the new physics deviations ($C_{LL}^{\rm new}, C_{LR}^{\rm new}$). The other vector interactions, denoted by C_{RL} and C_{RR}, are obtained by interchanging the chirality projections $L\leftrightarrow R$. There are four scalar-type interactions, C_{LRLR}, C_{RLLR}, C_{RLLR}, and C_{RLRL}. The remaining two denoted by C_T and C_{TE} correspond to tensor type. The indices L and R are chiral projections, $L=\frac{1}{2}(1-\gamma_5)$ and $R=\frac{1}{2}(1+\gamma_5)$. Then, we can get the differential branching ratio of the FCNC process $b\to s l^+ l^-$,

$$
\begin{aligned}
\frac{d\mathcal{B}}{ds}=\frac{1}{2m_b^8}\mathcal{B}_0\mathrm{Re}\ & [S_1(s)\{m_s^2|C_{SL}|^2+m_b^2|C_{BR}|^2\}+S_2(s) \\
&\times\{2m_b m_s C_{SL}C_{BR}^*\}+S_3(s)\{2m_s^2 C_{SL}(C_{LL}^*+C_{LR}^*) \\
&+2m_b m_s C_{BR}(C_{RL}^*+C_{RR}^*)\}+S_4(s)\{2m_b^2 C_{BR}(C_{LL}^* \\
&+C_{LR}^*)+2m_b m_s C_{SL}(C_{RL}^*+C_{RR}^*)\}+M_2(s)\{|C_{LL}|^2 \\
&+|C_{LR}|^2+|C_{RL}|^2+|C_{RR}|^2\}+M_6(s)\{-2(C_{LL}C_{RL}^*
\end{aligned}
$$

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$$+C_{LR}C_{RR}^{*})+(C_{LRLR}C_{RLLR}^{*}+C_{LRRL}C_{RLRL}^{*})\}+M_8(s)$$
$$\times\{|C_{LRLR}|^2+|C_{RLLR}|^2+|C_{LRRL}|^2+|C_{RLRL}|^2\}$$
$$+M_9(s)\{16|C_T|^2+64|C_{TE}|^2\}]. \tag{2}$$

Here, we ignore terms including lepton mass m_l, because we take only massless (anti-) lepton into consideration. A set of the kinematic functions $S_i(s)$ $(i=1,2,3,4,5,6)$ and $M_n(s)$ $(n=2,6,8)$ is shown in the Appendix. The normalization factor $\mathcal{B}_0$ is given by

$$\mathcal{B}_0\equiv\mathcal{B}_{sl}\frac{3\alpha^2}{16\pi^2}\frac{|V_{ts}^{*}V_{tb}|^2}{|V_{cb}|^2}\frac{1}{f(\widehat{m_c})\kappa(\widehat{m_c})}, \tag{3}$$

where the other factors $f(\widehat{m_c})$ and $\kappa(\widehat{m_c})$ are the phase-space factor and the $O(\alpha_s)$ QCD correction factor [19]. The factor $\mathcal{B}_{sl}$ denotes the branching ratio of the semileptonic decay, and we set it to 10.4%. We can also have the FB asymmetry from Eq. (1). Thus, by numerical analysis, we obtained useful information to pin down new physics beyond the standard model. However, we set all the new Wilson coefficients to real when we carried out the numerical analysis. This means that we assume that there is no new CP-violating source in the decay $B\to X_s l^+ l^-$. The CP asymmetry is sensitive to the imaginary part of the coefficients. Therefore, it is worth treating the CP asymmetry based on our previous analysis.

This paper is organized as follows. In Sec. II, we find how to obtain the general CP asymmetry, study the correlation between the asymmetry and the branching ratio to pin down the type of interactions, and give some discussions. We give a summary in Sec. III.

II. GENERAL *CP* ASYMMETRY

We assume the semileptonic decay $b\to c l^- \overline{\nu_l}$ is an approximately CP-conserving mode; in fact, experiments show they correspond with each other within about 10^{-2} [20]. The partonic approximation predicts no CP-violating asymmetry in the SM. That is, we can use the same normalization factor as Eq. (3) to express the branching ratio of $b\to s l^+ l^-$ and $\overline{b}\to\overline{s}l^+l^-$. For a general Wilson coefficient C_{XX}, we can define B_{XX}, λ_{XX}, and A_{XX} by

$$C_{XX}\equiv B_{XX}+\lambda_{XX}A_{XX}, \tag{4}$$

where λ_{XX} is the CP-violating phase and generally both B_{XX} and A_{XX} are complex. In the case of the SM, only the CKM matrix elements give the CP-violating weak phase and the strong phase appears through the QCD penguin correction. Conventionally, these effects are included in the Wilson coefficients $C_9^{\rm eff}$ of the vector-type current-current interaction [21]. Explicitly it is expressed by [21,22]

$$C_9^{\rm eff}=B_9+\lambda_u A_9, \tag{5}$$

where, without the $c\overline{c}$ long-distant contribution,

TABLE I. The partially integrated CP asymmetry for $(\rho,\eta)=(0.12,0.25)$, $(0.16,0.33)$, and $(0.27,0.40)$ and in the SM at $\mu=(m_b)_{\overline{MS}}$.

(ρ,η)	$\mathcal{A}_{CP}^{SM}$
(0.12,0.25)	0.85×10^{-3}
(0.16,0.33)	1.12×10^{-3}
(0.27,0.40)	1.36×10^{-3}

$$B_9=\left(1+\alpha\frac{w(s)}{\pi}\right)C_9^{NDR}+Y(s). \tag{6}$$

Only $\lambda_u\equiv(V_{ub}Vus^*)/(V_{tb}Vts^*)$ includes the CP-violating phase. Since $\lambda_u A_9$ is very small except for the $c\overline{c}$ resonance region, the SM predicts that the CP asymmetry is very negligible [15].

We must take $c\overline{c}$ resonance into consideration to discuss the branching ratio and the CP asymmetry [23], otherwise avoid the region where J/ψ and ψ' poles contribute [16]. In this paper, we take the latter stand. The residual region is the lower region before the J/ψ resonance or higher region after ψ' resonance [13]. We restrict our discussion to only the low invariant mass region, $1<s<8$ $({\rm GeV}^2)$, where $s\equiv(p_{l^+}+p_{l^-})^2$. We then introduce the partially integrated CP asymmetry $\mathcal{A}_{\rm CP}$ defined by

$$\mathcal{A}_{\rm CP}\equiv\frac{\mathcal{B}(B\to X_s l^+l^-)-\mathcal{B}(\overline{B}\to X_s l^+l^-)}{\mathcal{B}(B\to X_s l^+l^-)+\mathcal{B}(\overline{B}\to X_s l^+l^-)}\equiv\frac{\mathcal{N}_{\rm CP}}{\mathcal{D}_{\rm CP}}, \tag{7}$$

where $\mathcal{B}(B\to X_s l^+ l^-)$ is the partially integrated branching ratio for the process $B\to X_s l^+l^-$, defined by

$$\int_{1({\rm GeV}^2)}^{8}ds\frac{d\mathcal{B}(B\to X_s l^+l^-)}{ds}\sim3.73.\times10^{-6}$$
$$[\text{at }\mu=(m_b)_{\overline{MS}}].$$

In the same way, we define the partially integrated branching ratio for $\overline{B}\to X_s l^+l^-$. We set $(C_7^{\rm eff},C_9^{NDR},C_{10})=(-0.317,4.52,-4.29)$ for numerical calculation. We listed its value for the SM at the renormalization scale $\mu=(m_b)_{\overline{MS}}\equiv4.2$ GeV in Table I, where we set Wolfenstein's CKM parameters [24] to $(\rho,\eta)=(0.12,0.25)$, $(0.16,0.33)$, and $(0.27,0.40)$. We should note that there is a huge uncertainty about the CP asymmetry predicted by the SM before we discuss the sensitivity to new physics from our numerical results. The asymmetry in the SM is uncertain by almost 100% [16]. So, we must get at least a 10 times larger size as the SM prediction for the CP asymmetry to find the signal of new physics; otherwise we fail to do so. Then, from Eq. (1), we can get the numerator $\mathcal{N}_{CP}$ of the CP asymmetry by replacing ${\rm Re}(C_{XX}C_{YY}^{*})$ in the branching ratio given in Eq. (2) with

$$-2\,{\rm Im}(\lambda_{XX}){\rm Im}(B_{YY}^{*}A_{XX})-2\,{\rm Im}(\lambda_{YY}){\rm Im}(B_{XX}^{*}A_{YY})$$
$$-2\,{\rm Im}(\lambda_{XX}\lambda_{YY}^{*}){\rm Im}(A_{XX}A_{YY}^{*}),$$

and, for the dominator $\mathcal{D}_{CP}$, with

$$2\,\mathrm{Re}(B_{XX}B_{YY}^*)+2\,\mathrm{Re}(\lambda_{XX})\mathrm{Re}(B_{YY}^*A_{XX})$$
$$+2\,\mathrm{Re}(\lambda_{YY})\mathrm{Re}(B_{XX}^*A_{YY})+2\,\mathrm{Re}(\lambda_{XX}\lambda_{YY}^*)\mathrm{Re}(A_{XX}A_{YY}^*).$$

The resultant CP asymmetry takes the most general model-independent form. We show the explicit expression of this asymmetry in the Appendix. The CP asymmetry does not vanish, if and only if B_{XX} or A_{XX} has a different phase from A_{YY} and λ_{YY} or $\lambda_{XX}\lambda_{YY}^*$ has an imaginary part. Here, XX and YY denote types of interactions, whether they are the same type or not. However, in the most interesting models like the 2HDM [13,14] and the MSSM [1,11], the strong phase does not play such an important role to the CP asymmetry. Therefore, we assume that we can ignore a set of strong phases introduced by new physics [9]. Then, for new vector, scalar, and tensor-type interactions, we can redefine the Wilson coefficients as

$$C_{XX}=B_{XX}^{SM}+(\lambda_{XX}+\lambda_u)(A_9+A_{XX})\quad \text{for } XX=LL \text{ or } LR, \tag{8}$$

$$C_{XX}=\lambda_{XX}A_{XX}\quad \text{for others.} \tag{9}$$

Here, A_{XX}s are real and λ_{XX}s are phase factors defined by $\exp(i\phi_{XX})$, where $0\leqslant\phi_{XX}<2\pi$, and $B_{LL}^{SM}\equiv B_9-C_{10}$ and $B_{LR}^{SM}\equiv B_9+C_{10}$. In the same way, we can redefine C_{BR} and C_{SL}, and have other constraints from the measurement of $B\to X_s\gamma$,

$$4|C_7^{\mathrm{eff}}|^2(m_b^2+m_s^2)=m_b^2(|A_{SL}^N|^2+|A_{BR}|^2), \tag{10}$$

where $A_{SL}^N=(m_b/m_s)A_{SL}$ [18]. The definitions of A_{BR} and A_{SL} and ϕ_{BR} and ϕ_{SL} follow Eq. (9). Thus if there is an interference between such coefficients and the C_9^{eff} it can enlarge the CP asymmetry. Otherwise, the new interactions suppress the observable according to the above assumption. In this case, the explicit form of the partially integrated CP asymmetry is given by

$$\mathcal{A}_{CP}\equiv\frac{\int_{1\ \mathrm{GeV}^2}^{8}ds[d\mathcal{N}_{CP}(s)/ds]}{\int_{1\ \mathrm{GeV}^2}^{8}ds[d\mathcal{D}_{CP}(s)/ds]}\equiv\frac{\mathcal{N}_{CP}}{\mathcal{D}_{CP}}, \tag{11}$$

where

$$\begin{aligned}\frac{d\mathcal{N}_{CP}(s)}{ds}=-\frac{1}{m_b^8}\,&\mathcal{B}_0[S_3(s)\{2m_s^2[\mathrm{Im}(\lambda_{SL})\mathrm{Im}(A_{SL}B_9^*)+\mathrm{Im}(\lambda_{SL}\lambda_{LL}^*)\mathrm{Im}(A_{SL}A_9^*)+\mathrm{Im}(\lambda_{SL})\mathrm{Im}(A_{SL}B_9^*)\\&+\mathrm{Im}(\lambda_{SL}\lambda_{LR}^*)\mathrm{Im}(A_{SL}A_9^*)]\}+S_4(s)\{2m_b^2[\mathrm{Im}(\lambda_{BR})\mathrm{Im}(A_{BR}B_9^*)+\mathrm{Im}(\lambda_{BR}\lambda_{LL}^*)\mathrm{Im}(A_{BR}A_9^*)\\&+\mathrm{Im}(\lambda_{BR})\mathrm{Im}(A_{BR}B_9^*)+\mathrm{Im}(\lambda_{BR}\lambda_{LR}^*)\mathrm{Im}(A_{BR}A_9^*)]\}\\&+M_2(s)\{2(\mathrm{Im}(\lambda_{LL})\mathrm{Im}((B_9-C_{10})(A_9+A_{LL})^*)+\mathrm{Im}(\lambda_{LR})((B_9+C_{10})(A_9+A_{LR})^*))\}\\&+M_6(s)\{-2(\mathrm{Im}(\lambda_{RL})\mathrm{Im}((B_9^*-C_{10})A_{RL})+\mathrm{Im}(\lambda_{LL}\lambda_{RL}^*)\mathrm{Im}(A_9A_{RL}^*)\\&+\mathrm{Im}(\mathrm{Im}(\lambda_{RR})\mathrm{Im}((B_9^*+C_{10})A_{RR})+\mathrm{Im}(\lambda_{LR}\lambda_{RR}^*)\mathrm{Im}((A_9+A_{LR})A_{RR}^*))\}\end{aligned} \tag{12}$$

and

$$\begin{aligned}\frac{d\mathcal{D}_{CP}(s)}{ds}=\frac{1}{m_b^8}&\mathcal{B}_0[S_1(s)\{m_s^2|A_{SL}|^2+m_b^2|A_{BR}|^2\}+S_2(s)\{2m_bm_s\mathrm{Re}(\lambda_{SL}\lambda_{BR}^*)\mathrm{Re}(A_{SL}A_{BR}^*)\}+S_3(s)\{2m_s^2\{\mathrm{Re}(\lambda_{SL})\mathrm{Re}[A_{SL}(B_9\\&-C_{10})^*]+\mathrm{Re}[\lambda_{SL}\lambda_{LL}^*)\mathrm{Re}(A_{SL}(A_9+A_{LL})^*]+\mathrm{Re}(\lambda_{SL})\mathrm{Re}[A_{SL}(B_9+C_{10})^*]+\mathrm{Re}(\lambda_{SL}\lambda_{LR}^*)\mathrm{Re}[A_{SL}(A_9+A_{LR})^*]\}\\&+2m_bm_s[\mathrm{Re}(\lambda_{BR}\lambda_{RL}^*)\mathrm{Re}(A_{BR}A_{RL}^*)+\mathrm{Re}(\lambda_{BR}\lambda_{RR}^*)\mathrm{Re}(A_{BR}A_{RR}^*)]\}+S_4(s)\{2m_b^2\{\mathrm{Re}(\lambda_{BR})\mathrm{Re}[A_{BR}(B_9-C_{10})^*]\\&+\mathrm{Re}(\lambda_{BR}\lambda_{LL}^*)\mathrm{Re}(A_{BR}[A_9+A_{LL})^*]+\mathrm{Re}(\lambda_{BR})\mathrm{Re}[A_{BR}(B_9+C_{10})^*]+\mathrm{Re}(\lambda_{BR}\lambda_{LR}^*)\mathrm{Re}[A_{BR}(A_9+A_{LR})^*]\}\\&+2m_bm_s[\mathrm{Re}(\lambda_{SL}\lambda_{RL}^*)\mathrm{Re}(A_{SL}A_{RL}^*)+\mathrm{Re}(\lambda_{SL}\lambda_{RR}^*)\mathrm{Re}(A_{SL}A_{RR}^*)]\}+M_2(s)\{|B_9-C_{10}|^2+|A_9+A_{LL}|^2\\&+2\,\mathrm{Re}(\lambda_{LL})\mathrm{Re}[(B_9-C_{10})(A_9+A_{LL})^*]+|B_9+C_{10}|+|A_9+A_{LR}|^2+2\,\mathrm{Re}(\lambda_{LR})\mathrm{Re}[(B_9+C_{10})(A_9+A_{LR})^*]\\&+|A_{RL}|^2+|A_{RR}|^2\}+M_6(s)\{-2\{\mathrm{Re}(\lambda_{RL})\mathrm{Re}[(B_9-C_{10})^*A_{RL}]+\mathrm{Re}(\lambda_{LL}\lambda_{RL}^*)\mathrm{Re}[(A_9+A_{LL})A_{RL}^*]\\&+\mathrm{Re}(\lambda_{RR})\mathrm{Re}[(B_9+C_{10})^*A_{RR}]+\mathrm{Re}(\lambda_{LR}\lambda_{RR}^*)\mathrm{Re}[(A_9+A_{LR})A_{RR}^*]\}+[\mathrm{Re}(\lambda_{LRLR}\lambda_{RLLR}^*)\mathrm{Re}(A_{LRLR}A_{RLLR}^*)\\&+\mathrm{Re}(\lambda_{LRRL}\lambda_{RLRL}^*)\mathrm{Re}(A_{LRRL}A_{RLRL}^*)]\}+M_8(s)\{|A_{LRLR}|^2+|A_{RLLR}|^2+|A_{LRRL}|^2+|A_{RLRL}|^2\}\\&+M_9(s)\{16|A_T|^2+64|A_{TE}|^2\}].\end{aligned} \tag{13}$$

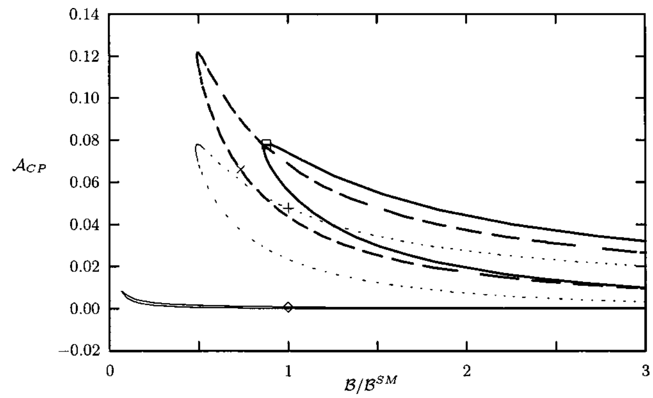

FIG. 1. The correlation between $\mathcal{B}/\mathcal{B}^{SM}$ and $\mathcal{A}_{CP}$ as A_{LL} moves, and $\phi_{LL}=0$ (thin solid line), $\pi/4$ (dotted line), $\pi/2$ (thick solid line), and $3\pi/4$ (dashed line). The marks $\diamond$, $+$, $\square$, and $\times$ show the prediction for $\phi_{LL}=0$, $\pi/4$, $\pi/2$, and $3\pi/4$ with $A_{LL}=0$.

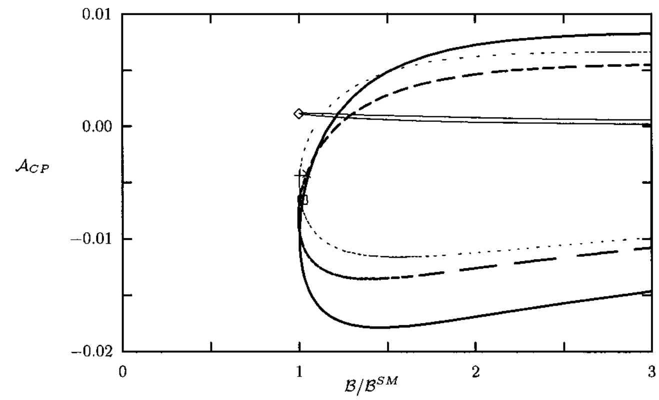

FIG. 2. The correlation between $\mathcal{B}/\mathcal{B}^{SM}$ and $\mathcal{A}_{CP}$ as A_{LR} moves, and $\phi_{LR}=0$ (thin solid line), $\pi/4$ (dotted line), $\pi/2$ (thick solid line), and $3\pi/4$ (dashed line). $\diamond$ shows the standard model prediction. $\diamond$, $+$, $\square$, and $\times$ show the prediction for $\phi_{LR}=0$, $\pi/4$, $\pi/2$, and $3\pi/4$ with $A_{LR}=0$.

Here, we omitted λ_u because it is very small.

We will analyze the partially integrated CP asymmetry defined by Eq. (7) and examine its sensitivity to each Wilson coefficient. For numerical estimation, we set $(\rho,\eta)=(0.16,0.33)$. At first, we investigate vector, scalar, and tensor-type interactions, which are collectively *new local interactions*. The results of Ref. [17] make us predict the sensitivity of the CP asymmetry to each Wilson coefficient. The branching ratio is the most sensitive to the vector-type interactions, especially C_{LL}, and the contribution due to C_{RL} and C_{RR} is positive. Only the C_{LL} and C_{LR} have the weak and strong phases, so we can expect that only two types of interactions can make CP asymmetry large, we can especially expect that the CP asymmetry is sizable by appropriate C_{LL}. However, C_{RL} and C_{RR} would suppress the CP asymmetry. The scalar and tensor-type interactions hardly interfere with each other or a vector-type interaction in the massless lepton limit. Thus, if a scalar or tensor-type interaction enters into our decay mode, it would suppress the CP asymmetry. In Figs. 1 and 2, the correlation between the branching ratio and the CP asymmetry when C_{LL} or C_{LR} moves is plotted. Because the flow of each interaction depends on the type of the interaction, we can pin down the type of interaction that contributes to the processes once we measure those observable. These show behavior as expected in the above discussion. We should pay attention to Fig. 1, which shows the CP asymmetry can get much larger as the branching ratio is predicted by the SM. It is because the partially integrated CP asymmetry for the SM is so suppressed that it is enlarged by 10^2. For $\phi_{LL}=\pi/4$, $\pi/2$, or $3\pi/4$, the asymmetry is the most enlarged when $A_{LL}\sim-1.2|C_{10}|$, 0 or $1.1|C_{10}|$. If we ignore the SM CP-violating contribution, A_9 and λ_u, C_{LL} enters into the asymmetry as in the following:

$$\frac{2\int ds\{\mathrm{Im}[M_2(B_9-C_{10})-2M_4C_7^{\mathrm{eff}}](A_9^*+A_{LL})\}\sin\phi_{LL}}{2m_b^8\mathcal{B}_{SM}+\int dsM_2|A_9+A_{LL}|^2+2\int ds\{\mathrm{Re}[M_2(B_9-C_{10})-2M_4C_7^{\mathrm{eff}}](A_9^*+A_{LL})\}\cos\phi_{LL}}, \tag{14}$$

where $M_2(s)$ and $M_4(s)$ are shown in the Appendix, and $2m_b^8\mathcal{B}_{SM}\sim 0.72$. By choosing an approximate set of A_{LL} and ϕ_{LL} to hold

$$\int dsM_2A_{LL}\sim-2\int dsM_2\,\mathrm{Re}(B_9-C_{10})\cos\phi_{LL},$$

the asymmetry can become 10^{-1}. That is, if there is new physics through C_{LL} with a weak phase, there is the possibility that we may pin down this type of interaction at the B factory in the near future, even if there is no contradiction with present experiments. Equation (14) shows the correlation is very sensitive to whether ϕ_{LL} is infinitesimal or not. Thus, the SM prediction point is far from other lines. In the same way, some A_{LR} and ϕ_{LR} enlarge the asymmetry and are sensitive to ϕ_{LR} but, because $B_9+C_{10}\ll B_9-C_{10}$, its contribution is smaller than A_{LL} and ϕ_{LL}. And, in order to see how much the coefficient A_{LR} contributes to the asymmetry, we check when the absolute value of the asymmetry becomes the maximum. By analogy with the analysis for A_{LL} and Eq. (14), we find that it has the largest value when, roughly,

$$2\int ds\,M_2\mathrm{Re}(A_9)\sim-\int ds\,M_2A_{LR},$$

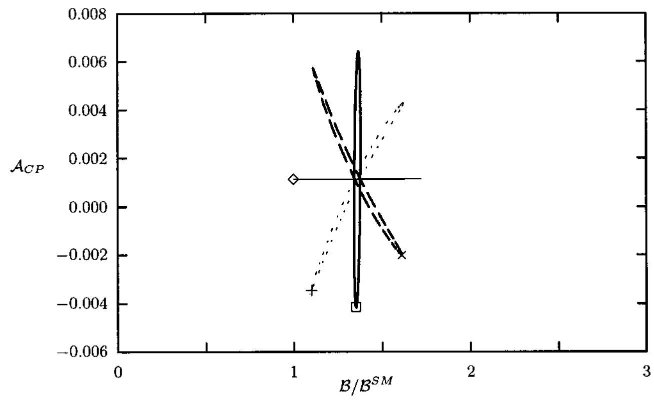

FIG. 3. The correlation between $\mathcal{B}/\mathcal{B}^{SM}$ and $\mathcal{A}_{CP}$ as θ moves, and $\phi_{BR}=0$ (thin solid line), $\pi/4$ (dotted line), $\pi/2$ (thick solid line), and $3\pi/4$ (dashed line), where $\tan\theta=A_{BR}/A_{SL}^N$. We set $\phi_{SL}=0$. And, for $A_{SL}=-2C_7^{\rm eff}$ and $A_{BR}=-2C_7^{\rm eff}$, plotted some marks, $\diamond$ ($\phi_{BR}=0$), $+$ ($\phi_{BR}=\pi/4$), $\square$ ($\phi_{BR}=\phi/2$), and $\times$ ($\phi_{BR}=3\pi/4$).

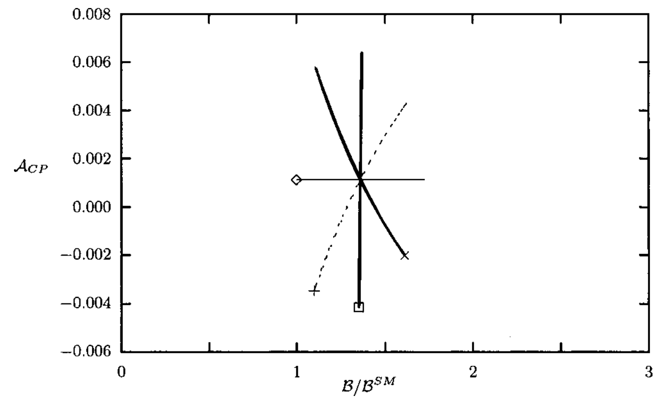

FIG. 4. The correlation between $\mathcal{B}/\mathcal{B}^{SM}$ and $\mathcal{A}_{CP}$ as θ moves, and $\phi_{NL}=0$ (thin solid line), $\pi/4$ (thin dotted line), $\pi/2$ (thick solid line), and $3\pi/4$ (thick solid line), where $\tan\theta=A_{BR}/A_{SM}^N$. We set $\phi_{NL}\equiv\phi_{SL}=\phi_{BR}$. For $A_{SL}=-2C_7^{\rm eff}$ and $A_{BR}=-2C_7^{\rm eff}$: $\diamond$ ($\phi_{NL}=0$), $+$ ($\phi_{NL}=\pi/4$), $\square$ ($\phi_{NL}=\phi/2$), and $\times$ ($\phi_{NL}=3\pi/4$).

numerically $A_{LR}\sim-1.4|C_{10}|$ or $-1.5|C_{10}|$ for $\phi_{LR}=\pi/4$ or $\pi/2$ and $3\pi/4$. [Note we ignored the term including $M_2{\rm Re}(B_9+C_{10})-2M_4C_7^{\rm eff}$ because it is much smaller than the remains in the dominator.]

For C_{RL} and C_{RR}, the terms from $M_2C_{RL}^2$ and $M_2C_{RR}^2$ disappear in the numerator, so that the other terms,

$$-M_6(s)(C_9^{\rm eff}-C_{10})C_{RL}^*, \tag{15}$$

$$-M_6(s)(C_9^{\rm eff}+C_{10})C_{RL}^*, \tag{16}$$

which we ignored when we discussed the sensitivity of C_{RL} and C_{RR} to the branching ratio, give significant effect to the CP asymmetry, so that the asymmetry may depend on ϕ_{RL} and ϕ_{RR}. Here, $M_6(s)$ is given in the Appendix. Equations (15) and (16) give similar contributions to the asymmetry except that it includes not only M_2 but M_6. Since $M_6\ll M_2$ due to strange quark mass m_s, its sensitivity is small. We can also consider the correlation where A_{RL} and A_{RR} are very small strong phases, that is

$$A_{RL}=A_9+A'_{RL}, \tag{17}$$

$$A_{RR}=A_9+A'_{RR}, \tag{18}$$

where A'_{RL} and A'_{RR} are real. In this case, the sign of the imaginary part of $(B_9-C_{10})(A_9+A'_{RL})$ and $(B_9+C_{10})(A_9+A'_{RR})$ yields the difference between the correlations, however, the sensitivity is still small.

For scalar and tensor interactions, in the massless lepton limit, Wilson coefficients appear only through the squared absolute. So, the asymmetry is almost independent of ϕ_S ($S=LRLR$, $LRRL$, $RLLR$, $RLRL$), ϕ_T, and ϕ_{TE} and it gets only more suppressed as A_S, A_T, or A_{TE} gets larger. Moreover, the sensitivity is very small because the corresponding kinematic functions include a factor m_l.

Next, consider only C_{BR} and C_{SL}, which is constrained by Eq. (10). Generally, without a strong phase, these coefficients are expressed by

$$C_{BR}=A_{BR}e^{i\phi_{BR}}, \quad C_{SL}=A_{SL}e^{i\phi_{SL}}, \tag{19}$$

where ϕ_{BR} and ϕ_{SL} are independent weak phases. As shown in Ref. [18], the partially integrated branching ratio $\mathcal{B}$ is more sensitive to C_{BR} than $C_{SL}^N\equiv(m_b/m_s)C_{SL}$ because of the strange quark mass m_s. This is true for the partially integrated CP asymmetry $\mathcal{A}_{CP}$. In other words, it is almost independent of the phase ϕ_{SL} in comparison with ϕ_{BR}. The asymmetry cannot be enlarged by A_{SL} (or A_{BR}) with $\phi_{BR}=0$. We can find this feature by comparing Fig. 3 with Fig. 4. In the former, we set ϕ_{SL} to 0; in the latter, however, we set $\phi_{SL}=\phi_{BR}\equiv\phi_{NL}$. By contrast with C_{SL}, the form of the correlation depends on C_{BR} considerably. Ignoring the SM contribution, in the case of $\phi_{SL}=\phi_{BR}=\phi_{NL}$, the asymmetry takes the form

$$\frac{8m_bC_7^{\rm eff}\left[m_s\int dsS_3\cos\theta{\rm Im}(B_9)+m_b\int dsS_4\sin\theta{\rm Im}(B_9)\right]\sin\phi_{NL}}{2m_b^8\mathcal{B}_{NL}+2m_b^8\mathcal{B}_L-8m_bC_7^{\rm eff}\left[m_s\int dsS_3\cos\theta{\rm Re}(B_9)+m_b\int ds\sin\theta S_4{\rm Re}(B_9)\right]\cos\phi_{NL}}, \tag{20}$$

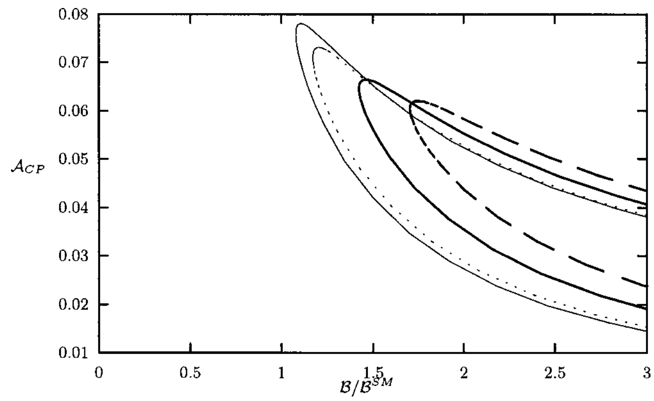

FIG. 5. The correlation of $\mathcal{B}/\mathcal{B}_0$ and $\mathcal{A}_{CP}/\mathcal{A}_{CP}^{SM}$ as A_{LL} moves for $\phi_{LL}=\pi/2$ and $\phi_{NL}=0$ (thin solid line), $\pi/4$ (dotted line), $\pi/2$ (thick solid line), and $3\pi/4$ (dashed line). Here the definition of ϕ_{NL} is the same as Fig. 4.

where $\mathcal{B}_{NL}$ and $\mathcal{B}_L$ are the partially integrated branching ratios. For the former, only nonvanishing new Wilson coefficients are A_{BR} and the latter has A_{SL} and $A_{BR}=A_{SL}=0$. We set $\tan\theta=A_{BR}/A_{SL}^N$ and ignored the higher-order terms about m_s/m_b. The definition of S_3 and S_4 is given in the Appendix. Since $\mathrm{Im}(B_9)\ll\mathrm{Re}(B_9)$, the partially integrated branching ratio is expressed by

$$\frac{1}{2m_b^8}\mathcal{B}_0\left\{\mathcal{B}_{NL}+\mathcal{B}_L-8m_bC_7^{\mathrm{eff}}\left[m_s\int dsS_3\mathrm{Re}(B_9)\cos\theta\right.\right.$$
$$\left.\left.+m_b\int dsS_4\mathrm{Re}(B_9)\sin\theta\right]\cos\phi_{NL}\right\}. \tag{21}$$

Equations (20) and (21) show that, when ϕ_{NL} rounds from 0 to 2π, so does the ellipse of the correlation, as shown in Fig. 4. The size of A_{BR}, and also A_{SL}, is not so significant to enlarge the partially integrated CP asymmetry. Thus, these two types of interactions do not give a great influence to the partially integrated CP asymmetry even if there is another type of new interaction, say C_{LL}. For example, when we set $\phi_{LL}=\pi/2$ and we check the dependency of ϕ_{NL} on the asymmetry, it does not largely change the form of the correlation between $\mathcal{B}$ and $\mathcal{A}$ as A_{LL} moves negligibly, as shown in Fig. 5, so we must note, if and only if very minute experiments are done.

III. SUMMARY

The model-independent analysis of the partially integrated CP asymmetry of the inclusive rare B decay $B\to X_sl^+l^-$ was presented. CP violation is one of the most interesting topics to research new physics and understand bariogenesis in the early universe, and many researchers have studied this observable through both the experimental and theoretical approaches. The process $B\to X_sl^+l^-$ is experimentally clean, and there is a possibility that this mode is found by KEKB and PEP-II B factories. Because $B\to X_sl^+l^-$ is a FCNC process, it is the most sensitive to the various extensions of the SM. Our analysis includes the full operator basis, i.e., 12 independent four-Fermi operators. In the SM, only three types of Wilson coefficients contribute to $B\to X_sl^+l^-$, and the partially integrated CP asymmetry has an order of 10^{-3}. We investigated the correlation of the partially integrated branching ratio and the partially integrated CP asymmetry, and then can conclude that only C_{LL}, the coefficient of the operator $(\overline{s}_L\gamma_\mu b_L\overline{l}_L\gamma^\mu l_L)$, can be a meaningful contribution to our process. This cause is the same as the branching ratio [17], i.e., the large interference between (B_9-C_{10}) and C_{LL}. Since $(B_9+C_{10})\ll(B_9-C_{19})$, the contribution of C_{LR}, the coefficient of the operator $(\overline{s}_L\gamma_\mu b_L\overline{l}_R\gamma^\mu l_R)$, is less than C_{LL}. However, the Wilson coefficients of the other new local interactions beyond SM work only to suppress the asymmetry, because we assumed there was no new strong phase, and then they have no interference with the SM interactions. In order to contrast with the left-right symmetric model, we made C_{RL} and C_{RR} have very small strong phases; however it changes the size of the CP asymmetry a little. As for C_{BR} and C_{SL}, the coefficients of the $B\to X_s\gamma$ operators, although the asymmetry depends largely on the weak phase ϕ_{BR} of C_{BR}, their size makes little contribution to the asymmetry. Thus, the dependency of two coefficients is much smaller than that of A_{LL}. Note that the branching ratio also depends on the ϕ_{BR}.

Our analysis contains the special cases such as the MSSM and the 2HDM. In the MSSM, a special case is $C_{BR}=C_{SL}=2C_7$, $C_{LL}=C_9^{\mathrm{eff}}-C_{10}$, and $C_{LR}=C_9^{\mathrm{eff}}+C_{10}$. This is expressed as an example in Fig. 4. Therefore, the asymmetry is very suppressed like the standard model, although the branching ratio can be large (or not). However, the model has the possibility of conversion of the sign of C_{10}. In this case, Figs. 1 and 2 show that the CP asymmetry may be enlarged. Large contributions to A_{CP} were pointed out by Ref. [10]. Figure 4 includes the rough character of 2HDM, where a new weak phase enters into C_{BR} and C_{SL} with the deviation from the SM prediction for the numerical values of C_{BR}, C_{SL}, C_{LL}, and C_{LR}. When $\sin\phi_{NL}$ is small, the asymmetry is suppressed; however, when $\sin\phi_{NL}$ is close to unity, it changes with the sign of C_{BR} [14]. Once the CP asymmetry is measured, we will be able to constrain the extended models by comparing the data with our numerical analysis. If we get the signature of the asymmetry in it, we can conclude that there is a new $(V-A)\otimes(V-A)$ interaction and/or a sizable strong coupling. Otherwise, the analysis of the present paper cannot constrain us within some models, so we have to wait for future experiments to get some information on the CP from $B\to X_sl^+l^-$.

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APPENDIX: KINEMATIC FUNCTIONS

We list a set of kinematic functions, which decide the behavior of the branching ratio and the CP asymmetry for the decay $b\to sl^+l^-$, and show the general expression of the direct CP asymmetry. The ratio is shown by Eq. (2). We follow Refs. [17,25] with regard to notation. That is, the functions are given by

$$S_1(s) = -\frac{4}{s}u(s)\left\{s^2 - \frac{1}{3}u(s)^2 - (m_b^2 - m_s^2)^2\right\},$$

$$S_2(s) = -16u(s)m_b m_s,$$

$$S_3(s) = 4u(s)(s + m_b^2 - m_s^2),$$

$$S_4(s) = 4u(s)(s - m_b^2 + m_s^2),$$

$$M_1(s) = (m_s^2 + m_b^2)S_1(s) + 2m_b m_s S_2(s),$$

$$M_2(s) = 2u(s)[-\tfrac{1}{3}u(s)^2 - s^2 + (m_b^2 - m_s^2)^2],$$

$$M_4(s) = m_s^2 S_3(s) + m_b^2 S_4(s),$$

$$M_6(s) = m_b m_s [S_3(s) + S_4(s)],$$

$$M_8(s) = 2u(s)(m_b^2 + m_s^2 - s)s,$$

$$M_9(s) = 2u(s)[-\tfrac{2}{3}u(s)^2 - 2(m_b^2 + m_s^2)s + 2(m_b^2 - m_s^2)^2], \quad \text{(A1)}$$

where we neglect lepton mass.

With the above functions, we can express the partially integrated CP asymmetry delivered from the matrix element [Eq. (1)], that is,

$$\mathcal{A}_{CP} \equiv \frac{\int_{1GeV^2}^{8} ds[d\mathcal{N}_{CP}(s)/ds]}{\int_{1GeV^2}^{8} ds[d\mathcal{D}_{CP}(s)/ds]} \equiv \frac{\mathcal{N}_{CP}}{\mathcal{D}_{CP}}, \quad \text{(A2)}$$

where

$$\begin{aligned}
\frac{d\mathcal{N}_{CP}(s)}{ds} = & -\frac{1}{m_b^8}\mathcal{B}_0[S_1(s)\{2m_s^2 \mathrm{Im}(\lambda_{SL})\mathrm{Im}(B_{SL}A_{SL}^*) + 2m_b^2 \mathrm{Im}(\lambda_{BR})\mathrm{Im}(B_{BR}A_{BR}^*)\} + S_2(s)\{2m_b m_s[\mathrm{Im}(\lambda_{SL})\mathrm{Im}(A_{SL}B_{BR}^*) \\
& + \mathrm{Im}(\lambda_{BR})\mathrm{Im}(B_{SL}^*A_{BR}) + \mathrm{Im}(\lambda_{SL}\lambda_{BR}^*)\mathrm{Im}(A_{SL}A_{BR}^*)]\} + S_3(s)\{2m_s^2[\mathrm{Im}(\lambda_{SL})\mathrm{Im}(A_{SL}B_{LL}^*) \\
& + \mathrm{Im}(\lambda_{LL})\mathrm{Im}(B_{SL}^*A_{LL}) + \mathrm{Im}(\lambda_{SL}\lambda_{LL}^*)\mathrm{Im}(A_{SL}A_{LL}^*) + \mathrm{Im}(\lambda_{SL})\mathrm{Im}(A_{SL}B_{LR}^*) + \mathrm{Im}(\lambda_{LR})\mathrm{Im}(B_{SL}^*A_{LR}) \\
& + \mathrm{Im}(\lambda_{SL}\lambda_{LR}^*)\mathrm{Im}(A_{SL}A_{LR}^*)] + 2m_b m_s(\mathrm{Im}(\lambda_{BR})\mathrm{Im}(A_{BR}B_{RL}^*) + \mathrm{Im}(\lambda_{RL})\mathrm{Im}(B_{BR}^*A_{RL}) \\
& + \mathrm{Im}(\lambda_{BR}\lambda_{RL}^*)\mathrm{Im}(A_{BR}A_{RL}^*) + \mathrm{Im}(\lambda_{BR})\mathrm{Im}(A_{BR}B_{RR}^*) + \mathrm{Im}(\lambda_{RR})\mathrm{Im}(B_{BR}^*A_{RR}) + \mathrm{Im}(\lambda_{BR}\lambda_{RR}^*)\mathrm{Im}(A_{BR}A_{RR}^*)]\} \\
& + S_4(s)\{2m_b^2[\mathrm{Im}(\lambda_{BR})\mathrm{Im}(A_{BR}B_{LL}^*) + \mathrm{Im}(\lambda_{LL})\mathrm{Im}(B_{BR}^*A_{LL}) + \mathrm{Im}(\lambda_{BR}\lambda_{LL}^*)\mathrm{Im}(A_{BR}A_{LL}^*) + \mathrm{Im}(\lambda_{BR})\mathrm{Im}(A_{BR}B_{LR}^*) \\
& + \mathrm{Im}(\lambda_{LR})\mathrm{Im}(B_{BR}^*A_{LR}) + \mathrm{Im}(\lambda_{BR}\lambda_{LR}^*)\mathrm{Im}(A_{BR}A_{LR}^*)] + 2m_b m_s[\mathrm{Im}(\lambda_{SL})\mathrm{Im}(A_{SL}B_{RL}^*) + \mathrm{Im}(\lambda_{RL})\mathrm{Im}(B_{SL}^*A_{RL}) \\
& + \mathrm{Im}(\lambda_{SL}\lambda_{RL}^*)\mathrm{Im}(A_{SL}A_{RL}^*) + \mathrm{Im}(\lambda_{SL})\mathrm{Im}(A_{SL}B_{RR}^*) + \mathrm{Im}(\lambda_{RR})\mathrm{Im}(B_{SL}^*A_{RR}) + \mathrm{Im}(\lambda_{SL}\lambda_{RR}^*)\mathrm{Im}(A_{SL}A_{RR}^*)]\} \\
& + M_2(s)\{2[\mathrm{Im}(\lambda_{LL})\mathrm{Im}(B_{LL}A_{LL}^*) + \mathrm{Im}(\lambda_{LR})\mathrm{Im}(B_{LR}A_{LR}^*) + \mathrm{Im}(\lambda_{RL})\mathrm{Im}(B_{RL}A_{RL}^*) + \mathrm{Im}(\lambda_{RR})\mathrm{Im}(B_{RR}A_{RR}^*)]\} \\
& + M_6(s)\{-2[\mathrm{Im}(\lambda_{LL})\mathrm{Im}(A_{LL}B_{RL}^*) + \mathrm{Im}(\lambda_{RL})\mathrm{Im}(B_{LL}^*A_{RL}) + \mathrm{Im}(\lambda_{LL}\lambda_{RL}^*)\mathrm{Im}(A_{LL}A_{RL}^*) + \mathrm{Im}(\lambda_{LR})\mathrm{Im}(A_{LR}B_{RR}^*) \\
& + \mathrm{Im}(\lambda_{RR})\mathrm{Im}(B_{LR}^*A_{RR}) + \mathrm{Im}(\lambda_{LR}\lambda_{RR}^*)\mathrm{Im}(A_{LR}A_{RR}^*)] + [\mathrm{Im}(\lambda_{LRLR})\mathrm{Im}(A_{LRLR}B_{RLLR}^*) \\
& + \mathrm{Im}(\lambda_{RLLR})\mathrm{Im}(B_{LRLR}^*A_{RLLR}) + \mathrm{Im}(\lambda_{LRLR}\lambda_{RLLR}^*)\mathrm{Im}(A_{LRLR}A_{RLLR}^*) + \mathrm{Im}(\lambda_{LRRL})\mathrm{Im}(A_{LRRL}B_{RLRL}^*) \\
& + \mathrm{Im}(\lambda_{RLRL})\mathrm{Im}(B_{LRRL}^*A_{RLRL}) + \mathrm{Im}(\lambda_{LRRL}\lambda_{RLRL}^*)\mathrm{Im}(A_{LRRL}A_{RLRL}^*)]\} + M_8(s)\{2[\mathrm{Im}(\lambda_{LRLR})\mathrm{Im}(B_{LRLR}A_{LRLR}^*) \\
& + \mathrm{Im}(\lambda_{RLLR})\mathrm{Im}(B_{RLLR}A_{RLLR}^*) + \mathrm{Im}(\lambda_{LRRL})\mathrm{Im}(B_{LRRL}A_{LRRL}^*) + \mathrm{Im}(\lambda_{RLRL})\mathrm{Im}(B_{RLRL}A_{RLRL}^*)]\} + M_9(s) \\
& \times\{32\mathrm{Im}(\lambda_T)\mathrm{Im}(B_T A_T^*) + 128\ \ \mathrm{Im}(\lambda_{TE})\mathrm{Im}(B_{TE}A_{TE}^*)\},
\end{aligned} \quad \text{(A3)}$$

and

$$\begin{aligned}
\frac{d\mathcal{D}_{CP}(s)}{ds} = & \, 2\frac{d\mathcal{B}(s)}{ds}\bigg|_{C_{XX}\rightarrow B_{XX}} + \frac{1}{m_b^8}\mathcal{B}_0[S_1(s)\{m_s^2[|A_{SL}|^2 + 2\ \mathrm{Re}(\lambda_{SL})\mathrm{Re}(B_{SL}A_{SL}^*)] + m_b^2[|A_{BR}|^2 + 2\ \mathrm{Re}(\lambda_{BR})\mathrm{Re}(B_{BR}A_{BR}^*)]\} \\
& + S_2(s)\{2m_b m_s[\mathrm{Re}(\lambda_{SL})\mathrm{Re}(A_{SL}B_{BR}^*) + \mathrm{Re}(\lambda_{BR})\mathrm{Re}(B_{SL}^*A_{BR}) + \mathrm{Re}(\lambda_{SL}\lambda_{BR}^*)\mathrm{Re}(A_{SL}A_{BR}^*)]\} + S_3(s) \\
& \times\{2m_s^2[\mathrm{Re}(\lambda_{SL})\mathrm{Re}(A_{SL}B_{LL}^*) + \mathrm{Re}(\lambda_{LL})\mathrm{Re}(B_{SL}^*A_{LL}) + \mathrm{Re}(\lambda_{SL}\lambda_{LL}^*)\mathrm{Re}(A_{SL}A_{LL}^*) + \mathrm{Re}(\lambda_{SL})\mathrm{Re}(A_{SL}B_{LR}^*) \\
& + \mathrm{Re}(\lambda_{LR})\mathrm{Re}(B_{SL}^*A_{LR}) + \mathrm{Re}(\lambda_{SL}\lambda_{LR}^*)\mathrm{Re}(A_{SL}A_{LR}^*)] + 2m_b m_s[\mathrm{Re}(\lambda_{BR})\mathrm{Re}(A_{BR}B_{RL}^*) + \mathrm{Re}(\lambda_{RL})\mathrm{Re}(B_{BR}^*A_{RL})
\end{aligned}$$

$$+\mathrm{Re}(\lambda_{BR}\lambda_{RL}^*)\mathrm{Re}(A_{BR}A_{RL}^*)+\mathrm{Re}(\lambda_{BR})\mathrm{Re}(A_{BR}B_{RR}^*)+\mathrm{Re}(\lambda_{RR})\mathrm{Re}(B_{BR}^*A_{RR})+\mathrm{Re}(\lambda_{BR}\lambda_{RR}^*)\mathrm{Re}(A_{BR}A_{RR}^*)]\}+S_4(s)$$
$$\times\{2m_b^2[\mathrm{Re}(\lambda_{BR})\mathrm{Re}(A_{BR}B_{LL}^*)+\mathrm{Re}(\lambda_{LL})\mathrm{Re}(B_{BR}^*A_{LL})+\mathrm{Re}(\lambda_{BR}\lambda_{LL}^*)\mathrm{Re}(A_{BR}A_{LL}^*)+\mathrm{Re}(\lambda_{BR})\mathrm{Re}(A_{BR}B_{LR}^*)$$
$$+\mathrm{Re}(\lambda_{LR})\mathrm{Re}(B_{BR}^*A_{LR})+\mathrm{Re}(\lambda_{BR}\lambda_{LR}^*)\mathrm{Re}(A_{BR}A_{LR}^*)]+2m_bm_s[\mathrm{Re}(\lambda_{SL})\mathrm{Re}(A_{SL}B_{RL}^*)+\mathrm{Re}(\lambda_{RL})\mathrm{Re}(B_{SL}^*A_{RL})$$
$$+\mathrm{Re}(\lambda_{SL}\lambda_{RL}^*)\mathrm{Re}(A_{SL}A_{RL}^*)+\mathrm{Re}(\lambda_{SL})\mathrm{Re}(A_{SL}B_{RR}^*)+\mathrm{Re}(\lambda_{RR})\mathrm{Re}(B_{SL}^*A_{RR})+\mathrm{Re}(\lambda_{SL}\lambda_{RR}^*)\mathrm{Re}(A_{SL}A_{RR}^*)]\}+M_2(s)$$
$$\times\{|A_{LL}|^2+2\,\mathrm{Re}(\lambda_{LL})\mathrm{Re}(B_{LL}A_{LL}^*)+|A_{LR}|^2+2\,\mathrm{Re}(\lambda_{LR})\mathrm{Re}(B_{LR}A_{LR}^*)+|A_{RL}|^2+2\,\mathrm{Re}(\lambda_{RL})\mathrm{Re}(B_{RL}A_{RL}^*)+|A_{RR}|^2$$
$$+2\,\mathrm{Re}(\lambda_{RR})\mathrm{Re}(B_{RR}A_{RR}^*)\}+M_6(s)\{-2[\mathrm{Re}(\lambda_{LL})\mathrm{Re}(A_{LL}B_{RL}^*)+\mathrm{Re}(\lambda_{RL})\mathrm{Re}(B_{LL}^*A_{RL})+\mathrm{Re}(\lambda_{LL}\lambda_{RL}^*)\mathrm{Re}(A_{LL}A_{RL}^*)$$
$$+\mathrm{Re}(\lambda_{LR})\mathrm{Re}(A_{LR}B_{RR}^*)+\mathrm{Re}(\lambda_{RR})\mathrm{Re}(B_{LR}^*A_{RR})+\mathrm{Re}(\lambda_{LR}\lambda_{RR}^*)\mathrm{Re}(A_{LR}A_{RR}^*)]+[\mathrm{Re}(\lambda_{LRLR})\mathrm{Re}(A_{LRLR}B_{RLLR}^*)$$
$$+\mathrm{Re}(\lambda_{RLLR})\mathrm{Re}(B_{LRLR}^*A_{RLLR})+\mathrm{Re}(\lambda_{LRLR}\lambda_{RLLR}^*)\mathrm{Re}(A_{LRLR}A_{RLLR}^*)+\mathrm{Re}(\lambda_{LRRL})\mathrm{Re}(A_{LRRL}B_{RLRL}^*)$$
$$+\mathrm{Re}(\lambda_{RLRL})\mathrm{Re}(B_{LRRL}^*A_{RLRL})+\mathrm{Re}(\lambda_{LRRL}\lambda_{RLRL}^*)\mathrm{Re}(A_{LRRL}A_{RLRL}^*)]\}+M_8(s)\{|A_{LRLR}|^2$$
$$+2\,\mathrm{Re}(\lambda_{LRLR})\mathrm{Re}(B_{LRLR}A_{LRLR}^*)+|A_{RLLR}|^2+2\,\mathrm{Re}(\lambda_{RLLR})\mathrm{Re}(B_{RLLR}A_{RLLR}^*)+|A_{LRRL}|^2$$
$$+2\,\mathrm{Re}(\lambda_{LRRL})\mathrm{Re}(B_{LRRL}A_{LRRL}^*)+|A_{RLRL}|^2+2\,\mathrm{Re}(\lambda_{RLRL})\mathrm{Re}(B_{RLRL}A_{RLRL}^*)\}+M_9(s)\{16[|A_T|^2$$
$$+2\,\mathrm{Re}(\lambda_T)\mathrm{Re}(B_TA_T^*)]+64[|A_{TE}|^2+2\,\mathrm{Re}(\lambda_{TE})\mathrm{Re}(B_{TE}A_{TE}^*)]\}. \tag{A4}$$

The first term $d\mathcal{B}/ds|_{C_{XX}\to B_{XX}}$ in Eq. (A4) is the differential branching ratio given by Eq. (2) after replacing all Wilson coefficients C_{XX} with B_{XX}, respectively.

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