Testing neutrino oscillations with τ lepton flavor violating decays in the left-right supersymmetric model

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We present a detailed analysis of the lepton flavor violating decays of the τ lepton to one loop order in the presence of neutrino mixing in the left-right supersymmetric model. We find that when the seesaw mechanism is responsible for nonzero neutrino masses and large neutrino mixings, the present neutrino data lead to an enhancement of $\tau\rightarrow\mu\gamma$ close to, or above, the experimental bound for all neutrino mixing schemes. By comparison, the branching ratio for $\tau\rightarrow e\gamma$ is expected to be several orders of magnitude smaller and dependent on the neutrino mixing scheme. Among the three lepton decays of the τ, the decay $\tau\rightarrow\mu e^+e^-$ is the largest, but still below the experimental bound by several orders of magnitude. This provides an interesting test for left-right supersymmetry and a strong incentive for improving the limit on the branching ratio of $\tau\rightarrow\mu\gamma$.

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I. INTRODUCTION

The generally held belief that the standard model of electroweak interactions is a low energy approximation of a more fundamental theory has received a lot of support recently with the discovery of neutrino oscillations [1]. The introduction of supersymmetry (SUSY) and of the supersymmetric counterpart of the standard model, the minimal supersymmetric standard model (MSSM), provides a solution to the naturalness problem of the radiative corrections to the Higgs boson mass. However, this model fails to explain other phenomena such as the weak mixing angle, the small mass (or masslessness) of the known neutrinos, the origin of CP violation, or the absence of rapid proton decay. It is hoped that extended gauge structures, introduced to provide an elegant framework for the unification of forces [2], will connect the standard model with more fundamental structures such as superstrings, while at the same time resolving the puzzles of the electroweak theory. The left-right supersymmetry (LRSUSY) is perhaps the most natural extension of the minimal model [3–6]. LRSUSY was originally seen as a natural way to suppress rapid proton decay and as a mechanism for providing small neutrino masses [5]. Left-right supersymmetry is based on the group $SU(2)_L\times SU(2)_R\times U(1)_{B-L}$, which would then break spontaneously to $SU(2)_L\times U(1)_Y$ [3]. In addition to being a plausible symmetry itself, LRSUSY models have the added attractive features that they can be embedded in a supersymmetric grand unified theory such as SO(10) [7]. Additional support for left-right theories is provided by building realistic brane worlds from type I strings, which involves left-right supersymmetry, with supersymmetry broken either at the string scale $M_{SUSY}\approx 10^{10}-10^{12}$ GeV, or at $M_{SUSY}\approx 1$ TeV, the difference having implications for gauge unification [8]. The consequences of the left-right supersymmetric model at colliders have been explored extensively [6,9]. It was shown that in most cases it could lead to enhanced production rates for charginos and neutralinos or allow for the production of light doubly charged Higgs bosons and Higgsinos [10]. LRSUSY also provides an elegant solution to the SUSY CP problem [11], and it places the seesaw mechanism of giving masses to neutrinos in a supersymmetric context.

Neutrino oscillations are the first observed breaking of global symmetry within the standard model. (Lepton number conservation is broken by neutrino masses; also lepton flavor violation is the result of neutrino oscillations.) The most natural explanation for the small left-handed neutrino masses leads one to assume the existence of a right-handed neutrino, with mass in the range 10^{12} GeV$\leqslant M_N \leqslant 10^{16}$ GeV and a seesaw mechanism, as in the LRSUSY. Breaking of symmetry in the neutral leptonic sector has implications for the charged leptonic sector, where experiments measuring lepton flavor violation (LFV) are expected to be able to increase sensitivity by four or more orders of magnitude [12]. These experiments have great potential to uncover many exotic phenomena and probe electroweak symmetry breaking. A great deal of attention has been paid to the decay $\mu\rightarrow e\gamma$ and to μ-e conversion as the dominant lepton flavor violation, because of the tight experimental bounds on these processes. However, if one takes into account the convincing result that the atmospheric neutrino anomaly is indicative of ν_μ-ν_τ oscillations, the theoretical expectation would be that there must be large lepton flavor violation mixing between the second and third generation sleptons. Then one would expect branching ratios for the process $\tau\rightarrow\mu\gamma$ and related LFV processes that test second and third generation mixings to be important.

In this paper we investigate the lepton flavor violation decay of the τ in the LRSUSY model. In LRSUSY lepton number is a local symmetry. We include in our analysis the results for the solar and atmospheric neutrinos and calculate the τ LFV branching ratios corresponding to the popular scenarios for neutrino oscillations (large angle, small angle, and vacuum oscillations). We do this for two reasons. One is that LRSUSY allows for some new contributions in some cases, and gives different dominant contributions in other cases, from models discussed so far [13]. The other reason is

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that τ flavor violating decays can be used to restrict the parameter space of the LRSUSY model. This paper is organized as follows. We review the LRSUSY model in Sec. II and discuss the sources of flavor violation in Sec. III, after which we give a complete list of the contributions to the decay $\tau\to l\gamma$ and $\tau\to 3l$ in Sec. IV. Our numerical analysis and discussion are included in Sec. V, and we conclude in Sec. VI.

II. THE LEFT-RIGHT SUPERSYMMETRIC MODEL

The LRSUSY model, based on $SU(2)_L\times SU(2)_R \times U(1)_{B-L}$, has matter doublets for both left- and right-handed fermions and the corresponding left- and right-handed scalar partners (sleptons and squarks) [5]. In the gauge sector, corresponding to $SU(2)_L$ and $SU(2)_R$, there are triplet gauge bosons $(W^{+,-},W^0)_L$, $(W^{+,-},W^0)_R$, and a singlet gauge boson V corresponding to $U(1)_{B-L}$, together with their superpartners. The Higgs sector of this model consists of two Higgs bidoublets $\Phi_u(\frac{1}{2},\frac{1}{2},0)$ and $\Phi_d(\frac{1}{2},\frac{1}{2},0)$, which are required to give masses to the up and down quarks. The phenomenology of the doublet Higgs is similar to the nonsupersymmetric left-right model [4], except that the second pair of Higgs doublet fields, which provides new contributions to the flavor changing neutral currents, must be heavy, in the 5–10 TeV range, effectively decoupling from the low energy spectrum [14]. The spontaneous symmetry breaking of the group $SU(2)_R\times U(1)_{B\text{-}L}$ to the hypercharge symmetry group $U(1)_Y$ is accomplished by the vacuum expectation values of a pair of Higgs triplet fields $\Delta_L(1,0,2)$ and $\Delta_R(0,1,2)$, which transform as the adjoint representation of $SU(2)_R$. The choice of the triplets (versus four doublets) is preferred because with this choice a large Majorana mass can be generated (through the seesaw mechanism) for the right-handed neutrino and a small one for the left-handed neutrino [4]. In addition to the triplets $\Delta_{L,R}$, the model must contain two additional triplets $\delta_L(1,0,-2)$ and $\delta_R(0,1,-2)$, with quantum number $B-L=-2$ to insure cancellation of the anomalies that would otherwise occur in the fermionic sector. Given their strange quantum numbers, the δ_L and δ_R do not couple to any of the particles in the theory, so their contribution is negligible for any phenomenological studies.

The superpotential for the LRSUSY is

$$W=\mathbf{h}_q^{(i)}Q^T\tau_2\Phi_i\tau_2Q^c+\mathbf{h}_l^{(i)}L^T\tau_2\Phi_i\tau_2L^c+i(\mathbf{h}_{LR}L^T\tau_2\Delta_L L + \mathbf{h}_{LR}L^{cT}\tau_2\Delta_R L^c)+M_{LR}[\mathrm{Tr}(\Delta_L\delta_L+\Delta_R\delta_R)] + \mu_{ij}\mathrm{Tr}(\tau_2\Phi_i^T\tau_2\Phi_j)+W_{NR} \quad (1)$$

where W_{NR} denotes (possible) nonrenormalizable terms arising from higher scale physics or Planck scale effects [10]. The presence of these terms ensures that, when the SUSY breaking scale is above M_{W_R}, the ground state is R-parity conserving [15].

As in the standard model, in order to preserve $U(1)_{EM}$ gauge invariance, only the neutral Higgs fields acquire nonzero vacuum expectation values (VEV). These values are

$$\langle\Delta_L\rangle=\begin{pmatrix}0 & 0\\ v_L & 0\end{pmatrix},\quad \langle\Delta_R\rangle=\begin{pmatrix}0 & 0\\ v_R & 0\end{pmatrix},\quad \text{and}$$

$$\langle\Phi\rangle=\begin{pmatrix}\kappa & 0\\ 0 & \kappa' e^{i\omega}\end{pmatrix}.$$

$\langle\Phi\rangle$ causes the mixing of W_L and W_R bosons with the CP violating phase ω. In order to simplify, we will take the VEV's of the Higgs fields as $\langle\Delta_L\rangle=0$ and

$$\langle\Delta_R\rangle=\begin{pmatrix}0 & 0\\ v_R & 0\end{pmatrix},\quad \langle\Phi_u\rangle=\begin{pmatrix}\kappa_u & 0\\ 0 & 0\end{pmatrix},\quad \text{and}$$

$$\langle\Phi_d\rangle=\begin{pmatrix}0 & 0\\ 0 & \kappa_d\end{pmatrix}.$$

Choosing $v_L=\kappa'=0$ satisfies the more loosely required hierarchy $v_R \gg \max(\kappa,\kappa') \gg v_L$ and also the required cancellation of flavor changing neutral currents. The Higgs fields acquire nonzero $VEV's$ to break both parity and $SU(2)_R$. In the first stage of breaking, the right-handed gauge bosons W_R and Z_R acquire masses proportional to v_R and become much heavier than the usual (left-handed) neutral gauge bosons W_L and Z_L, which pick up masses proportional to κ_u and κ_d at the second stage of breaking.

The supersymmetric sector of the model, while preserving left-right symmetry, has six singly charged charginos, corresponding to $\tilde{\lambda}_L, \tilde{\lambda}_R, \tilde{\phi}_u, \tilde{\phi}_d, \tilde{\Delta}_L^-$, and $\tilde{\Delta}_R^-$. The model also has 11 neutralinos, corresponding to $\tilde{\lambda}_Z$, $\tilde{\lambda}_{Z'}$, $\tilde{\lambda}_V$, $\tilde{\phi}_{1u}^0$, $\tilde{\phi}_{2u}^0$, $\tilde{\phi}_{1d}^0$, $\tilde{\phi}_{2d}^0$, $\tilde{\Delta}_L^0$, $\tilde{\Delta}_R^0$, $\tilde{\delta}_L^0$, and $\tilde{\delta}_R^0$. It has been shown that in the scalar sector the left triplet Δ_L couplings can be neglected in phenomenological analyses of muon and tau decays [16]. Although Δ_L is not necessary for symmetry breaking [6], and is introduced only for preserving left-right symmetry, both Δ_L^{--} and its right-handed counterpart Δ_R^{--} play very important roles in phenomenological studies of the LRSUSY model. We include them both in our formal expressions, but only the Δ_R contribution in the numerical analysis.

III. SOURCES OF FLAVOR VIOLATION IN LRSUSY

The sources of flavor violation in the LRSUSY model come from either the Yukawa potential or from the trilinear scalar coupling.

The interaction of fermions with scalar (Higgs) fields has the following form:

$$\mathcal{L}_Y=\mathbf{h}_u\overline{Q_L}\Phi_uQ_R+\mathbf{h}_d\overline{Q_L}\Phi_dQ_R+\mathbf{h}_\nu\overline{L_L}\Phi_uL_R+\mathbf{h}_e\overline{L_L}\Phi_dL_R+\text{H.c.},$$

$$\mathcal{L}_M=i\mathbf{h}_{LR}(L_L^TC^{-1}\tau_2\Delta_LL_L+L_R^TC^{-1}\tau_2\Delta_RL_R)+\text{H.c.} \quad (2)$$

where $\mathbf{h}_u$, $\mathbf{h}_d$, $\mathbf{h}_\nu$, and $\mathbf{h}_e$ are the Yukawa couplings for the up and down quarks and neutrino and electron, respectively, and $\mathbf{h}_{LR}$ is the coupling for the triplet Higgs bosons. LR symmetry requires all $\mathbf{h}$ matrices to be Hermitian in the generation space and the $\mathbf{h}_{LR}$ matrix to be symmetric. The Yukawa matrices have physical and geometrical significance and cannot be rotated away. Geometrically, they represent misalignment between the particle and sparticle bases in flavor space, the physical significance of which is that they cause flavor violation. In addition soft supersymmetry breaking terms that generate masses for the charged slepton fields also induce LFV. The SUSY breaking term for the Higgs bosons and lepton sector in LRSUSY is given by

$$-\mathcal{L}_{soft}=-[\mathbf{A}_l^i\mathbf{h}_l^{(i)}\tilde{L}^T\tau_2\Phi_i\tau_2\tilde{L}^c+i\mathbf{A}_{LR}\mathbf{h}_{LR}(\tilde{L}^T\tau_2\Delta_L\tilde{L}+\tilde{L}^{cT}\tau_2\Delta_R\tilde{L}^c)+m_\Phi^{(ij)2}\Phi_i^\dagger\Phi_j]+[(m_L^2)_{ij}\tilde{l}_{Li}^\dagger\tilde{l}_{Lj}+(m_R^2)_{ij}\tilde{l}_{Ri}^\dagger\tilde{l}_{Rj}+(M_N^2)_{ij}\tilde{N}_i^*\tilde{N}_j^*]+M_{LR}^2[\mathrm{Tr}(\Delta_R\delta_R)+\mathrm{Tr}(\Delta_L\delta_L)]+B\mu_{ij}\Phi_i\Phi_j, \quad (3)$$

where the $\mathbf{A}$ matrices (A_u, A_d, A_ν, and A_e) are of similar form to the Yukawa couplings and provide additional sources of flavor violation, B is a mass term, and $\tilde{N}$ is the scalar component of the right-handed neutrino supermultiplet. The intergenerational slepton mixing ($\tilde{e}$, $\tilde{\mu}$, and $\tilde{\tau}$) and also left-right slepton mixing ($\tilde{e}_L,\tilde{e}_R,\tilde{\mu}_L,\tilde{\mu}_R,\tilde{\tau}_L,\tilde{\tau}_R$) cause the off-diagonal nature of the matrices, and are therefore responsible for flavor violation. We discuss these next.

In the scalar matter sector, the LRSUSY model contains two left-handed and two right-handed scalar fermions as partners of the ordinary leptons, which themselves come in left- and right-handed doublets. In general the left- and right-handed scalar leptons will mix together. The slepton mass matrix, which arises as a result of the renormalization group evolution from the Λ_{GUT} scale, is, incorporating some elements of the left-right symmetry [17],

$$(m_{\tilde{l}}^2)_{eff}=\begin{pmatrix} m_L^2+D_L & 0 & 0 & \mathcal{A}_e & 0 & 0\\ 0 & m_L^2+D_L & (\tilde{m}_L)_{32}^2 & 0 & \mathcal{A}_\mu & 0\\ 0 & (\tilde{m}_L)_{32}^2 & m_L^2+D_L & 0 & 0 & \mathcal{A}_\tau\\ \mathcal{A}_e & 0 & 0 & m_R^2+D_R & (\tilde{m}_R)_{32}^2 & 0\\ 0 & \mathcal{A}_\mu & 0 & (\tilde{m}_R)_{32}^2 & m_R^2+D_R & 0\\ 0 & 0 & \mathcal{A}_\tau & 0 & 0 & m_R^2+D_R \end{pmatrix}, \quad (4)$$

where $\mathcal{A}_l=A_lm_l+m_l\mu\tan\beta$, $(l=e,\mu,\tau)$, $D_L=M_{Z_L}^2(T_3/2+\sin^2\theta_W)\cos2\beta+M_{Z_R}^2\sin^2\theta_W/\sin2\theta_W\cos2\beta$, and $D_R=-M_{Z_L}^2\sin^2\theta_W\cos2\beta+M_{Z_R}^2(T_3/2-\sin^2\theta_W/\cos2\theta_W)\sin2\beta$.

The full mass for left- and right-handed sneutrino has a complicated 12×12 matrix structure [19]:

$$(m_{\tilde{\nu}\tilde{N}}^2)=\begin{pmatrix} m_L^2+m_D^*m_D^T & 0 & m_D^*M^T & \mathcal{A}_\nu^*m_D^*\\ 0 & m_L^2+m_Dm_D^\dagger & \mathcal{A}_\nu^*m_D & m_DM^\dagger\\ M^*m_D^T & m_D^\dagger\mathcal{A}_\nu^* & m_\nu^2+M^*M^T+m_D^*m_D^\dagger & A_N^*M^*\\ \mathcal{A}_\nu m_D^T & Mm_D^* & A_NM & m_\nu^2+MM^\dagger+m_Dm_D^\dagger \end{pmatrix} \quad (5)$$

where $\mathcal{A}_\nu=A_\nu+\mu\cot\beta$ and m_D and M are, repectively, the Dirac and the Majorana mass matrices. From this we can construct an effective 6×6 matrix for the light neutrinos using the seesaw mechanism:

$$(m_{\tilde{\nu}}^2)_{eff}=\begin{pmatrix} m_L^2-\mathcal{A}'_\nu(A_\nu-2A_N)(m_DM^{-2}m_D^\dagger) & \mathcal{A}'^*_\nu(m_DM^{-1}m_D^\dagger)\\ \mathcal{A}'_\nu(m_DM^{-1}m_D^\dagger)^* & m_L^2-\mathcal{A}'_\nu(A_\nu-2A_N)(m_DM^{-2}m_D^\dagger) \end{pmatrix} \quad (6)$$

where $\mathcal{A}'_\nu\sim2A_\nu+A_N+2\mu\cot\beta$. Note that in the LR model the left-handed neutrino mass is allowed to be nonzero, but can be made small through the seesaw mechanism, as long as the right-handed neutrino is very heavy (masses of order 10^{10} TeV or so are consistent with the 1 eV limit on the left-handed neutrino mass). Despite the presence of the two scalar neutrinos, the mixing between the right-handed and the left-handed sneutrinos is small, due to the seesaw mechanism in the sfermion sector. The left-right elements of the sneutrino mass matrix are proportional to the Dirac neutrino mass, which can be significant. But the right-right element of the sneutrino mass matrix is very heavy, so the mixing of sneutrinos will be suppressed by the inverse M_N^2, the right-handed neutrino mass. However, as opposed to the charged slepton sector, in the light sneutrino sector the Dirac terms do not induce considerable mixing [20]. The off-diagonal terms in the sneutrino mass matrix mix almost degenerate states and affect flavor violating decays less than the charged slepton mixing.

Next we consider the implications of these flavor changing mechanisms in LRSUSY on the lepton-flavor violating decays $\tau \to l\gamma$ and the related LFV processes $\tau \to 3l$.

IV. THE AMPLITUDE FOR THE PROCESSES $\tau \to l\gamma$ AND $\tau \to 3l$

A. $\tau \to \mu$ $(e) + \gamma$

The amplitude of the $\tau \to l\gamma$ transition can be written in the form of the usual dipole-type interaction:

$$\mathcal{M}_{\tau \to l\gamma} = \frac{1}{2}\bar{\psi}_l (A_L P_L + A_R P_R)\sigma^{\mu\nu}F_{\mu\nu}\psi_\tau. \qquad (7)$$

It leads to the branching ratio

$$\Gamma_{\tau \to l\gamma} = \frac{1}{16\pi^2}(|A_L|^2 + |A_R|^2)m_\tau^5. \qquad (8)$$

Comparing it with the standard decay width $\Gamma_\tau = (1/192\pi^3)G_F^2 m_\tau^5$ and using the recent experimental constraint on the branching ratio [18]

$$BR(\tau \to \mu\gamma) \leqslant 1.1 \times 10^{-6}, \qquad (9)$$

we get the following limit on the dipole amplitude:

$$|d| = \sqrt{(|A_L|^2 + |A_R|^2)/2} < 5.2(8.2) \times 10^{-24} \;\; e \;\; \mathrm{cm} \qquad (10)$$

for $\tau \to \mu\gamma$ ($\tau \to e\gamma$).

Because the scalar mass splittings are required to be small, we will parametrize the scalar mass eigenvalues by the average masses m_L^2 and m_R^2 and the mass splittings $\delta\tilde{m}_L^2$ and $\delta\tilde{m}_R^2$. We keep only the leading contribution in both the mass splittings and the mixing parameters. In addition to scalar mass splitting, LFV decays will depend also on gaugino-Higgsino mixing.

We now consider the implications of these mixings in the the lepton flavor violating decay $\tau \to l\gamma$. The argument of the loop functions (defined below) is $r_{pk} = M_k^2/m_p^2$ where k represents the chargino or neutralino, and p represents the slepton or sneutrino. The chargino and neutralino masses enter the theory via their mass eigenvalues and mixing matrices. Following [21], we employ the following notation: the U,N matrices rotate the gaugino or Higgsino interaction basis into the neutralino or chargino mass basis. N^0 is the matrix for the neutralinos; U^+ is the matrix for the charginos Ψ_i^+; and U^- is for the charginos Ψ_i^-. $U_{\Delta_{L,R}}^{--}$ and $U_{\Delta_{L,R}}^{++}$ are mixing matrices for the doubly charged $\tilde{\Delta}_{L,R}$ and $\tilde{\delta}_{L,R}$ Higgsinos.

The electroweak gauginos and Higgsinos are all spin-1/2 weakly interacting charged particles which mix once the symmetry is broken. In the left-right supersymmetric model, the chargino matrix is a 5×5, nonsymmetric, non-Hermitian matrix M^c, from the Lagrangian (we include here, for the sake of minimizing the number of parameters, only the right-handed triplet Higgsinos)

$$\mathcal{L}_{ch} = -\frac{1}{2}(\Psi^+ \Psi^-)\begin{pmatrix} 0 & M^{cT} \\ M^c & 0 \end{pmatrix}\begin{pmatrix} \Psi^+ \\ \Psi^- \end{pmatrix} + \mathrm{H.c.} \qquad (11)$$

where

$$\Psi^+ = (-i\lambda_L^+, -i\lambda_R^+, \tilde{\Phi}_u^+, \tilde{\Phi}_d^+, \tilde{\delta}_R^+), \qquad (12)$$

$$\Psi^- = (-i\lambda_L^-, -i\lambda_R^-, \tilde{\Phi}_u^-, \tilde{\Phi}_d^-, \tilde{\Delta}_R^-), \qquad (13)$$

and

$$M^c = \begin{pmatrix} M_L & 0 & 0 & \sqrt{2}M_{W_L}\sin\beta & 0 \\ 0 & M_R & 0 & \sqrt{2}M_{W_L}\sin\beta & \dfrac{(M_{W_R}^2 - M_{W_L}^2)^{1/2}}{\sqrt{2}} \\ \sqrt{2}M_{W_L}\cos\beta & \sqrt{2}M_{W_L}\cos\beta & 0 & \mu & 0 \\ 0 & 0 & \mu & 0 & 0 \\ 0 & -\dfrac{(M_{W_R}^2 - M_{W_L}^2)^{1/2}}{2} & 0 & 0 & M_{LR} \end{pmatrix} \qquad (14)$$

where $\tan\beta = \kappa_d/\kappa_u$, M_L and M_R are the gaugino masses in the left- and right-handed sectors and $\chi_i^- = U_{ij}^- \Psi_j^-$, $\chi_i^+ = U_{ij}^+ \Psi_j^+ (i,j = 1, \ldots ,5)$.

As in the MSSM, we need two unitary matrices $U^{(-)}$ and $U^{(+)}$ to diagonalize M^c:

$$M_D = U^{(-)*}M^c U^{(+)-1}. \qquad (15)$$

The eigenvalues of M^c can be either positive or negative, whereas we require M_D to have only non-negative entries. We shall use numerical expressions for U_{ij}^+ and U_{ij}^- obtained in [21].

The neutralino Lagrangian can be written in matrix form as

$$\mathcal{L}_n = -\frac{1}{2}(\Psi^0)^T M^n (\Psi^0) + \text{H.c.} \tag{16}$$

using the basis (of neutralinos that couple to leptons or sleptons)

$$\Psi^0 = (-i\lambda_B^0 \cos\theta_W, -i\lambda_L^0, -i\lambda_R^0 \sin\theta_W, \tilde{\Phi}_u^0, \tilde{\Phi}_d^0, \tilde{\Delta}_R^0, \tilde{\delta}_R^0). \tag{17}$$

The neutralino mixing matrix is in general a complex symmetric matrix given by

$$M^N = \begin{pmatrix} M_V + M_R \tan^2\theta_W & 0 & 2(M_R - M_V) & C_1 & -C_2 & C_4 & -C_4 \\ 0 & M_L & 0 & -C_3 & C_2 & 0 & 0 \\ 2(M_R - M_V) & 0 & M_V + \dfrac{M_R}{\tan^2\theta_W} & \dfrac{M_Z \sin\theta_W \cos\beta}{\tan^2\theta_W} & C_2 & -C_5 & C_5 \\ C_1 & -C_3 & \dfrac{M_Z \cos\theta_W \sin\beta}{\tan^2\theta_W} & -\mu & 0 & 0 & 0 \\ -C_2 & C_2 & C_2 & 0 & -\mu & 0 & 0 \\ C_4 & 0 & -C_5 & 0 & 0 & 0 & M_{LR} \\ -C_4 & 0 & C_5 & 0 & 0 & M_{LR} & 0 \end{pmatrix} \tag{18}$$

where

$$C_1 = M_Z \sin\theta_W \cos\beta, \tag{19}$$

$$C_2 = M_Z \cos\theta_W \sin\beta, \tag{20}$$

$$C_3 = M_Z \cos\theta_W \cos\beta, \tag{21}$$

$$C_4 = 2M_{W_R} \cos\theta_W, \tag{22}$$

$$C_5 = 2M_{W_R} \sin^2\theta_W / \sqrt{\cos 2\theta_W}. \tag{23}$$

We define the masss eigenstates to be

$$\chi_i^0 = N_{il}\Psi_l, \quad i,l = 1, \ldots, 7, \tag{24}$$

where N_{il} is a unitary matrix that diagonalizes the neutralino mass matrix:

$$M_D^N = N^* M^N N^{-1} \tag{25}$$

with M_D^N the diagonal neutralino mass matrix. Again, we use the numerical expressions for N_{ij} obtained in [21].

The contributions to the decay $\tau \to l + \gamma$ in the left-right supersymmetric model are presented in the diagrams of Fig. 1. The evaluation of these graphs includes the elements of LFV outlined in the previous section. We write the amplitudes for the decays as

$$A_L = A_{Lf} + A_{Lg} + A_{Lv} + A_{Lj} + A_{Ll} + A_{Ln} + A_{L\Delta_f}, \tag{26}$$

$$A_R = A_{Rf} + A_{Rg} + A_{Rv} + A_{Rj} + A_{Rl} + A_{Rn} + A_{R\Delta_f}. \tag{27}$$

Here A_L represents the left-handed contribution and A_R the contribution from the right-handed sector, as given below.

We present first the individual contributions for graphs with chirality flip on the external fermion line. For charginos, left-handed fermions, with an external chirality flip [Fig. 1(a)]:

$$A_{Lf} = -\frac{g^2}{16\pi^2}\frac{1}{2}(U_{W_L k})^2 \left[\frac{(\tilde{m}_L)^2_{32}}{m^2_{\tilde{\nu}_\mu} - m^2_{\tilde{\nu}_\tau}}\right]\left\{\frac{f(x_{k\nu_\mu})}{m^2_{\tilde{\nu}_\mu}} - \frac{f(x_{k\nu_\tau})}{m^2_{\tilde{\nu}_\tau}}\right\}. \tag{28}$$

For charginos, right-handed fermions, with an external chirality flip [Fig. 1(b)]:

$$A_{Rf} = -\frac{g^2}{16\pi^2}\frac{1}{2}(U_{W_R k})^2 \left[\frac{(\tilde{m}_R)^2_{32}}{m^2_{\tilde{N}_\mu} - m^2_{\tilde{N}_\tau}}\right] \times \left\{\frac{f(x_{kN_\mu})}{m^2_{\tilde{N}_\mu}} - \frac{f(x_{kN_\tau})}{m^2_{\tilde{N}_\tau}}\right\}. \tag{29}$$

For neutralinos, left-handed fermions, with an external chirality flip [Fig. 1(c)]:

$$A_{Lg} = \frac{g^2}{16\pi^2}\frac{1}{2}(N^0_{W_L k} + \tan\theta_W^2 N^0_{Bk})^2 \left[\frac{(\tilde{m}_L)^2_{32}}{m^2_{\tilde{\mu}_L} - m^2_{\tilde{\tau}_L}}\right] \times \left\{\frac{g(x_{k\mu_L})}{m^2_{\tilde{\mu}_L}} - \frac{g(x_{k\tau_L})}{m^2_{\tilde{\tau}_L}}\right\}. \tag{30}$$

For neutralinos, right-handed fermions, with an external chirality flip [Fig. 1(d)]:

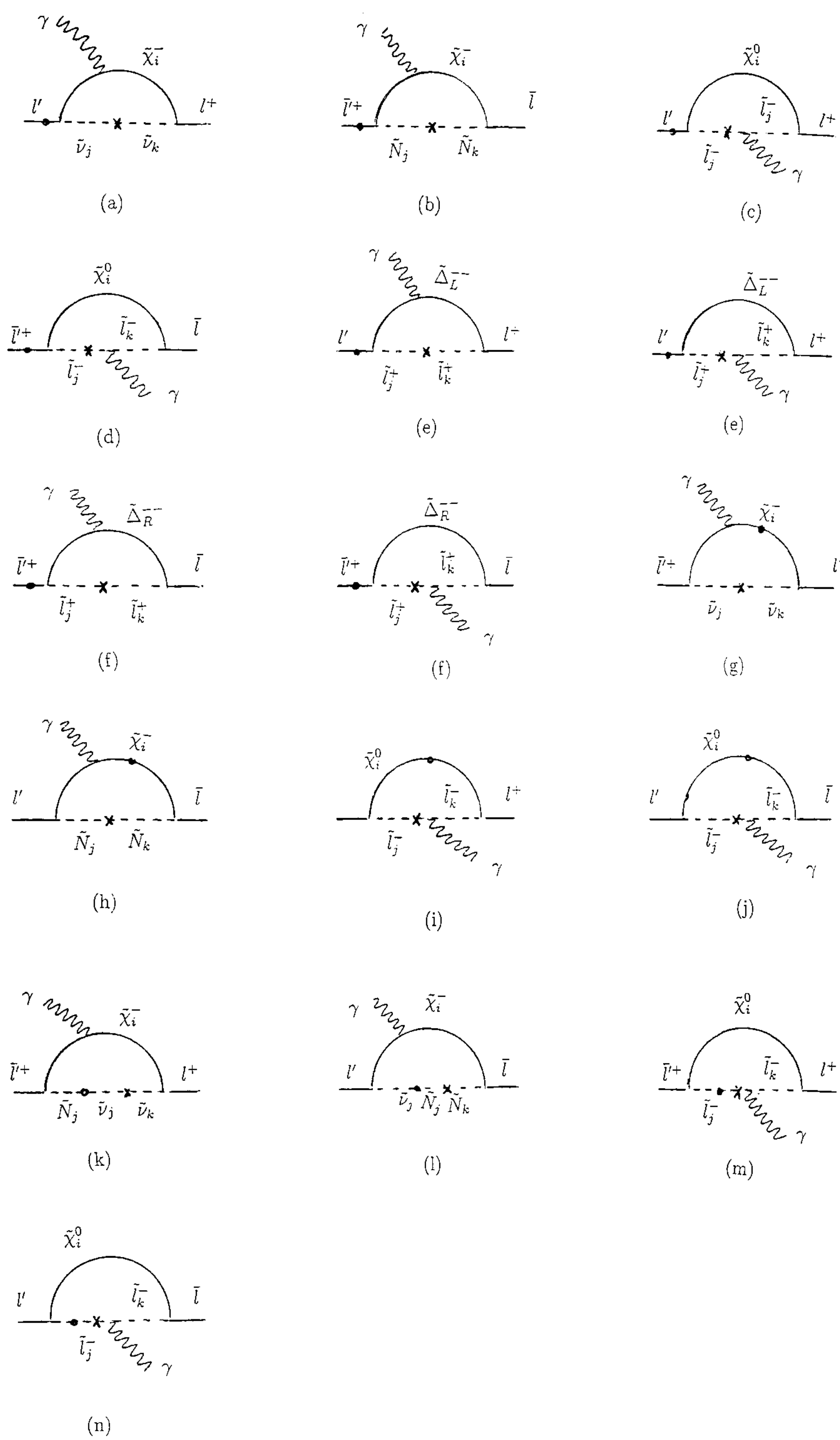

FIG. 1. One-loop contributions to the decay $l' \rightarrow l\gamma$: (a) charginos, left-handed fermions, with an external chirality flip; (b) charginos, right-handed fermions, with an external chirality flip; (c) neutralinos, left-handed fermions, with an external chirality flip; (d) neutralinos, right-handed fermions, with an external chirality flip; (e) doubly-charged Higgsinos, left-handed fermions, with an external chirality flip; (f) doubly-charged Higgsinos, right-handed fermions, with an external chirality flip; (g) charginos, left-handed fermions, with a vertex chirality flip; (h) charginos, right-handed fermions, with a vertex chirality flip; (i) neutralinos, left-handed fermions, with a vertex chirality flip; (j) neutralinos, right-handed fermions, with a vertex chirality flip; (k) charginos, left-handed fermions, with an internal line chirality flip; (l) charginos, right-handed fermions, with an internal line chirality flip; (m) neutralinos, left-handed fermions, with an internal line chirality flip; (n) neutralinos, right-handed fermions, with an internal line chirality flip. Here $\tilde{\chi}_i^{\pm}$ represents a chargino state and i runs from 1 to 8; $\tilde{\chi}_i^0$ represents a neutralino state and i runs from 1 to 11; and $\tilde{\Delta}_{L,R}^{--}$ represents the doubly charged Higgsino state; $\tilde{l}$, $\tilde{\nu}$, and $\tilde{N}$ represent slepton, left-handed sneutrino, and right-handed sneutrino fields, respectively. The crosses represent slepton or sneutrino flavor mixing and the dots chirality flips.

$$A_{Rg}=\frac{g^2}{16\pi^2}\frac{1}{2}(N^0_{W_Rk}-2\tan\theta_W^2 N^0_{Bk})^2\left[\frac{(\widetilde{m}_R)^2_{32}}{m^2_{\tilde{\mu}_R}-m^2_{\tilde{\tau}_R}}\right]\times\left\{\frac{g(x_{k\mu_R})}{m^2_{\tilde{\mu}_R}}-\frac{g(x_{k\tau_R})}{m^2_{\tilde{\tau}_R}}\right\}. \tag{31}$$

For doubly charged Higgsinos, left-handed fermions, with an external chirality flip [Fig. 1(e)]:

$$A_{L\Delta_f}=-\frac{h_{LR\tau}h_{LR\mu}}{16\pi^2}(U_{\Delta_L^{++}})^2\left[\frac{(\widetilde{m}_L)^2_{32}}{m^2_{\tilde{\mu}_L}-m^2_{\tilde{\tau}_L}}\right]\times\left\{\frac{f(x_{k\mu_L})+2g(x_{k\mu_L})}{m^2_{\tilde{\mu}_L}}-\frac{f(x_{k\tau_L})+2g(x_{k\tau_L})}{m^2_{\tilde{\tau}_L}}\right\}. \tag{32}$$

For doubly charged Higgsinos, right-handed fermions, with an external chirality flip [Fig. 1(f)]:

$$A_{R\Delta_f}=-\frac{h_{LR\tau}h_{LR\mu}}{16\pi^2}(U_{\Delta_R^{++}})^2\left[\frac{(\widetilde{m}_R)^2_{32}}{m^2_{\tilde{\mu}_R}-m^2_{\tilde{\tau}_R}}\right]\times\left\{\frac{f(x_{k\mu_R})+2g(x_{k\mu_R})}{m^2_{\tilde{\mu}_R}}-\frac{f(x_{k\tau_R})+2g(x_{k\tau_R})}{m^2_{\tilde{\tau}_R}}\right\}. \tag{33}$$

Next, we present the expressions for the graphs where chirality is flipped at the vertex. For most of the parameter space, these graphs are dominant over the graphs where the chiralty is flipped externally, unless they involve heavy sneutrinos.

For charginos, left-handed fermions, with a vertex chirality flip [Fig. 1(g)]:

$$A_{Lv}=\frac{g^2}{16\pi^2}\frac{M_{\chi_k}}{\sqrt{2}M_W\cos\beta}(U^-_{H_k}U^+_{W_{Lk}})\left[\frac{(\widetilde{m}_L)^2_{32}}{m^2_{\tilde{\nu}_\mu}-m^2_{\tilde{\nu}_\tau}}\right]\times\left\{\frac{h(x_{k\nu_\mu})}{m^2_{\tilde{\nu}_\mu}}-\frac{h(x_{k\nu_\tau})}{m^2_{\tilde{\nu}_\tau}}\right\}. \tag{34}$$

For charginos, right-handed fermions, with a vertex chirality flip [Fig. 1(h)]:

$$A_{Rv}=\frac{g^2}{16\pi^2}\frac{M_{\chi_k}}{\sqrt{2}M_W\cos\beta}(U^-_{H_k}U^+_{W_{Rk}})\left[\frac{(\widetilde{m}_R)^2_{32}}{m^2_{\tilde{N}_\mu}-m^2_{\tilde{N}_\tau}}\right]\times\left\{\frac{h(x_{kN_\mu})}{m^2_{\tilde{N}_\mu}}-\frac{h(x_{kN_\tau})}{m^2_{\tilde{N}_\tau}}\right\}. \tag{35}$$

For neutralinos, left-handed fermions, with a vertex chirality flip [Fig. 1(i)]:

$$A_{Lj}=\frac{g^2}{16\pi^2}\frac{1}{2}(N^0_{W_Lk}+\tan\theta_W^2 N^0_{Bk})N^0_{H_k}\left[\frac{(\widetilde{m}_L)^2_{32}}{m^2_{\tilde{\mu}_L}-m^2_{\tilde{\tau}_L}}\right]\times\left\{\frac{j(x_{k\mu_L})}{m^2_{\tilde{\mu}_L}}-\frac{j(x_{k\tau_L})}{m^2_{\tilde{\tau}_L}}\right\}. \tag{36}$$

For neutralinos, right-handed fermions, with a vertex chirality flip [Fig. 1(j)]:

$$A_{Rj}=\frac{g^2}{16\pi^2}\frac{1}{2}(N^0_{W_Rk}-2\tan\theta_W^2 N^0_{Bk})N^0_{H_k}\left[\frac{(\widetilde{m}_R)^2_{32}}{m^2_{\tilde{\mu}_R}-m^2_{\tilde{\tau}_R}}\right]\times\left\{\frac{j(x_{k\mu_R})}{m^2_{\tilde{\mu}_R}}-\frac{j(x_{k\tau_R})}{m^2_{\tilde{\tau}_R}}\right\}. \tag{37}$$

The right-handed chargino contribution with chirality flipped on an external line or at the vertex is negligible since it involves heavy sneutrinos.

Finally, we give the expressions for the case in which the chirality is flipped on the internal slepton line. Here essentially all the contribution comes from the neutralino graphs, because the chargino graphs (both left- and right-handed) are suppressed by factors of the heavy sneutrino mass in the denominator.

For charginos, left-handed fermions, with an internal line chirality flip [Fig. 1(k)]:

$$\begin{aligned}A_{Ll}=\frac{g^2}{16\pi^2}M_{\chi_k}(U^-_{W_{Rk}}U^+_{W_{Lk}})(\widetilde{m}_L)^2_{32}(\mathcal{A}_\tau)&\times\left\{\frac{1}{m^2_{\tilde{N}_\tau}-m^2_{\tilde{\nu}_\tau}}\frac{1}{m^2_{\tilde{N}_\tau}-m^2_{\tilde{\nu}_\mu}}\frac{h(x_{kN_\tau})}{m^2_{\tilde{N}_\tau}}\right.\\&+\frac{1}{m^2_{\tilde{\nu}_\tau}-m^2_{\tilde{N}_\tau}}\frac{1}{m^2_{\tilde{\nu}_\tau}-m^2_{\tilde{\nu}_\mu}}\frac{h(x_{k\nu_\tau})}{m^2_{\tilde{\nu}_\tau}}\\&\left.+\frac{1}{m^2_{\tilde{\nu}_\mu}-m^2_{\tilde{N}_\tau}}\frac{1}{m^2_{\tilde{\nu}_\mu}-m^2_{\tilde{\nu}_\tau}}\frac{h(x_{k\nu_\mu})}{m^2_{\tilde{\nu}_\mu}}\right\}.\end{aligned} \tag{38}$$

For charginos, right-handed fermions, with an internal line chirality flip [Fig. 1(l)]:

$$A_{Rl}=\frac{g^2}{16\pi^2}M_{\chi_k}(U^-_{W_{Lk}}U^+_{W_{Rk}})(\widetilde{m}_R)^2_{32}(\mathcal{A}_\tau)$$
$$\times\left\{\frac{1}{m^2_{\tilde{\nu}_\tau}-m^2_{\tilde{N}_\tau}}\frac{1}{m^2_{\tilde{\nu}_\tau}-m^2_{\tilde{N}_\mu}}\frac{h(x_{k\nu_\tau})}{m^2_{\tilde{\nu}_\tau}}\right.$$
$$+\frac{1}{m^2_{\tilde{N}_\tau}-m^2_{\tilde{\nu}_\tau}}\frac{1}{m^2_{\tilde{N}_\tau}-m^2_{\tilde{N}_\mu}}\frac{h(x_{kN_\tau})}{m^2_{\tilde{N}_\tau}}$$
$$\left.+\frac{1}{m^2_{\tilde{N}_\mu}-m^2_{\tilde{\nu}_\tau}}\frac{1}{m^2_{\tilde{N}_\mu}-m^2_{\tilde{N}_\tau}}\frac{h(x_{kN_\mu})}{m^2_{\tilde{N}_\mu}}\right\}. \tag{39}$$

For neutralinos, left-handed fermions, with an internal line chirality flip [Fig. 1(m)]:

$$A_{Ln}=\frac{g^2}{16\pi^2}\frac{1}{4}(N^0_{W_R k}-2\tan\theta_W N^0_{Bk})(N^0_{W_L k}+\tan\theta^2_W N^0_{Bk})$$
$$\times(\widetilde{m}_L)^2_{32}(\mathcal{A}_\tau)\left\{\frac{1}{m^2_{\tilde{\tau}_R}-m^2_{\tilde{\tau}_L}}\frac{1}{m^2_{\tilde{\tau}_R}-m^2_{\tilde{\mu}_L}}\frac{j(x_{k\tau_R})}{m^2_{\tilde{\tau}_R}}\right.$$
$$+\frac{1}{m^2_{\tilde{\tau}_L}-m^2_{\tilde{\tau}_R}}\frac{1}{m^2_{\tilde{\tau}_L}-m^2_{\tilde{\mu}_L}}\frac{j(x_{k\tau_L})}{m^2_{\tilde{\tau}_L}}$$
$$\left.+\frac{1}{m^2_{\tilde{\mu}_L}-m^2_{\tilde{\tau}_R}}\frac{1}{m^2_{\tilde{\mu}_L}-m^2_{\tilde{\tau}_L}}\frac{j(x_{k\mu_L})}{m^2_{\tilde{\mu}_L}}\right\}. \tag{40}$$

For neutralinos, right-handed fermions, with an internal line chirality flip [Fig. 1(n)]:

$$A_{Rn}=\frac{g^2}{16\pi^2}\frac{1}{4}(N^0_{W_L k}+\tan\theta^2_W N^0_{Bk})(N^0_{W_R k}-2\tan\theta_W N^0_{Bk})$$
$$\times(\widetilde{m}_R)^2_{32}(\mathcal{A}_\tau)\left\{\frac{1}{m^2_{\tilde{\tau}_L}-m^2_{\tilde{\tau}_R}}\frac{1}{m^2_{\tilde{\tau}_L}-m^2_{\tilde{\mu}_R}}\frac{j(x_{k\tau_L})}{m^2_{\tilde{\tau}_L}}\right.$$
$$+\frac{1}{m^2_{\tilde{\tau}_R}-m^2_{\tilde{\tau}_L}}\frac{1}{m^2_{\tilde{\tau}_R}-m^2_{\tilde{\mu}_R}}\frac{j(x_{k\tau_R})}{m^2_{\tilde{\tau}_R}}$$
$$\left.+\frac{1}{m^2_{\tilde{\mu}_R}-m^2_{\tilde{\tau}_L}}\frac{1}{m^2_{\tilde{\mu}_R}-m^2_{\tilde{\tau}_R}}\frac{j(x_{k\mu_R})}{m^2_{\tilde{\mu}_R}}\right\}. \tag{41}$$

Note that chirality cannot be flipped internally in the graphs with doubly charged Higgsinos.

The $\tau\to l+\gamma$ loop functions are

$$f(x)=\frac{1}{12(1-x)^4}(x^3-6x^2+3x+2+6x\log x), \tag{42}$$

$$g(x)=\frac{1}{12(1-x)^4}(2x^3+3x^2-6x+1-6x^2\log x), \tag{43}$$

$$h(x)=-\frac{1}{2(1-x)^3}(x^2-4x+3+2\log x), \tag{44}$$

$$j(x)=\frac{1}{2(1-x)^3}(-x^2+1+2x\log x). \tag{45}$$

B. Three-body lepton flavor violating decays of the τ

These decays are important sources of LFV, and connected to the decay $\tau\to l\gamma$. It is also hoped that the experimental limits obtained so far might improve. The bounds on these decays are [22]

$$BR(\tau^-\to e^+\mu^-\mu^-)<2.9\times10^{-6}, \tag{46}$$

$$BR(\tau^-\to e^-e^-e^+)<1.5\times10^{-6}, \tag{47}$$

$$BR(\tau^-\to e^-\mu^-\mu^+)<1.8\times10^{-6}, \tag{48}$$

$$BR(\tau^-\to e^-e^-\mu^+)<1.5\times10^{-6}, \tag{49}$$

$$BR(\tau^-\to \mu^-\mu^-\mu^+)<1.9\times10^{-6}, \tag{50}$$

$$BR(\tau^-\to \mu^-e^-e^+)<1.7\times10^{-6}. \tag{51}$$

The branching ratios for these three-body decays can be expressed as

$$BR(l\to l_1l_2l_3)=\frac{g^2}{(16\pi)^2}\frac{m_l}{\Gamma_l}\frac{m_W^4}{m_{\tilde{l}}^4}|\mathcal{F}_{ll_1l_2l_3}|^2. \tag{52}$$

The transition amplitude of the decay $l(p)\to l_1(p_1)l_2(p_2)\overline{l_3}(p_3)$ receives contributions from γ- (and Z-)mediated graphs shown in Fig. 1 as well as from box diagrams. We distinguish three different cases, depending on the flavors of the final states. These three different amplitudes are conveniently written down as follows:

$$BR(l^-\to l_1^-l_2^-l_3^+,l_1\neq l_3,l_2=l_3)$$
$$=\frac{\left[\sum_{i,j,k=1,2,3}h_{l l_i}h_{l_j l_k}\right]^2}{(4\pi)^4}\frac{m_W^4}{m_{\tilde{l}}^4}\frac{m_l}{\Gamma_l}\left\{|F^{ll_1l_2l_2}_{box}+F^{ll_1}_Z\right.$$
$$-2s_w^2(F_Z^{ll_1}-F_\gamma^{ll_1})|^2+4s_w^4|F_Z^{ll_1}-F_\gamma^{ll_1}|^2$$
$$+8s_w^2\Re[(F_Z^{ll_1}+F^{ll_1l_2l_2}_{box})G_\gamma^{ll_1*}]$$
$$-32s_w^4\Re[(F_Z^{ll_1}-F_\gamma^{ll_1})G_\gamma^{ll_1*}]$$
$$\left.+32s_w^4|G_\gamma^{ll_1}|^2\left[\ln\frac{m_l^2}{m_{l_2}}-3\right]\right\}, \tag{53}$$

where $s_w = \sin\theta_w$ and F_γ^{ll}, $G_\gamma^{ll_1}$ are form factors associated with the one-loop functions f and g for the photon vertex, $F_Z^{ll_1}$ is the form factor associated with the Z vertex, and $F_{box}^{ll_1l_2l_3}$ is the form factor associated with the box diagram. All these composite form factors are defined explicitly in [16]. The decays in this category are $\tau^- \to e^-\mu^-\mu^+$ and $\tau^- \to e^-\mu^-e^+$. The second type of decay is

$$BR(l^- \to l_1^- l_2^- l_3^+, l_3 = l_1 = l_2) = \frac{\left[\sum_{i,j,k=1,2,3} h_{ll_i} h_{l_j l_k}\right]^2}{(4\pi)^4} \frac{m_W^4}{m_{\tilde{l}}^4} \frac{m_l}{\Gamma_l} \times \Bigg\{ 2\left|\frac{1}{2}F_{box}^{ll_1l_1l_1} + F_Z^{ll_1} - 2s_w^2(F_Z^{ll_1} - F_\gamma^{ll_1})\right|^2 + 4s_w^4|F_Z^{ll_1} - F_\gamma^{ll_1}|^2 + 16 s_w^2 \Re\left[\left(F_Z^{ll_1} + \frac{1}{2}F_{box}^{ll_1l_1l_1}\right) G_\gamma^{ll_1 *}\right] - 48 s_w^4 \Re[(F_Z^{ll_1} - F_\gamma^{ll_1}) G_\gamma^{ll_1 *}] + 32 s_w^4 |G_\gamma^{ll_1}|^2 \left[\ln\frac{m_l^2}{m_{l_1}^2} - \frac{11}{4}\right]\Bigg\}. \quad (54)$$

Decays of this type are $\tau^- \to \mu^-\mu^-\mu^+$ and $\tau^- \to e^-e^-e^+$. Finally, the third type of decay can only proceed through box diagrams, and we have

$$BR(l^- \to l_1^- l_2^- l_3^+, l_2 \neq l_3, l_1 \neq l_3) = \frac{\left[\sum_{i,j,k=1,2,3} h_{ll_i} h_{l_j l_k}\right]^2}{(4\pi)^4} \frac{m_W^4}{m_{\tilde{l}}^4} \frac{m_l}{\Gamma_l} |F_{box}^{ll_1l_2l_3}|^2. \quad (55)$$

The decays in this category are $\tau^- \to e^-e^-\mu^+$ and $\tau^- \to \mu^-\mu^-e^+$. We can approximate the branching ratios for the first two types of decay as

$$BR(l^- \to l_1^- l_2^- l_3^+, l_1 \neq l_3, l_2 = l_3) = \frac{\left[\sum_{i,j,k=1,2,3} h_{ll_i} h_{l_j l_k}\right]^2}{(4\pi)^4} \frac{m_W^4}{m_{\tilde{l}}^4} \frac{m_l}{\Gamma_l} 32 s_w^4 |G_\gamma^{ll_1}|^2 \left[\ln\frac{m_l^2}{m_{l_2}^2} - 3\right] \quad (56)$$

and

$$BR(l^- \to l_1^- l_2^- l_3^+, l_1 = l_2 = l_3) = \frac{\sum_{i,j,k=1,2,3} h_{ll_i} h_{l_j l_k}}{(4\pi)^2} \frac{m_W^4}{m_{\tilde{l}}^4} \frac{m_l}{\Gamma_l} 32 s_w^4 |G_\gamma^{ll_1}|^2 \left[\ln\frac{m_l^2}{m_{l_1}^2} - \frac{11}{4}\right] \quad (57)$$

whereas decays of the form $l^- \to l_1^- l_2^- l_3^+, l_2 \neq l_3, l_1 \neq l_3$ can occur through only the box diagram and their contribution is much smaller than the other cases.

V. NUMERICAL RESULTS AND DISCUSSION

In the previous sections we presented analytical expressions for the branching ratios of $\tau \to l\gamma$ and $\tau \to 3l$ in the LRSUSY model. Unfortunately, the large number of parameters makes an exact solution impossible. Before making some approximations, we will discuss all the (real) parameters involved. The electroweak sector of LRSUSY contains three gauge couplings and three gaugino masses. The Higgs sector of the Lagrangian contains the scalar masses m_{Φ_u}, m_{Φ_d}, and M_{LR} as well as the parameters μ_{ij} and B. The remaining part of the Lagrangian contains, in the flavor sector, fermion Yukawa matrices for both left- and right-handed fermions, four trilinear scalar coupling matrices, and scalar mass matrices.

The flavor violating decays are sensitive to the universal grand unified theory (GUT) parameters m_0 (the scalar mass), the trilinear coupling A, the value and the sign of the Higgs mixing parameter μ, the value of $\tan\beta$, the values of the left- and right-handed gaugino masses M_L and M_R, and h_{LR}, the doubly charged Higgsino Yukawa coupling.

In order to protect the decay $\tau \to l\gamma$ from a large contribution from the A term, we assume that all the A terms are *approximately* proportional and that the scalar masses are approximately universal. We assume that the SUSY breaking parameters associated with the supersymmetric Yukawa couplings or masses are proportional to the Yukawa coupling constants or masses, and are given as

$$(m_L^2)_{ij} = (m_R^2)_{ij} = (m_\nu^2)_{ij} = \delta_{ij} m_0^2,$$

$$m_{\tilde{\Phi}_1}^2 = m_{\tilde{\Phi}_2}^2 = m_0^2,$$

$$A_\nu^{ij} = f_{\nu_{ij}} a_0, \quad A_l^{ij} = f_{l_{ij}} a_0,$$

$$B_\nu^{ij} = M_{\nu_i\nu_j} b_0, \quad B_\Phi = \mu b_0. \quad (58)$$

We also include radiative corrections, which we can parametrize using the logarithmic approximation [23]. The supersymmetry breaking masses of sleptons are given by the renormalization group equations. Since the Yukawa couplings and the A terms are flavor violating, they will induce LFV terms in the off-diagonal mass matrices. The soft mass parameters $(\tilde{m}_L)_{32}^2$ and $(\tilde{m}_R)_{32}^2$ will evolve differently, such that the slepton masses receive corrections proportional to the Dirac mass.

The neutrino mix through $\nu_\alpha = K_{\alpha i}\nu_i$, where α denotes the flavor e, μ, τ, and i denotes mass eigenvalues 1,2,3. If we assume that the atmospheric data are fitted by $\sin^2 2\theta_{atm} = 1$ the mixing matrix can be parametrized as

$$K_{\alpha i}=\begin{pmatrix} c & -s & K_{e3} \\ \frac{s}{\sqrt{2}} & \frac{c}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ \frac{s}{\sqrt{2}} & \frac{c}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix}. \tag{59}$$

In the above formula $s=\sin\theta_{sol}$, $c=\cos\theta_{sol}$. (If $\sin^2 2\theta_{atm}=8/9$ the matrix is similar; indeed we need not consider it separately.) In this pattern the solar neutrino deficit is explained either by the small mixing angle Mikheyev-Smirnov-Wolfenstein (MSW) (oscillation enhanced by the solar core) solution (MSW-SA) $\sin\theta_{sol}\approx 5.5\times 10^{-3}$, $\sin\theta_{e\tau}$ and $\sin\theta_{e\mu}$ small; or by the large mixing angle MSW solution (MSW-LA) $\sin^2 2\theta_{sol}\approx 1$. The other solution is for oscillations without help from the solar core, called vacuum oscillations (VO) (or the just-so solution) and it requires $\sin^2 2\theta_{sol}\approx 0.75$. If one includes the Liquid Scintillation Neutrino Detecter (LSND) data, one might need to introduce a sterile neutrino as the fourth neutrino. There are cogent arguments that the only schemes consistent with all the experiments are those that include two pairs of neutrinos with nearly degenerate masses separated by a gap of order 1 eV. This scenario has a 4×4 neutrino mass matrix in which the pairs ν_e-ν_s and ν_μ-ν_τ mix maximally within doublets but mixing between doublets is weak; scenarios with more than one sterile neutrino also exist, but we will not discuss them here [24]. This solution (LSND) requires $\sin^2 2\theta_{sol}\approx 0.003$–$0.03$. In all cases K_{e3} is very small and constrained by the CHOOZ data to be $|K_{e3}|\leqslant 0.2$ [25]. The restrictions on mass splittings coming from these mixings are [24]

$$(\Delta m^2)_{MSW\text{-}LA}\sim 3.5\times 10^{-3}\ \ \mathrm{eV}^2,$$
$$(\Delta m^2)_{MSW\text{-}SA}\sim 5\times 10^{-6}\ \ \mathrm{eV}^2,$$
$$(\Delta m^2)_{VO}\sim 10^{-10}\ \ \mathrm{eV}^2,$$
$$(\Delta m^2)_{LSND}\sim 0.2-2\ \ \mathrm{eV}^2. \tag{60}$$

The limits on neutrino masses come from several considerations. First, there are the direct bounds: $m_{\nu_e}\leqslant 5$ eV, $m_{\nu_\mu}\leqslant 170$ keV, and $m_{\nu_\tau}\leqslant 18$ MeV. However, cosmological restrictions coming from the critical density for neutrino hot dark matter impose neutrino masses whose sum cannot exceed a few eV, $\Sigma\nu_i\leqslant 6$ eV. This would allow the heaviest neutrinos to have masses of order 2 eV, if all three neutrinos are degenerate in mass. Most theoretical fits to the data indicate, however, a tau neutrino of mass 1 eV [24].

Clearly, from the point of view of muon decays, the most interesting scenario is either the MSW-LA or VO scenario; these allow for significant mixing between the e-μ sector if $|K_{e3}|$ is allowed to differ from zero. If, however, $|K_{e3}|\approx 0$, which is a common assumption in the literature, we expect the most pronounced lepton flavor violation to occur in the τ-μ sector and therefore result in a large, possibly observable, $\tau\to\mu\gamma$ branching ratio. The branching ratio for $\tau\to e\gamma$ would be suppressed by comparison.

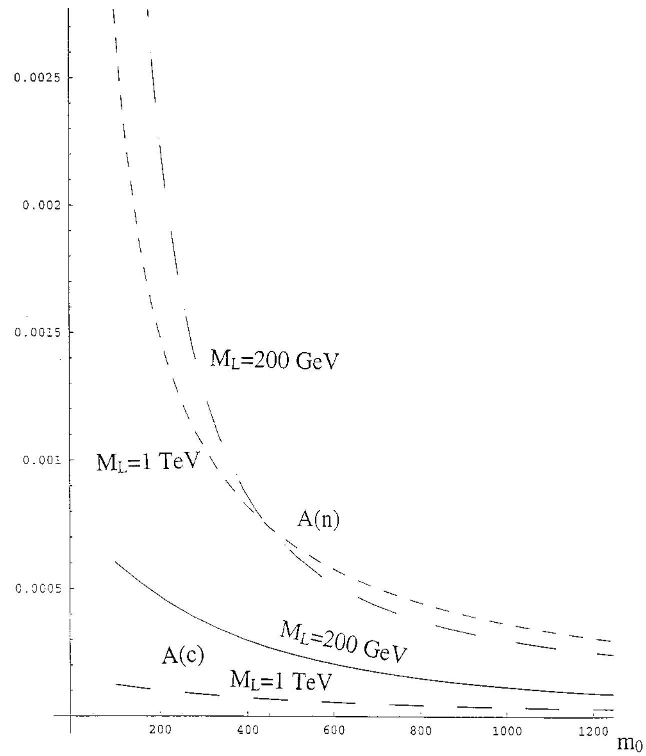

FIG. 2. Chargino and neutralino contributions to the branching amplitude for the decay $\tau\to\mu\gamma$ as a function of the universal scalar mass parameter m_0 for $\tan\beta=3$ for light and heavy left-handed gaugino masses. The curves are marked: (solid curve) chargino contribution for $M_L=200$ GeV, (large dashed curve) chargino contribution for $M_L=1$ TeV, (dot dashed curve) neutralino contribution for $M_L=200$ GeV, (small dashed curve) neutralino contribution for $M_L=1$ TeV. We take $M_R=10$ TeV in all of our plots.

The neutralino contribution to the branching ratio is dominant throughout a large part of the parameter space as seen in Fig. 2. We plot the absolute values of the relative contributions of the chargino and neutralino for the light M_L ($M_L=200$ GeV) and heavy M_L ($M_L=1$ TeV) scenarios, as function of the universal scalar mass m_0. (The contribution from the doubly charged Higgsino is much smaller and off the graph, justifying, from the point of view of τ decays, neglecting the contribution from the left-handed triplet Higgsino.) The chargino contribution should be different from the one in MSSM with right-handed neutrino because of the contribution of the right-handed gaugino. However, the right-handed chargino contribution is suppressed by the decoupling of the right-handed sneutrinos from the low energy spectrum and the neutralino contribution dominates for most of the spectrum. The left-handed chargino contribution is similar to that in the MSSM with right-handed neutrinos, except that often there are three light charginos. For the neutralino contribution, in most cases there are at least three light neutralino states. The situation is relatively insensitive to the right-handed M_R scale, for $M_R\approx 10$–100 TeV, since the number of light neutralino states is the same in this M_R range. The amplitude for $\tau\to\mu\gamma$ is also relatively insensitive to the various neutrino mixing scenarios, since it depends on the ν_τ-ν_μ mixing, which is expected to be large in all

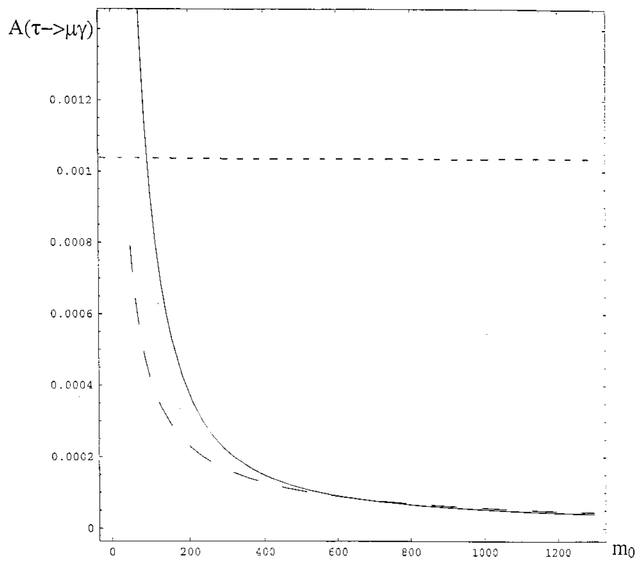

FIG. 3. Total branching amplitude for the decay $\tau \rightarrow \mu\gamma$ as a function of the universal scalar mass m_0 for both the light and heavy gaugino mass scenarios. We set $\tan\beta = 3$ and $M_R = 10$ TeV. The curves are marked: (solid curve) the branching amplitude for $M_L = 200$ GeV and (large dashed curve) the branching amplitude for $M_L = 1$ TeV. The experimental bound is 1.04×10^{-3} (small dashed curve).

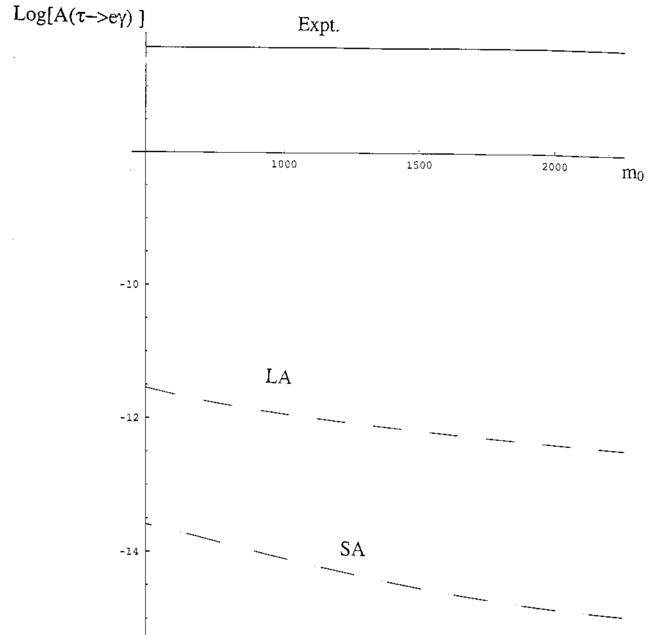

FIG. 4. Logarithmic plot of the branching amplitude for $\tau \rightarrow e\gamma$ as a function of the universal scalar mass for the large angle neutrino mixing scenario (LA, the dashed curve); and the small angle neutrino mixing scenario (SA, the dot dashed curve). The experimental limit is plotted for comparison as the solid line.

schemes. The comparison with the experimental value is shown in Fig. 3 for both light chargino (low M_L, solid curve) and heavy chargino (high M_L, dashed curve) scenarios. The predicted amplitude is very close to the experimental bound (1.04×10^{-3}) for light supersymmetric masses, and indeed it exceeds the bound for scalar masses $m_0 \leqslant 200$ GeV, for the light left-handed gaugino scenario. Even for $M_L = 1$ TeV a signal should be detected for light scalar masses with only a slightly improved precision in the measurement of the branching ratio of $\tau \rightarrow \mu\gamma$. In contrast, the branching ratio $\tau \rightarrow e\gamma$ is very sensitive to the neutrino mixing schemes, as seen in Fig. 4 for the large angle (dashed curve) and small angle (dot dashed curve) scenarios. Unfortunately, in both the LA (or VO) and the SA mixing schemes, the amplitude is several orders of magnitude smaller than the experimental bound, as shown in the graph by the solid line; thus $\tau \rightarrow e\gamma$ does not appear to be a promising area for testing LFV's. Figures 5 and 6 show the dependence of the amplitude of branching ratio for $\tau \rightarrow \mu\gamma$ on other parameters of the model. In Fig. 5 we plot the dependence on the left-handed gaugino mass M_L for two values of the universal scalar mass. For $m_0 = 100$ GeV and $M_L \leqslant 200$ GeV, the predicted branching ratio (solid curve) exceeds the present experimental bound. This situation seems to be alleviated for $m_0 = 500$ GeV. Figure 6 plots the dependence of the amplitude on $\tan\beta$ for two values of scalar mass: $m_0 = 200$ GeV and $m_0 = 700$ GeV. The dependence of the branching ratio on $\tan\beta$ comes from mixing matrix elements in the chargino-neutralino sectors and increases rapidly with $\tan\beta$, such that the restrictions on the parameter space for large $\tan\beta$ are very stringent. Indeed, for $m_0 = 200$ GeV and $M_L = 200$ GeV plot points with $\tan\beta \geqslant 10$ are ruled out experi-

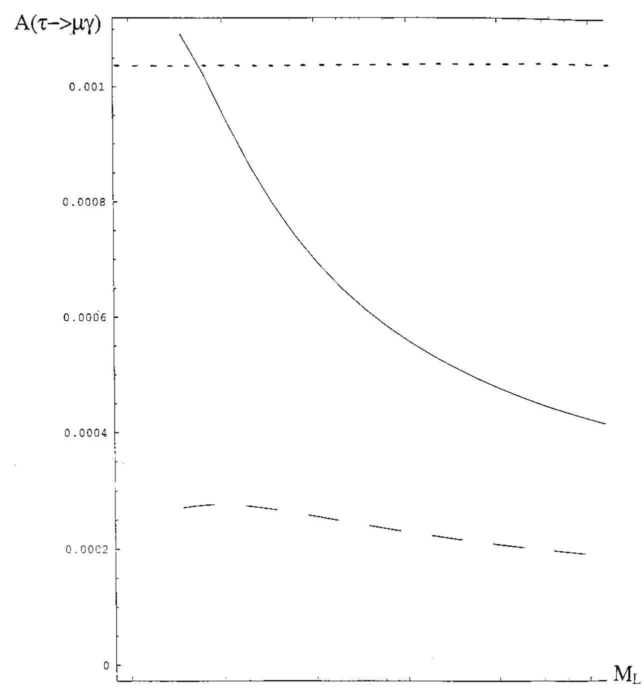

FIG. 5. Total branching amplitude for the decay $\tau \rightarrow \mu\gamma$ as a function of the left-handed gaugino mass M_L for the light and intermediate scalar mass scenarios. We set $\tan\beta = 3$ and $M_R = 10$ TeV. The curves are marked: (solid curve) the branching amplitude for $m_0 = 100$ GeV and (large dashed curve) the branching amplitude for $m_0 = 500$ GeV. The experimental bound is 1.04×10^{-3} (small dashed curve).

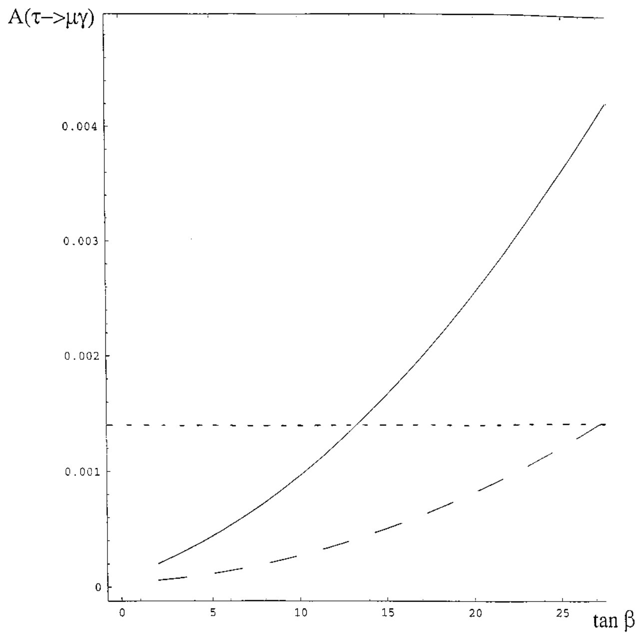

FIG. 6. Total branching amplitude for the decay $\tau\to\mu\gamma$ as a function of $\tan\beta$ for the light and heavy scalar mass scenarios. We set $M_L=200$ GeV and $M_R=10$ TeV. The curves are marked: (solid curve) the branching amplitude for $m_0=200$ GeV and (large dashed curve) the branching amplitude for $m_0=700$ GeV. The experimental bound is 1.04×10^{-3} (small dashed curve).

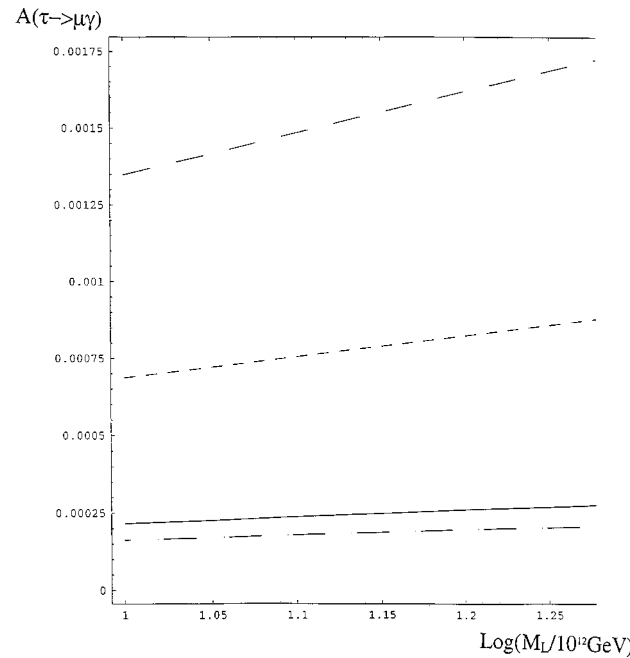

FIG. 7. Total branching amplitude for the decay $\tau\to\mu\gamma$ as a function of the right-handed neutrino mass $\log(M_N/10^{12})$ for the light and heavy gaugino mass scenarios; and for small and intermediate $\tan\beta$. We set $m_0=300$ GeV and $M_R=10$ TeV. The curves are marked: (solid curve) the branching amplitude for $M_L=200$ GeV and $\tan\beta=3$ GeV; (dot dashed curve) the branching amplitude for $M_L=1$ TeV and $\tan\beta=3$; (large dashed curve) the branching amplitude for $M_L=200$ GeV and $\tan\beta=15$; (small dashed curve) the branching amplitude for $M_L=1$ TeV and $\tan\beta=15$.

mentally. Finally, we show in Fig. 7 the dependence of the branching ratio on the mass of the (third generation) right-handed neutrino M_N for a variety of M_L, the left-handed gaugino mass, and $\tan\beta$ scenarios. This mass is restricted by the seesaw mechanism to be in the 10^{12}–10^{15} GeV region. For the parameter space considered, the graphs are somewhat sensitive to $\log(M_N/10^{12})$; the branching amplitude seems to be more sensitive to low values of the mass parameters and large values of $\tan\beta$ and it exceeds the experimental bound for large $\tan\beta=15$, $m_0=300$ GeV, and $M_L=200$ GeV for all values of the right-handed neutrino mass; but this is more of a consequence of the $\tan\beta$ than of the M_N dependence.

Next we discuss the numerical estimates for the three-body decay of the τ. The formulas given for the decays $\tau\to3l$ simplify if we compare them with the branching ratios of $\tau\to l\gamma$:

$$BR(\tau\to3\mu)\cong BR(\tau\to\mu\gamma)\frac{\alpha}{3\pi}\left\{\log\frac{m_\tau^2}{m_\mu^2}-\frac{11}{4}+\frac{3}{8}\left(\frac{1}{1-F(x_{ki})}\right)\right\}, \tag{61}$$

$$BR(\tau\to\mu e^+e^-)\cong BR(\tau\to\mu\gamma)\frac{\alpha}{3\pi}\left\{\log\frac{m_\tau^2}{m_e^2}-3+\frac{1}{4}\left(\frac{1}{1-F(x_{ki})}\right)\right\}, \tag{62}$$

$$BR(\tau\to e\mu^+\mu^-)\cong BR(\tau\to e\gamma)\frac{\alpha}{3\pi}\left\{\log\frac{m_\tau^2}{m_\mu^2}-3+\frac{1}{4}\left(\frac{1}{1-F(x_{ki})}\right)\right\}, \tag{63}$$

$$BR(\tau\to3e)\cong BR(\tau\to e\gamma)\frac{\alpha}{3\pi}\left\{\log\frac{m_\tau^2}{m_e^2}-\frac{11}{4}+\frac{3}{8}\left(\frac{1}{1-F(x_{ki})}\right)\right\} \tag{64}$$

with $F(x_{ki})$ a correction function given below:

$$F(x_{ki})=\sum_{ki}\left[\frac{m_{\chi_k^0}}{m_\tau}N_{ki}^LN_{ki}^Rf_N(x_{ki})+\frac{m_{\chi_k^\pm}}{m_\tau}C_{ki}^LC_{ki}^Rf_C(x_{ki})\right], \tag{65}$$

$$f_N(x_{ki})=\frac{1-3x+2x\log x}{13-60x+111x^2-106x^3-6x^2(6x-1)\log x}, \tag{66}$$

$$f_C(x_{ki})=\frac{3-4x+x^2+2\log x}{98-267x+210x^2-41x^3+6(12-7x)\log x}. \tag{67}$$

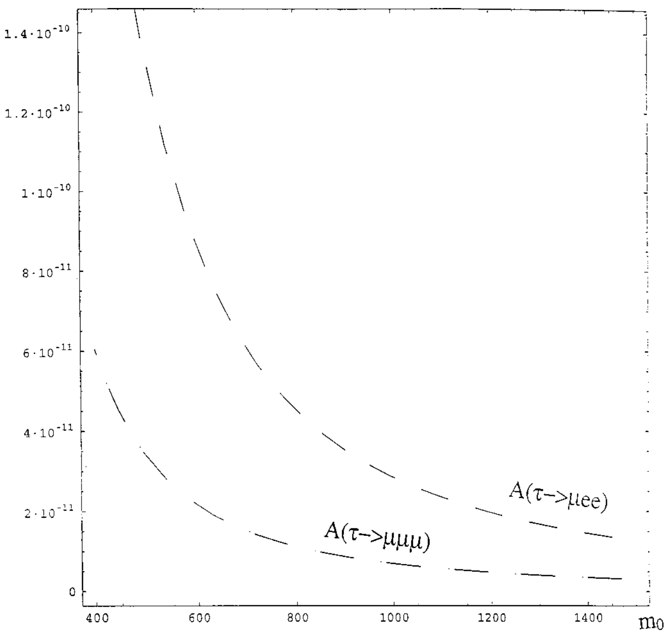

FIG. 8. Total branching ratio for the decay $\tau^- \to \mu^- e^+ e^-$ (large dashed curve) and $\tau^- \to \mu^- \mu^+ \mu^-$ (dot dashed curve) as a function of the universal scalar mass m_0 for the light left-handed gaugino mass scenario $M_L = 200$ GeV. We set $\tan\beta = 3$ and $M_R = 10$ TeV.

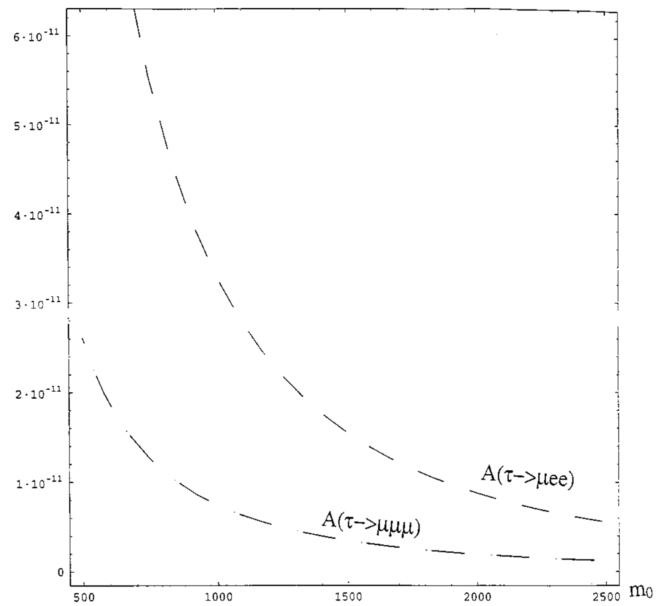

FIG. 9. Same as Fig. 8, but for $M_L = 1$ TeV.

We neglect the contribution of the doubly charged Higgsino, as it amounts to only a few percentage of the total branching ratio. Figures 8 and 9 show the branching ratios for $\tau \to \mu\mu\mu$ and $\tau \to \mu e^+ e^-$ as a function of the universal scalar mass m_0 for the, respectively, light ($M_L = 200$ GeV) and heavy ($M_L = 1$ TeV) left-handed gaugino mass scenarios. For a wide range of the parameter space, these branching ratios are expected to be smaller than $\mathcal{O}(10^{-9})$ for regions in which $\tau \to \mu\gamma$ is in the experimentally allowed region, the largest being $\tau \to \mu e^+ e^-$. We predict

$$BR(\tau \to \mu e^+ e^-) \leqslant 2 \times 10^{-10}, \tag{68}$$

$$BR(\tau \to 3\mu) \leqslant 7 \times 10^{-11}, \tag{69}$$

and $BR(\tau \to \mu e^+ e^-), BR(\tau \to \mu e^+ e^-) \leqslant 10^{-14}$. Even in the case in which the predicted branching ratio for $\tau \to \mu\gamma$ is close to the experimental bound, the three-body decays are much smaller. The decays that occur only through the box diagram ($\tau \to \mu^- \mu^- e^+$ and $\tau \to \mu^- e^- e^-$) require an improvement factor of n^2 in the measurement of the branching ratio for a corresponding improvement factor of n for the $\tau \to l\gamma$ branching ratio.

VI. CONCLUSION

The solar and atmospheric neutrino experiements have renewed interest in lepton flavor violation. If the neutrinos mix, they will induce LFV decays in the charged lepton sector. Various neutrino mixing schemes accommodate neutrino oscillations by allowing a large mixing angle between ν_τ and ν_μ; however they vary in their estimate of the ν_e-ν_μ mixing angle. One would expect therefore large flavor violations to be produced in the decays of the τ lepton. We present here an analysis of these decays in a model that incorporates the seesaw mechanism in a fully left-right supersymmetric gauge structure. LFV decays of the τ present new features compared to the MSSM with right-handed neutrinos or extended group structures such as SU(5). The prediction of the model is that the branching ratio for the decay $\tau \to \mu\gamma$ is large, sometimes as large as, or exceeding, the present experimental bound. It is very sensitive to variations in m_0, the universal scalar mass, and $\tan\beta$ and somewhat sensitive to variations in the mass of the left-handed gaugino M_L. But the branching ratio is stable against variations in neutrino mixing schemes and depends only weakly on the mass of the (third generation) right-handed neutrino. Although the model depends on many parmeters, regions of light scalar masses ($m_0 \leqslant 100$ GeV) and light gauginos ($M_L \leqslant 100$ GeV) are ruled out for all values of the other parameters within all three neutrino mixing schemes investigated (large angle, small angle, or vacuum oscillations). The restriction is more stringent for intermediate values of $\tan\beta \sim 10$–15. An increase in the present experimental precision by a factor of 10–100 would restrict the parameter space even further. By contrast the branching ratio for $\tau \to e\gamma$ is several orders of magnitude smaller and sensitive to whether the neutrinos mix under the large angle or just-so (LA or VO) or small angle (SA) scenarios. The relative strength of the two decays of the τ is a very sensitive indicator of the neutrino mixing scheme. Tau decays may be a better indicator of LFV than the μ decays $\mu \to e\gamma$ or μ-e conversion [26], and they should be, if the present understanding of the neutrino mixing is correct. Of the three-body lepton flavor violating decays of the τ, the largest branching ratio is obtained for the decay $\tau \rightarrow \mu e^+ e^-$, but unfortunately still several orders of magnitude below the present bound. In conclusion, the LR-SUSY model makes definite predictions for τ decays that can be tested if LFV decays are discovered, or if the experimental limits can be improved in the near future.

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