D-branes as solitons of an $\mathcal{N}=1$, $D=10$ noncommutative gauge theory

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(Received 21 November 2000; published 23 July 2001)

We consider a Dp-brane within a D9-brane in the presence of a B field whose polarization is *transverse* to the Dp-brane. To be definite, we take a D3-D9 system. It is observed that the system has the same pattern of supersymmetry breaking as that of a soliton of the six-dimensional noncommutative gauge theory that is obtained by dimensional reduction of an $\mathcal{N}=1,D=10$ gauge theory. These results indicate that the soliton solution is the low energy realization of a D3-brane in a D9-brane with a transverse B field and, hence, can be viewed as a generalization of the previous results in the literature where similar observations were made for lower codimensional cases.

DOI: 10.1103/PhysRevD.64.046006 PACS number(s): 11.25.Sq, 11.30.Pb

I. INTRODUCTION

D-branes appear when one T dualizes open strings with fully Neumann boundary conditions. Since the duality connects two equivalent descriptions, it is worth asking whether open string theory has a different way of incorporating D-branes. The most natural candidate that could provide such an incorporation may be the soliton solutions of the low energy effective action of open string theory.

Progress toward different realizations of D-branes has been made[1] in the framework of open bosonic string field theory [3] and noncommutative gauge theory [4,5]. In particular, it was argued that D-branes can be constructed as solitons of open string field theory [6–8]. A similar claim has also been made for the soliton solutions of noncommutative gauge theories [9–15]. In this paper, we study the pattern of supersymmetry breaking of such noncommutative solitons.

To be definite, we consider a D3-D9 system although various other systems can be analyzed similarly. (Various properties of generic p-p' systems were studied in [16] that have some related discussions.[2]) We consider an oriented open superstring description of D-branes [17,18] using the Green-Schwarz (GS) formalism [19]. A constant B field is turned on with the only nonzero components *transverse* to the D3-brane. The boundary conditions of the system impose a constraint on the parameters of the supersymmetry transformations. The constraint equation in an appropriate limit is to be compared with the corresponding gauge theory result.

The low energy limit of open superstring theory is an $\mathcal{N}=1$ gauge theory in $D=10$. We dimensionally reduce the theory to $D=6$. With the same B-field configuration as that of the D3-D9 system, one gets six-dimensional noncommutative gauge theory. Following [12], soliton solutions are written down. By a trivial lifting procedure, they can also be considered as solutions of the ten-dimensional gauge theory that one started with. We examine the supersymmetry preserving conditions and observe that the supersymmetry breaking pattern is equivalent to the constraint equation obtained for the D3-D9 system. This can be viewed as a generalization of the previous results in the literature where similar observations were made for lower codimensional cases.

The rest of the paper is organized as follows. In Sec. II, we consider a D3-D9-brane system with the only nonzero components transverse to the D3-brane. The boundary condition at one end of the open string is different from the one at the other end. This makes it necessary, for any possible residual supersymmetry, to impose an extra constraint among supersymmetry parameters in addition to the ordinary one that breaks one-half of the supersymmetry. In Sec. III, we consider an $\mathcal{N}=1$ supersymmetric gauge theory in $D=10$ and dimensionally reduce it to six dimensions that have the nonzero B components. Therefore the six-dimensional theory is noncommutative. We show that the supersymmetry breaking pattern for the soliton solutions is equivalent to that of the D3-D9 system. Although the supersymmetry gets completely broken for a generic configuration of the B field, one can choose the values of the B components such that the system has residual supersymmetry. It is shown in Sec. IV that there are no tachyons for those values of the B field. We end with conclusions in Sec. V.

II. OPEN SUPERSTRING ANALYSIS

Consider an open superstring in a flat ten-dimensional background. We turn on a constant B field and impose Dirichlet boundary conditions appropriate for a given D-brane configuration. We briefly review [17,18] for a D3-brane case where the authors considered a B field polarized along the brane direction. Then we move to the case of a transverse B field.

In the Green-Schwarz formulation, the action with a B field is given by[3]

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[1]Earlier discussions of D-branes as instantons can be found, for example, in [1,2].

[2]We thank E. Witten for bringing this paper to our attention.

[3]We denote the coordinate by $M=(\mu,m)$, $\mu=0,\ldots,3$, $m=4,\ldots,9$.

$$S=-\frac{1}{2\pi}\int d^2\sigma[\sqrt{-g}g^{ij}\Pi_i{}^M\Pi_j{}^N\eta_{MN}$$
$$+2i\epsilon^{ij}\partial_iX^M(\overline{\theta^1}\Gamma_M\partial_j\theta^1-\overline{\theta^2}\Gamma_M\partial_j\theta^2)-2\epsilon^{ij}(\overline{\theta^1}\Gamma^M\partial_i\theta^1)$$
$$\times(\overline{\theta^2}\Gamma_M\partial_j\theta^2)+\epsilon^{ij}\partial_iX^M\partial_jX^NB_{MN}]. \tag{1}$$

By taking the variations with respect to X and θ, one can obtain the following boundary terms:

$$\delta X_M(\Pi_\sigma{}^M-i\overline{\theta^1}\Gamma^M\partial_\tau\theta^1+i\overline{\theta^2}\Gamma^M\partial_\tau\theta^2)+\delta X_M\partial_\tau X^NB_N{}^M$$
$$-i(\overline{\theta^A}\Gamma_M\delta\theta^A)\Pi_\sigma{}^M+i\partial_\tau X^M(\overline{\theta^1}\Gamma_M\delta\theta^1-\overline{\theta^2}\Gamma_M\delta\theta^2)$$
$$+(\overline{\theta^1}\Gamma^M\delta\theta^1\overline{\theta^2}\Gamma_M\partial_\tau\theta^2-\overline{\theta^2}\Gamma^M\delta\theta^2\overline{\theta^1}\Gamma_M\partial_\tau\theta^1)|_{\sigma=0,\pi}$$
$$=0. \tag{2}$$

With the B field whose nonzero components are only along the brane directions, the equation above becomes

$$\delta X_M(\Pi_\sigma{}^M-i\overline{\theta^1}\Gamma^M\partial_\tau\theta^1+i\overline{\theta^2}\Gamma^M\partial_\tau\theta^2)+\delta X_\mu\partial_\tau X^\nu B_\nu{}^\mu$$
$$-i(\overline{\theta^A}\Gamma_M\delta\theta^A)\Pi_\sigma{}^M+i\partial_\tau X^M(\overline{\theta^1}\Gamma_M\delta\theta^1-\overline{\theta^2}\Gamma_M\delta\theta^2)$$
$$+(\overline{\theta^1}\Gamma^M\delta\theta^1\overline{\theta^2}\Gamma_M\partial_\tau\theta^2-\overline{\theta^2}\Gamma^M\delta\theta^2\overline{\theta^1}\Gamma_M\partial_\tau\theta^1)|_{\sigma=0,\pi}$$
$$=0. \tag{3}$$

The X variation vanishes if one imposes the following boundary conditions:

$$\partial_\tau X^m=0,\quad \partial_\sigma X^\mu+\partial_\tau X^\nu B_\nu{}^\mu=0,$$
$$\overline{\theta^1}\Gamma^\mu(\partial_\sigma+\partial_\tau)\theta^1+\overline{\theta^2}\Gamma^\mu(\partial_\sigma-\partial_\tau)\theta^2=0. \tag{4}$$

The part containing $\delta\theta$ can be rewritten as

$$i\partial_\sigma X^m(\overline{\theta^1}\Gamma_m\delta\theta^1+\overline{\theta^2}\Gamma_m\delta\theta^2)+i\partial_\tau X^\mu[(1+B)_{\mu\nu}\overline{\theta^2}\Gamma^\nu\delta\theta^2$$
$$-(1-B)_{\mu\nu}\overline{\theta^1}\Gamma^\nu\delta\theta^1]+\overline{\theta^1}\Gamma_M\delta\theta^1\overline{\theta^1}\Gamma^M\partial_\tau\theta^1$$
$$-\overline{\theta^2}\Gamma_M\delta\theta^2\overline{\theta^2}\Gamma^M\partial_\tau\theta^2=0. \tag{5}$$

This can be satisfied by imposing the following relation between the two θ's:

$$\theta^2=P(B_\parallel)\theta^1\quad\text{at}\quad\sigma=0,\pi, \tag{6}$$

where

$$P(B_\parallel)\equiv e^{(-Y^{\mu\nu}\Gamma_{\mu\nu}\sigma_3/2)}i\sigma_2\Gamma^{0\cdots3}, \tag{7}$$

with

$$Y=\frac{1}{2}\ln\left(\frac{1-B}{1+B}\right). \tag{8}$$

Now, we consider the D3-D9 system having a B field along the transverse directions to D3. The boundary conditions are as follows:

σ	X^0	X^1	X^2	X^3	X^4	X^5	X^6	X^7	X^8	X^9
0	N	N	N	N	D	D	D	D	D	D
π	N	N	N	N	ND	ND	ND	ND	ND	ND

where ND denotes the boundary condition of the type $\partial_\sigma X+\partial_\tau X\cdot B$. It is convenient to examine the boundary conditions at $\sigma=0$ and $\sigma=\pi$ separately.

A. Boundary conditions at $\sigma=0$

First consider boundary conditions at $\sigma=0$. We require the following conditions:

$$\partial_\tau X^m=0,$$
$$\partial_\sigma X^\mu=0,$$
$$\overline{\theta^1}\Gamma^M(\partial_\sigma+\partial_\tau)\theta^1+\overline{\theta^2}\Gamma^M(\partial_\sigma-\partial_\tau)\theta^2=0. \tag{9}$$

Two remarks are in order: first, it is important that the B field does not appear in the equations above whereas it does appear in the boundary conditions at $\sigma=\pi$. This difference is what induces an additional constraint on supersymmetry parameters as will be discussed below. Second, although we can replace Γ^M in the third equation by Γ^μ, we keep the full Γ matrices because at $\sigma=\pi$ we cannot make the same replacement. With these conditions, it is easy to show that the δX part vanishes. The remaining $\delta\theta$ part is as follows:

$$-i\partial_\sigma X^m(\overline{\theta^1}\Gamma_m\delta\theta^1+\overline{\theta^2}\Gamma_m\delta\theta^2)+i\partial_\tau X^\mu[\delta_{\mu\nu}\overline{\theta^1}\Gamma^\nu\delta\theta^1$$
$$-\delta_{\mu\nu}\overline{\theta^2}\Gamma^\nu\delta\theta^2]+\overline{\theta^1}\Gamma_M\delta\theta^1\overline{\theta^1}\Gamma^M\partial_\tau\theta^1$$
$$-\overline{\theta^2}\Gamma_M\delta\theta^2\overline{\theta^2}\Gamma^M\partial_\tau\theta^2=0, \tag{10}$$

which implies that a relation between θ^1 and θ^2 is

$$\theta^2=P_{\sigma=0}(B_\perp)\theta^1, \tag{11}$$

where

$$P_{\sigma=0}(B_\perp)=i\sigma_2\Gamma_{4\cdots9}. \tag{12}$$

B. Boundary conditions at $\sigma=\pi$

The boundary conditions that remove the δX part are

$$\partial_\sigma X^\mu=0,$$
$$\partial_\sigma X^m+\partial_\tau X^nB_n^m=0,$$
$$\overline{\theta^1}\Gamma^M(\partial_\sigma+\partial_\tau)\theta^1+\overline{\theta^2}\Gamma^M(\partial_\sigma-\partial_\tau)\theta^2=0. \tag{13}$$

Unlike Eq. (9), Eq. (13) contains the B field, which will, in turn, modify the constraint equation (11) accordingly. The remaining $\delta\theta$ part is

$$i\partial_\sigma X^\mu(\overline{\theta^1}\Gamma_\mu\delta\theta^1-\overline{\theta^2}\Gamma_\mu\delta\theta^2)+i\partial_\tau X^m[(-1+B)_{mn}\overline{\theta^2}\Gamma^n\delta\theta^2 +(1+B)_{mn}\overline{\theta^1}\Gamma^n\delta\theta^1]+\overline{\theta^1}\Gamma_M\delta\theta^1\overline{\theta^1}\Gamma^M\partial_\tau\theta^1 -\overline{\theta^2}\Gamma_M\delta\theta^2\overline{\theta^2}\Gamma^M\partial_\tau\theta^2=0. \quad (14)$$

The modified relation between the θ's is

$$\theta^2=P_{\sigma=\pi}(B_\perp)\theta^1, \quad (15)$$

where

$$P_{\sigma=\pi}(B_\perp)=e^{(-Y^{mn}\Gamma_{mn}\sigma_3/2)}i\sigma_2\Gamma^{4\cdots 9}, \quad (16)$$

with

$$Y=\frac{1}{2}\ln\left(\frac{1+B}{1-B}\right). \quad (17)$$

The action (1) is invariant under the following supersymmetry transformation:

$$\delta X^M=i\overline{\epsilon^A}\Gamma^M\theta^A,$$
$$\delta\theta=\epsilon^A. \quad (18)$$

In order for these transformations to be consistent with the boundary conditions discussed above, it must be satisfied that

$$\epsilon^2=P_{\sigma=0}(B_\perp)\epsilon^1, \quad \epsilon^2=P_{\sigma=\pi}(B_\perp)\epsilon^1. \quad (19)$$

This implies

$$Y^{mn}\Gamma_{mn}\epsilon^1=0, \quad Y^{mn}\Gamma_{mn}\epsilon^2=0. \quad (20)$$

In the limit $\alpha'\to 0$ discussed in [20], the leading term of Eq. (17) is

$$Y^{mn}\Gamma_{mn}\sim\left(\frac{1}{B}\right)^{mn}\Gamma_{mn}. \quad (21)$$

It follows that we have the following two conditions:

$$\epsilon_2=\mathrm{i}\sigma^2\Gamma_{4,\ldots,9}\epsilon_1,$$
$$\left(\frac{1}{B}\right)^{mn}\Gamma_{mn}\epsilon_1=0. \quad (22)$$

The first equation breaks one-half of the supersymmetry, leaving 16 supercharges. The second equation breaks further, leaving no supersymmetry in general. We can apply the above discussion, e.g., to the case of D5-D9 by replacing $\Gamma_{4,\ldots,9}$ with $\Gamma_{6,\ldots,9}$. The corresponding open string theory is supersymmetric only for B's having special properties. For the D5-D9 case, the supersymmetry preserving B is self-dual or anti-self-dual and in this case the system preserves eight supercharges.

We also require that the boundary conditions for the bosonic fields X's be compatible with the supersymmetry transformations and obtain further constraints for the fermions θ's. It is easy to check that the boundary conditions for fermions at both ends are

$$(\partial_\sigma+\partial_\tau)\theta_1=0, \quad (\partial_\sigma-\partial_\tau)\theta_2=0. \quad (23)$$

These conditions are consistent with the light cone gauge and they solve the last conditions in Eqs. (9) and (13).

III. GAUGE THEORY ANALYSIS

In this section, we consider an $\mathcal{N}=1$ Abelian gauge theory in ten dimensions among which six directions are noncommutative. We use then the methods developed in [10–15] for noncommutative spaces to construct solitons in the six-dimensional noncommutative gauge theory. We compare the supersymmetry breaking pattern for these solutions with the one obtained in the previous section. We observe that they are equivalent to each other.

The action for an $\mathcal{N}=1$ Abelian gauge theory[4] in $D=10$ is

$$-S=\int d^{10}x\sqrt{G}\left(\frac{1}{4}F_{MN}F^{MN}+\frac{1}{2}\overline{\Psi}\Gamma^M D_M\Psi\right). \quad (24)$$

Since we take the same B-field configuration as in the previous section, six dimensions become noncommutative. The noncommutativity is parametrized by

$$[x^4,x^5]\equiv i\Theta^{45}\equiv i\theta^1, \quad [x^6,x^7]\equiv i\Theta^{67}\equiv i\theta^2,$$
$$[x^8,x^9]\equiv i\Theta^{89}\equiv i\theta^3. \quad (25)$$

It is also convenient to introduce a complex coordinate system defined by

$$z^1=\frac{x^4+\mathrm{i}x^5}{\sqrt{2}}, \quad z^2=\frac{x^6+\mathrm{i}x^7}{\sqrt{2}}, \quad z^3=\frac{x^8+\mathrm{i}x^9}{\sqrt{2}} \quad (26)$$

and define the creation and annihilation operators

$$a_p^\dagger\equiv i\Theta^{-1}_{p\bar{q}}\bar{z}^{\bar{q}}, \quad a_{\bar{p}}\equiv -i\Theta^{-1}_{\bar{p}q}z^q, \quad p=1,2,3. \quad (27)$$

Here we have chosen the annihilation and creation operators in accordance with the positivity of the $\theta_{1,2,3}$. Namely, for positive θ's the above notation is valid, whereas for negative θ's we have to switch the operators. We assume from now on that our θ's are all positive. In the ten-dimensional action, we replace the integration over the six noncommutative dimensions with a trace over the Hilbert space enacted by the creation and annihilation operators. We also replace the covariant derivative and the gauge field strengths on the noncommutative space in an appropriate way:

[4] For an ordinary Abelian gauge theory, it is a partial derivative ∂_M that acts on Ψ. For a theory of noncommutative spaces the fields do not commute and it is necessary to replace the partial derivative by a covariant derivative.

$$
\begin{aligned}
D_p &\to -[C_p,],\\
F_{p\bar{q}} &\to \mathrm{i}[C_p, \overline{C_{\bar{q}}}] - \Theta^{-1}_{p\bar{q}},\\
F_{pq} &\to -\mathrm{i}[C_p, C_q],\\
F_{\bar{p}\bar{q}} &\to -\mathrm{i}[\overline{C_{\bar{p}}}, \overline{C_{\bar{q}}}],
\end{aligned} \tag{28}
$$

where $C_p = -\mathrm{i}A_p + a_p^\dagger$. We consider, for simplicity, that the metric on the noncommutative directions takes the simple form $G_{p\bar{q}} = \delta_{p\bar{q}}$. We make the above replacements in the ten-dimensional action and obtain

$$
\begin{aligned}
S = &-\frac{(2\pi)^3\sqrt{-\det\Theta}}{g_{YM}^2}\int \sqrt{G}\, d^4x\, \mathrm{Tr}\bigg(\frac{1}{4}F_{\mu\nu}F^{\mu\nu}\\
&+\frac{1}{2}([C_p, \overline{C_{\bar{q}}}] + \mathrm{i}\Theta^{-1}_{p\bar{q}})([C_q, \overline{C_{\bar{p}}}] + \mathrm{i}\Theta^{-1}_{q\bar{p}}) - \frac{1}{2}[C_p, C_q]\\
&\times[\overline{C_{\bar{p}}}, \overline{C_{\bar{q}}}] + D_\mu C_p D^\mu \overline{C_{\bar{p}}} + \frac{1}{2}\bar{\psi}\Gamma^\mu D_\mu\psi + \frac{1}{2}\bar{\psi}\Gamma_p[\overline{C_{\bar{p}}}, \psi]\\
&-\frac{1}{2}\bar{\psi}\Gamma_{\bar{p}}[C_p, \psi]\bigg).
\end{aligned} \tag{29}
$$

The supersymmetry transformations in six dimensions can be obtained from those of the ten-dimensional theory by the same replacements we used for the action:

$$
\begin{aligned}
\delta A_\mu &= \frac{1}{2}\bar{\eta}\Gamma_\mu\psi,\\
\delta C_p &= \frac{1}{2}\bar{\eta}\Gamma_p\psi,\\
\delta\psi &= -\frac{1}{4}(F_{\mu\nu}\Gamma^{\mu\nu} + 2D_\mu C_p\Gamma^{\mu p} + F_{mn}\Gamma^{mn})\eta,
\end{aligned} \tag{30}
$$

where $m, n = p, \bar{p}$ in the last equation. These transformations have the same form even when we leave the metric G arbitrary.

We consider now the solitonic solutions presented in [12] and establish when these solutions preserve supersymmetry. The solutions have nontrivial values for C_p and do not depend on the commutative coordinates:

$$
C_p = S^\dagger a_p^\dagger S, \quad \overline{C_{\bar{p}}} = S^\dagger a_p S. \tag{31}
$$

The operator S has the property that $SS^\dagger = 1$ and $S^\dagger S = 1 - P_0$, where P_0 is the projector on the vacuum $|0\rangle$ $(a_p|0\rangle = 0$ for any $p)$. As such, it is a static solution and has a single unit of topological charge. The solution has field strength in the noncommutative directions: $F_{p\bar{q}} = -P_0\Theta^{-1}_{p\bar{q}}$. Solutions with arbitrary positive charge m are obtained by simply replacing S with S^m. The discussion for the single charged solutions applies equally well for $m > 1$. The first two equations of Eqs. (30) are trivially satisfied. The first and second terms of the ψ variation vanish once the solution is substituted. Therefore the only nontrivial part is the last term:

$$
\delta\psi = -\frac{1}{4}F_{p\bar{q}}\Gamma^{p\bar{q}}\epsilon = 0. \tag{32}
$$

Since the indices in Eqs. (22) are contracted by using the closed string metric g_{mn}, we need to convert Eq. (32) into an expression where the indices are also contracted with respect to the closed string metric. Using the solution, one can rewrite Eq. (32) as

$$
\delta\psi = \frac{1}{4}P_0\Theta^{-1}{}_{p\bar{q}}E^p{}_c E^{\bar{q}}{}_{\bar{d}}\Gamma^{c\bar{d}}\epsilon, \tag{33}
$$

where c and $\bar{d}$ (p and $\bar{q}$) are flat (curved) indices. E_c^p represents the transverse components of the inverse 10-beins of the open string metric. They are related to the corresponding components of the closed string 10-beins, e_p^c, by [20]

$$
E_p^c = \mathrm{i}(2\pi\alpha')B_{p\bar{s}}e^{\bar{s}c}. \tag{34}
$$

Substitution of Eq. (34) into Eq. (33) leads to

$$
\delta\psi = \frac{1}{4\pi^2\alpha'^2}P_0\left(\frac{1}{B}\right)^{mn}\Gamma_{mn}\epsilon = 0, \tag{35}
$$

where now the contractions are with respect to the closed string metric. Therefore this condition is equivalent to the one obtained in the previous section.

The result shows that in gauge theory we can construct supersymmetric solutions by the above method only in the case where the noncommutative parameters satisfy some particular relations. For generic Θ's the solutions do not preserve supersymmetry. In the case of four noncommutative dimensions, by a similar analysis, we conclude that such instantons preserve supersymmetry only for self-dual or anti-self-dual noncommutative parameters.

IV. FLUCTUATIONS

For properly chosen values of Θ's, one has residual supersymmetry. By studying fluctuations around the solitonic configuration, we show here that the tachyon does not appear. From the action (29) we can write the potential for the bosonic fields C's:

$$
\begin{aligned}
V(C) = \frac{1}{2}\mathrm{Tr}\{&([C_p, C_{\bar{q}}] + i\Theta^{-1}_{p\bar{q}})([C_q, C_{\bar{p}}] + i\Theta^{-1}_{q\bar{p}})\\
&-[C_p, C_q][C_{\bar{p}}, C_{\bar{q}}]\},
\end{aligned} \tag{36}
$$

where we use summation over repeated indices p, $\bar{p}$. The above potential can be used also for the other cases of interest: D0-D2 [12] and D0-D4 if we consider $p, q = 1$ and $p, q = 1,2$, respectively. We consider fluctuations $C_p = C_p^0 + \delta C_p$ around the solitonic solution, $C_p^0 = S^\dagger a_p^\dagger S$, and decompose them in terms of the projection operators P_0 and $1 - P_0$:

$$\delta C_p = A_p + \bar{T}_p + W_p + S^\dagger D_p S,$$

$$A_p = P_0 \delta C_p P_0, \quad \bar{T}_p = (1-P_0)\,\delta C_p P_0, \tag{37}$$

$$W_p = P_0 \delta C_p (1-P_0), \quad S^\dagger D_p S = (1-P_0)\,\delta C_p (1-P_0),$$
$$S = |(0,\ldots,0)\rangle\langle(1,0,\ldots,0)| + \cdots. \tag{38}$$

There is a corresponding decomposition for the Hermitian conjugates of these. Focusing only on the part of this potential that can give a tachyon, we need only the first term in the expression of S. The first order term in fluctuations is zero and the second order term splits into two independent terms, one for T's and W's and the other for D's. We keep the one for T's and W's because the tachyonic mode will come from this:

$$V_2(T,W) = \mathrm{Tr}[2i\Theta^{-1}_{qp}(W_p \bar{W}_{\bar{q}} - T_{\bar{q}} \bar{T}_p) + (C^0_p \bar{C}^0_{\bar{p}} + \bar{C}^0_{\bar{p}} C^0_p) \times (\bar{W}_{\bar{q}} W_q + \bar{T}_q T_{\bar{q}}) - \bar{C}^0_{\bar{p}} \bar{C}^0_{\bar{q}} \bar{T}_p W_q - C^0_p C^0_q \bar{W}_{\bar{p}} T_{\bar{q}} - \bar{C}^0_{\bar{p}} C^0_q \bar{T}_p T_{\bar{q}} - C^0_p \bar{C}^0_{\bar{q}} \bar{W}_{\bar{p}} W_q]. \tag{39}$$

Using the simplified notations $\theta_{1,2,3}$, we obtain the following potential in second order:

$$V_2(T,W) = \mathrm{Tr}\Big(-(W_p S^\dagger a_{\bar{p}} + T_{\bar{p}} S^\dagger a^\dagger_p)(a^\dagger_q S \bar{W}_{\bar{q}} + a_{\bar{q}} S \bar{T}_q) + T_{\bar{p}} S^\dagger (a^\dagger_k a_{\bar{k}} + a_{\bar{k}} a^\dagger_k) S \bar{T}_p + W_p S^\dagger (a^\dagger_k a_{\bar{k}} + a_{\bar{k}} a^\dagger_k) S \bar{W}_{\bar{p}} + \frac{2}{\theta_k}(W_k \bar{W}_{\bar{k}} - T_{\bar{k}} \bar{T}_k) \Big). \tag{40}$$

We introduce in the expression for the potential the matrix components for the operators T's and W's:

$$T_p = T^1_p |(0,0,\ldots,)\rangle\langle\langle(1,0,\ldots,)| + \cdots,$$
$$\bar{W}_p = \bar{W}^1_p |(1,0,\ldots,)\rangle\langle(0,0,\ldots,)| + \cdots, \tag{41}$$

where by the ellipsis we mean other states of the type $|0\rangle\langle\,|$ or $|\,\rangle\langle 0|$, respectively. As can be seen from Eq. (40), all the modes except T^1_p will have positive mass. The mass terms for T^1_p modes are

$$\sum_p |T^1_p|^2 \left(\sum_k \frac{1}{\theta_k} - \frac{2}{\theta_p} \right). \tag{42}$$

We observe that for the D0-D2 system studied in [12] ($p=1$) we always have a tachyon. The condition for not having a tachyon in the case of D0-D4 is that $\theta_1 = \theta_2$, namely, the self-duality condition for Θ. For our case, the conditions are $\Sigma^3_k (1/\theta_k) - 2/\theta_p \geqslant 0$ for any p. The supersymmetry condition for our solution implies indeed that there is no tachyon in the system.

V. CONCLUSIONS

The best studied example of the AdS conformal field theory (CFT) correspondence [21–23], the duality between IIB supergravity, and $\mathcal{N}=4$, $d=4$ super Yang-Mills (SYM) theory was motivated in [26] by taking the viewpoint that there are two different but dual string theory descriptions of the same objects, D3-branes. The viewpoint seems to point toward a duality between the two stringy descriptions themselves, i.e., between a type IIB closed superstring description and an oriented open superstring description.

It was suggested in the same paper that in each description open strings, closed strings, and D3 branes may not all appear "explicitly." For example, it was speculated that in the type IIB closed string description the presence of open strings on the D-branes might be only associated with the size of the nonextremality of a D-brane soliton solution. Similar subtleties may lie in the open string description for the realization of closed strings. A related discussion can be found in [24,25] for open string field theory in tachyonic vacua.

As far as D-branes are concerned, open string theory has an efficient way of realizing them: they appear as Dirichlet boundary conditions.[5] As is well known, Dirichlet boundary conditions are obtained through T duality. Since the duality connects two equivalent descriptions, it is natural to ask whether open string theory has another way of realizing D-branes.

With this motivation, we have considered the open string description of the D3-D9 system in the Green-Schwarz formulation with the nonzero components of the B field transverse to the D3-brane. The boundary conditions of the D3-D9 system give an extra condition for residual supersymmetry. Then we considered the supersymmetric gauge theory in ten dimensions and its dimensional reduction to six spatial dimensions. Turning on a constant B field in six dimensions, the gauge theory becomes noncommutative. We have shown that the conditions for residual supersymmetry are equivalent, in an appropriate limit, to that of the D3-D9 system mentioned above. This can be viewed as a generalization of the previous results in the literature concerning lower codimensional cases where various (noncommutative) solitons were identified as D-branes.

Note Added. Sometime after our paper was published, a paper [29] appeared that has some overlap with our results.

[5]It was argued in [26] with evidence [27,28] that in the context of AdS/CFT D-branes may not merely be boundary conditions but provide a curved background for open strings to propagate in. However, we did not, in this article, compare the open string description of D-branes with the realization of them as a supergravity solution, but have remained within the open string and gauge theory description. Therefore, the background was taken to be flat for simplicity.

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