Compactified $D=11$ supermembranes and symplectic noncommutative gauge theories

I. Martín*
Departamento de Física, Universidad Simón Bolívar, Venezuela
and Theoretical Physics Group, Imperial College, London University, United Kingdom

J. Ovalle†
Departamento de Física, Universidad Simón Bolívar, Venezuela

A. Restuccia‡
Departamento de Física, Universidad Simón Bolívar, Venezuela
and Department of Mathematics, King's College, London University, United Kingdom

(Received 30 January 2001; published 5 July 2001)

It is shown that a double compactified $D=11$ supermembrane with nontrivial wrapping may be formulated as a symplectic noncommutative gauge theory on the world volume. The symplectic noncommutative structure is intrinsically obtained from the symplectic two-form on the world volume defined by the minimal configuration of its Hamiltonian. The gauge transformations on the symplectic fibration are generated by the area preserving diffeomorphisms on the world volume. Geometrically, this gauge theory corresponds to a symplectic fibration over a compact Riemann surface with a symplectic connection.

DOI: 10.1103/PhysRevD.64.046001 PACS number(s): 11.25.Mj, 11.25.Hf

I. INTRODUCTION

Noncommutative geometry in string theory with a nonzero B field [1] has recently been discussed by several authors [2–5]. The relation of the noncommutative Yang-Mills Lagrangian to the Born-Infeld one was considered in [6,7] and general aspects of noncommutative gauge fields have been discussed in [8–22]. In [16], the change of variables from ordinary to noncommutative Yang-Mills fields was explicitly found and the equivalence between the Born-Infeld action for ordinary Yang-Mills fields in the presence of a B field and the Born-Infeld action for noncommutative Yang-Mills fields was proved.

In this work, we follow a different approach. We relate the double compactified $D=11$ closed supermembrane [23] dual [24–26] to a symplectic noncommutative gauge theory on the world volume minimally coupled to seven scalar fields representing the transverse coordinates to the brane.

We first show that there is a natural symplectic structure for the double compactified supermembrane with nontrivial wrapping on the target space. It is defined by the minimal configurations of the Hamiltonian [25,26]. In fact, the solutions when interpreted in terms of connection one-forms over principal bundles satisfy the global condition

$$*F(A)=n. \tag{1}$$

On the other hand, it is known that for a given a symplectic structure over a manifold there always exists a globally defined deformation of the Poisson brackets. Moreover, even for Poisson manifolds it is possible to define star brackets [27] which lead to a noncommutative geometry. Having this idea in mind, we show that the Hamiltonian for the dual of the the double compactified supermembrane corresponds exactly to a super-Maxwell theory of a symplectic connection on a symplectic fibration, the fiber being the space generated by transverse coordinates and its conjugate momenta to the brane in the target phase space. It is noticed that a deformation of the given geometrical structure in this theory will lead in a straightforward way to a noncommutative (in the manner of Moyal) gauge theory. The reformulation of the compactified $D=11$ supermembrane dual in terms of noncommutative gauge theories provides a different point of view to analyze fundamental properties of the supermembrane as discussed in [28,29].

The steps taken in our formulation are as follows: we first construct the Hamiltonian for the doubly compactified supermembrane dual. The Hamiltonian minima are smooth configurations corresponding to U(1) connections globally defined over the brane world volume. The curvature of these connections is a nondegenerate two-form that gives rise to a well-defined symplectic structure. The second step in our construction is then to introduce symplectic connections with their covariant derivatives in the compactified directions. The Hamiltonian reduces then to an exact symplectic noncommutative super-Maxwell theory interacting with scalar fields.

In our construction of the Hamiltonian, we deal with a canonical analysis of the dual theory to the compactified supermembrane. This approach allows us to study directly the global aspects related to the nontrivial wrapping of the membrane on the torus. The Hamiltonian for a generic supermembrane was first performed in [30]; however, the abovementioned global aspects were not tackled in it.

II. HAMILTONIAN FORMULATION OF THE DOUBLE COMPACTIFIED $D=11$ SUPERMEMBRANE DUAL

We consider the compactified $D=11$ closed supermembrane dual obtained in [24] and [26]. The bosonic part of its action is given by

*Email address: isbeliam@usb.ve; isbeliam@ic.ac.uk
†Email address: jovalle@usb.ve
‡Email address: arestu@usb.ve

$$S(\gamma,X,A)=-\frac{1}{2}\int_{\Sigma x R} d^3\xi\sqrt{-\gamma}\left(\gamma^{ij}\partial_i X^m\partial_j X^n\eta_{mn}\right.$$
$$\left.+\frac{1}{2}\gamma^{ij}\gamma^{kl}F^r_{ik}F^r_{jl}-1\right), \tag{2}$$

where X^m, $m=1,\dots,D-q$ denote the maps from the world volume $\Sigma\times R$ to the target space $M_{D-q}x\underbrace{S^1x\dots xS^1}_{q}$, Σ been a compact (closed) Riemann surface. A^r_i, $r=1,\dots,q$, denotes the components of the q U(1) connection one-forms over $\Sigma\times R$. Here γ^{ij} is the auxiliary metric. We will be interested in the cases $q=1$ and $q=2$, the single and double compactified cases. The case $q=0$ corresponds to the supermembrane action over M_{11}. The action (2) for the $q=1$ case is dual to the supermembrane with target space $M_{10}\times S^1$, while the action for $q=2$ is dual to the supermembrane with target space $M_9\times S^1\times S^1$. The equivalence between the actions under duality transformations is valid off shell. The functional integral formulations may be proved to be formally equivalent in both cases.

To obtain the Hamiltonian formulation of the theory, we consider in the usual way, the Arnowitt-Deser-Misner (ADM) decomposition of the metric

$$\gamma_{ab}=\beta_{ab},\quad \gamma^{ab}=\beta^{ab}-N^aN^bN^{-2},$$
$$\gamma^{0a}=N^aN^{-2},\quad \gamma_{0a}=\beta_{ab}N^b, \tag{3}$$
$$\gamma_{00}=-N^2+\beta_{ab}N^aN^b,\quad \gamma^{00}=-N^{-2}$$

and define β^{ab} by $\beta^{ab}\beta_{bc}=\delta^a_c$.

The light-cone gauge fixing conditions are

$$X^+=P^+_0\mathcal{T},\quad P^+=P^+_0\sqrt{W}, \tag{4}$$

where $\mathcal{T}$ is the time coordinate on the world volume and W the determinant of the metric over Σ introduced through the gauge fixing condition only.

After elimination of X^- and P^- one obtains [26] the Hamiltonian density

$$\mathcal{H}=\frac{1}{2}\frac{1}{\sqrt{W}}\left(P^MP_M+\beta+\frac{1}{2}\beta\beta^{ac}\beta^{bd}F^r_{ab}F^r_{cd}\right)-A^r_0\phi_r+\Lambda\phi, \tag{5}$$

where A^r_0 and Λ are the Lagrange multipliers associated with the first class constraints

$$\phi_r\equiv\partial_a\Pi^a_r=0, \tag{6}$$

$$\phi\equiv\epsilon^{ab}\partial_b\left[\frac{\partial_aX^MP_M+\Pi^c_rF^r_{ac}}{\sqrt{W}}\right]=0, \tag{7}$$

ϕ being the generator of the area preserving diffeomorphism. There is also a global constraint arising from the elimination of X^-; this is

$$\oint_c\left(\frac{\partial_aX^MP_M+\Pi^b_rF^r_{ab}}{\sqrt{W}}\right)d\xi^a=2\pi n_c, \tag{8}$$

where c is a basis of homology of dimension 1. Here n_c are integers associated with c.

β_{ab} is the auxiliarly metric satisfying

$$\beta_{ab}=(1-\beta^{-1}\Pi^a_r\Pi^b_r\beta_{ab})^{-1}(\partial_aX^M\partial_bX_M$$
$$-\beta^{-1}\Pi^c_r\Pi^d_r\beta_{ca}\beta_{db}), \tag{9}$$

where β is the determinant of the matrix β_{ab}.

P_M are the conjugate momenta associated with X^M. The index M refers to the transverse coordinates in the light-cone decomposition of the target space. Equations (7) and (8) arise from the integrability condition on the resolution for X^- and the further assumption that X^- winds up over S^1 with winding numbers n_c.

It is interesting to notice that the Hamiltonian density (5) depends on the auxiliarly metric only through its determinant β. In fact,

$$\beta F^r_{ab}F^r_{cd}\beta^{ac}\beta^{bd}=\frac{1}{2}W(*F^r)^2 \tag{10}$$

where

$$*F^r\equiv\frac{\epsilon^{ab}}{\sqrt{W}}F^r_{ab} \tag{11}$$

is the Hodge dual to the curvature two-form F^r.

The determinant β may be obtained from Eq. (9) after some calculations; it has the following expressions:

$$\beta=\det(\partial_aX^M\partial_bX_M),\quad M=1,\dots,9, \tag{12}$$

for the $q=0$ case [28],

$$\beta=\det(\partial_aX^M\partial_bX_M)+(\Pi^a\partial_aX^M)^2,\quad M=1,\dots,8, \tag{13}$$

for the $q=1$ case, and

$$\beta=\det(\partial_aX^M\partial_bX_M)+(\Pi^a_r\partial_aX^M)^2+\frac{1}{4}(\Pi^a_r\Pi^b_s\epsilon_{ab}\epsilon^{rs})^2,$$
$$r=1,2,\quad M=1,\dots,7, \tag{14}$$

for the $q=2$ case.

The Hamiltonian densities obtained after replacing Eqs. (13) and (14) into Eq. (5) may also be constructed in a more direct way from the Hamiltonian density of the supermembrane in the light-cone-gauge (LCG) by using duality in the canonical approach directly without starting from the covariant formulation. Let us analyze briefly this point. We consider the canonical action of the supermembrane in the LCG [28] with target space $M_9\times S^1\times S^1$; its bosonic part is

$$\mathcal{H}_{SM}=\frac{1}{2}\frac{1}{\sqrt{W}}[P^M P_M+\det(\partial_a X^M \partial_b X_M)]+\Lambda \epsilon^{ab}\partial_b\left(\frac{\partial_a X^M P_M}{\sqrt{W}}\right),\quad M=1,\ldots,9. \tag{15}$$

Consider, first, one of the compactified coordinates taking values over S^1. It must satisfy

$$\oint_c dX=2\pi n_c. \tag{16}$$

The terms involving that map in the canonical action are

$$\left\langle P\dot{X}-\frac{1}{2}\frac{1}{\sqrt{W}}(P^2+\epsilon^{ac}\epsilon^{bd}\partial_a X\partial_b X\partial_c X^N\partial_d X^N)+\partial_b\Lambda\epsilon^{ab}\partial_a X\frac{P}{\sqrt{W}}\right\rangle, \tag{17}$$

where X^N is different from X. We may then construct an equivalent constrained term

$$\left\langle PL_0-\frac{1}{2}\frac{1}{\sqrt{W}}(P^2+\epsilon^{ac}\epsilon^{bd}L_a L_b\partial_c X^N\partial_d X^N)+\partial_b\Lambda\epsilon^{ab}L_a\frac{P}{\sqrt{W}}\right\rangle \tag{18}$$

subject to

$$\epsilon^{ca}\partial_c L_a=0,\quad \partial_a L_0-\partial_0 L_a=0. \tag{19}$$

We may introduce them into the action (17) through the use of Lagrange multipliers, which we will denote A_0 and $\epsilon^{ab}A_b$, respectively. We then recognize that the conjugate momenta to A_b is

$$\Pi^b\equiv\epsilon^{ab}L_a. \tag{20}$$

After the elimination of L_0 we get

$$p=\epsilon^{ab}\partial_a A_b. \tag{21}$$

Equation (17) subject to Eq. (19) reduces then to

$$\left\langle \Pi^b\dot{A}_b-\frac{1}{2}\frac{1}{\sqrt{W}}[(\epsilon^{ab}\partial_a A_b)^2+\Pi^c\Pi^d\partial_c X^N\partial_d X_N]-A_0\partial_c\Pi^c-\Lambda\Pi^b\partial_b\left(\frac{\epsilon^{cd}\partial_c A_d}{\sqrt{W}}\right)\right\rangle. \tag{22}$$

The terms (22) contribute together with the terms independent of X and P in Eq. (15) to give exactly the same expression of the Hamiltonian density (5) and (13):

$$\mathcal{H}_D=\frac{1}{2}\frac{1}{\sqrt{W}}\left(P^M P_M+\det(\partial_a X^M\partial_b X_M)+(\Pi^a\partial_a X^M)^2+\frac{1}{4}W(*F)^2\right)-A_0\partial_c\Pi^c+\Lambda\epsilon^{ab}\partial_b\times\left(\frac{\partial_a X^M P_M+\Pi^c F_{ac}}{\sqrt{W}}\right),\quad M=1,\ldots,8. \tag{23}$$

Equation (23) is also the Hamiltonian density arising from the canonical formulation of the Born-Infeld action [31]; it describes then the $D2$-brane in ten dimensions in the case of open supermembranes. If we now repeat the above procedure for the second compactified coordinate, we obtain the following Hamiltonian density:

$$\mathcal{H}=\frac{1}{2}\frac{1}{\sqrt{W}}\left(P^M P_M+\det(\partial_a X^M\partial_b X_M)+(\Pi^a_r\partial_a X^M)^2+\frac{1}{4}(\Pi^a_r\Pi^b_s\epsilon_{ab}\epsilon^{rs})^2+\frac{1}{4}W(*F^r)^2\right)-A^r_0\partial_c\Pi^c_r+\Lambda\epsilon^{ab}\partial_b\left(\frac{\partial_a X^M P_M+\Pi^c_r F^r_{ac}}{\sqrt{W}}\right), \tag{24}$$

in complete agreement with Eqs. (5) and (14) which were obtained from the canonical analysis of the covariant formulation of the theory. The equivalence between $\mathcal{H}_{SM}$, $\mathcal{H}_D$, and $\mathcal{H}$ may be then established from the duality equivalence between the covariant formulations of the theories or, more directly, from the duality equivalence of the gauge fixed canonical formulations in the LCG. The relation becomes nontrivial because the procedure of going from the covariant formulation to the LCG one involves the elimination of the auxiliarly metric which is an on-shell step while the duality equivalence are off-shell ones; they can be formally performed on the functional integral.

III. MINIMAL CONFIGURATIONS OF THE HAMILTONIAN

We will now analyze more in detail Eq. (24). Its supersymmetric extension may be obtained in a straightforward way from the supermembrane Hamiltonian in the LCG by the procedure described above, we will write the resulting expression at the end of the analysis. We may solve explicitly the constraints on Π^c_r, obtaining

$$\Pi^c_r=\epsilon^{cb}\partial_b\Pi_r,\quad r=1,2. \tag{25}$$

Defining the two-form ω in terms of Π_r as

$$\omega=\partial_a\Pi_r\partial_b\Pi_s\epsilon^{rs}d\xi^a\wedge d\xi^b, \tag{26}$$

the condition of nontrivial membrane winding imposes a restriction on it, namely,

$$\oint_{\Sigma}\omega=2\pi n. \tag{27}$$

With this condition on ω, Weil's theorem ensures that there will always exist an associated U(1) principal bundle over Σ and a connection on it such that ω is its curvature. The minimal configurations for the Hamiltonian (24) may be expressed in terms of such connections.

In [26] the minimal configurations of the Hamiltonian of the double compactified supermembrane were obtained. In spite of the fact that the explicit expression (24) was not then written, all minimal configurations were found. They correspond to $\Pi_r = \hat{\Pi}_r$, satisfying

$$*\hat{\omega}=\epsilon^{ab}\partial_a\hat{\Pi}_r\partial_b\hat{\Pi}_s\epsilon^{rs}=n\sqrt{W}, \quad n\neq 0. \tag{28}$$

The explicit expressions for $\hat{\Pi}_r$ are [26]

$$\hat{\Pi}_1=\pm 1+\tanh\phi, \quad \hat{\Pi}_2=\varphi. \tag{29}$$

Here ϕ and φ are the real and imaginary parts of an Abelian integral, respectively. This Abelian integral is defined over the Riemann surface Σ of genus g. ϕ is a single valued harmonic function and φ a multivalued one. For more details on the deduction of these monopole solutions on the Riemann surface see [33].

As mentioned before, they correspond to U(1) connections on nontrivial principal bundles over Σ. The principal bundle is characterized by the integer n corresponding to an irreducible winding of the supermembrane [25]. Moreover, the semiclassical approximation of the Hamiltonian density around the minimal configuration was shown to agree with the Hamiltonian density of super-Maxwell theory on the world sheet, minimally coupled to the seven scalar fields representing the coordinates transverse to the world volume of the super-brane.

IV. SYMPLECTIC NONCOMMUTATIVE FORMULATION

Let us now analyze the geometrical structure of the constructed Hamiltonian. We notice that the minimal configurations of the Hamiltonian introduce a natural symplectic structure in the theory through the nondegenerate two-form $\hat{\omega}$:

$$\hat{\omega}=\partial_a\hat{\Pi}_r\partial_b\hat{\Pi}_s\epsilon^{rs}d\xi^a\wedge d\xi^b. \tag{30}$$

Also, $\hat{\Pi}^a_r$ is an invertible matrix. It allows one to define the metric W_{ab} on the world volume,

$$W_{ab}=2\partial_a\hat{\Pi}_r\partial_b\hat{\Pi}_r. \tag{31}$$

Its determinant takes the value

$$\det W_{ab}=n^2W, \tag{32}$$

and its inverse is given by

$$n^2W^{ab}=\frac{\epsilon^{ac}}{\sqrt{W}}\frac{\epsilon^{bd}}{\sqrt{W}}W_{cd}=\frac{2}{\sqrt{W}}\hat{\Pi}^a_r\hat{\Pi}^b_r. \tag{33}$$

Furthermore, we introduce the covariant derivative D_a with respect to this metric W_{ab}; it then follows that

$$D_aW=0, \quad D_a\hat{\Pi}^b_r=0. \tag{34}$$

We now define the rotated covariant derivatives in terms of tangent space coordinates in the compactified directions:

$$D_r\equiv\frac{\hat{\Pi}^a_r}{\sqrt{W}}D_a. \tag{35}$$

We may now perform a canonical transformation in order to introduce a symplectic connection $\mathcal{A}_r$ in our formalism. The kinetic term

$$\langle\Pi^a_r\dot{A}^r_a\rangle \tag{36}$$

may then be rewritten as

$$\langle\Pi^a_r\dot{A}^r_a\rangle=\langle\epsilon^{ab}\partial_bA^r_a\dot{\Pi}_r\rangle=\langle\Pi^r\dot{\mathcal{A}}_r\rangle, \tag{37}$$

where we have introduced

$$\Pi^r\equiv\epsilon^{ab}\partial_bA^r_a, \tag{38}$$

$$\mathcal{A}_r\equiv\Pi_r-\mathcal{C}_r, \tag{39}$$

where $\mathcal{C}_r$ is a time independent geometrical object, which will be defined shortly. They satisfy the following Poisson bracket relation:

$$\{\mathcal{A}_r(\xi),\Pi^r(\xi')\}_P=\delta(\xi,\xi'). \tag{40}$$

The symplectic noncommutative derivative $\mathcal{D}_r$ may be defined now as

$$\mathcal{D}_r\equiv D_r+\{\mathcal{A}_r,\} \tag{41}$$

where the brackets $\{\bullet,\bullet\}$ are defined as follows:

$$\{\bullet,\bullet\}\equiv\frac{2\epsilon^{sr}}{n}D_r\bullet D_s\bullet=\frac{\epsilon^{ba}}{\sqrt{W}}D_a\bullet D_b\bullet, \quad n\neq 0. \tag{42}$$

We remark that these symplectic noncommutative derivatives behave as symplectic connections on a symplectic fibration over Σ with the phase space $(X^M,P^M)(\xi)$ being the fiber. The gauge transformations generated by the first class constraint (area preserving diffeomorphisms in the base manifold Σ) preserve the Poisson brackets in the fiber. The symplectic noncommutative derivatives preserve, in turn, the same structure; i.e., the symplectic noncommutative derivatives of the fields transform under gauge transformations in the same way as the fields and the holonomies generated by the symplectic connections preserve the Poisson brackets in the fiber. These properties may be checked out by straightforward calculations. In particular, $\delta\mathcal{A}_r=\mathcal{D}_r\xi$ under infinitesimal gauge transformations with parameter ξ.

Without loss of generality we rewrite Eq. (39) as

$$\Pi_r = \mathcal{A}_r + \hat{\Pi}_r, \tag{43}$$

where $\mathcal{C}_r$ is defined as $\hat{\Pi}_r$.

We then have, for the terms in Eq. (24),

$$\frac{1}{\sqrt{W}}\Pi_r^a \partial_a X^M = \frac{1}{\sqrt{W}}\hat{\Pi}_r^a \partial_a X^M + \frac{\epsilon^{ab}}{\sqrt{W}}\partial_b \mathcal{A}_r \partial_a X^M$$
$$= D_r X^M + \{\mathcal{A}_r, X^M\} = \mathcal{D}_r X^M, \tag{44}$$

$$\det \partial_a X^M \partial_b X_M = \frac{1}{2}\partial_a X^M \partial_c X^N \partial_b X_M \partial_d X_N \epsilon^{ac}\epsilon^{bd}$$
$$= \frac{1}{2} W\{X^M, X^N\}^2, \tag{45}$$

$$\Pi_r^a \Pi_s^b \epsilon_{ab}\epsilon^{rs} = n\sqrt{W} - 2\sqrt{W} D_r \mathcal{A}_s \epsilon^{rs} + \epsilon^{bc}\partial_b \mathcal{A}_r \partial_c \mathcal{A}_s \epsilon^{rs}$$
$$= n\sqrt{W} - \epsilon^{rs}\sqrt{W}(D_r \mathcal{A}_s - D_s \mathcal{A}_r + \{\mathcal{A}_r, \mathcal{A}_s\})$$
$$= (n - {*\mathcal{F}})\sqrt{W}, \tag{46}$$

where

$$*\mathcal{F} \equiv \epsilon^{rs}\mathcal{F}_{rs},$$
$$\mathcal{F}_{rs} \equiv D_r \mathcal{A}_s - D_s \mathcal{A}_r + \{\mathcal{A}_r, \mathcal{A}_s\}. \tag{47}$$

Finally, the generator of area preserving diffeomorphisms,

$$\phi \equiv \epsilon^{ab}\partial_b\left(\frac{\partial_a X^M P_M + \Pi_r^c F_{ac}^r}{\sqrt{W}}\right), \tag{48}$$

may be expressed as

$$-\phi = \mathcal{D}_r \Pi^r + \{X^M, P_M\}. \tag{49}$$

The Hamiltonian density (24) may then be rewritten

$$H = \int_\Sigma \mathcal{H} = \int_\Sigma \frac{1}{2\sqrt{W}}\Big[(P^M)^2 + (\Pi^r)^2 + \frac{1}{2}W\{X^M, X^N\}^2$$
$$+ W(\mathcal{D}_r X^M)^2 + \frac{1}{2}W(\mathcal{F}_{rs})^2\Big]$$
$$+ \int_\Sigma\left[\frac{1}{8}\sqrt{W}n^2 - \Lambda(\mathcal{D}_r \Pi^r + \{X^M, P_M\})\right], \tag{50}$$

where the following global condition has been imposed:

$$\int_\Sigma *\mathcal{F}\sqrt{W} d^2\xi = 0. \tag{51}$$

The Hamiltonian (50) may be extended to include the fermionic terms of the supersymmetric theory. They may be obtained from the Hamiltonian of the supermembrane in [28] by the dual approach discussed previously. They are

$$\int_\Sigma \sqrt{W}(\Lambda\{\bar{\theta}\Gamma_-, \theta\} - \bar{\theta}\Gamma_-\Gamma_r \mathcal{D}_r \theta + \bar{\theta}\Gamma_-\Gamma_M\{X^M, \theta\}), \tag{52}$$

where θ is the Majorana spinor of the original formulation of the supermembrane in the LCG in $D=11$ which may be decomposed in terms of a complex eight-component spinor of $SO(7)\times U(1)$.

The Hamiltonian (50) corresponds then exactly to a symplectic noncommutative super-Maxwell theory on the world volume minimally coupled to seven scalar fields X^M, $M=1,\dots,7$. The generator of area preserving diffeomorphisms becomes the generator of gauge transformations. In distinction to the star product defined in [16] which depends on a constant large background antisymmetric field of the string which couples to the U(1) gauge fields of the D-brane, the symplectic noncommutative product here is intrinsically constructed from minimal configurations of the Hamiltonian density which are unique (up to closed one-forms) for each given n and related to the natural symplectic structure of the world volume Riemann surface. This theory may be interpreted geometrically as a symplectic fibration over a Riemann surface, with fiber given by the symplectic phase space manifold generated by the transverse coordinate to the brane in the target space, its symplectic structure being preserved under the symplectomorphism induced by the first class constraint of the theory. The connection $\mathcal{D}_r$ is a symplectic connection on this symplectic fibration; i.e., the associated holonomies preserve the symplectic structure in the fibers [32]. Whether this symplectic fibration with a symplectic connection could be globally extended in a consistent manner to a type of Moyal noncommutative gauge theory is an open question. As commented on before, one can always globally deform the Poisson brackets in the fibration base space to Moyal brackets, but it is not necessarily true that the symplectic structure on the fiber could be extended in the same way and, moreover, be preserved under holonomies.

V. CONCLUSIONS

We have formulated the double compactified $D=11$ supermembrane dual with nontrivial irreducible winding as a symplectic noncommutative super-Maxwell theory, i.e., as an exact symplectic fibration over a compact Riemann surface with a symplectic connection, the connection dynamics being governed by a Hamiltonian that resembles that of a Maxwell theory. We emphasize that our construction is globally defined. Also, we remark that the symplectic noncommutative gauge theory we have introduced relies on the nonsingular minimal configuration of the Hamiltonian (24), where the assumption $n\neq 0$ is essential. The minimal configuration obtained in [26] corresponds to the monopole connection one-forms over Riemann surfaces [33] which may also be obtained from a suitable pullback to Σ of the connection one-forms on the Hopf fibering over CP_n [34]. Its curvature is a nondegenerate closed two-form defining a natural symplectic structure over Σ. The equivalence between the Hamiltonian (24) of the double compactified $D=11$ supermembrane dual and the Hamiltonian (50) of the symplectic noncommutative geometry is exact.

ACKNOWLEDGMENTS

I.M. and A.R. thank the kind hospitality of the Imperial College's Theoretical Group and the King's College Mathematics Department, respectively, where part of this work was done.

[1] A. Connes, M.R. Douglas, and A. Schwarz, J. High Energy Phys. **02**, 003 (1998).
[2] Douglas and C. Hull, J. High Energy Phys. **02**, 008 (1998).
[3] Y.-K.E. Cheung and M. Krogh, Nucl. Phys. **B528**, 185 (1998).
[4] C.-S. Chu and P.-M. Ho, Nucl. Phys. **B550**, 151 (1999); **B568**, 497 (2000).
[5] V. Schomerus, J. High Energy Phys. **06**, 030 (1999).
[6] C. Hofman and E. Verlinde, J. High Energy Phys. **12**, 010 (1998); Nucl. Phys. **B547**, 157 (1999).
[7] B. Pioline and A. Schwarz, J. High Energy Phys. **05**, 021 (1999).
[8] P. Ho and Y. Wu, Phys. Lett. B **398**, 52 (1997); P.-M. Ho, Y.-Y. Wu, and Y.-S. Wu, Phys. Rev. D **58**, 026006 (1998); P. Ho, Phys. Lett. B **434**, 41 (1998); P. Ho and Y. Wu, Phys. Rev. D **58**, 066003 (1998).
[9] M. Li, ''Comments on Supersymmetric Yang-Mills Theory on a Noncommutative Torus,'' hep-th/9802052.
[10] T. Kawano and K. Okuyama, Phys. Lett. B **433**, 29 (1998).
[11] F. Lizzi and R.J. Szabo, Chaos, Solitons Fractals **10**, 445 (1999); G. Landi, F. Lizzi, and R.J. Szabo, Commun. Math. Phys. **200**, 603 (1999); F. Lizzi and R.J. Szabo, ''Noncommutative Geometry and String Duality,'' hep-th/9904064.
[12] D. Bigatti, Phys. Lett. B **451**, 324 (1999).
[13] D. Bigatti and L. Susskind, Phys. Rev. D **62**, 066004 (2000).
[14] A. Hashimoto and N. Itzhaki, Phys. Lett. B **465**, 142 (1999).
[15] L. Cornalba and R. Schiappa, ''Matrix Theory Star Products from the Born-Infeld Action,'' hep-th/9907211.
[16] N. Seiberg and E. Witten, J. High Energy Phys. **09**, 032 (1999).
[17] M.M. Sheikh-Jabbari, ''Noncommutative open string theories and their dualities,'' hep-th/0101045.
[18] Y. Matsuo and Y. Shibusa, J. High Energy Phys. **02**, 006 (2001).
[19] I. Rudychev, J. High Energy Phys. **04**, 015 (2001).
[20] K. Ohta, hep-th/0101082.
[21] R. Amorim and J. Barcelos-Neto, hep-th/0101196.
[22] H. Girotti, M. Gomes, V. Rivelles, and A.J. da Silva, Nucl. Phys. **B587**, 299 (2000).
[23] E. Bergshoeff, E. Sezgin, and P.K. Townsend, Phys. Lett. B **189**, 75 (1987).
[24] P.K. Townsend, Phys. Lett. B **350**, 184 (1995); **373**, 68 (1996).
[25] I. Martín, A. Restuccia, and R. Torrealba, Nucl. Phys. **B521**, 117 (1998).
[26] I. Martín, J. Ovalle, and A. Restuccia, Phys. Lett. B **472**, 77 (2000).
[27] J. Vey, Comment. Math. Helv. **50**, 421 (1975); M. Kontsevich, ''Deformation quantization of Poisson manifolds,'' q-alg/9709040.
[28] B. de Wit, M. Lüsher, and H. Nicolai, Nucl. Phys. **B320**, 135 (1989).
[29] B. de Wit, K. Peeters, and J.C. Plefka, Nucl. Phys. B (Proc. Suppl.) **62**, 405 (1998).
[30] B. de Wit, J. Hoppe, and H. Nicolai, Nucl. Phys. **B305**, 545 (1988).
[31] R. Manvelyan, A. Melikyan, and R. Mkrtchyan, Phys. Lett. B **425**, 277 (1998).
[32] V. Guillemin, E. Lerman, and S. Sternberg, *Symplectic Fibrations and Multiplicity Diagrams* (Cambridge University Press, Cambridge, England, 1997).
[33] I. Martín and A. Restuccia, Lett. Math. Phys. **39**, 379 (1997).
[34] H. Porta and L. Recht, J. Math. Anal. Appl. **118**(2), 547 (1986); Rev. Mat. Iber. **2**, 397 (1986).