

Gauge symmetry enhancement and radiatively induced mass in the large N nonlinear sigma model

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We consider a hybrid of nonlinear sigma models in which two complex projective spaces are coupled with each other under a duality. We study the large N effective action in 1+1 dimensions. We find that some of the dynamically generated gauge bosons acquire radiatively induced masses which, however, vanish along the self-dual points where the two couplings characterizing each complex projective space coincide. These points correspond to the target space of the Grassmann manifold along which the gauge symmetry is enhanced, and the theory favors the non-Abelian ultraviolet fixed point.

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The nonlinear sigma models (NLSM's) in which the dynamical fields take values in some target manifolds have been a subject of extensive research in theoretical physics due to their wide range of physical applications and their relevance with geometrical aspects of quantum theory [1,2]. Especially, the large- N analysis [3] of this model has proved to exhibit many remarkable physical properties, such as dynamical generation of gauge bosons, nonperturbative renormalizability, dimensional transmutation, and phase transitions in the lower dimensional space-time [4–6].

One of the well studied NLSM is the complex projective $CP(N)$ model [7] where the target space is given by the complex projective space $CP(N) \equiv SU(N)/SU(N-1) \times U(1)$. The purpose of this paper is to investigate the large- N limit of the NLSM for some other target space and to re-examine the issue of the dynamical generation of non-Abelian gauge bosons in 1+1 dimensions. Especially, we first study the specific target space given by the Grassmann coset space $Gr(N,2) \equiv SU(N)/SU(N-2) \times U(2)$ [8]. It turns out that this NLSM can be written as a hybrid of two $CP(N)$ models coupled to each other with the same coupling constant for each complex projective space and the interaction terms respect the dual exchange symmetry between the two sectors [see Eq. (1)]. We observe that there exists a manifest dual symmetry between the two sectors, and the Grassmann manifold corresponds to a self-dual case with equal coupling constants. If we start from different coupling constants for each complex space for the generality, this leads to the target space belonging to the so-called flag manifold [9] $\mathcal{M} = SU(N)/SU(N-2) \times U(1) \times U(1)$. The dynamically generated gauge bosons would have $U(1) \times U(1)$ gauge symmetry. These observations lead to our main motivation for this work, that is, a study of self-duality in the coupling constant space and subsequent enhancement of gauge symmetry. In order to investigate this issue, we analyze the large- N mass gap equations, and renormalization

group (RG) properties in the coupling constant spaces. We also explicitly compute the large- N effective action. We find that some of the dynamically generated gauge bosons acquire radiatively induced finite mass terms and gauge noninvariant interaction away from the self-dual points, leading to a local $U(1) \times U(1)$ symmetry. However, they vanish at the self-dual points enhancing the gauge symmetry to a non-Abelian $U(2)$ symmetry. The ultraviolet (UV) fixed point corresponds to a special self-dual point and the theory prefers the non-Abelian phase in the UV limit. Even though the dynamical generation of non-Abelian gauge bosons for the Grassmann target space has been discussed before [1,10], the way in which the enhancement of gauge symmetry at the fixed point occurs through the RG evolution has not been addressed so far.

Let us consider a Lagrangian which is given by

$$\begin{aligned} \mathcal{L}_0 = & \frac{1}{g_1^2} |(\partial_\mu + iA_\mu)\psi_1|^2 + \frac{1}{g_2^2} |(\partial_\mu + iB_\mu)\psi_2|^2 \\ & + \frac{1}{4} \left(\frac{g_1}{g_2} + \frac{g_2}{g_1} \right) C_\mu^* C^\mu - i \frac{1}{\sqrt{g_1 g_2}} C_\mu^* \psi_1^\dagger \partial^\mu \psi_2 \\ & - i \frac{1}{\sqrt{g_1 g_2}} C_\mu \psi_2^\dagger \partial^\mu \psi_1, \end{aligned} \quad (1)$$

where ψ_1 and ψ_2 are two orthonormal complex N vectors such that $\psi_i^\dagger \psi_j = \delta_{ij}$ ($i, j = 1, 2$). The above Lagrangian describes two $CP(N)$ models each described by ψ_1 and ψ_2 with coupling constants g_1 and g_2 , respectively, coupled through derivative coupling. There is a manifest dual symmetry between sectors 1 and 2, A_μ and B_μ , and C_μ and C_μ^* . When $g_1 = g_2$, the above model corresponds to the nonlinear sigma model with the target space of Grassmann manifold $Gr(N,2) = SU(N)/SU(N-2) \times U(2)$. Let us write the above Lagrangian in the more conventional form in terms of the $N \times 2$ matrix Z :

$$Z = [\psi_1, \psi_2], \quad \leftrightarrow \quad Z^\dagger = \begin{bmatrix} \psi_1^\dagger \\ \psi_2^\dagger \end{bmatrix}. \quad (2)$$

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We first introduce new sets of coupling constants defined by $g \equiv \sqrt{g_1 g_2}$ and $r \equiv g_2 / g_1$. Then, we consider

$$\mathcal{L} = \frac{1}{g^2} \text{tr}[(D_\mu Z)^\dagger (D^\mu Z) - \lambda(Z^\dagger Z - R)], \quad (3)$$

where we collected the orthonormal constraints into a 2×2 Hermitian matrix λ which transforms as an adjoint representation under the local $U(2)$ transformation, and the R matrix given by

$$\lambda = \begin{bmatrix} \lambda_1 & \lambda_3 \\ \lambda_3^* & \lambda_2 \end{bmatrix}, \quad R = \begin{bmatrix} r & 0 \\ 0 & r^{-1} \end{bmatrix}, \quad (4)$$

with a real positive r [11]. The covariant derivative is defined as $D_\mu Z \equiv \partial_\mu Z - Z \tilde{A}_\mu$ with a 2×2 anti-Hermitian matrix gauge potential $\tilde{A}_\mu \equiv -i \tilde{A}_\mu^a T^a$ associated with the local $U(2)$ symmetry. Each component of \tilde{A}_μ is assigned as

$$\tilde{A}_\mu = -i \begin{bmatrix} A_\mu & \frac{1}{2} C_\mu \\ \frac{1}{2} C_\mu^* & B_\mu \end{bmatrix}. \quad (5)$$

The on-shell equivalence between Lagrangians Eqs. (1) and (3) will be discussed shortly. The Lagrangian Eq. (3) is invariant under the local $U(2)$ transformation for $r=1$, whereas the R with $r \neq 1$ explicitly breaks the $U(2)$ gauge symmetry down to $U(1)_A \times U(1)_B$, where $U(1)_A$ and $U(1)_B$ are generated by $(T^0 \pm T^3)$, respectively. Thus the symmetry of our model is $[SU(N)]_{\text{global}} \times [U(2)]_{\text{local}}$ for $r=1$, while $[SU(N)]_{\text{global}} \times [U(1)_A \times U(1)_B]_{\text{local}}$ for $r \neq 1$. Invoking the hidden local symmetry [12], we infer that the theory with $r \neq 1$ corresponds to NLSM on the flag manifold $\mathcal{M} = SU(N)/SU(N-2) \times U(1) \times U(1)$.

In order to carry out the path integration in the large- N limit, we arrange the Lagrangian (3) in terms of the $2N \times 2N$ matrix form

$$\mathcal{L} = \frac{1}{g^2} [\psi_1^\dagger, \psi_2^\dagger] (M^T \otimes I) \begin{bmatrix} \psi_1 \\ \psi_2 \end{bmatrix} + \frac{r}{g^2} \lambda_1 + \frac{1}{r g^2} \lambda_2, \quad (6)$$

where I is an $N \times N$ unit matrix and M is a 2×2 matrix operator given by

$$M \equiv G^{-1} - \Gamma(\tilde{A}), \quad (7)$$

$$G^{-1} \equiv -\square - \lambda = \begin{bmatrix} -\square - \lambda_1 & -\lambda_3 \\ -\lambda_3^* & -\square - \lambda_2 \end{bmatrix}, \quad (8)$$

$$\Gamma(\tilde{A}) \equiv -\tilde{A}_\mu \hat{\partial}^\mu + \tilde{A}_\mu \tilde{A}^\mu, \quad (9)$$

where the differential operator $\hat{\partial}^\mu \equiv \partial^\mu - \tilde{\partial}^\mu$ must be regarded as not operating on the gauge potential \tilde{A}_μ . One can show that imposing the on-shell constraints $\psi_1^\dagger \psi_1 = r$, $\psi_2^\dagger \psi_2 = r^{-1}$, $\psi_1^\dagger \psi_2 = \psi_2^\dagger \psi_1 = 0$, and in terms of rescaling given by

$$\frac{\psi_1}{g} \rightarrow \frac{\psi_1}{g_1}, \quad \frac{\psi_2}{g} \rightarrow \frac{\psi_2}{g_2}, \quad \frac{C_\mu}{g} \rightarrow C_\mu, \quad \frac{C_\mu^*}{g} \rightarrow C_\mu^*, \quad (10)$$

the Lagrangian Eq. (6) reduces Eq. (1). We note that in Eq. (6), we never used the on-shell constraints so that the quadratic term of $C_\mu^* C_\mu$ has been absorbed into the matrix M .

Let us focus on the two dimensions from here on. Detailed analysis in other dimensions will be reported elsewhere [13]. The large- N effective action is given by path integrating Z and Z^\dagger , or equivalently ψ_1 , ψ_1^\dagger , ψ_2 , and ψ_2^\dagger . We obtain

$$S_{\text{eff}} = \int_x \mathcal{L} + iN \ln \text{Det } M. \quad (11)$$

According to the Coleman-Mermin-Wagner theorem, which states that there is no spontaneous breaking of any continuous global symmetry in dimensions two or less, we can set the vacuum expectation values of ψ_1 and ψ_2 equal to zero from the beginning in the effective potential such that

$$V_{\text{eff}} = -\frac{1}{N\Omega} S_{\text{eff}}[\psi_{1,2}=0, \lambda_{1,2,3}=m_{1,2,3}^2, \tilde{A}_\mu=0], \quad (12)$$

where Ω denotes the space-time volume. Then we obtain the large- N effective potential as

$$V_{\text{eff}} = -\frac{m_1^2}{Ng^2} r - \frac{m_2^2}{Ng^2} r^{-1} - i\Omega^{-1} \ln \text{Det } G^{-1}, \quad (13)$$

from which the gap equations are schematically given as follows:

$$\frac{\partial V_{\text{eff}}}{\partial m_3^2} = -\int \frac{d^2 k}{(2\pi)^2} \frac{2m_3^2}{(k^2 + m_+^2)(k^2 + m_-^2)} = 0, \quad (14)$$

$$\frac{\partial V_{\text{eff}}}{\partial m_1^2} = -\frac{1}{Ng^2} r + \int \frac{d^2 k}{(2\pi)^2} \frac{k^2 + m_2^2}{(k^2 + m_+^2)(k^2 + m_-^2)} = 0, \quad (15)$$

$$\frac{\partial V_{\text{eff}}}{\partial m_2^2} = -\frac{1}{Ng^2} r^{-1} + \int \frac{d^2 k}{(2\pi)^2} \frac{k^2 + m_1^2}{(k^2 + m_+^2)(k^2 + m_-^2)} = 0. \quad (16)$$

Here the loop momenta are Euclidean and m_\pm^2 are given in terms of $m_{1,2,3}^2$ by

$$m_+^2 + m_-^2 = m_1^2 + m_2^2, \quad m_+^2 m_-^2 = m_1^2 m_2^2 - m_3^4. \quad (17)$$

Since Eq. (14) simply states $m_3=0$, we can choose for example $m_+^2 = m_1^2$, $m_-^2 = m_2^2$ after setting $m_3=0$ in Eq. (17). Then the gap equations are given by two decoupled sets of equations expressed by

$$0 = \frac{1}{Ng_1^2} - \frac{1}{4\pi} \ln \frac{\Lambda^2}{m_1^2}, \quad (18)$$

$$0 = \frac{1}{Ng_2^2} - \frac{1}{4\pi} \ln \frac{\Lambda^2}{m_2^2}, \quad (19)$$

where Λ is the cutoff of the theory. The above equations yield two mass scales given by

$$m_i^2 = \Lambda^2 \exp \left[-\frac{4\pi}{Ng_i^2} \right] \quad (i=1,2). \quad (20)$$

Imposing the cutoff independence of the mass scales, $\Lambda dm_i/d\Lambda = 0$ leads to the Callan-Symanzik β functions

$$\beta_i(g_i) = \frac{dg_i}{d \ln \Lambda} = -\frac{Ng_i^3}{4\pi}, \quad (21)$$

which show the asymptotic free behaviors of both couplings and a UV fixed point at the origin of the coupling constant space (g_1, g_2) . Note that when $m_1 = m_2$, we have $g_1 = g_2$ and the corresponding nonlinear sigma model is defined on the Grassmann manifold.

Let us discuss the dynamical generation of gauge bosons in our model. It has been discussed before that if we start from the same coupling constants $r=1$ in the Lagrangian Eq. (6), the effective action generates non-Abelian gauge bosons with a local $U(2)$ symmetry [10], rendering all four gauge bosons A, B, C , and C^* massless. Our main objective here is to compute the radiatively induced mass terms for the gauge bosons in the generic case where $g_1 \neq g_2$, hence $m_1 \neq m_2$. The large- N effective action Eq. (11) is schematically expanded such that

$$S_{\text{eff}} = \int_x \mathcal{L} + iN \ln \text{Det } G^{-1} - iN \sum_{n=1}^{\infty} \frac{1}{n} \text{Tr}[G\Gamma(\tilde{A})]^n. \quad (22)$$

The boson propagator G becomes a diagonal 2×2 matrix due to the gap equation solution $m_3 = 0$. We neglect the fluctuation fields coming from $\lambda_{1,2,3}$ around $m_{1,2,3}^2$. The diagrams which arise from $n=1,2,3,4$ can contribute to the Yang-Mills action. The mass term comes from $n=1$ and $n=2$. For $n=2$, we have three diagrams with two, three, and four point functions. The two point vacuum polarization function provides the gauge bosons with the kinetic terms and the mass term for C, C^* fields in the case $m_1 \neq m_2$. The contributions from both $n=1$ and $n=2$ are explicitly given by the integral

$$\begin{aligned} & -iN \frac{1}{2} \text{Tr}[G\tilde{A}_\mu \hat{\partial}^\mu G\tilde{A}_\nu \hat{\partial}^\nu] - iN \text{Tr}[G\tilde{A}_\mu \tilde{A}^\mu] \\ & = \frac{N}{2} \sum_{ij} \int_x \tilde{A}_{ij}^\mu(x) \Pi_{\mu\nu}^{ij}(i\partial_x) \tilde{A}_{ji}^\nu(x), \end{aligned} \quad (23)$$

where

$$\begin{aligned} \Pi_{\mu\nu}^{ij}(p) = & - \int \frac{d^2k}{i(2\pi)^2} \frac{(2k+p)_\mu(2k+p)_\nu}{(k^2-m_i^2)[(k+p)^2-m_j^2]} \\ & + \int \frac{d^2k}{i(2\pi)^2} \frac{2g_{\mu\nu}}{k^2-m_i^2}. \end{aligned} \quad (24)$$

This vacuum polarization can be explicitly computed to yield

$$\Pi_{\mu\nu}^{ij}(p) = \left(g_{\mu\nu} - \frac{p_\mu p_\nu}{p^2} \right) \Pi_T^{ij}(p) + \left(\frac{p_\mu p_\nu}{p^2} \right) \Pi_L^{ij}(p), \quad (25)$$

where the transverse function Π_T and the longitudinal one Π_L are obtained as

$$\Pi_T^{ij}(p) \equiv \frac{1}{2\pi} \left[\ln \frac{m_j}{m_i} - \int_0^1 dx \ln \frac{K^{ij}}{m_i^2} \right], \quad (26)$$

$$\Pi_L^{ij}(p) \equiv \frac{(m_j^2 - m_i^2)^2}{4\pi p^2} \left[\frac{2}{m_j^2 - m_i^2} \ln \frac{m_j}{m_i} - \int_0^1 dx \frac{1}{K^{ij}} \right], \quad (27)$$

with $K^{ij} \equiv xm_j^2 + (1-x)m_i^2 - x(1-x)p^2$. Moreover we see that

$$\Pi_T^{ij}(p) = c^{ij} + p^2 f_T^{ij}(p), \quad \Pi_L^{ij}(p) = c^{ij} + p^2 f_L^{ij}(p), \quad (28)$$

where the same constant c^{ij} arises in the leading terms of both Π_T and Π_L , and is given by

$$c^{ij} = \frac{1}{2\pi} \left[1 - \frac{m_j^2 + m_i^2}{m_j^2 - m_i^2} \ln \frac{m_j}{m_i} \right]. \quad (29)$$

We note that, despite its appearance, c^{ij} vanishes for $m_i = m_j$. Then the vacuum polarization can be written as

$$\Pi_{\mu\nu}^{ij}(p) = c^{ij} g_{\mu\nu} + (p^2 g_{\mu\nu} - p_\mu p_\nu) f_T^{ij}(p) + p_\mu p_\nu f_L^{ij}(p), \quad (30)$$

where both c^{ij} and f_L^{ij} vanish when $i=j$ so as to provide the A (B) boson with the $U(1)_A$ ($U(1)_B$) gauge invariant kinetic term. On the other hand, they remain nonzero when $i \neq j$ and provide the C boson with the mass given by

$$\begin{aligned} M_C &= \sqrt{\frac{-c^{12}}{f_T^{12}(0)}} \\ &= |m_1^2 - m_2^2| \sqrt{\frac{(m_1^2 + m_2^2) \ln(m_1/m_2) + m_2^2 - m_1^2}{(m_1^4 - m_2^4)/2 - 2m_1^2 m_2^2 \ln(m_1/m_2)}}. \end{aligned} \quad (31)$$

This is one of the main results of our paper. We note that the above mass does not vanish when $m_1 \neq m_2$, which in turn implies $g_1 \neq g_2$ from the mass gap equations (18) and (19). It is also symmetric under the exchange of m_1 and m_2 . When

$r=1$ ($m_1=m_2$), both c^{12} and f_L^{12} become zero. The three point function with one seagull does not contribute to the Yang-Mills action. The four-point vertex with two seagulls also contributes to the kinetic term of Yang-Mills action with other contributions from $n=3,4$.

Combining the relevant diagrams up to $n=4$, we obtain the Yang-Mills effective action for $m_1=m_2=m$ given by [10]

$$\mathcal{L}_{\text{eff}} = \frac{N}{48\pi m^2} \text{tr} F_{\mu\nu} F^{\mu\nu}(\tilde{A}), \quad (32)$$

where $F_{\mu\nu}(\tilde{A}) \equiv \partial_{[\mu} \tilde{A}_{\nu]} + [\tilde{A}_{\mu}, \tilde{A}_{\nu}]$ is the gauge field strength of \tilde{A}_{μ} . When $m_1 \neq m_2$, the effective action contains interactions that are not $U(2)$ gauge invariant. These terms and the nature of their interactions will be reported elsewhere [13]. In passing, we observe that the large- N effective action is renormalizable because the only UV divergence is the one that arises in the gap equation and the other possible UV divergences in the vacuum polarization function are forbidden by the gauge symmetry. The renormalization conditions Eqs. (18) and (19) are enough to realize the UV finite large- N theory. The higher order corrections in $1/N$ expansion can be systematically renormalized by using the counter terms, which are provided by the large- N effective action.

We have performed the large- N path integral of a coupled $CP(N)$ model with dual symmetry and have analyzed the vacuum structure and renormalization in $1+1$ dimensions. The large- N gap equation analysis yields two decoupled gap equations whose solution ensures the renormalizability. We also have computed the effective action, and have found that some of the dynamically generated gauge bosons acquire radiatively induced finite mass terms away from the self-dual points, and the gauge symmetry is reduced to its subgroup

[14]. We note that the theory favors the conformal fixed point and the non-Abelian phase in the ultraviolet limit. Also the classical dual symmetry is not broken by the nonperturbative radiative corrections.

We would like to emphasize that the mass generation of C gauge bosons is a genuine quantum effect away from the self-dual points. The finite mass term is determined unambiguously and is independent of the gauge invariant regularization scheme employed. In our scheme, the C boson mass arises from a purely finite term of Π_T of Eq. (28). This unambiguity is in contrast with some other radiative corrections in quantum field theory which are finite but undetermined [15].

We could extend our model to describe other types of symmetry reduction patterns and to include supersymmetry. For example, if we envisage the Grassmann space $\text{Gr}(N, n_1 + n_2) = SU(N)/SU(N-n_1-n_2) \times U(n_1+n_2)$ and the flag manifold $\mathcal{M}' = SU(N)/SU(N-n_1-n_2) \times U(n_1) \times U(n_2)$, the NLSM describes the reduction of $U(n_1+n_2)$ gauge symmetry into $U(n_1) \times U(n_2)$. This type of reduction and radiative mass generations may provide an alternative approach to the Higgs mechanism in the theories beyond the standard model or in the effective field theory of QCD in the context of the hidden local symmetry [1].

Finally, we mention possible relevance of our work with string theory. We recall that the gauge symmetry enhancement [16] and target space duality [17] in string theory have attracted an extensive study recently. Target space in the large- N limit could be unrealistic as space-time. Nevertheless, our results could provide us with some insight to study these subjects for strings moving on curved backgrounds.

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