Can the SO(10) model with two Higgs doublets reproduce the observed fermion masses?

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It is usually considered that the SO(10) model with one **10** and one **126** Higgs scalar cannot reproduce the observed quark and charged lepton masses. Against this conventional conjecture, we find solutions of the parameters which can give the observed fermion mass spectra. The SO(10) model with one **10** and one **120** Higgs scalar is also discussed.

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I. INTRODUCTION

The grand unification theory (GUT) is very attractive as a unified description of the fundamental forces in nature. Especially, the SO(10) model is the most attractive to us when we take the unification of the quarks and leptons into consideration. However, in order to reproduce the observed quark and lepton masses and mixings, usually a lot of Higgs scalars are brought into the model. We think that nature is simple. What is of the greatest interest to us is to know the minimum number of the Higgs scalars which can give the observed fermion mass spectra. A model with one Higgs scalar is obviously ruled out for the description of the realistic quark and lepton mass spectra. Then, how is a model with two different types of Higgs scalars (e.g., **10** and **126** scalars)?

In the SO(10) GUT scenario, a model with one **10** and one **126** Higgs scalar leads to the relation [1]

$$M_e = c_u M_u + c_d M_d , \tag{1.1}$$

where M_e, M_u and M_d are the charged lepton, up-quark, and down-quark mass matrices, respectively. It is widely accepted that there will be almost no solution of c_u and c_d which give the observed fermion mass spectra. The reason is as follows: We take a basis on which the up-quark mass matrix M_u is diagonal ($M_u = D_u$). Then, the relation Eq. (1.1) is expressed as

$$\widetilde{M}_e = c_u D_u + c_d \widetilde{M}_d . \tag{1.2}$$

Considering that $\widetilde{M}_d$ is almost diagonal and the mass hierarchy of the up-quark sector is much more severe than that of the down-quark sector, we observe that the contribution to the first and the second generation part of $\widetilde{M}_e$ from the up-quark part D_u is negligible so that it is proportional to that of $\widetilde{M}_d$. Thus, the relation Eq. (1.1) which predicts $m_e/m_\mu \simeq m_d/m_s$ does not reproduce the observed hierarchical structure of the down-quark and charged lepton masses [2] such as predicted by Georgi-Jarlskog mass relations $m_b = m_\tau$, $m_s = m_\mu/3$ and $m_d = 3m_e$ at the GUT scale [3]. However, the above conclusion is a somewhat impatient one. (i) It is too simplified to regard $\widetilde{M}_d$ as almost diagonal. (ii) We must check the possibility that the mass relations are satisfied with the opposite signs, i.e., $m_b = \pm m_\tau$, $m_s = \pm m_\mu/3$ and $m_d = \pm 3m_e$. (iii) The mass values at the GUT scale, which are evaluated from the observed values by using the renormalization group equations, show sizable deviations from the Georgi-Jarskog relations. The purpose of the present paper is to investigate systematically whether there are solutions of c_u and c_d which give the realistic quark and lepton masses or not.

II. OUTLINE OF THE INVESTIGATION

In the SO(10) GUT model with one **10** and one **126** Higgs scalar, the down-quark and down-lepton mass matrices M_d and M_e are given by

$$M_d = M_0 + M_1 , \quad M_e = M_0 - 3M_1 , \tag{2.1}$$

where M_0 and M_1 are mass matrices which are generated by the **10** and **126** Higgs scalars ϕ_{10} and ϕ_{126}, respectively. Inversely, we obtain

$$M_0 = \tfrac{1}{4}(3M_d + M_e), \quad M_1 = \tfrac{1}{4}(M_d - M_e). \tag{2.2}$$

On the other hand, the up-quark mass matrix M_u is given by

$$M_u = c_0 M_0 + c_1 M_1 , \tag{2.3}$$

where

$$c_0 = v_0^u / v_0^d = \langle \phi_{10}^{u0} \rangle / \langle \phi_{10}^{d0} \rangle ,$$

$$c_1 = v_1^u / v_1^d = \langle \phi_{126}^{u0} \rangle / \langle \phi_{126}^{d0} \rangle , \tag{2.4}$$

and ϕ^u and ϕ^d denote Higgs scalar components which couple with up- and down-quark sectors, respectively. Therefore, by using the relations Eq. (2.2), we obtain the relation

$$M_e = c_d M_d + c_u M_u, \tag{2.5}$$

where

$$c_d = -\frac{3c_0 + c_1}{c_0 - c_1}, \quad c_u = \frac{4}{c_0 - c_1}. \tag{2.6}$$

For convenience, first we investigate the case where the matrices M_u, M_d, and M_e are symmetrical matrices at the unification scale because we assume that they are generated by the **10** and **126** Higgs. Then, we can diagonalize those by unitary matrices U_u, U_d, and U_e, respectively, as

$$U_u^T M_u U_u = D_u, \quad U_d^T M_d U_d = D_d, \quad U_e^T M_e U_e = D_e, \tag{2.7}$$

where D_u, D_d, and D_e are diagonal matrices. Since the Cabibbo-Kobayashi-Maskawa (CKM) matrix V is given by

$$V = U_u^T U_d^*, \tag{2.8}$$

the relation Eq. (2.5) is rewritten as follows:

$$(U_e^\dagger U_u)^T D_e (U_e^\dagger U_u) = c_d V D_d V^T + c_u D_u. \tag{2.9}$$

At present, we almost know the experimental values of D_e, D_u, and $VD_dV^\dagger$. Therefore, we obtain the three independent equations:

$$\mathrm{Tr}\, D_e D_e^\dagger = |c_d|^2\, \mathrm{Tr}[(VD_dV^T + \kappa D_u)(VD_dV^T + \kappa D_u)^\dagger], \tag{2.10}$$

$$\mathrm{Tr}(D_e D_e^\dagger)^2 = |c_d|^4\, \mathrm{Tr}[((VD_dV^T + \kappa D_u)(VD_dV^T + \kappa D_u)^\dagger)^2], \tag{2.11}$$

$$\det D_e D_e^\dagger = |c_d|^6 \det[(VD_dV^T + \kappa D_u)(VD_dV^T + \kappa D_u)^\dagger], \tag{2.12}$$

where $\kappa = c_u/c_d$. By eliminating the parameter c_d, we have two equations for the parameter κ:

$$\frac{(m_e^2 + m_\mu^2 + m_\tau^2)^3}{m_e^2 m_\mu^2 m_\tau^2} = \frac{(2.10)^3}{(2.12)}, \tag{2.13}$$

$$\frac{(m_e^2 + m_\mu^2 + m_\tau^2)^2}{2(m_e^2 m_\mu^2 + m_\mu^2 m_\tau^2 + m_\tau^2 m_e^2)} = \frac{(2.10)^2}{(2.10)^2 - (2.11)}, \tag{2.14}$$

where $(2.10)^3$, for instance, means the right-hand side of Eq. (2.10) to the third power. Let us denote the parameter values of κ evaluated from Eqs. (2.13) and (2.14) as κ_A and κ_B, respectively. If κ_A and κ_B coincide with each other, then we have the possibility that the SO(10) GUT model can reproduce the observed quark and lepton mass spectra. If κ_A and κ_B do not do so, the SO(10) model with one **10** and one **126** Higgs scalar is ruled out, and we must bring more Higgs scalars into the model. Of course, in the numerical evaluation, the values κ_A and κ_B will have sizable errors, because the observed values D_e, D_u, D_d, and V have experimental errors, and the values at the GUT scale also have errors. The values κ_A and κ_B are not so sensitive to the renormalization group equation effect (evolution effect), because those are almost determined only by the mass ratios. (More details will be discussed in Sec. III.) Therefore, we will evaluate κ_A and κ_B by using the center values at $\mu = m_Z$ in Sec. IV. If we find $\kappa_A \simeq \kappa_B$, we will give a further detailed numerical study only for this case.

III. EVOLUTION EFFECT

The relations Eqs. (2.13) and (2.14) hold only at the unification scale $\mu = \Lambda_X$ On the other hand, we know only the experimental values of the fermion masses m_f and CKM matrix parameters V_{ij} at the electroweak scale $\mu = m_Z$. For a model that does not have any intermediate energy scales, we can straightforwardly estimate the values of m_f and V_{ij} at $\mu = \Lambda_X$ from those at $\mu = m_Z$ by the one-loop renormalization equation

$$\frac{dY_f}{dt} = \frac{1}{16\pi^2}(T_f - G_f + H_f)Y_f, \tag{3.1}$$

where T_f, G_f, and H_f denote contributions from fermion-loop corrections, vertex corrections due to the gauge bosons, and vertex corrections due to the Higgs boson(s), respectively. Therefore, we can directly check the relations Eqs. (2.13) and (2.14) by substituting the observable quantities m_f and V_{ij} at $\mu = \Lambda_X$. However, for a model which has an intermediate energy scale such as a non-supersymmetric (SUSY) model, the values of m_f and V_{ij} at $\mu = \Lambda_X$ are highly model-dependent, so that a check of Eqs. (2.13) and (2.14) cannot be done so straightforwardly.

In this section, we will show that we can approximately check Eqs. (2.13) and (2.14) by using the values of m_f and V_{ij} at $\mu = m_Z$, without knowing the explicit values of m_f and V_{ij} at $\mu = \Lambda_X$, as long as the evolutions of m_f and V_{ij} are not singular.

It is well known that in such a conventional model the evolution effects are approximately described as [4]

$$\begin{aligned}
&\frac{m_u^0/m_t^0}{m_u/m_t} \simeq \frac{m_c^0/m_t^0}{m_c/m_t} \simeq 1 + \varepsilon_u,\\
&\frac{m_d^0/m_b^0}{m_d/m_b} \simeq \frac{m_s^0/m_b^0}{m_s/m_b} \simeq 1 + \varepsilon_d,\\
&\frac{|V_{ub}^0|}{|V_{ub}|} \simeq \frac{|V_{cb}^0|}{|V_{cb}|} \simeq \frac{|V_{td}^0|}{|V_{td}|} \simeq \frac{|V_{ts}^0|}{|V_{ts}|} \simeq 1 + \varepsilon_d,\\
&\frac{m_u^0/m_c^0}{m_u/m_c} \simeq \frac{m_d^0/m_s^0}{m_d/m_s} \simeq \frac{|V_{us}^0|}{|V_{us}|} \simeq \frac{|V_{cd}^0|}{|V_{cd}|} \simeq 1,
\end{aligned} \tag{3.2}$$

where m_q^0 and V_{ij}^0 (m_q and V_{ij}) denote the values at $\mu = \Lambda_X$ ($\mu = m_Z$). The relations, Eq. (3.2), hold only for a model where the Yukawa coupling constant of top quark, $y_t \equiv (Y_u)_{33}$, satisfies $y_t \gg (Y_d)_{ij}$ $(i,j = 1,2,3)$. The relations Eq. (3.2) also hold even in a model that has an intermediate energy scale Λ_I because, for example, when we denote $(m_u/m_t)_{\mu=\Lambda_X}/(m_u/m_t)_{\mu=\Lambda_I}$ and $(m_u/m_t)_{\mu=\Lambda_I}/$

$(m_u/m_t)_{\mu=m_Z}$ as $1+\varepsilon_{u1}$ and $1+\varepsilon_{u2}$, respectively, we can obtain $(m_u/m_t)_{\mu=\Lambda_X}/(m_u/m_t)_{\mu=m_Z}\simeq 1+\varepsilon_u$ with $\varepsilon_u=\varepsilon_{u1}+\varepsilon_{u2}$.

By using the approximate relations Eq. (3.2) the diagonalized up-quark mass matrix D_u^0 at $\mu=\Lambda_X$ is presented as

$$D_u^0=m_t^0\begin{pmatrix} m_u^0/m_t^0 & 0 & 0\\ 0 & m_c^0/m_t^0 & 0\\ 0 & 0 & 1\end{pmatrix} \simeq m_t^0\begin{pmatrix} m_u/m_t & 0 & 0\\ 0 & m_c/m_t & 0\\ 0 & 0 & 1\end{pmatrix}\begin{pmatrix} 1+\varepsilon_u & 0 & 0\\ 0 & 1+\varepsilon_u & 0\\ 0 & 0 & 1\end{pmatrix} =\frac{m_t^0}{m_t}(1+\varepsilon_u S)D_u, \tag{3.3}$$

where

$$S=\begin{pmatrix} 1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 0\end{pmatrix}. \tag{3.4}$$

Similarly, the matrix D_d^0 is given by

$$D_d^0\simeq\frac{m_b^0}{m_b}(1+\varepsilon_d S)D_d. \tag{3.5}$$

The CKM matrix V^0 at $\mu=\Lambda_X$ is given by

$$V^0\simeq\begin{pmatrix} 1 & V_{us} & V_{ub}(1+\varepsilon_d)\\ V_{cd} & 1 & V_{cb}(1+\varepsilon_d)\\ V_{td}(1+\varepsilon_d) & V_{ts}(1+\varepsilon_d) & 1\end{pmatrix} \simeq(\mathbf{1}+\varepsilon_d S_3)V(\mathbf{1}+\varepsilon_d S_3)-2\varepsilon_d S_3, \tag{3.6}$$

where $S_3=\mathbf{1}-S$ and $\mathbf{1}$ is a 3×3 unit matrix. By using the relations Eqs. (3.4)–(3.6), we can obtain the approximate expression

$$V^0D_d^0V^{0T}\simeq\frac{m_b^0}{m_b}[(1+\varepsilon_d)VD_dV^T-\varepsilon_d m_b S_3], \tag{3.7}$$

where we have used the observed hierarchical relations among the quark mass ratios and CKM matrix parameters. Therefore, the matrix $VD_dV^T+\kappa D_u$ in Eqs. (2.10)–(2.12) is given by

$$K^0\equiv V^0D_d^0V^{0T}+\kappa^0D_u^0\simeq(1+\varepsilon_d)\frac{m_b^0}{m_b}(VD_dV^T+\kappa D_u -\varepsilon_d m_b S_3+\varepsilon_u\kappa D_u S), \tag{3.8}$$

where

$$\kappa=\frac{m_t^0/m_t}{m_b^0/m_b}\frac{\kappa^0}{1+\varepsilon_d}. \tag{3.9}$$

Since the solutions κ are on the order of 10^{-2} as we show in Sec. IV, we can neglect the term $\kappa D_u S$ compared with VD_dV^T [note that in order to neglect the component $(D_uS)_{11}$ it is essential that the sign of m_d/m_s is positive, because $(VD_dV^T)_{11}\simeq m_d+V_{us}^2m_s$ and $V_{us}^2\simeq|m_d/m_s|$]. On the other hand, for such a small value of κ, the term m_bS_3 cannot be neglected compared with the term κD_u. However, for a small value of ε_d, we can find that the solutions κ are substantially not affected by the term $\varepsilon_d m_b S_3$. As a result, we obtain the approximate expression

$$K^0\simeq(1+\varepsilon_d)\frac{m_b^0}{m_b}(VD_dV^T+\kappa D_u). \tag{3.10}$$

Therefore, Eqs. (2.13) and (2.14) at $\mu=\Lambda_X$, i.e.,

$$\frac{[(m_e^0)^2+(m_\mu^0)^2+(m_\tau^0)^2]^3}{(m_e^0)^2(m_\mu^0)^2(m_\tau^2)^2}=\frac{[\mathrm{Tr}(K^0K^{0\dagger}])]^3}{\det(K^0K^{0\dagger})}, \tag{3.11}$$

$$\frac{[(m_e^0)^2+(m_\mu^0)^2+(m_\tau^0)^2]^2}{2[(m_e^0)^2(m_\mu^0)^2+(m_\mu^0)^2(m_\tau^0)^2+(m_\tau^0)^2(m_e^0)^2]} =\frac{[\mathrm{Tr}(K^0K^{0\dagger})]^2}{[\mathrm{Tr}(K^0K^{0\dagger})]^2-\mathrm{Tr}(K^0K^{0\dagger})^2}, \tag{3.12}$$

are approximately replaced by the relations at $\mu=m_Z$:

$$\frac{(m_e^2+m_\mu^2+m_\tau^2)^3}{m_e^2m_\mu^2m_\tau^2}=\frac{[\mathrm{Tr}(KK^\dagger)]^3}{\det(KK^\dagger)}, \tag{3.13}$$

$$\frac{(m_e^2+m_\mu^2+m_\tau^2)^2}{2(m_e^2m_\mu^2+m_\mu^2m_\tau^2+m_\tau^2m_e^2)}=\frac{[\mathrm{Tr}(KK^\dagger)]^2}{\mathrm{Tr}[(KK^\dagger)]^2-\mathrm{Tr}(KK^\dagger)^2}, \tag{3.14}$$

where

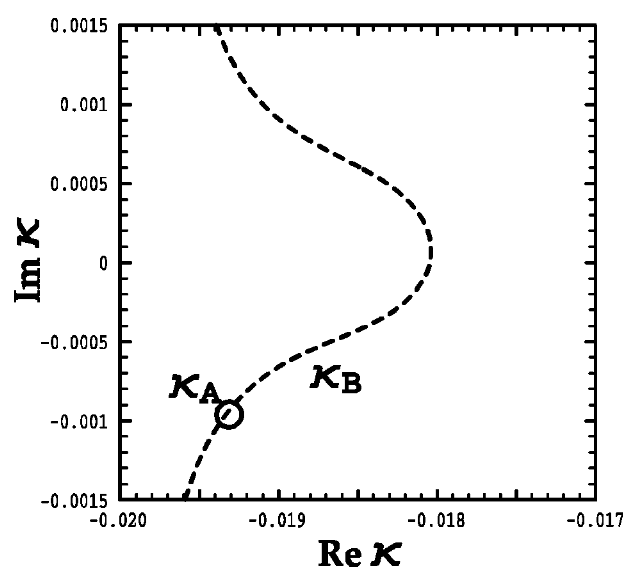

FIG. 1. The relations between Eqs. (2.13) and (2.14) on the complex plane of κ. The solid (dotted) line shows the solution of Eqs. (2.13) [Eq. (2.14)].

TABLE I. The combinations of the signs of (m_t, m_c, m_u), (m_b, m_s, m_d) and (m_τ, m_μ, m_e). The notation $(m_t, m_c, m_u)=(+-+)$ denotes $m_t>0$, $m_c<0$ and $m_u>0$. Equations (2.13) and (2.14) are not affected by the signs of charged leptons.

Num.	(m_t, m_c, m_u)	(m_b, m_s, m_d)	(m_τ, m_μ, m_e)
(a)	$(+\,-\,+)$	$(+\,-\,-)$	$(+\,\pm\,\pm)$
(b)	$(+\,-\,-)$	$(+\,-\,-)$	$(+\,\pm\,\pm)$

$$K = VD_dV^\dagger + \kappa D_u, \tag{3.15}$$

and κ is given by Eq. (3.9). This means that when we find the solution κ at $\mu=m_Z$, the solution at $\mu=\Lambda_X$ also exists, no matter whether the model is a SUSY one or a non-SUSY one. Then, we can obtain the value κ^0 at $\mu=\Lambda_X$ from the relation Eq. (3.9) with the solution κ at $\mu=m_Z$.

IV. NUMERICAL STUDY AT $\mu=m_Z$

As mentioned in Sec. III, if the solution κ exists at the energy scale $\mu=m_Z$, the one at $\mu=\Lambda_X$ also exists. Therefore, we investigate the relations Eqs. (2.13) and (2.14) at $\mu=m_Z$. Note that Eqs. (2.13) and (2.14) are realized by the GUT scale because Eq. (2.7) is broken at $\mu=m_Z$. In the present section, tentatively, we assume that the Yukawa coupling constant Y_{10} and Y_{126} at $\mu=m_Z$ keep their forms symmetrical, so that we can put the observed values D_u, D_d, and V at $\mu=m_Z$ into the relations Eqs. (2.13) and (2.14). For the fermion masses at $\mu=m_Z$, we use the following values [5]:

$$m_t=181\pm13 \text{ GeV}, \quad m_b=3.00\pm0.11 \text{ GeV},$$
$$m_c=677^{+56}_{-61} \text{ MeV}, \quad m_s=93.4^{+11.8}_{-13.0} \text{ MeV},$$
$$m_u=2.33^{+0.42}_{-0.45} \text{ MeV}, \quad m_d=4.69^{+0.60}_{-0.66} \text{ MeV}, \tag{4.1}$$
$$m_\tau=1746.7\pm0.3 \text{ MeV},$$
$$m_\mu=102.75138\pm0.00033 \text{ MeV},$$
$$m_e=0.48684727\pm0.00000014 \text{ MeV}.$$

The input values for the CKM matrix parameters have been taken as [6]

$$\theta_{12}=0.219-0.226, \quad \theta_{23}=0.037-0.043,$$
$$\theta_{13}=0.002-0.005, \tag{4.2}$$

where

$$V=\begin{pmatrix} c_{13}c_{12} & c_{13}s_{12} & s_{13}e^{-i\delta} \\ -c_{23}s_{12}-s_{23}c_{12}s_{13}e^{i\delta} & c_{23}c_{12}-s_{23}s_{12}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{23}s_{12}-c_{23}c_{12}s_{13}e^{i\delta} & -s_{23}c_{12}-c_{23}s_{12}s_{12}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix}, \tag{4.3}$$

with $c_{ij}\equiv\cos\theta_{ij}$ and $s_{ij}\equiv\sin\theta_{ij}$. The calculation has been performed allowing all the combinations of the quark mass signatures. Here it should be noted that since m_u is much smaller than m_c and m_t, the difference of the sign of m_u scarcely makes a change of allowed regions. In this calculation, we have selected θ_{23} and δ as input parameters and m_s, c_d, and κ as output parameters because the calculation is sensitive to these parameters. We give the numerical results in Fig. 1. Here, except for m_s, θ_{23}, and δ, we have adopted the center values of Eq. (4.1) as input values. Moving θ_{23} at intervals of 0.0005 rad and fixing $\delta=60°$, we search the solutions where κ_A and κ_B become coincident. Our numerical analysis shows that the solutions exist in the combinations of Table I. In Table II, we show the nearest solution of m_s, θ_{23}, and δ to the center values of Eq. (4.1).

In the following we perform data fitting for the case of the top line of Table II. Equations (2.10)–(2.12) can constrain only the absolute value of c_d. The argument of the parameter c_d may be decided by taking the neutrino sector into consideration in the future. For the time being, we set $c_d\equiv|c_d|e^{i\sigma}=e^{0.107i}$ so that c_0 becomes a real number:

$$c_0=\frac{1-c_d}{c_u}=34.7, \tag{4.4}$$

$$c_1=-\frac{3+c_d}{c_u}=101.8-10.8i. \tag{4.5}$$

In this case, the mass matrices in MeV are

$$M_0=\frac{3VD_dV^T+c_d(\kappa D_u+VD_dV^T)}{4}$$
$$=\begin{pmatrix} -12.4-0.7i & -23.0-1.8i & 9.6-13.2i \\ -23.0-1.8i & -91.5-3.9i & 194.0+10.5i \\ 9.6-13.2i & 194.0+10.5i & 1874.9-180.0i \end{pmatrix}, \tag{4.6}$$

$$M_1=\frac{VD_dV^T-c_d(\kappa D_u+VD_dV^T)}{4}=\begin{pmatrix} 4.19+0.69i & 7.68+1.43i & -3.72+4.09i \\ 7.68+1.43i & 24.14+3.88i & -65.05-10.48i \\ -3.72+4.09i & -65.05-10.48i & 1119.67+179.98i \end{pmatrix}. \tag{4.7}$$

Here, using the condition $\sqrt{|v_0^u|^2+|v_0^d|^2+|v_1^u|^2+|v_1^d|^2}=246$ GeV, we can get VEV's as

$$v_0^d=\frac{246\,[\mathrm{GeV}]}{\sqrt{(|c_0|^2+1)+(|c_1|^2+1)|\rho|^2}}, \tag{4.8}$$

with $\rho\equiv v_1^d/v_0^d$. Then, the Yukawa couplings about **10** and **126** become

$$Y_{10}=\frac{M_0}{v_0^d},\quad Y_{126}=\frac{M_1}{v_1^d}. \tag{4.9}$$

We consider that the model should be calculable perturbativly. We can see that every element of the Yukawa coupling constants Eq. (4.9) is smaller than one if we take a suitable value of $|\rho|$.

V. 10 AND 120

In the SO(10) GUT scenario, we can also discuss the model with one **10** and one **120** by the same method. The Yukawa couplings of **10** and **120** are symmetric and antisymmetric, respectively. If we consider the case where the Yukawa coupling constants of **10** are real and **120** pure imaginary, we can make them Hermitian, i.e., $Y_{10}^\dagger=Y_{10}$ and $Y_{120}^\dagger=Y_{120}$. Therefore, by considering the real vacuum expectation values v_{10} and v_{120}, we can obtain the Hermitian mass matrices M_u, M_d, and M_e:

$$M_d=M_0+M_2,\quad M_e=M_0-3M_2,$$
$$M_u=c_0M_0+c_2M_2. \tag{5.1}$$

Then, we can diagonalize those by unitary matrices U_u, U_d, and U_e as

$$U_u^\dagger M_uU_u=D_u,\quad U_d^\dagger M_dU_d=D_d,\quad U_e^\dagger M_eU_e=D_e. \tag{5.2}$$

Since the CKM matrix V is given by

$$V=U_u^\dagger U_d, \tag{5.3}$$

the relation Eq. (5.1) is rewritten as follows:

$$(U_u^\dagger U_e)D_e(U_u^\dagger U_e)^\dagger=c_dVD_dV^\dagger+c_uD_u. \tag{5.4}$$

As stated previously, we almost know the experimental values of D_e, D_u, and $VD_dV^\dagger$. Therefore, we obtain the independent three equations:

$$\mathrm{Tr}\,D_e=c_d[\mathrm{Tr}\,D_d+\kappa\,\mathrm{Tr}\,D_u], \tag{5.5}$$

$$\mathrm{Tr}\,D_e^2=c_d^2[\mathrm{Tr}\,D_d^2+2\kappa\,\mathrm{Tr}\,(D_uVD_dV^\dagger)+\kappa^2\,\mathrm{Tr}\,D_u^2], \tag{5.6}$$

$$\det D_e=c_d^3\det(VD_dV^\dagger+\kappa D_u), \tag{5.7}$$

where $\kappa=c_u/c_d$. For the parameter κ, we have two equations:

$$\frac{m_e^2+m_\mu^2+m_\tau^2}{(m_e+m_\mu+m_\tau)^2}=\frac{\mathrm{Tr}\,D_d^2+2\kappa\,\mathrm{Tr}\,(D_uVD_dV^\dagger)+\kappa^2\mathrm{Tr}\,D_u^2}{(\mathrm{Tr}\,D_d+\kappa\,\mathrm{Tr}\,D_u)^2}, \tag{5.8}$$

$$\frac{m_em_\mu m_\tau}{(m_e+m_\mu+m_\tau)^3}=\frac{\det(VD_dV^\dagger+\kappa D_u)}{(\mathrm{Tr}\,D_d+\kappa\,\mathrm{Tr}\,D_u)^3}. \tag{5.9}$$

Equations (5.8) and (5.9) are simpler than Eqs. (2.13) and (2.14). c_d and κ are real since we have assumed the M_u, M_d, and M_e to be Hermitian. So the calculation is easier than the case for **10** and **126**. The numerical results are listed in Tables III–IV.

VI. SUMMARY AND DISCUSSION

In conclusion, we have investigated whether or not an SO(10) model with two Higgs scalars can reproduce the observed mass spectra of the up- and down-quark sectors and charged lepton sector. What is of great interest is to see

TABLE II. Four sets of parameters giving good data fitting at $\mu=m_Z$ for one **10** and one **126** Higgs scalar. (a) and (b) correspond to the mass signatures in Table I, and the upper and lower lines to the two intersections in Fig. 1.

	Input			Output	
	$\|\theta_{23}\|$[rad]	$\delta[°]$	m_s[MeV]	$\|c_d\|$	κ
(a)	0.0420	60.0	76.3	3.15698	$-0.01928-0.00089i$
	0.0420	60.0	76.3	3.03577	$-0.01937-0.00101i$
(b)	0.0420	60.0	76.3	3.13307	$-0.01929-0.00092i$
	0.0420	60.0	76.3	3.00558	$-0.01939-0.00105i$

TABLE III. The combinations of the signs of $(m_t,\ m_c,m_u)$, (m_b,m_s,m_d), and (m_τ,m_μ,m_e) for one **10** and one **120** Higgs scalar.

Num.	(m_t,m_c,m_u)	(m_b,m_s,m_d)	(m_τ,m_μ,m_e)
(a-1)	$(+\ -\ +)$	$(+\ -\ -)$	$(+\ +\ +)$
(a-2)	$(+\ -\ +)$	$(+\ -\ -)$	$(+\ +\ -)$
(b-1)	$(+\ -\ -)$	$(+\ -\ -)$	$(+\ +\ +)$
(b-2)	$(+\ -\ -)$	$(+\ -\ -)$	$(+\ +\ -)$

TABLE IV. Four sets of parameters giving good data fitting at $\mu = m_Z$ for one **10** and one **120** Higgs scalar. (a-i) and (b-i) correspond to the mass signatures in Table III.

	Input			Output	
	$\lvert\theta_{23}\rvert$[rad]	$\delta[°]$	m_s[MeV]	c_d	κ
(a-1)	0.0415	60.0	79.551	0.05905	-0.01957
(a-2)	0.0415	60.0	79.238	0.06124	-0.01942
(b-1)	0.0415	60.0	79.673	0.05855	-0.01960
(b-2)	0.0415	60.0	79.316	0.06080	-0.01945

whether or not we can find reasonable values of the parameters c_u and c_d, which satisfy the SO(10) relation Eq. (2.5) or not. For the case with one **10** and one **126** scalar, in a parameter $\kappa = c_u / c_d$, we have obtained two equations (2.13) and (2.14) which hold at the unification scale $\mu = \Lambda_X$ and which are described in terms of the observable quantities (the fermion masses and CKM matrix parameters). We have sought the approximate solution of κ by using the observed fermion masses and CKM matrix parameters at $\mu = m_Z$, instead of the observable quantities at $\mu = \Lambda_X$. Although we have found no solution for real κ, we have found four solutions for complex κ which satisfy Eqs. (2.13) and (2.14) within the experimental errors. Similarly, we have found four solutions for a model with one **10** and one **120** scalar. It should be worthwhile noting that the solutions in the latter model are real. The latter model is very attractive because the origin of the CP violation is attributed only to the **120** scalar. In both models, we can make the magnitudes of all the Yukawa coupling constants smaller than one, so that the models are safely calculable under the perturbation theory.

By the way, note that the numerical results are very sensitive to the values of m_s and θ_{23}. For numerical fittings, it is favorable that the strange quark mass m_s is somewhat smaller than the center value $m_s = 93.4$ MeV which is quoted in Ref. [5].

Also note that the relative sign of m_d to m_s in each solution is positive, i.e, $m_d / m_s > 0$ as seen in Tables I and III. It is well known that a model with a texture $(M_d)_{11} = 0$ on the nearly diagonal basis of the up-quark mass matrix M_u leads to the relation $|V_{us}| = \sqrt{-m_d / m_s}$ [7], where the relative sign is negative, i.e., $m_d / m_s < 0$. On the contrary, we can conclude that in the SO(10) model with two Higgs scalars, we cannot adopt a model with the texture $(M_d)_{11} = 0$.

In the present paper, we have demonstrated that the unified description of the quark and charged lepton masses in the SO(10) model with two Higgs scalars is possible. However, we have not referred to the neutrino masses. Concerning this problem, Brahmachari and Mohapatra have recently showed that one **10** and one **126** model is incompatible with the large ν_μ-ν_τ mixing angle [8]. Since there are many possibilities for the neutrino mass generation mechanism, we are optimistic about this problem too. Investigating whether an SO(10) model with two Higgs scalars can give a unified description of quark and lepton masses including neutrino masses and mixings is our next big task.

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