

# Quark-hadron duality in structure functions

Nathan Isgur,<sup>1</sup> Sabine Jeschonnek,<sup>1</sup> W. Melnitchouk,<sup>1,2</sup> and J. W. Van Orden<sup>1,3</sup>

<sup>1</sup>*Jefferson Lab, 12000 Jefferson Avenue, Newport News, Virginia 23606*

<sup>2</sup>*Special Research Centre for the Subatomic Structure of Matter, Adelaide University, Adelaide 5005, Australia*

<sup>3</sup>*Department of Physics, Old Dominion University, Norfolk, Virginia 23529*

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While quark-hadron duality is well established experimentally, the current theoretical understanding of this important phenomenon is quite limited. To expose the essential features of the dynamics behind duality, we use a simple model in which the hadronic spectrum consists of narrow resonances made of valence quarks. We qualitatively reproduce the features of duality as seen in electron scattering data within our model. We show that in order to observe duality, it is essential to use the appropriate scaling variable and scaling function. In addition to its great intrinsic interest in connecting the quark-gluon and hadronic pictures, an understanding of quark-hadron duality could lead to important benefits in extending the applicability of scaling into previously inaccessible regions.

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## I. INTRODUCTION

### A. Background

Duality is a much used and much abused concept. In some cases it is used to describe an equivalence between quark- and hadron-based pictures which is trivial; in others an equivalence which is impossible. In almost all cases, the conceptual framework in which duality is discussed and used is either hopelessly muddled or hopelessly abstract. Nevertheless, the data indicate that some extremely interesting and potentially very important “duality” phenomena are occurring at low energy.

We begin by making the trivial observation that any hadronic process can be correctly described in terms of quarks and gluons, assuming that quantum chromodynamics (QCD) is the correct theory for strong interactions. While this statement is obvious, it rarely has practical value, since in most cases we can neither perform nor interpret a full QCD calculation. We will refer to the above statement that any hadronic process can be described by a full QCD calculation as “degrees of freedom duality”; if one could perform and interpret the calculations, it would not matter at all which set of states—hadronic states or quark and gluon states—was used.

On the other hand, there are rare cases where the average of hadronic observables is described by a perturbative QCD (PQCD) calculation. We reserve the use of the term “duality” to describe these rare correspondences, in contrast to the trivial “degrees of freedom duality” described above. In these rare cases, a quark-gluon calculation leads to a very simple description of some phenomenon even though this phenomenon “materializes” in the form of hadrons. Deep inelastic scattering is the prototypical example, and the one on which we focus here. These rare examples are all characterized by a special choice of kinematic conditions which serve to expose the “bare” quarks and gluons of the QCD Lagrangian. In the case of deep inelastic scattering, the kinematics are such that the struck quark receives so much energy over such a small space-time region that it behaves like

a free particle during the essential part of its interaction. This leads to the compellingly simple picture that the electron-nucleon cross section is determined in this kinematic region by free electron-quark scattering, i.e. duality is exact for this process in this kinematic regime.

For inclusive inelastic electron scattering from a proton in the scaling region, the cross section is determined by the convolution of a nonperturbative and currently difficult to calculate parton distribution function with an electron-quark scattering cross section determined by perturbative QCD. For semileptonic decays of heavy quarks, e.g.,  $\bar{B} \rightarrow X_c l \bar{\nu}_l$ , one can prove using pQCD that the decay rate is determined by that of the underlying heavy quark, in this case obtained from the process  $b \rightarrow c l \bar{\nu}_l$  [1]. In  $e^+e^- \rightarrow \text{hadrons}$ , it is the underlying  $e^+e^- \rightarrow q\bar{q}$  process that applies because of pQCD. However, while duality applies to all of these phenomena, we will see that even in these special processes we must invoke an averaging procedure to identify the hadronic results with the quark-gluon predictions.

An important application of quark-hadron duality is QCD sum rules; see, e.g., Ref. [2]. There, properties of the ground state are calculated by matching results obtained in the parton picture, using the operator product expansion, and in the hadron picture, using dispersion relations, in a certain kinematic region where the ground state is dominant. The region of applicability of duality in QCD sum rules was studied extensively in various channels and structure functions, and has been studied more recently in quantum mechanical models [3].

In addition to its need of an averaging procedure, it is easy to see that the pQCD picture of inelastic electron scattering must fail for  $Q^2 \rightarrow 0$ . For duality to hold for the nucleon structure functions in this case, the elastic electric proton and neutron form factors, which take the value of the nucleon charges for  $Q^2 \rightarrow 0$ , would have to be reproduced by electron scattering off the corresponding  $u$  and  $d$  quarks. This is possible for the proton since the squares of the charges of two  $u$  quarks and one  $d$  quark add up to 1 [4]. However, for

the neutron, the squared quark charges cannot add up to 0, so it is clear that local duality in inclusive inelastic electron scattering from a neutron must fail for  $Q^2 \rightarrow 0$ . Also, we know that duality must fail for polarized structure functions at low  $Q^2$ , as the Ellis-Jaffe (EJ) sum rule and the Gerassimov-Drell-Hearn (GDH) sum rule, which can be written as integrals over  $g_1(\nu, Q^2)$  at different  $Q^2$ , are negative (GDH sum rule for  $Q^2=0$ ) and positive (EJ sum rule at  $Q^2$  of several  $\text{GeV}^2$ ), respectively [5].

Thus duality in inelastic electron scattering has to hold in the scaling regime and must in general break down at low energy. Obviously, a very interesting question is what happens in between these regimes, i.e., how does duality break down? This paper answers this question, which is not only interesting in itself, but also crucial for practical, quantitative applications of duality.

### B. Introducing local averaging and our model

We begin by discussing the issue of averaging. If duality is relevant at all at low energy, then it is quite obvious that we need to perform some sort of average: the smooth, analytical pQCD prediction cannot in general correspond exactly to the generally highly structured hadronic data. For low energies this requirement is universally accepted; however, even in the “scaling” region one must average in principle. To see this, consider QCD in the large- $N_c$  limit [6]. We can do this because no element of the pQCD results for deep inelastic scattering depends on the number of colors. However, in this limit the hadronic spectrum consists entirely of infinitely narrow noninteracting resonances [7], i.e., there are only infinitely narrow spikes in the  $N_c \rightarrow \infty$  hadronic world. Since the quark level calculation still yields a smooth scaling curve, and the kinematic conditions for being in the scaling region are unchanged as  $N_c \rightarrow \infty$ , we see that we must average even in the scaling region. While in Nature the resonances have fairly broad decay widths, so that the averaging takes place automatically in the data, the large- $N_c$  limit shows us that averaging is always required in principle. It is thus clearly important to be able to define this averaging procedure, e.g., how large the intervals must be and which resonances have to be included.

It is easy to see that this procedure will not be universal, and will certainly not simply be that the resonances one-by-one locally average the pQCD-derived scaling curve: the averaging method will depend on the process and on the target. Consider, as an illustration of these points, the case of a spinless quark and antiquark with charges  $e_1$  and  $e_2$  and equal masses bound into a nonrelativistic  $q_1\bar{q}_2$  system. The inelastic electron scattering rate calculated at the quark level in leading twist will then be proportional to  $e_1^2 + e_2^2$ . Since the elastic state will be produced with a rate proportional to  $(e_1 + e_2)^2$ , it clearly cannot in general be locally dual to the scaling curve [8]. How then is duality realized in this system? Consider the charge operator  $\sum_i e_i e^{i\vec{q} \cdot \vec{r}_i}$ ; from the ground state it excites even partial wave states with an amplitude proportional to  $e_1 + e_2$  and odd ones with an amplitude proportional to  $e_1 - e_2$ . Thus the resonances build up a cross section of the form  $\alpha_1(e_1 + e_2)^2 + \alpha_2(e_1 - e_2)^2$

$+ \alpha_3(e_1 + e_2)^2 + \dots$ , and one can see by explicit calculations in models that (up to phase space factors) the cross terms in this sum will cancel to give a cross section proportional to  $e_1^2 + e_2^2$  once averaged over nearby even and odd parity resonances. It is clear that such target- and process-dependence is worthy of study. However, in this paper we will restrict ourselves to a model with  $e_2=0$  so that local duality might apply even at low  $Q^2$  [9].

The question of the validity of low energy duality, i.e., duality in electron scattering at finite beam energies in inelastic electron scattering after suitable averaging, is as old as the first inclusive electron scattering experiments themselves. It began with the seminal paper of Bloom and Gilman [10], which made the observation that the inclusive  $F_2$  structure function in the resonance region at low  $Q^2$  generally oscillates about and averages to a global scaling curve which describes high  $Q^2$  data. More recently, interest in Bloom-Gilman duality was revived with the collection of high precision data on the  $F_2$  structure function from Jefferson Lab [11]. These data not only confirmed the existence of the Bloom-Gilman duality to rather low values of  $Q^2$ , but also seem to demonstrate that for the proton the equivalence of the averaged resonance and scaling structure functions holds also for each resonance so that duality also exists locally.

Here we present a model for the study of quark-hadron duality in electron scattering that uses only a few basic ingredients. That is, in addition to requiring that our model be relativistic, we assume confinement, and assume that it is sufficient to consider only valence quarks (this latter simplification being underwritten, as mentioned previously, by the large- $N_c$  limit). In addition, since our model is designed to explore conceptual issues and not to be compared to data, and since we postpone addressing spin-dependent issues to later work, for simplicity we also take the quarks, electrons and photons to be scalars. A model with these features will not give a realistic description of any data, but it should allow us to study the critical questions of when and why duality holds. While this model is extremely simple, we see no impediment to extending it to describe a more realistic situation since we find that duality arises from the most basic properties of our model.

We make several more convenient simplifications. Although it is our aim to study duality in electron scattering from the nucleon, i.e., from a three-quark system, as a first step we study these issues in what is effectively a one quark system by considering such a quark to be confined to an infinitely massive antiquark. In the case of scalar quarks considered here, we can therefore describe the system by the Klein-Gordon equation. We also select for our confining potential one which is linear in  $r$ , namely  $V^2(\vec{r}) = \alpha r^2$ , where  $\alpha$  is a generalized, relativistic string constant. This choice allows us to obtain analytic solutions, without which the required numerical work for this study would be daunting. Indeed, the energy eigenvalues,  $E_N = \sqrt{2\sqrt{\alpha}(N+3/2) + m^2}$ , where  $m$  is the mass of interacting quark, can be readily obtained by noting the similarity to the Schrödinger equation for a non-relativistic harmonic oscillator potential: the solu-

tions for the wave functions are the same as for the non-relativistic case.

In Sec. II we construct the structure function out of resonances described by form factors, each of which individually gives vanishing contributions at large momenta, and show that it both scales and, when suitably averaged, is equal to the “free quark” result. An analysis in terms of structure function moments is presented in Sec. III. In Sec. IV we examine the onset of scaling, and the appearance of Bloom-Gilman duality, while in Sec. V we discuss the connection of Bloom-Gilman duality with duality in heavy quark systems. Finally, in Sec. VI we summarize our results and mention some possible directions for future research.

## II. QUARK-HADRON DUALITY IN THE SCALING LIMIT

The differential cross section for inclusive inelastic scattering of a “scalar electron” via the exchange of a “scalar photon” is

$$\frac{d\sigma}{dE_f d\Omega_f} = \frac{g^4}{16\pi^2} \frac{E_f}{E_i} \frac{1}{Q^4} \mathcal{W}, \quad (1)$$

where the scalar coupling constant  $g$  carries the dimension of a mass, and the factor multiplying the scalar structure function  $\mathcal{W}$  corresponds to the Mott cross section. In a model where the only excited states are infinitely narrow resonances,  $\mathcal{W}$  is given entirely by a sum of squares of transition form factors weighted by appropriate kinematic factors,

$$\mathcal{W}(\nu, \vec{q}^2) = \sum_{N=0}^{N_{\max}} \frac{1}{4E_0 E_N} |F_{0N}(\vec{q})|^2 \delta(E_N - E_0 - \nu), \quad (2)$$

where  $\vec{q} \equiv \vec{p}_i - \vec{p}_f$ , the form factor  $F_{0N}$  represents a transition from the ground state to a state characterized by the principal quantum number  $N$ , and the sum over states  $N$  goes up to the maximum  $N_{\max}$  allowed kinematically. Note that for fixed, positive  $Q^2 \equiv \vec{q}^2 - \nu^2$ ,  $N_{\max} = \infty$ .

The excitation form factors can be derived using the recurrence relations of the Hermite polynomials. One finds

$$F_{0N}(\vec{q}^2) = \frac{1}{\sqrt{N!}} i^N \left( \frac{|\vec{q}|}{\sqrt{2}\beta} \right)^N \exp(-\vec{q}^2/4\beta^2), \quad (3)$$

where  $\beta = \alpha^{1/4}$ . As a precursor to our discussion of duality, we note that it will be a necessary condition for duality that these form factors (or more generally those corresponding to some other model potential) can represent the pointlike free quark. It is in fact the case that  $\sum_{N=0}^{N_{\max}} |F_{0N}(\vec{q})|^2 \rightarrow 1$  as  $N_{\max} \rightarrow \infty$ , a relation which follows from the completeness of the confined wave functions. Incidentally, an examination of the convergence of this sum as a function of  $|\vec{q}|^2$  is sufficient to make the point that reproducing the behavior of a free quark requires more and more resonances as  $|\vec{q}|^2$  increases (details of this will be discussed in a forthcoming publication).

Scaling in the presence of confining final state interactions was previously investigated in Refs. [12–15], where similar conclusions are reached. This suggests that scaling may indeed be a trivial feature of a large class of simple quantum mechanical models. Some sense of how this can occur can be obtained by considering some of the properties of the relativistic oscillator model used in this paper. In particular, consider the properties of the square of the form factors. For a fixed principal quantum number  $N$ , the form factor has a maximum in  $|\vec{q}|$  at  $\vec{q}_N^2 = 2\beta^2 N$ . Using  $\nu_N = E_N - E_0$  and  $E_N = \sqrt{2\beta^2 N + E_0^2}$ , it can be shown that

$$\nu_N = \frac{Q_N^2}{2E_0}, \quad (4)$$

where  $Q_N^2 = \vec{q}_N^2 - \nu_N^2$ . So the position of the peak in the averaged structure function occurs at  $u_{Bj} = m/E_0$  where  $u_{Bj} = Q^2/2m\nu$  is a scaled Bjorken scaling variable  $u_{Bj} \equiv (M/m)x_{Bj}$  which takes into account that as the mass of the antiquark  $M_{\bar{Q}} \rightarrow \infty$ , the light constituent quark will carry only a fraction of order  $m/E_0$  of the hadron's infinite-momentum-frame momentum. Furthermore, for fixed  $\vec{q}$  the structure function falls off smoothly for energy transfers away from the peak value. The width of this peak as a function of energy transfer also becomes constant for large  $|\vec{q}|$ .

Now consider the integral of the structure function

$$\begin{aligned} \Sigma(\vec{q}^2) &= \int_0^\infty d\nu \mathcal{W}(\nu, \vec{q}^2) \\ &= \sum_{nlm} \frac{1}{4E_0 E_N} \langle \psi_{000} | \rho(-\vec{q}) | \psi_{nlm} \rangle \langle \psi_{nlm} | \rho(\vec{q}) | \psi_{000} \rangle, \end{aligned} \quad (5)$$

where  $N = 2(n-1) + l$  with  $n = 1, 2, 3, \dots$ , and where  $\rho(\vec{q}) = e^{i\vec{q} \cdot \vec{x}}$ . Since the form factor sum for a fixed  $\vec{q}$  peaks about  $E_{N_{\max}} = \sqrt{\vec{q}^2 + E_0^2}$ , we can substitute  $E_N \rightarrow E_{N_{\max}}$  and then sum over the complete set of final states to obtain

$$\Sigma(\vec{q}^2) \cong \frac{1}{4E_0 E_{N_{\max}}} \cong \frac{1}{4E_0 q} \quad (6)$$

for large momentum transfer. Therefore, if we define the scaling function as  $S \equiv |\vec{q}| \mathcal{W}$ , as will be done below, the area under the scaling function becomes constant at large momentum transfer.

Since the scaling function peaks at fixed  $u_{Bj}$ , smoothly falls about the peak, and has fixed width and constant area at large momentum transfer, the model scales. It is a common misconception that the presence of scaling implies that the final states must become plane waves. In fact, the argument above makes it clear that scaling occurs when the structure function becomes independent of the final states as in the closure approximation used here.

To see duality clearly both experimentally and theoretically, one needs to go beyond the Bjorken scaling variable  $x_{Bj}$  and the scaling function  $S_{Bj} = \nu \mathcal{W}$  that goes with it. This is because in deriving Bjorken's variable and scaling function, one not only assumes  $Q^2$  to be larger than any mass

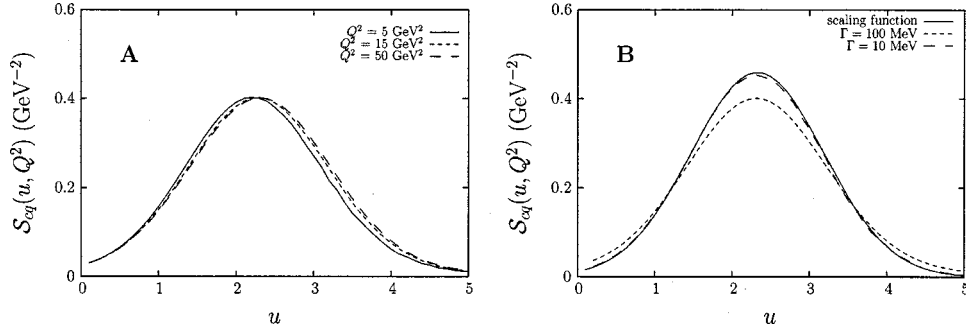


FIG. 1. The high energy scaling behavior of  $S_{cq}$  as a function of  $u$  for various values of  $Q^2$ . In panel A we have used  $\Gamma = 100$  MeV to give the impression of real resonances even though this large value distorts the scaling curve somewhat; for any width equal to or smaller than this, the distortion is rather innocuous, and for  $\Gamma \rightarrow 0$ , the structure function approaches the scaling function in Eq. (11), as shown in panel B.

scale in the problem, but also that high  $Q^2$  (pQCD) dynamics controls the interactions. However, duality has its onset in the region of low to moderate  $Q^2$ , and there masses and violations of asymptotic freedom do play a role. Bloom and Gilman used a new, *ad hoc* scaling variable  $\omega'$  [10] in an attempt to deal with this fact. In most contemporary data analyses, the Nachtmann variable [16,17] is used together with  $S_{Bj}$ . Nachtmann's variable contains the target mass as a scale, but neglects quark masses. For our model, the constituent quark mass (assumed to arise as a result of spontaneous chiral symmetry breaking) is vital at low energy, and a scaling variable that treats both target and quark masses is desirable. Such a variable was derived more than twenty years ago by Barbieri *et al.* [18] to take into account the masses of heavy quarks; we use it here given that after spontaneous chiral symmetry breaking the nearly massless light quarks have become massive constituent quarks, calling it  $x_{cq}$ :

$$x_{cq} = \frac{1}{2M} (\sqrt{\nu^2 + Q^2} - \nu) \left( 1 + \sqrt{1 + \frac{4m^2}{Q^2}} \right). \quad (7)$$

The scaling function associated with this variable is given by

$$S_{cq} \equiv |\vec{q}| \mathcal{W} = \sqrt{\nu^2 + Q^2} \mathcal{W}. \quad (8)$$

This scaling function and variable were derived for scalar quarks which are free, but have a momentum distribution. The derivation of a new scaling variable and function for bound quarks will be published elsewhere. Numerically, this scaling variable does not differ very much from the one in Eq. (7). Of course all versions of the scaling variable must converge to  $x_{Bj}$  and all versions of the scaling function must converge toward  $S_{Bj}$  for large enough  $Q^2$ . One can also easily verify that in the limit  $m \rightarrow 0$  one obtains the Nachtmann scaling variable from Eq. (7). In the following, we use the variable  $x_{cq}$  and the scaling function  $S_{cq}$ .

We are now ready to look at scaling and duality in our model. Since the target has mass  $M \rightarrow \infty$ , it is convenient to rescale the scaling variable  $x_{cq}$  by a factor  $M/m$ :

$$u \equiv \frac{M}{m} x_{cq}. \quad (9)$$

The variable  $u$  takes values from 0 to a maximal,  $Q^2$  dependent value, which can go to infinity. The high energy scaling behavior of the appropriately rescaled structure function  $S_{cq}$  is illustrated in Fig. 1.

The structure function has been evaluated using the phenomenologically reasonable parameters  $m = 0.33$  GeV and  $\alpha^{1/4} = 0.4$  GeV, though we remind the reader not to compare our results, which might resemble electron scattering from a  $B$  meson, to nucleon data. To display it in a visually meaningful manner, the energy-dependent  $\delta$ -function has been smoothed out by introducing an unphysical Breit-Wigner shape with an arbitrary but small width  $\Gamma$ , chosen for purposes of illustration

$$\delta(E_N - E_0 - \nu) \rightarrow \frac{\Gamma}{2\pi} \frac{f}{(E_N - E_0 - \nu)^2 + (\Gamma/2)^2}, \quad (10)$$

where the factor  $f = \pi / ((\pi/2) + \arctan[2(E_N - E_0)/\Gamma])$  ensures that the integral over the  $\delta$ -function is identical to that over the Breit-Wigner shape. The curves in Fig. 1 show that scaling sets in rather rapidly. The resonances show up as bumpy structures in the low  $Q^2$  region (which will be discussed in Sec. IV below), a trace of which is visible for the  $Q^2 = 5$  GeV<sup>2</sup> curve.

By taking the continuum limit for the energy and applying Stirling's formula, one can obtain an analytic expression for the scaling curve, valid in the scaling region, for the transition of the quark from the ground state to the sum of all excited states:

$$S_{cq}(u) = \frac{1}{4\sqrt{\pi}\beta E_0} \exp\left(-\frac{(E_0 - mu)^2}{\beta^2}\right). \quad (11)$$

Of course we still need to verify that this scaling curve as seen in Fig. 1 found by summing over hadrons is the same as the one which we would obtain from deep inelastic scattering off the quark, i.e., if we were to switch off the potential in the final state. In this case, the tower of hadronic states is replaced by the free quark continuum. Duality predicts that the results should be the same in the scaling limit, and by direct calculation we confirm this.



### III. MOMENTS OF STRUCTURE FUNCTIONS

Bloom-Gilman duality relates structure functions at low and high  $Q^2$  averaged over appropriate intervals of the hadronic mass  $W$ . As a quantitative measure of this feature of the data, one conventionally examines the  $Q^2$  dependence of moments of structure functions. The moments offer the cleanest connection with the operator product expansion of QCD, and provide a natural connection between duality in the high- and low- $Q^2$  regions. By considering the moments, we also remove artifacts introduced through the smoothing procedure described above for the structure function itself.

The moments of the structure function  $\mathcal{S}_{cq}(u, Q^2)$  are defined as

$$M_n(Q^2) = \int_0^{u_{\max}} du u^{n-2} \mathcal{S}_{cq}(u, Q^2), \quad (12)$$

where  $u_{\max}$  corresponds to the maximum value of  $u$  which is kinematically accessible at a given  $Q^2$ . Evaluating the moments of the structure function [Eq. (8)] explicitly one has (provided the kinematics allow us to access all excited states),

$$M_n(Q^2) = \left(\frac{r}{2m}\right)^{n-1} \sum_{N=0}^{\infty} (\sqrt{\nu_N^2 + Q^2} - \nu_N)^{n-1} \times \frac{E_0}{E_N} |F_{0N}(\sqrt{\nu_N^2 + Q^2})|^2, \quad (13)$$

where  $\nu_N = E_N - E_0$  and  $r = 1 + \sqrt{1 + 4m^2/Q^2}$ . The higher moments, i.e.  $n=4, 6, \dots$ , tend to emphasize the resonance region, as for fixed  $Q^2$ , the resonances are found at larger values of the scaling variable. The elastic contribution to the moments is

$$M_n^{\text{elastic}}(Q^2) = \left(\frac{r}{2m}\right)^{n-1} Q^{n-1} |F_{00}(Q^2)|^2 = u_0^{n-1} |F_{00}(Q^2)|^2, \quad (14)$$

where  $u_0(Q^2)$  is the position in  $u$  of the ground state. Note that  $M_n^{\text{elastic}}(Q^2)$  becomes independent of  $n$  in the limit  $Q^2 \rightarrow 0$ , approaching the value  $1/4E_0^2$  and that the inelastic contributions to the moments vanish for vanishing  $Q^2$ .

In Fig. 2 we show the  $n=2, 4, 6$  and  $8$  moments  $M_n$  as a function of  $Q^2$ . While all moments appear qualitatively similar, as would be expected the higher moments become independent of  $Q^2$  only at larger values of  $Q^2$ . The lowest moment  $M_2$  is very close to its asymptotic value at  $Q^2 = 5 \text{ GeV}^2$ , while the highest shown moment,  $M_8$ , is still slightly increasing at the highest  $Q^2$  shown. This is qualitatively consistent with the expectation from the operator product expansion discussed in Ref. [19], where it was argued that the effective expansion parameter in the twist expansion  $\sim n/Q^2$ , so that for higher moments  $n$  the higher twist terms survive to larger values of  $Q^2$ .

Unfortunately, these moments do not have such useful interpretations here as they do in deep inelastic scattering. For example, the analog of the Gross-Llewellyn Smith sum

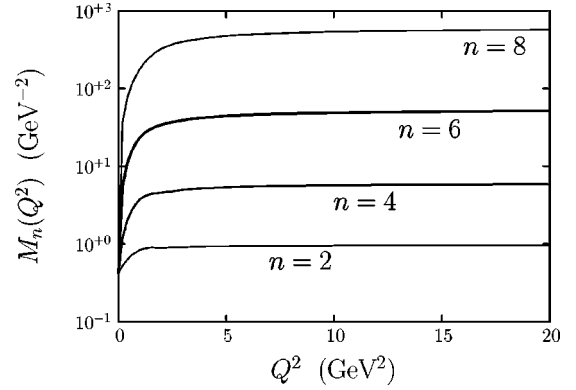


FIG. 2. Some moments  $M_n$  as a function of  $Q^2$ .

rule is not applicable here because the scalar current which couples to our quark is not conserved. Nonetheless, the moments in Fig. 2 do serve to demonstrate that scaling is a natural consequence of our model, and illustrate the relative onset of scaling for different moments.

### IV. ONSET OF SCALING AND BLOOM-GILMAN DUALITY

After studying the scaling behavior of the structure functions in our model at high  $Q^2$  and the moments over a range of 4-momentum transfers, we now study the structure functions at low  $Q^2$ . There, resonances are visibly dominant over a wide range in the scaling variable, not only in the large  $N_c$  limit, but also in Nature. Here, we consider a target where only one quark carries all the charge of the system, so there is no forced breakdown of duality at  $Q^2=0 \text{ GeV}^2$  of the type noted earlier for the neutron. Still, one cannot expect that the perturbative QCD result will describe even averaged hadronic observables well at very low  $Q^2$ : these are after all strong interactions.

If local duality holds, one might expect the resonance “spikes” to oscillate around the scaling curve and to average to it, once  $Q^2$  is large enough. (We remind the reader that while scaling in deep-inelastic electron scattering from the nucleon is known from experiment to set in by  $Q^2 \sim 2 \text{ GeV}^2$ , the target considered here corresponds to an infinitely heavy “meson” composed of scalar quarks interacting with a scalar current, so one should not expect numerically realistic results, only qualitative ones.) Figure 3 shows the onset of scaling for the structure function  $\mathcal{S}_{cq}$  as a function of  $u$ , as  $Q^2$  varies from 0.5 to 2  $\text{GeV}^2$ . As in Fig. 1, for each of the resonances (excluding the elastic peak) the energy  $\delta$ -function has been smoothed out using the Breit-Wigner method with a width  $\Gamma=100 \text{ MeV}$ . The elastic peak is displayed with the arbitrarily chosen width  $\Gamma=30 \text{ MeV}$ . With increasing  $Q^2$ , each of the resonances moves out toward higher  $u$ , as dictated by kinematics. At  $Q^2=0$ , the elastic peak is the only allowed state and contributes about 44% of the asymptotic value of  $M_2$ . It remains rather prominent for  $Q^2=0.5 \text{ GeV}^2$ , though most of  $M_2$  is by this point built up of excited states, and it becomes negligible for  $Q^2 \geq 2.0 \text{ GeV}^2$ . Remarkably, the curves at lower  $Q^2$  do tend to oscillate (at least qualitatively) around the scaling curve, as

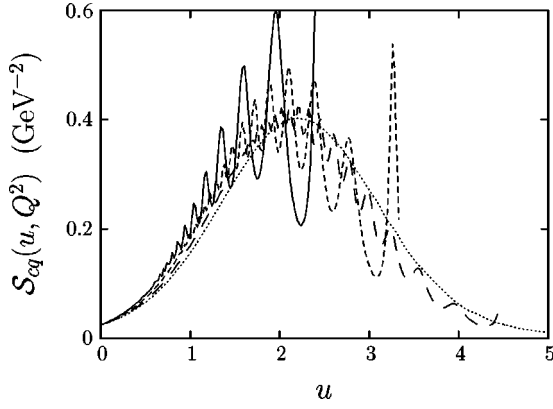


FIG. 3. Onset of scaling for the structure function  $S_{cq}$  as a function of  $u$  for  $Q^2=0.5$   $\text{GeV}^2$  (solid curve),  $Q^2=1$   $\text{GeV}^2$  (short-dashed curve),  $2$   $\text{GeV}^2$  (long-dashed curve) and  $5$   $\text{GeV}^2$  (dotted curve). Although off-scale, the elastic peak at  $Q^2=0.5$   $\text{GeV}^2$  accounts for about 22% of the area under the scaling curve.

is observed in proton data. Note that these curves are at fixed  $Q^2$ , but sweep over all  $\nu$ . In a typical low energy experiment,  $\nu$  will also be limited; in such circumstances these curves still apply, but they are cut off at the minimum value of  $u$  that is kinematically allowed. For another perspective on these curves, note that  $|\vec{q}|^2 = Q^2 + \nu^2$ , so for fixed  $Q^2$ , as  $\nu$  is increased so that more and more highly excited states are created, the struck quark is being hit harder and harder.

In contrast, the structure function  $S_{Bj}$  when plotted as a function of the scaled Bjorken variable  $u_{Bj}$  shows very poor duality between its low- and high- $Q^2$  behaviors, as seen in Fig. 4. One of the reasons for this failure is that  $x_{Bj}$  and  $S_{Bj}$  know nothing about the constituent quark mass, while low energy free quark scattering certainly does, so the corresponding pQCD cross section calculated neglecting the quark mass is simply wrong at low energy.

## V. DUALITY IN SEMILEPTONIC DECAYS OF HEAVY QUARKS

We have seen that low-energy (Bloom-Gilman) duality is displayed by our model in terms of the appropriate low-energy variable  $u$  and described some of the physics behind this duality (completeness of the bound state wave functions to expand a plane wave and an approximate closure based on the required expansion states being in a narrow band of  $\nu$  relative to those that are kinematically allowed). To obtain a deeper understanding of the physics behind low energy duality, it is instructive to compare and contrast duality in electron scattering with that in heavy quark decays. We will begin by carefully examining duality in heavy-light systems, where it is exact in the heavy quark limit even down to zero recoil, and where the mechanisms behind this exact duality are very clear.

Duality in heavy quark systems is easily understood intuitively. Consider a  $Q^*\bar{q}$  system where  $m_{Q^*} \gg \Lambda_{QCD}$ , and imagine that  $Q^*$  can decay to  $Q$  by emitting a scalar particle  $\phi$   $Q^* \rightarrow Q + \phi$ . (Note that in this case it is the heavy quark

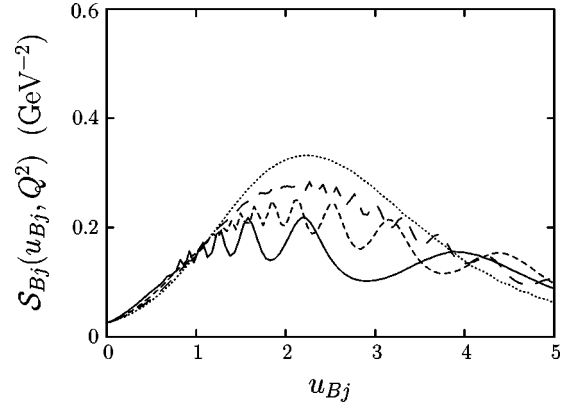


FIG. 4. Onset of scaling for the structure function  $S_{Bj}$  as a function of  $u_{Bj}$  for  $Q^2=0.5$   $\text{GeV}^2$  (solid curve),  $Q^2=1$   $\text{GeV}^2$  (short-dashed curve),  $2$   $\text{GeV}^2$  (long-dashed curve) and  $5$   $\text{GeV}^2$  (dotted curve).

that interacts with the current and not the light quark as in our model.) At the free quark level, the decay of  $Q^*$  at rest will produce the  $\phi$  with a single sharp kinetic energy  $T_{free}$  and corresponding  $Q$  recoil velocity  $\vec{v}$ . (We use the standard variables  $T_{free}$  and  $\vec{v}$ , but others, like the  $\phi$  recoil momentum, could be chosen.) In reality, since the heavy quarks are bound into mesons,  $\phi$  will (in the narrow resonance approximation) emerge from the decay at rest of the initial meson's ground state  $(Q^*\bar{q})_0$  with any of the sharp kinetic energies allowed by the processes  $(Q^*\bar{q})_0 \rightarrow (Q\bar{q})_n + \phi$  as determined by the strong interaction spectra of these two mesonic systems. Since in the heavy quark limit  $m_{(Q^*\bar{q})_n} - m_{(Q\bar{q})_n} \simeq m_{Q^*} - m_Q$ ,  $m_{(Q^*\bar{q})_n} \simeq m_{Q^*}$ , and  $m_{(Q\bar{q})_n} \simeq m_Q$ , the hadronic spectral lines are guaranteed to cluster around  $T_{free}$ , and to coincide with it exactly as  $m_Q \rightarrow \infty$ . Moreover, since  $m_{Q^*}, m_Q \gg \Lambda_{QCD}$ , one can show using an analog of the operator product expansion [20] that the strong interactions can be neglected in calculating the total decay rate (i.e., the heavy quarks  $Q^*$  and  $Q$  are so heavy that the decay proceeds as though the quarks were free). Thus the sum of the strengths of the spectral lines clustering around  $T_{free}$  is the free quark strength: there is perfect low energy duality as  $m_{Q^*}, m_Q \rightarrow \infty$ .

What is now especially interesting is to unravel this duality to understand how the required "conspiracy" of spectral line strengths arises physically. Because the heavy quark is so massive, if it would as a free particle recoil with a velocity  $\vec{v}$ , then this velocity would be changed only negligibly by the strong interaction since in the heavy quark limit it carries off a negligible kinetic energy, but a momentum much larger than  $\Lambda_{QCD}$ . In the rest frame of the recoiling meson, this configuration requires that the two constituents have a relative momentum  $\vec{q}$  which grows with  $\vec{v}$ . Thus the strong interaction dynamics is identical to that of our model in which the relative momentum  $\vec{q}$  is supplied by the scattered electron. Moreover, in this case, with duality exact at all energies, we can reconstruct exactly how it arises. What one sees is remarkably simple [21,22]. At low  $\vec{v}$  corresponding to low

$\vec{q}$ , only the ground state process  $(Q^*\bar{q})_0 \rightarrow (Q\bar{q})_0 + \phi$  occurs. Since the masses and matrix elements for the transitions  $(Q^*\bar{q})_0 \rightarrow (Q\bar{q})_0 + \phi$  and  $Q^* \rightarrow Q + \phi$  are identical (the elastic form factor goes identically to unity as  $\vec{q} \rightarrow 0$ ), the hadronic and quark spectral lines and strengths are also identical and duality is valid at  $|\vec{q}|^2 = 0$ ! Next consider duality at a different kinematic point (which one might reach by choosing a smaller  $\phi$  mass) where  $\vec{v}$  and therefore  $\vec{q}$  have increased. The elastic form factor will fall, so its spectral line (which is still found at exactly the new value of  $T_{free}$  in the heavy quark limit) will carry less strength. However, once  $\vec{q}$  differs from zero, excited states  $(Q\bar{q})_n$  can be created, and indeed are created with a strength that exactly compensates for the loss of elastic rate. These excited state spectral lines also coincide with  $T_{free}$  and duality is once again exact. Indeed, no matter how large  $|\vec{q}|^2$  becomes, all of the excited states produce spectral lines at  $T_{free}$  with strengths that sum to that of the free quark spectral line.

Heavy quark theory also allows one to go beyond the heavy quark limit to the case of quarks of finite mass. In this case, of course, one finds that duality violation occurs, but that it is formally suppressed by two powers of  $\Lambda_{QCD}/m_Q$  [20,23], with the spectral lines now clustered about  $T_{free}$  but not coinciding with it. A remarkable feature of this duality violation is that the spectral line strengths differ from those of the heavy quark limit in ways that tend to compensate for the duality-violating phase space effects from the spread of spectral lines around  $T_{free}$ . An additional source of duality violation is that some of the high mass resonances that are required for exact duality are kinematically forbidden, since for finite heavy quark masses  $m_{Q^*} - m_Q$  is finite.

From this discussion it is clear that the strong interaction dynamics of heavy-light decays is the same as that of scattering a probe off of the  $Q$  of a  $Q\bar{q}$  system [24]: what is relevant is that the system must in each case respond to a relative momentum kick  $\vec{q}$ . Needless to say, one must still carefully organize the kinematics to expose duality: in a decay to a fixed mass  $\phi$  only a single magnitude  $|\vec{q}|^2$  is produced at the quark level, while in electron scattering a large range of  $|\vec{q}|^2$  and  $\nu$  is produced by a given electron beam.

Given these connections, it is relevant to note that in addition to the obvious conceptual relevance of heavy-light systems, model studies indicate that in these systems heavy quark behavior continues to hold qualitatively even for  $m_Q \sim m$ . These models are, as one might expect, similar to ours, which displays the same clustering of spectral lines, the same tendency for excited state spectral lines to compensate for the fall with  $|\vec{q}|^2$  of lighter states, and the same sources of duality violation such as kinematically forbidden states and mismatches between the mass of the recoiling hadrons and the struck quark. We believe that these elements of the dynamics are clearly in operation, and that we have understood through our model that the qualitative applicability of duality for real systems should indeed extend all of the way down to zero recoil as seen in Nature.

## VI. SUMMARY AND OUTLOOK

We have presented a simple, quantum-mechanical model in which we were able to reproduce the features of Bloom-Gilman duality qualitatively. The model assumptions we made are the most basic ones possible: we assumed relativistic, confined, valence scalar quarks and treated the hadrons in the infinitely narrow resonance approximation. To simplify the situation further, we did not consider a three quark “nucleon” target, but a target made up by an infinitely heavy antiquark and a light quark. The present work does not attempt to quantitatively describe any data, but to give qualitative insight into the physics of duality.

Our work complements previous work on duality, where the experimental data were analyzed in terms of the operator product expansion [19,25]. There, it was observed that at moderate  $Q^2$ , the higher twist corrections to the lower moments of the structure function are small. The higher twist corrections arise due to initial and final state interactions of the quarks and gluons. Hence the average value of the structure function at moderate  $Q^2$  is not very different from its value in the scaling region. While true, this statement is merely a rephrasing in the language of the operator product expansion of the experimentally observed fact that the resonance curve averages to the scaling curve. However, the operator product expansion does not explain why a certain correction is small or why there are cancellations: the expansion coefficients which determine this behavior are not predicted. The confirmation of these coefficients will eventually come from a numerical solution of QCD on the lattice, but an *understanding* of the physical mechanism that leads to the small values of the expansion coefficient will almost certainly only be found in the framework of a model like ours.

For example, one clear lesson from our study of duality is that the commonly made sharp distinction between the “resonance region,” corresponding to an invariant mass  $W < 2$  GeV for scattering from a proton, and the deep inelastic region, where  $W > 2$  GeV, is completely artificial.

Finally, we remind the reader that our model, with all the charge on a single quark, with scalar currents, and with no spin degrees of freedom, leaves much to be done in model-building. The next step is to use more realistic currents. While making the calculations more complicated, coupling to the conserved quark current will allow one to study the  $Q^2$  evolution of the Gross-Llewellyn Smith and momentum sum rules. To use a spin- $\frac{1}{2}$  target will also be a useful step forward, but it may require foregoing the great advantages of the analytic solutions of the Klein-Gordon equation. As we have emphasized, the local duality seen here *cannot* be expected for more complicated targets and processes, and pursuing this issue is also clearly very important [9]. Here we have taken a first small step which nevertheless has been enough to strongly suggest that for these more realistic models and more general processes there will be a generalization of local averaging—a theoretically well-defined procedure for integrating over regions of  $x_{c\bar{q}}$ —which will also display low energy duality. If so, we will not only have understood quark-hadron duality, we will also have opened the door to



extending studies of a variety of structure functions into previously unreachable kinematic regimes.

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