Interference effects in the decay $\phi \to \pi^0 \pi^0 \gamma$ and the coupling constant $g_{\phi\sigma\gamma}$

A. Gokalp* and O. Yilmaz[†]

Physics Department, Middle East Technical University, 06531 Ankara, Turkey
(Received 7 May 2001; revised manuscript received 7 June 2001; published 10 August 2001)

We study the radiative decay $\phi \to \pi^0 \pi^0 \gamma$ within the framework of a phenomenological approach in which the contributions of σ mesons, ρ mesons, and f_0 mesons are considered. We analyze the interference effects between different contributions and, utilizing the experimental branching ratio for this decay, we estimate the coupling constant $g_{\phi\sigma\gamma}$.

I. INTRODUCTION

The radiative decays of ϕ mesons are valuable sources of information in low-energy hadron physics in areas such as the quark model, SU(3) symmetry, and the Okubo-Zweig-Iizuka (OZI) rule. In particular, radiative ϕ meson decays $\phi \to \pi^0 \pi^0 \gamma$ and $\phi \to \pi^0 \eta \gamma$ can provide insight into the structure and properties of low-mass scalar resonances, since these decays primarily proceed through processes involving scalar resonances such as $\phi \to f_0(980)\gamma$ and $\phi \to a_0(980)\gamma$, with subsequent decays into $\pi^0 \pi^0 \gamma$ and $\pi^0 \eta \gamma$ [1,2]. On the experimental side, the Novosibirsk SND [3] and CMD-2 [4] Collaborations recently reported very accurate results on these decays with the following branching ratios: $\mathrm{BR}(\phi \to \pi^0 \pi^0 \gamma) = (1.221 \pm 0.098 \pm 0.061) \times 10^{-4}$, $\mathrm{BR}(\phi \to \pi^0 \eta \gamma) = (0.88 \pm 0.14 \pm 0.09) \times 10^{-4}$ [4], and $\mathrm{BR}(\phi \to \pi^0 \pi^0 \gamma) = (0.92 \pm 0.08 \pm 0.06) \times 10^{-4}$, $\mathrm{BR}(\phi \to \pi^0 \eta \gamma) = (0.9 \pm 0.24 \pm 0.1) \times 10^{-4}$ [4], where the first error is statistical and the second one is systematic.

The low-mass scalar mesons with vacuum quantum numbers have a fundamental importance in understanding low-energy QCD phenomenology and the symmetry breaking mechanisms in QCD. In addition to $f_0(980)$ and $a_0(980)$, the existence of the scalar-isoscalar σ meson as a broad $\pi\pi$ resonance, which has long been controversial, seems to be established. An increasing number of theoretical and experimental analyses find a σ-pole position near $(500 - i250)$ MeV [5,6]. Direct experimental evidence seems to emerge from the $D^+ \to \sigma\pi^+ \to 3\pi$ decay channel observed by the Fermilab (E791) Collaboration, where the σ meson is seen as a clear dominant peak with $M_\sigma = 478$ MeV and $\Gamma_\sigma = 324$ MeV [7]. On the other hand, the nature and the quark substructure of these scalar mesons have not been established yet, whether they are conventional $q\bar{q}$ states [8], $\pi\bar{\pi}$ in the case of σ [9] and $K\bar{K}$ in the case of f_0 and a_0 [10] molecules, or exotic multiquark $q^2\bar{q}^2$ states [11,12] has been a subject of debate. It has been noted that the radiative decay of ϕ mesons to scalar mesons can differentiate among various models of their structure [1].

The radiative decay process $\phi \to \pi^0 \pi^0 \gamma$ among other radiative decay processes of the type $V^0 \to P^0 P^0 \gamma$ where V and

P belong to the lowest multiplets of vector (V) and pseudoscalar (P) mesons was studied by Fajfer and Oakes [13] using a low-energy effective Lagrangian approach with gauged Wess-Zumino terms. They considered the contributions in which the virtual vector meson states dominate, and obtained the branching ratio for this decay as $\mathrm{BR}(\phi \to \pi^0 \pi^0 \gamma) = 3.46 \times 10^{-5}$. The contribution of intermediate vector mesons to the decays $V^0 \to P^0 P^0 \gamma$ was later considered by Bramon *et al.* [14] using standard Lagrangians obeying SU(3) symmetry, and they obtained the result $\mathrm{BR}(\phi \to \pi^0 \pi^0 \gamma) = 1.2 \times 10^{-5}$ for the branching ratio of the $\phi \to \pi^0 \pi^0 \gamma$ decay. Bramon *et al.* [15] later considered these decays within the framework of chiral effective Lagrangians, and using chiral perturbation theory they calculated the branching ratios for various decays of the type $V^0 \to P^0 P^0 \gamma$ at the one-loop level. They showed that the one-loop contributions are finite and to this order no counterterms are required. In this approach the decay $\phi \to \pi^0 \pi^0 \gamma$ proceeds through the charged-kaon loop and they obtained the contribution of charged-kaon loops to this decay as $\Gamma(\phi \to \pi^0 \pi^0 \gamma) = 224$ eV, which is much larger than the intermediate vector meson state (VMD) contribution due to the OZI rule. Considering the amplitudes of both VMD and kaon loop contributions, they obtained for the decay rate the value $\Gamma(\phi \to \pi^0 \pi^0 \gamma) = 269$ eV; moreover, they noted that OZI allowed kaon loops dominate both the photonic spectrum and the decay rate. Radiative decays of ϕ mesons were also investigated by Marco *et al.* [16] employing the techniques of chiral unitary theory developed earlier by Oller [17]. Using a chiral unitary approach, they included the final state interactions of two pions by summing the kaon loops through the Bethe-Salpeter equation. They obtained the branching ratio for the decay $\phi \to \pi^0 \pi^0 \gamma$ as $\mathrm{BR}(\phi \to \pi^0 \pi^0 \gamma) = 0.8 \times 10^{-4}$. Moreover, they obtained the photon distribution as a function of the invariant mass of the $\pi^0 \pi^0$ system and compared it with SND data. They noted that the shape of the experimental spectrum is relatively well reproduced with the $\phi \to f_0 \gamma$ contribution since the f_0 meson is the important scalar resonance appearing in the $K^+ K^- \to \pi^0 \pi^0$ amplitude. However, they also noted an appreciable strength for a possible $\phi \to \sigma\gamma$ contribution in the spectrum.

In this work, we follow a phenomenological approach and study the radiative $\phi \to \pi^0 \pi^0 \gamma$ decay by considering the ρ-pole vector meson dominance amplitude as well as scalar σ-pole and f_0-pole amplitudes. By employing the experi-

---

*Electronic address: agokalp@metu.edu.tr
[†]Electronic address: oyilmaz@metu.edu.tr
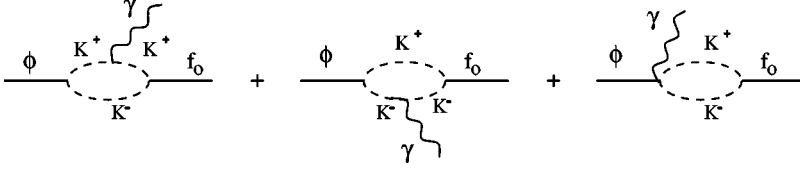
FIG. 1. Diagrams for the decay $\phi \to f_0 \gamma$.

mental value for this decay rate, we estimate the coupling constant $g_{\phi\sigma\gamma}$. This coupling constant is an important physical input for studies of ϕ meson photoproduction experiments on nucleons near threshold [18].

II. FORMALISM

In our analysis of the radiative decay $\phi \to \pi^0 \pi^0 \gamma$, we proceed within a phenomenological framework and we do not make any assumption about the structure of the f_0 meson. We note that the ϕ and f_0 mesons both couple strongly to the $K^+ K^-$ system, and therefore in our phenomenological approach we describe the ϕKK and $f_0 KK$ vertices by the effective Lagrangians

$$\mathcal{L}_{\phi KK}^{int.} = -g_{\phi KK} \phi^\mu (K^+ \partial_\mu K^- - K^- \partial_\mu K^+) \quad (1)$$

and

$$\mathcal{L}_{f_0 KK}^{int.} = g_{f_0 KK} M_{f_0} K^+ K^- f_0, \quad (2)$$

respectively, which also serve to define the coupling constants $g_{\phi KK}$ and $g_{f_0 KK}$. The effective Lagrangian for the ϕKK vertex is the one that results from the standard chiral Lagrangians in lowest order of chiral perturbation theory [19]. The decay rate for the $\phi \to K^+ K^-$ decay resulting from this Lagrangian is

$$\Gamma(\phi \to K^+ K^-) = \frac{g_{\phi KK}^2}{48\pi} M_\phi \left[1 - \left(\frac{2M_K}{M_\phi} \right)^2 \right]^{3/2}. \quad (3)$$

Utilizing the experimental value for the branching ratio BR$(\phi \to K^+ K^-) = 0.492 \pm 0.007$ for the decay $\phi \to K^+ K^-$ [20], we determine the coupling constant $g_{\phi KK}$ as $g_{\phi KK} = 4.59 \pm 0.05$. Furthermore, as a result of the strong coupling of both ϕ and f_0 to $K^+ K^-$, independent of the nature and dynamical structure of f_0, there is an amplitude for the decay $\phi \to f_0 \gamma$ to proceed through the charged kaon loop which we show diagrammatically in Fig. 1, where the last diagram assures gauge invariance [1,21]. The amplitude of the radiative decay $\phi \to f_0 \gamma$ that follows from the diagrams in Fig. 1 is

$$\mathcal{M}(\phi \to f_0 \gamma) = u^\mu \epsilon^\nu (p_\nu k_\mu - g_{\mu\nu} p \cdot k)$$
$$\times \frac{e g_{\phi KK} (g_{f_0 KK} M_{f_0})}{2\pi^2 M_K^2} I(a,b) \quad (4)$$

where (u, p) and (ϵ, k) are the polarizations and four-momenta of the ϕ meson and the photon, respectively, and $a = M_\phi^2 / M_K^2$, $b = M_{f_0}^2 / M_K^2$. The function $I(a,b)$ has been calculated in different contexts [2,17,22], and it is given as

$$I(a,b) = \frac{1}{2(a-b)} - \frac{2}{(a-b)^2} \left[f\left(\frac{1}{b} \right) - f\left(\frac{1}{a} \right) \right]$$
$$+ \frac{a}{(a-b)^2} \left[g\left(\frac{1}{b} \right) - g\left(\frac{1}{a} \right) \right] \quad (5)$$

where

$$f(x) = \begin{cases} -[\arcsin(1/2\sqrt{x})]^2, & x > \frac{1}{4} \\ \frac{1}{4}[\ln(\eta_+ / \eta_-) - i\pi]^2, & x < \frac{1}{4}, \end{cases} \quad (6)$$

$$g(x) = \begin{cases} (4x-1)^{1/2} \arcsin(1/2\sqrt{x}), & x > \frac{1}{4} \\ \frac{1}{2}(1-4x)^{1/2}[\ln(\eta_+ / \eta_-) - i\pi], & x < \frac{1}{4}, \end{cases}$$

$$\eta_\pm = \frac{1}{2x}[1 \pm (1-4x)^{1/2}].$$

The decay width for the radiative decay $\phi \to f_0 \gamma$ can then be obtained from the amplitude $\mathcal{M}(\phi \to f_0 \gamma)$ as

$$\Gamma(\phi \to f_0 \gamma) = \frac{\alpha}{6(2\pi)^4} \frac{M_\phi^2 - M_{f_0}^2}{M_\phi^3} g_{\phi KK}^2 (g_{f_0 KK} M_{f_0})^2$$
$$\times |(a-b) I(a,b)|^2 \quad (7)$$

from which, by using the experimental value BR$(\phi \to f_0 \gamma) = (3.4 \pm 0.4) \times 10^{-4}$ [20], we obtain the coupling constant $g_{f_0 KK}$ as $g_{f_0 KK} = 4.13 \pm 1.42$.

In our phenomenological approach, we assume that the radiative decay $\phi \to \pi^0 \pi^0 \gamma$ proceeds through the reactions $\phi \to \rho^0 \pi^0 \to \pi^0 \pi^0 \gamma$, $\phi \to \sigma \gamma \to \pi^0 \pi^0 \gamma$, and $\phi \to f_0 \gamma \to \pi^0 \pi^0 \gamma$. We write the total amplitude as the sum of the amplitudes of each reaction and this way we take the interference between different reactions into account. In order to describe the reaction $\phi \to f_0 \gamma \to \pi^0 \pi^0 \gamma$ we again note that both ϕ and f_0 couple strongly to $K^+ K^-$; furthermore, f_0 also couples strongly to $\pi^0 \pi^0$. Therefore, we assume that this reaction proceeds by a two-step mechanism with f_0 coupling to ϕ with intermediate $K\bar{K}$ states. We depict the processes contributing to the $\phi \to \pi^0 \pi^0 \gamma$ decay amplitude diagrammatically in Fig. 2, where the last diagram in Fig. 2(c) results from the minimal coupling for gauge invariance.

We describe the $\phi\sigma\gamma$ vertex by the effective Lagrangian
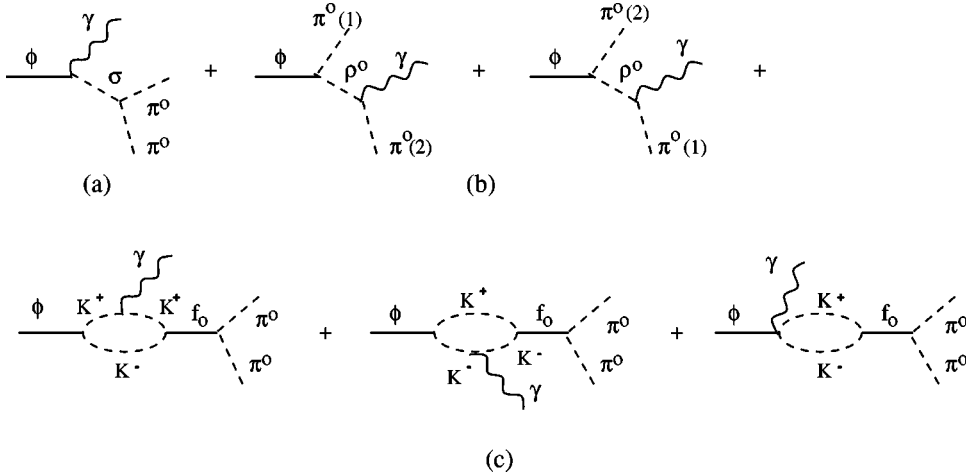
FIG. 2. Diagrams for the decay $\phi \to \pi^0 \pi^0 \gamma$.

$$\mathcal{L}_{\phi\sigma\gamma}^{int.} = \frac{e}{M_\phi} g_{\phi\sigma\gamma}[\partial^\alpha \phi^\beta \partial_\alpha A_\beta - \partial^\alpha \phi^\beta \partial_\beta A_\alpha]\sigma, \qquad (8)$$

which also defines the coupling constant $g_{\phi\sigma\gamma}$. For the $\sigma\pi\pi$ vertex we use the effective Lagrangian

$$\mathcal{L}_{\sigma\pi\pi}^{int} = \frac{1}{2} g_{\sigma\pi\pi} M_\sigma \vec{\pi} \cdot \vec{\pi} \sigma. \qquad (9)$$

The decay width of the σ meson that follows from this Lagrangian is given as

$$\Gamma_\sigma \equiv \Gamma(\sigma \to \pi\pi) = \frac{g_{\sigma\pi\pi}^2}{4\pi} \frac{3M_\sigma}{8} \left[1 - \left(\frac{2M_\pi}{M_\sigma}\right)^2\right]^{1/2}. \quad (10)$$

In our calculation, for given values of M_σ and Γ_σ we determine the coupling constant $g_{\sigma\pi\pi}$ by using the expression for Γ_σ. The $\phi\rho\pi$ vertex in Fig. 2(b) is conventionally described by the effective Lagrangian

$$\mathcal{L}_{\phi\rho\pi}^{int} = g_{\phi\rho\pi} \epsilon^{\mu\nu\alpha\beta} \partial_\mu \phi_\nu \partial_\alpha \rho_\beta \pi. \qquad (11)$$

The coupling constant $g_{\phi\rho\pi}$ was determined by Achasov and Gubin [23] as $g_{\phi\rho\pi} = 0.811 \pm 0.081$ GeV^{-1} using data on the decay $\phi \to \rho\pi \to \pi^+ \pi^- \pi^0$ [20]. The $\rho\pi\gamma$ vertex in Fig. 2(b) is described by the effective Lagrangian

$$\mathcal{L}_{\rho\pi\gamma}^{int} = \frac{e}{M_\rho} g_{\rho\pi\gamma} \epsilon^{\mu\nu\alpha\beta} \partial_\mu \rho_\nu \partial_\alpha A_\beta \pi. \qquad (12)$$

The coupling constant $g_{\rho\pi\gamma}$ is then obtained from the experimental partial width of the radiative decay $\rho \to \pi^0 \gamma$ [20] as $g_{\rho\pi\gamma} = 0.69 \pm 0.35$. Finally, we describe the $f_0 \pi^0 \pi^0$ vertex by the effective Lagrangian

$$\mathcal{L}_{f_0\pi\pi}^{int} = \frac{1}{2} g_{f_0\pi\pi} M_{f_0} \vec{\pi} \cdot \vec{\pi} f_0 \qquad (13)$$

and the decay width Γ_f for the decay $f_0 \to \pi\pi$ that results from this effective Lagrangian is given by a similar expression as for Γ_σ. For a given value of Γ_f we use this expression to determine the coupling constant $g_{f_0\pi\pi}$. Furthermore, in our calculation of invariant amplitudes we make the re-

placement $M \to M - \frac{1}{2} i\Gamma$ in f_0, σ, and ρ propagators and use the experimental values $\Gamma_\rho = 150.2 \pm 0.8$ MeV [20] for the ρ meson and $\Gamma_\sigma = 324 \pm 21$ MeV [7] for the σ meson. However, the mass $M_{f_0} = 980$ MeV of the f_0 meson is very close to the $K^+ K^-$ threshold, and this induces a strong energy dependence in the width Γ_{f_0} of the f_0 meson. In order to take that into account different expressions for Γ_{f_0} can be used. A first possibility is to use an energy dependent width for f_0,

$$\Gamma_{f_0}(q^2) = \Gamma_{\pi\pi}^{f_0}(q^2) \ \theta(\sqrt{q^2} - 2M_\pi) + \Gamma_{K\bar{K}}^{f_0}(q^2)$$
$$\times \theta(\sqrt{q^2} - 2M_K), \qquad (14)$$

where q^2 is the four-momentum squared of the virtual f_0 meson. In this expression the width $\Gamma_{\pi\pi}^{f_0}(q^2)$ is given as

$$\Gamma_{\pi\pi}^{f_0}(q^2) = \Gamma_{\pi\pi}^{f_0} \frac{M_{f_0}^2}{q^2} \sqrt{\frac{q^2 - 4M_\pi^2}{M_{f_0}^2 - 4M_\pi^2}} \qquad (15)$$

and $\Gamma_{K\bar{K}}^{f_0}(q^2)$ by a similar expression. We use the experimental value $\Gamma_{\pi\pi}^{f_0} = 40 - 100$ MeV [20] and we calculate $\Gamma_{K\bar{K}}^{f_0}$ from the effective Lagrangian given in Eq. (2). Another and widely accepted possibility is known from the work of Flatté [24]. This amounts to extending the expression for $\Gamma_{K\bar{K}}^{f_0}(q^2)$ below the $K\bar{K}$ threshold where $\sqrt{q^2 - 4M_K^2}$ is replaced by $i\sqrt{4M_K^2 - q^2}$ and thus $\Gamma_{K\bar{K}}^{f_0}(q^2)$ becomes purely imaginary. In our work, we consider both possibilities.

In terms of the invariant amplitude $\mathcal{M}(E_\gamma, E_1) = \mathcal{M}_a + \mathcal{M}_b + \mathcal{M}_c$ where $\mathcal{M}_a$, $\mathcal{M}_b$, and $\mathcal{M}_c$ are the invariant amplitudes resulting from the diagrams (a), (b), and (c) in Fig. 2, respectively, the differential decay probability for $\phi \to \pi^0 \pi^0 \gamma$ for an unpolarized ϕ meson at rest is then given as

$$\frac{d\Gamma}{dE_\gamma dE_1} = \frac{1}{(2\pi)^3} \frac{1}{8M_\phi} |\mathcal{M}|^2, \qquad (16)$$

where E_γ and E_1 are the photon and pion energies, respectively. We perform an average over the spin states of the ϕ

meson and a sum over the polarization states of the photon. The decay width $\Gamma(\phi \to \pi^0 \pi^0 \gamma)$ is then obtained by integration:

$$\Gamma = \frac{1}{2} \int_{E_{\gamma,min}}^{E_{\gamma,max}} dE_\gamma \int_{E_{1,min}}^{E_{1,max}} dE_1 \frac{d\Gamma}{dE_\gamma dE_1} \qquad (17)$$

where now the factor $(\frac{1}{2})$ is included because of the $\pi^0 \pi^0$ in the final state. The minimum photon energy is $E_{\gamma,min} = 0$ and the maximum photon energy is given as $E_{\gamma,max} = (M_\phi^2 - 4M_\pi^2)/2M_\phi = 474$ MeV. The maximum and minimum values for the pion energy E_1 are given by

$$\frac{1}{2(2E_\gamma M_\phi - M_\phi^2)} \{ -2E_\gamma^2 M_\phi + 3E_\gamma M_\phi^2 - M_\phi^3$$

$$\pm E_\gamma \sqrt{(-2E_\gamma M_\phi + M_\phi^2)(-2E_\gamma M_\phi + M_\phi^2 - 4M_\pi^2)} \}.$$

III. RESULTS AND DISCUSSION

The experimental full width of f_0 is $\Gamma_{f_0} = 40 - 100$ MeV [20]. Since the coupling constant $g_{\phi\sigma\gamma}$ depends on the value of the width of f_0, in order to estimate its effect on the coupling constant we take this width as $\Gamma_{f_0} = 70 \pm 30$ MeV and $M_{f_0} = 980$ MeV in our calculation. If we use the form for $\Gamma_{K\bar{K}}^{f_0}(q^2)$ that was proposed by Flatté [24], we are not able to reproduce the form of the invariant mass spectrum for the $\phi \to \pi^0 \pi^0 \gamma$ decay. In this case the enhancement in the invariant mass spectrum appears in the central part rather than around the f_0 pole. A similar problem was also encountered by Bramon *et al.* [25] in their study of the role of $a_0(980)$ exchange in the $\phi \to \pi^0 \eta \gamma$ decay. Therefore, in the analysis presented below, for $\Gamma_{f_0}(q^2)$ we use the form given in Eq. (14). Through the decay rate that results from the Lagrangian given in Eq. (13) describing the $f_0 \pi \pi$ vertex we obtain the coupling constant $g_{f_0 \pi \pi}$ as $g_{f_0 \pi \pi} = 1.58$. In order to determine the coupling constant $g_{\phi\sigma\gamma}$, we follow a procedure that we used in our previous work where we estimated the coupling constants $g_{\rho\sigma\gamma}$ [26] and $g_{\omega\sigma\gamma}$ [27]. We use the experimental value of the branching ratio for the radiative decay $\phi \to \pi^0 \pi^0 \gamma$ [3] in our calculation for this decay rate, and in this way we arrive at a quadratic equation for the coupling constant $g_{\phi\sigma\gamma}$, the coefficient of the quadratic term resulting from the σ meson contribution shown in Fig. 2(a), and the coefficient of the linear term from the interference of the σ meson amplitude with the ρ meson and f_0 meson amplitudes shown in Figs. 2(b) and 2(c), respectively. Therefore our analysis produces for a set of values of the σ meson parameters M_σ and Γ_σ two values for the coupling constant $g_{\phi\sigma\gamma}$. We choose for the σ meson parameters the values $M_\sigma = 478 \pm 17$ MeV and $\Gamma_\sigma = 324 \pm 21$ MeV as suggested by the recent Fermilab (E791) experiment [7], resulting for the coupling constant $g_{\sigma\pi\pi}$ in the value $g_{\sigma\pi\pi} = 5.25 \pm 0.32$ using the expression for the decay rate $\Gamma(\sigma \to \pi\pi)$ given in Eq. (10). In this way we obtain for the coupling constant $g_{\phi\sigma\gamma}$ the values $g_{\phi\sigma\gamma} = 0.064 \pm 0.008$ and $g_{\phi\sigma\gamma} = 0.025 \pm 0.009$. We
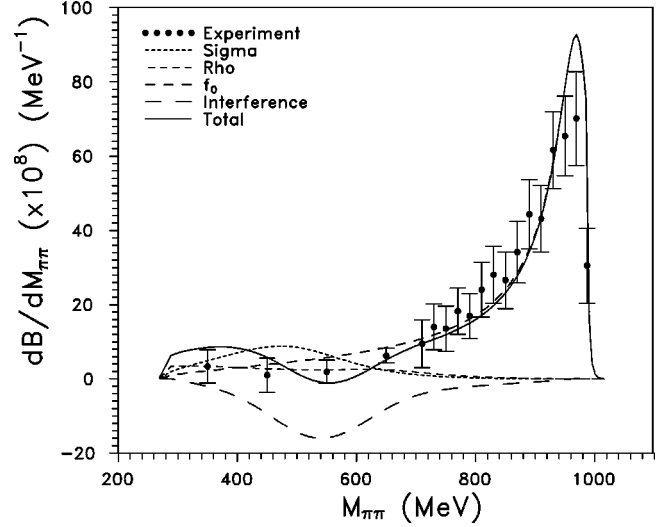

FIG. 3. The $\pi^0 \pi^0$ invariant mass spectrum for the decay $\phi \to \pi^0 \pi^0 \gamma$ for $g_{\phi\sigma\gamma} = 0.064$. The contributions of different terms are indicated.

then study the invariant mass distribution for the reaction $\phi \to \pi^0 \pi^0 \gamma$. In Fig. 3 we plot the distribution $d\text{BR}/dM_{\pi\pi} = (M_{\pi\pi}/M_\phi) d\text{BR}/dE_\gamma$ for the radiative decay $\phi \to \pi^0 \pi^0 \gamma$ in our phenomenological approach choosing the coupling constant $g_{\phi\sigma\gamma} = 0.064$ and in Fig. 4 we show the same distribution for $g_{\phi\sigma\gamma} = 0.025$ as a function of the invariant mass $M_{\pi\pi}$ of the $\pi^0 \pi^0$ system. In these figures we also indicate the contributions coming from the different reactions $\phi \to \sigma\gamma \to \pi^0 \pi^0 \gamma$, $\phi \to \rho^0 \pi^0 \to \pi^0 \pi^0 \gamma$, and $\phi \to f_0 \gamma \to \pi^0 \pi^0 \gamma$, as well as the contribution of the total amplitude which includes the interference terms as well. The difference between the two total contribution curves is mainly in the interference region $M_{\pi\pi} < 0.7$ GeV above which the f_0 amplitude dominates the spectrum. From the analysis of the spectrum obtained with the coupling constants $g_{\phi\sigma\gamma} = 0.064$
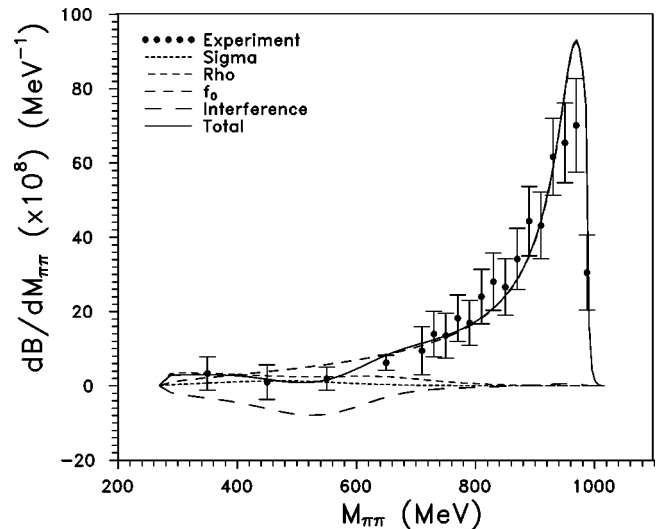

FIG. 4. The $\pi^0 \pi^0$ invariant mass spectrum for the decay $\phi \to \pi^0 \pi^0 \gamma$ for $g_{\phi\sigma\gamma} = 0.025$. The contributions of different terms are indicated.
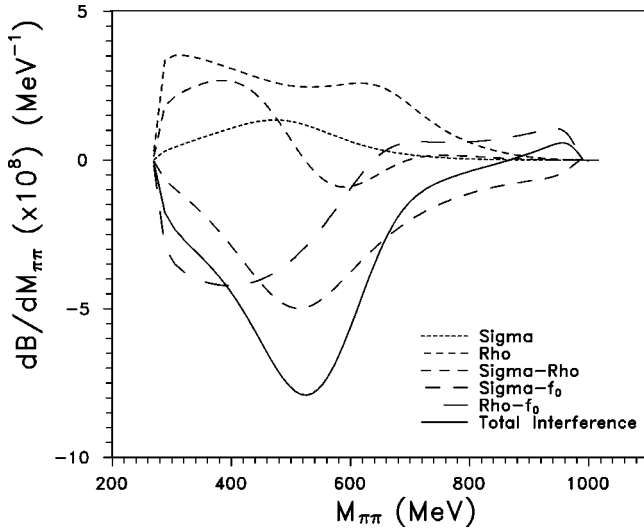
FIG. 5. The contributions of ρ- and σ- amplitudes and interference terms to the invariant mass spectrum of the decay $\phi \to \pi^0 \pi^0 \gamma$ for $g_{\phi\sigma\gamma} = 0.025$.

and $g_{\phi\sigma\gamma} = 0.025$ in Figs. 3 and 4, respectively, we may decide in favor of the latter value of the coupling constant $g_{\phi\sigma\gamma}$ and we may state that the experimental data within the framework of our phenomenological approach suggest that $g_{\phi\sigma\gamma} = 0.025 \pm 0.009$. In Fig. 5, we show the contributions of different amplitudes and the contributions of the interference terms in the interference region $M_{\pi\pi} < 0.7$ GeV for $g_{\phi\sigma\gamma} = 0.025$.

On the other hand, the photoproduction of ρ^0 mesons on proton targets near threshold can be described at low momentum transfers by the single one-meson exchange model. Friman and Soyeur [28] showed that in this picture the ρ^0 meson photoproduction cross section on protons is given mainly by σ-exchange. They calculated the $\rho\sigma\gamma$ vertex assuming vector meson dominance of the electromagnetic current, and their result when described using an effective Lagrangian for the $\rho\sigma\gamma$ vertex gives the value $g_{\rho\sigma\gamma} \approx 2.71$ for this coupling constant. Later, Titov et al. [18] in their study of the structure of the ϕ meson photoproduction amplitude based on the one-meson exchange and Pomeron-exchange mechanism used the coupling constant $g_{\phi\sigma\gamma}$ which they calculated from the above value of $g_{\rho\sigma\gamma}$ by invoking unitary symmetry arguments as $g_{\phi\sigma\gamma} \approx 0.047$. Our result for this coupling constant $g_{\phi\sigma\gamma} = 0.025 \pm 0.009$ is not in total agreement with this value. We note that in order to derive their result Titov et al. assumed that σ, f_0, and a_0 are members of a unitary nonet. However, assignments of scalar states into various unitary nonets are not without problems and other possible assignments than used by Titov et al. have also been suggested [29]. In our work we do not make any assumption about the quark substructure of σ and f_0 mesons and describe their couplings in a phenomenological framework.

Achasov and Gubin [23] performed a fit to the experimental data and they obtained the following values: the branching ratio with interference $\text{BR}(\phi \to (f_0\gamma + \pi^0\rho^0) \to \pi^0\pi^0\gamma) = 1.26 \times 10^{-4}$, the branching ratio of the signal $\text{BR}(\phi \to f_0\gamma \to \pi^0\pi^0\gamma) = 1.01 \times 10^{-4}$, the branching ratio of the background $\text{BR}(\phi \to \rho^0\pi^0 \to \pi^0\pi^0\gamma) = 0.18 \times 10^{-4}$. If we use the coupling constant $g_{\phi\sigma\gamma} = 0.025$ and $M_\sigma = 478$ MeV, $\Gamma_\sigma = 324$ MeV, we obtain for the branching ratios for different contributing reactions to the radiative decay $\phi \to \pi^0\pi^0\gamma$ the values $\text{BR}(\phi \to f_0\gamma \to \pi^0\pi^0\gamma) = 1.29 \times 10^{-4}$, $\text{BR}(\phi \to \sigma\gamma \to \pi^0\pi^0\gamma) = 0.04 \times 10^{-4}$, $\text{BR}(\phi \to \rho^0\pi^0 \to \pi^0\pi^0\gamma) = 0.14 \times 10^{-4}$, $\text{BR}(\phi \to (f_0\gamma + \pi^0\rho^0) \to \pi^0\pi^0\gamma) = 1.34 \times 10^{-4}$, $\text{BR}(\phi \to (f_0\gamma + \sigma\gamma) \to \pi^0\pi^0\gamma) = 1.16 \times 10^{-4}$ and for the total interference term $\text{BR}(\text{interference}) = -0.25 \times 10^{-4}$. Our results are in reasonable agreement with those obtained in the analysis of Achasov and Gubin [23], with the difference that our results include contributions coming from the σ-pole amplitude as well as from ρ^0- and f_0-pole amplitudes.

As we already noted, the spectrum for the decay $\phi \to \pi^0\pi^0\gamma$ is dominated by the f_0 amplitude, and the contribution coming from the σ amplitude can be appreciably noticed only in the region $M_{\pi\pi} < 0.7$ GeV through interference effects. Our analysis of these interference effects and our calculation for the decay rate of the radiative decay $\phi \to \pi^0\pi^0\gamma$ suggest the value of the coupling constant $g_{\phi\sigma\gamma}$ as $g_{\phi\sigma\gamma} = 0.025 \pm 0.009$.

[1] N. N. Achasov and V. N. Ivanchenko, Nucl. Phys. **B315**, 465 (1989).

[2] F. E. Close, N. Isgur, and S. Kumana, Nucl. Phys. **B389**, 513 (1993).

[3] M. N. Achasov et al., Phys. Lett. B **485**, 349 (2000).

[4] R. R. Akhmetshin et al., Phys. Lett. B **462**, 380 (1999).

[5] N. A. Törnqvist, invited talk at the conference "Possible Existence of the Light σ Resonance and its Implications to Hadron Physics," Kyoto, Japan, 2000, KEK Proceedings 2000-4, 2000, pp. 224–231, Soryushiron Kenkyu (Kyoto) **102**, E224–E231 (2001), hep-ph/0008135.

[6] N. A. Törnqvist and A. D. Polasa, invited talk at the conference "Heavy Quark at Fixed Target," Rio de Janeiro, Brazil, 2000, to appear in Frascati Physics Series, hep-ph/0011107.

[7] E791 Collaboration, E. M. Aitala et al., Phys. Rev. Lett. **86**, 770 (2001).

[8] N. A. Törnqvist and M. Ross, Phys. Rev. Lett. **76**, 1575 (1996).

[9] J. A. Oller and E. Oset, Nucl. Phys. **A620**, 438 (1997); **A652**, 407(E) (1999); Phys. Rev. D **60**, 074023 (1999); M. Jamin, J. A. Oller, and A. Pich, Nucl. Phys. **B587**, 331 (2000); J. A. Oller, E. Oset, and J. R. Pelaez, Phys. Rev. Lett. **80**, 3452 (1998); Phys. Rev. D **59**, 074001 (1999).

[10] J. Weinstein and N. Isgur, Phys. Rev. D **41**, 2236 (1990).

[11] R. L. Jaffe, Phys. Rev. D **15**, 267 (1977); **17**, 1444 (1978).

[12] N. N. Achasov, Nucl. Phys. **A675**, 279 (2000).

[13] S. Fajfer and R. J. Oakes, Phys. Rev. D **42**, 2392 (1990).

[14] A. Bramon, A. Grau, and G. Pancheri, Phys. Lett. B **283**, 416 (1992).

[15] A. Bramon, A. Grau, and G. Pancheri, Phys. Lett. B **289**, 97 (1992).

[16] E. Marco, S. Hirenzaki, E. Oset, and H. Toki, Phys. Lett. B **470**, 20 (1999).

[17] J. A. Oller, Phys. Lett. B **426**, 7 (1998).

[18] A. I. Titov, T.-S. H. Lee, H. Toki, and O. Streltrova, Phys. Rev. C **60**, 035205 (1999).

[19] F. Klingl, N. Kaiser, and W. Weise, Z. Phys. A **356**, 193 (1996).

[20] Particle Data Group, D. E. Groom *et al.*, Eur. Phys. J. C **15**, 1 (2000).

[21] V. E. Markushin, Eur. Phys. J. A **8**, 389 (2000).

[22] J. L. Lucio M. and J. Pestieau, Phys. Rev. D **42**, 3253 (1990); **43**, 2447(E) (1991).

[23] N. N. Achasov and V. V. Gubin, Phys. Rev. D **63**, 094007 (2001).

[24] S. M. Flatté, Phys. Lett. **63B**, 224 (1976).

[25] A. Bramon, R. Escribano, J. L. Lucio M., M. Napsuciale, and G. Pancheri, Phys. Lett. B **494**, 221 (2000).

[26] A. Gokalp and O. Yilmaz, Phys. Rev. D **62**, 093018 (2000).

[27] A. Gokalp and O. Yilmaz, Phys. Lett. B **494**, 69 (2000).

[28] B. Friman and M. Soyeur, Nucl. Phys. **A600**, 477 (1996).

[29] C. M Shakin and Huangsheng Wang, Phys. Rev. D **63**, 074017 (2001).