High spin gauge fields and two-time physics

Itzhak Bars*
CIT-USC Center for Theoretical Physics & Department of Physics, University of Southern California, Los Angeles, California 90089-2535

Cemsinan Deliduman†
Feza Gürsey Institute, Çengelköy 81220, İstanbul, Turkey

(Received 19 March 2001; published 19 July 2001)

All possible interactions of a point particle with background electromagnetic, gravitational and higher-spin fields are considered in the two-time physics worldline formalism in $(d,2)$ dimensions. This system has a counterpart in a recent formulation of two-time physics in noncommutative field theory with local Sp(2) symmetry. In either the worldline or field theory formulation, a general Sp(2) algebraic constraint governs the interactions, and determines the equations that the background fields of any spin must obey. The constraints are solved in the classical worldline formalism ($\hbar \to 0$ limit) as well as in the field theory formalism (all powers of $\hbar$). The solution in both cases coincide for a certain 2T to 1T holographic image which describes a relativistic particle interacting with background fields of any spin in $(d$-1,1) dimensions. Two disconnected branches of solutions exist, which seem to have a correspondence as massless states in string theory, one containing low spins in the zero Regge slope limit, and the other containing high spins in the infinite Regge slope limit.

DOI: 10.1103/PhysRevD.64.045004 PACS number(s): 11.30.Ly, 02.20.Tw, 02.40.Gh, 11.25.Hf

I. INTRODUCTION

Local Sp(2) symmetry, and its supersymmetric generalization, is the principle behind two-time (2T) physics [1–9]. For a spinless particle, in the worldline formalism with local Sp(2) symmetry, the action is

$$S=\int d\tau[\partial_\tau X^M P_M - A^{ij}Q_{ij}(X,P)], \tag{1}$$

where $A^{ij}(\tau)=A^{ji}(\tau)$ with $i,j=1,2$, is the Sp(2) gauge potential. This action is local Sp(2) invariant (see Ref. [4] and below) provided the three $Q_{ij}(X,P)$ are any general phase space functions that satisfy the Sp(2) algebra under the Poisson brackets

$$\{Q_{ij},Q_{kl}\}=\varepsilon_{jk}Q_{il}+\varepsilon_{ik}Q_{jl}+\varepsilon_{jl}Q_{ik}+\varepsilon_{il}Q_{jk}. \tag{2}$$

The antisymmetric $\varepsilon_{ij}=-\varepsilon_{ji}$ is the invariant metric of Sp(2) that is used to raise or lower indices.

The goal of this paper is to determine all possible Q_{ij} as functions of phase space X^M, P^M that satisfy this algebra. The solution will be given in the form of a power series in momenta which identify the background fields $A_M(X)$ and $Z_{ij}^{M_1M_2\cdots M_s}(X)$:

$$Q_{ij}(X,P)=\sum_{s=0}^{\infty} Z_{ij}^{M_1M_2\cdots M_s}(X)[P_{M_1}+A_{M_1}(X)] \times[P_{M_2}+A_{M_2}(X)]\cdots[P_{M_s}+A_{M_s}(X)]. \tag{3}$$

*Email address: bars@physics.usc.edu
†Email address: cemsinan@gursey.gov.tr

These $(d+2)$-dimensional fields will describe the particle interactions with the Maxwell field, gravitational field, and higher-spin fields, when interpreted in d dimensions as a particular 2T to 1T holographic d-dimensional picture of the higher $(d+2)$-dimensional theory. Furthermore, as is the usual case in 2T physics, there are a large number of 2T to 1T holographic d-dimensional pictures of the same $(d+2)$-dimensional system. For any fixed background the resulting d-dimensional dynamical systems are interpreted as a unified family of 1T dynamical systems that are related to each other by duality type Sp(2) transformations. This latter property is one of the novel unification features offered by 2T physics.

In previous investigations the general solution up to maximum spin $s=2$ was determined [4]. The most general solution with higher-spin fields for arbitrary spin s is given here. Such $Q_{ij}(X,P)$ are then the generators of local Sp(2), with transformations of the coordinates (X^M,P_M) given by

$$\delta_\omega X^M=-\omega^{ij}(\tau)\frac{\partial Q_{ij}}{\partial P_M},\quad \delta_\omega P^M=\omega^{ij}(\tau)\frac{\partial Q_{ij}}{\partial X_M}, \tag{4}$$

where the $\omega^{ij}(\tau)=\omega^{ji}(\tau)$ are the local Sp(2) gauge parameters [4]. When the Sp(2) gauge field $A^{ij}(\tau)$ transforms as usual in the adjoint representation

$$\delta_\omega A^{ij}=\partial_\tau\omega^{ij}+[\omega,A]^{ij}, \tag{5}$$

the action (1) is gauge invariant $\delta_\omega S=0$, provided the background fields $Z_{ij}^{M_1M_2\cdots M_s}(X)$ and $A_M(X)$ are such that $Q_{ij}(X,P)$ satisfy the Sp(2) algebra above. Thus, through the requirement of local Sp(2), the background fields are restricted by certain differential equations that will be derived and solved in this paper. As we will see, the solution permits certain unrestricted functions that are interpreted as background fields of any spin in *two lower dimensions*.

An important aspect for the physical interpretation is that the equations of motion for the gauge field $A^{ij}(\tau)$ restricts the system to phase space configurations that obey the classical on-shell condition

$$Q_{ij}(X,P)=0. \tag{6}$$

The meaning of this equation is that physical configurations are gauge invariant and correspond to singlets of Sp(2). These equations have an enormous amount of information and provide a unification of a large number of one-time physics systems in the form of a single higher-dimensional theory. One-time dynamical systems appear then as holographic images of the unifying bulk system that exists in one extra timelike and one extra spacelike dimensions. We will refer to this property as 2T to 1T holography.

This holography comes about because there are nontrivial solutions to Eq. (6) only if the spacetime includes two timelike dimensions with signature $(d,2)$. By Sp(2) gauge fixing, two dimensions are eliminated, and d dimensions are embedded inside $d+2$ dimensions in ways that are distinguishable from the point of view of the remaining timelike dimension. This provides the holographic images that are interpreted as distinguishable one-time dynamics. Thus one obtains a multitude of nontrivial solutions with different physical interpretations from the point of view of one time physics. Hence, for each set of fixed background fields that obey the local Sp(2) conditions, the 2T physics action above unifies various one-time physical systems (i.e., their actions, equations of motion, etc.) into a single 2T physics system.

To find all possible actions, one must first find all possible solutions of $Q_{ij}(X,P)$ that satisfy the off-shell Sp(2) algebra before imposing the singlet condition.

The simplest example is given by [2]

$$Q_{11}=X\cdot X,\quad Q_{12}=X\cdot P,\quad Q_{22}=P\cdot P. \tag{7}$$

This form satisfies the Sp(2) algebra for any number of dimensions X^M,P^M, $M=1,2,\ldots,D$, and any signature for the flat metric η_{MN} used in the dot products $X\cdot P=X^MP^N\eta_{MN}$, etc. However, the on-shell condition (6) has nontrivial solutions only and only if the metric η_{MN} has signature $(d,2)$ with two timelike dimensions: if the signature were Euclidean the solutions would be trivial $X^M=P^M=0$; if there would be only one timelike dimension, then there would be no angular momentum $L^{MN}=0$ since X^M,P^M would both be lightlike and parallel to each other; and if there were more than two timelike dimensions the solutions would have ghosts that could not be removed by the available Sp(2) gauge symmetry. Hence two timelike dimensions is the only nontrivial physical case allowed by the Sp(2) singlet condition (i.e., gauge invariance).

The general classical worldline problem that we will solve in this paper has a counterpart in noncommutative field theory (NCFT) with local Sp(2) symmetry as formulated recently in Ref. [9]. The solution that we give here provides also a solution to the noncommutative (NC) field equations of motions that arise in that context. We note that in NCFT the same field $Q_{ij}(X,P)$ emerges as the local Sp(2) covariant left-derivative including the gauge field. The field strength is given by

$$G_{ij,kl}=[Q_{ij},Q_{kl}]_\star-i\hbar(\varepsilon_{jk}Q_{il}+\varepsilon_{ik}Q_{jl}+\varepsilon_{jl}Q_{ik}+\varepsilon_{il}Q_{jk}), \tag{8}$$

where the star commutator $[Q_{ij},Q_{kl}]_\star=Q_{ij}\star Q_{kl}-Q_{kl}\star Q_{ij}$ is constructed using the Moyal star product

$$Q_{ij}\star Q_{kl}=\exp\left[\frac{i\hbar}{2}\eta^{MN}\left(\frac{\partial}{\partial X^M}\frac{\partial}{\partial \tilde{P}^N}-\frac{\partial}{\partial P^M}\frac{\partial}{\partial \tilde{X}^N}\right)\right]\times Q_{ij}(X,P)Q_{kl}(\tilde{X},\tilde{P})|_{X=\tilde{X},P=\tilde{P}}. \tag{9}$$

Although there are general field configurations in NCFT that include nonlinear field interactions [9], we will concentrate on a special solution of the NCFT equations of motion. The special solution is obtained when $G_{ij,kl}=0$ [i.e., Q_{ij} satisfies the Sp(2) algebra under star commutators], and Q_{ij} annihilates a wave function $\Phi(X,P)$ that is interpreted as a singlet of Sp(2)

$$\frac{1}{i\hbar}[Q_{ij},Q_{kl}]_\star=\varepsilon_{jk}Q_{il}+\varepsilon_{ik}Q_{jl}+\varepsilon_{jl}Q_{ik}+\varepsilon_{il}Q_{jk}, \tag{10}$$

$$Q_{ij}\star\Phi=0. \tag{11}$$

These field equations are equivalent to the first quantization of the worldline theory in a quantum phase space formalism (as opposed to the more traditional pure position space or pure momentum space formalism).

Compared to the Poisson brackets that appear in Eq. (2) the star commutator is an infinite series in powers of $\hbar$. It reduces to the Poisson brackets in the classical limit $\hbar\to 0$:

$$\frac{1}{i\hbar}[Q_{ij},Q_{kl}]_\star\to\{Q_{ij},Q_{kl}\}. \tag{12}$$

Therefore, any solution for $Q_{ij}(X,P)$ of the form (3) that satisfies the Poisson bracket Sp(2) algebra (2) is normally expected to be only an approximate semiclassical solution of the NCFT equations (10),(11) that involve the star product (9). However, we find a much better than expected solution: by choosing certain gauges of the Sp(2) gauge symmetry in the NCFT approach, we learn that the classical solution of the Poisson bracket algebra in Eq. (2) is also an exact solution of the star commutator algebra (10) to all orders of $\hbar$.

We will see that the solution has two disconnected branches of background fields. The first branch has only low spins $s\leqslant 2$ including the gravitational field. The second branch has only high spins $s\geqslant 2$ starting with the gravitational field. These appear to have a correspondence to the massless states in string theory at extreme limits of the string tension $T\sim 1/\alpha'\to 0,\infty$. Indeed when the Regge slope α' goes to zero by fixing the graviton state only the low spin $s\leqslant 2$ massless states survive, and when the Regge slope α' goes to infinity there are an infinite number of high spin massless states. The high spin fields that we find here correspond to those massless states obtained from the graviton trajectory $s \geqslant 2$.

The paper is organized as follows. In Sec. II we discuss an infinite dimensional canonical transformation symmetry of the equations. In Sec. III, we use the symmetry to simplify the content of the background fields that appear in the expansion (3), and we impose the Poisson bracket algebra (2) to determine the field equations that must be satisfied by the remaining background fields. In Sec. IV, we discuss two special coordinate systems to solve the field equations. Then we impose the gauge invariance condition $Q_{ij}=0$, and interpret the holographic image as a relativistic particle in d dimensions x^{μ}, moving in the background of fields of various spins, including a scalar field $u(x)$, gauge field $A_{\mu}(x)$, gravitational field $g^{\mu\nu}(x)$, and higher-spin fields $g^{\mu_1\mu_2\cdots\mu_s}(x)$ for any spin s. We also derive the gauge transformation rules of the higher-spin fields in d dimensions, and learn that there are two disconnected branches. In Sec. V we show that the classical solution is also an exact quantum solution of the star product system that emerges in NCFT with local Sp(2) symmetry. In Sec. VI, we conclude with some remarks.

Our 2T approach to higher-spin fields makes connections to other methods in the literature. One connection occurs in a special Sp(2) gauge (Sec. IV) which links to Dirac's formulation of SO(d,2) conformal symmetry by using $(d+2)$-dimensional fields to represent d-dimensional fields [10]. In our paper this method is extended to all high spin fields as a particular 2T to 1T holographic picture. The d-dimensional system of this holographic picture has an overlap with a description of higher-spin fields given in Ref. [11], which is probably related to the approach of Vasiliev *et al.* (see Ref. [12], and references therein). It was shown in Ref. [11] that our special 1T holographic picture, when translated to the second order formalism (as opposed to the phase space formalism), is a completion of the de Wit–Freedman action [13] for a spinless relativistic particle interacting with higher-spin background fields.

II. INFINITE DIMENSIONAL SYMMETRY

Although our initial problem is basically at the classical level, we will adopt the idea of the associative star product, in the $\hbar \to 0$ limit, as a convenient formalism. In this way our discussion will be naturally extended in Sec. V to the case of NCFT which will be valid for any $\hbar$. The Poisson bracket is written in terms of the star product (9) as a limit of the form

$$\{A,B\}=\lim_{\hbar\to 0}\frac{1}{i\hbar}(A\star B-B\star A)=\frac{\partial A}{\partial X^M}\frac{\partial B}{\partial P_M}-\frac{\partial B}{\partial X^M}\frac{\partial A}{\partial P_M}. \tag{13}$$

Consider the following transformation of any function of phase space $A(X,P)$:

$$A(X,P)\to\tilde{A}(X,P)=\lim_{\hbar\to 0}e^{-i\varepsilon/\hbar}\star A\star e^{i\varepsilon/\hbar} \tag{14}$$

$$=A+\{\varepsilon,A\}+\frac{1}{2!}\{\varepsilon,\{\varepsilon,A\}\}+\frac{1}{3!}\{\varepsilon,\{\varepsilon,\{\varepsilon,A\}\}\}+\cdots \tag{15}$$

for any $\varepsilon(X,P)$. If the $\hbar\to 0$ is not applied, every Poisson brackets on the right-hand side is replaced by the star commutator. It is straightforward to see that the Poisson brackets of two such functions transform as

$$\{A,B\}\to\{\tilde{A},\tilde{B}\}=\{(\lim_{\hbar\to 0}e^{-i\varepsilon/\hbar}\star A\star e^{i\varepsilon/\hbar}),(\lim_{\hbar\to 0}e^{-i\varepsilon/\hbar}\star B\star e^{i\varepsilon/\hbar})\} \tag{16}$$

$$=\lim_{\hbar\to 0}e^{-i\varepsilon/\hbar}\star\{A,B\}\star e^{i\varepsilon/\hbar}. \tag{17}$$

In particular, the phase space variables X^M,P_M transform into

$$\tilde{X}^M=\lim_{\hbar\to 0}e^{-i\varepsilon/\hbar}\star X^M\star e^{i\varepsilon/\hbar},\quad \tilde{P}_M=\lim_{\hbar\to 0}e^{-i\varepsilon/\hbar}\star P_M\star e^{i\varepsilon/\hbar}, \tag{18}$$

and one can easily verify that the canonical Poisson brackets remain invariant

$$\{X^M,P_N\}=\{\tilde{X}^M,\tilde{P}_N\}=\delta^M_N\,. \tag{19}$$

So, the transformation we have defined is the most general canonical transformation. In particular, for infinitesimal ε, one has

$$\delta_\varepsilon X^M=-\partial\varepsilon/\partial P_M\,,\quad \delta_\varepsilon P_M=\partial\varepsilon/\partial X^M, \tag{20}$$

which is again recognized as a general canonical transformation with generator $\varepsilon(X,P)$. The generator $\varepsilon(X,P)$ contains an infinite number of parameters, so this set of transformations form an infinite-dimensional group. There is a resemblance between Eq. (20) and the expressions in Eq. (4) but note that those include general τ dependence in $\omega^{ij}(\tau)$ and therefore are quite different.

Under general canonical transformations (18) the particle action (1) transforms as

$$S_{Q_{ij}}(X,P)\to S_{Q_{ij}}(\tilde{X},\tilde{P})=\int d\tau[\partial_\tau\tilde{X}^M\tilde{P}_M-A^{ij}Q_{ij}(\tilde{X},\tilde{P})]. \tag{21}$$

The first term is invariant $\int d\tau(\partial_\tau\tilde{X}^M\tilde{P}_M)=\int d\tau(\partial_\tau X^M P_M)$. This is easily verified for infinitesimal $\varepsilon(X,P)$ since $\delta_\varepsilon(\partial_\tau X^M P_M)$ is a total derivative

$$\delta_\varepsilon(\partial_\tau X^M P_M)=\partial_\tau\left(\varepsilon-P\cdot\frac{\partial\varepsilon}{\partial P}\right).$$

However, the full action (1) is not invariant. Instead, it is mapped to a new action with a new set of background fields $\tilde{Q}_{ij}(X,P)$ given by

$$Q_{ij}(\tilde{X},\tilde{P})=\tilde{Q}_{ij}(X,P)=\lim_{\hbar\to 0} e^{-i\varepsilon/\hbar}\star Q_{ij}(X,P)\star e^{i\varepsilon/\hbar}, \tag{22}$$

$$S_{Q_{ij}}(X,P)\to S_{Q_{ij}}(\tilde{X},\tilde{P})=S_{\tilde{Q}_{ij}}(X,P) = \int d\tau[\partial_\tau X^M P_M - A^{ij}\tilde{Q}_{ij}(X,P)]. \tag{23}$$

After taking into account Eq. (17) we learn that the new $\tilde{Q}_{ij}(X,P)$ also satisfies the Sp(2) algebra (2) if the old one $Q_{ij}(X,P)$ did. Thus, the new action $S_{\tilde{Q}_{ij}}(X,P)$ is again invariant under the local Sp(2) symmetry (4),(5) using the new generators $\tilde{Q}_{ij}$, and is therefore in the class of actions we are seeking.

Thus, if we find a given solution for the background fields $Z_{ij}^{M_1M_2\cdots M_s}(X)$ and $A_M(X)$ in Eq. (3) such that $Q_{ij}(X,P)$ satisfy the Sp(2) algebra (2), we can find an infinite number of new solutions $\tilde{Z}_{ij}^{M_1M_2\cdots M_s}(X)$ and $\tilde{A}_M(X)$ by applying to $Q_{ij}(X,P)$ the infinite-dimensional canonical transformation (22) for any function of phase space $\varepsilon(X,P)$. We may write this function in a series form similar to Eq. (3) to display its infinite number of local parameters $\varepsilon_s^{M_1M_2\cdots M_s}(X)$

$$\varepsilon(X,P)=\sum_{s=0}^{\infty}\varepsilon_s^{M_1M_2\cdots M_s}(X)(P_{M_1}+A_{M_1}) \times(P_{M_2}+A_{M_2})\cdots(P_{M_s}+A_{M_s}). \tag{24}$$

Although this set of transformations is not a symmetry of the worldline action for a fixed set of background fields, it is evidently a symmetry in the space of actions for all possible background fields, by allowing those fields to transform. It is also an automorphism symmetry of the Sp(2) algebra (2), and of the on-shell singlet condition (6) which identifies the physical sector. Furthermore, in the NCFT setting of Ref. [9] this is, in fact, the local Sp(2) symmetry with $\varepsilon(X,P)$ playing the role of the local gauge parameter in NC gauge field theory. We will use this information to simplify our task of finding the general solution.

III. IMPOSING THE POISSON BRACKET ALGEBRA

By taking into account the infinite dimensional symmetry of the previous section, we can always map a general $Q_{11}(X,P)$ to a function of only X^M

$$\tilde{Q}_{11}=\lim_{\hbar\to 0} e^{-i\varepsilon/\hbar}\star Q_{ij}(X,P)\star e^{i\varepsilon/\hbar}=W(X). \tag{25}$$

Conversely, given $\tilde{Q}_{11}=W(X)$ we may reconstruct the general $Q_{ij}(X,P)$ by using the inverse transformation

$$Q_{11}(X,P)=\lim_{\hbar\to 0} e^{i\varepsilon/\hbar}\star W(X)\star e^{-i\varepsilon/\hbar}. \tag{26}$$

There is enough symmetry to map $W(X)$ to any desired nonzero function of X^M that would permit the reconstruction (26) of the general $Q_{11}(X,P)$, but we postpone this freedom until a later stage (see next section) in order to first exhibit a more general setting.

After fixing $\tilde{Q}_{11}=W(X)$, there is a remaining subgroup of transformations $\varepsilon(X,P)$ for which the Poisson bracket $\{\varepsilon,W\}$ vanishes, and hence $W(X)$ is invariant under it. Using the form of Eq. (24) we see that the subgroup corresponds to those transformations that satisfy the condition

$$\frac{\partial W}{\partial X^{M_1}}\varepsilon_{s\geq 1}^{M_1M_2\cdots M_s}(X)=0,\quad \text{any } \varepsilon_0(X). \tag{27}$$

This subgroup can be used to further simplify the problem. To see how, let us consider the expansions

$$\tilde{Q}_{11}(X,P)=W(X), \tag{28}$$

$$\tilde{Q}_{12}(X,P)=\sum_{s=0}^{\infty} V_s^{M_1M_2\cdots M_s}(P_{M_1}+A_{M_1}) \times(P_{M_2}+A_{M_2})\cdots(P_{M_s}+A_{M_s}), \tag{29}$$

$$\tilde{Q}_{22}(X,P)=\sum_{s=0}^{\infty} G_s^{M_1M_2\cdots M_s}(P_{M_1}+A_{M_1}) \times(P_{M_2}+A_{M_2})\cdots(P_{M_s}+A_{M_s}), \tag{30}$$

where $G_s^{M_1M_2\cdots M_s}(X)$ and $V_s^{M_1M_2\cdots M_s}(X)$ are fully symmetric local tensors of rank s.

We can completely determine the coefficients $V_s^{M_1M_2\cdots M_s}$ in terms of W and $G_s^{M_1M_2\cdots M_s}$ by imposing one of the Sp(2) conditions $\{\tilde{Q}_{11},\tilde{Q}_{22}\}=4\tilde{Q}_{12}$

$$V_{s-1}^{M_2M_3\cdots M_s}=\frac{s}{4}\frac{\partial W}{\partial X^{M_1}}G_s^{M_1M_2\cdots M_s}. \tag{31}$$

Furthermore, by imposing another Sp(2) condition $\{\tilde{Q}_{11},\tilde{Q}_{12}\}=2\tilde{Q}_{11}$ we find

$$\frac{\partial W}{\partial X^{M_1}}V_1^{M_1}=2W,\quad \frac{\partial W}{\partial X^{M_1}}V_{s\geq 2}^{M_1M_2\cdots M_s}=0. \tag{32}$$

Now, for such $V_s^{M_1M_2\cdots M_s}$, by using the remaining subgroup symmetry (27) we can transform to a frame in which all $V_s^{M_1M_2\cdots M_s}$ for $s\geq 2$ vanish. By comparing the expressions (27),(32) and counting parameters we see that this must be possible. To see it in more detail, we derive the infinitesimal transformation law for $G_s^{M_1M_2\cdots M_s}$ and A_M from

$$\delta Q_{22}=(\partial Q_{22}/\partial A)\delta A+\sum_s(\partial Q_{22}/\partial G_s)\delta G_s=\{\varepsilon,Q_{22}\} \tag{33}$$

by expanding both sides in powers of $(P+A)$ and comparing coefficients. We write the result in symbolic notation by suppressing the indices

$$\delta A_M = \partial_M \tilde{\varepsilon}_0 - \mathcal{L}_{\varepsilon_1} A_M, \tag{34}$$

$$\delta G_0 = -\varepsilon_1 \cdot \partial G_0, \tag{35}$$

$$\delta G_1 = -\mathcal{L}_{\varepsilon_1} G_1 - (\varepsilon_2 \cdot \partial G_0 + \varepsilon_2 F G_1), \tag{36}$$

$$\begin{aligned}\delta G_{s\geqslant 2} = -\mathcal{L}_{\varepsilon_1} G_s &- (\varepsilon_2 \cdot \partial G_{s-1} - G_{s-1} \cdot \partial \varepsilon_2 + \varepsilon_2 F G_s)\\ &- \cdots - (\varepsilon_s \cdot \partial G_1 - G_1 \cdot \partial \varepsilon_s + \varepsilon_s F G_2)\\ &- (\varepsilon_{s+1} \cdot \partial G_0 + \varepsilon_{s+1} F G_1).\end{aligned} \tag{37}$$

In $\delta G_{s\geqslant 2}$ the ellipses represent terms of the form $(\varepsilon_k \cdot \partial G_{s-k+1} - G_{s-k+1} \cdot \partial \varepsilon_k + \varepsilon_k F G_{s-k+2})$ for all $2<k<s$. Here F_{MN} is the gauge field strength

$$F_{MN}(X) = \partial_M A_N(X) - \partial_N A_M(X), \tag{38}$$

$\mathcal{L}_{\varepsilon_1} G_s$ is the Lie derivative of the tensor $G_s^{M_1 M_2 \cdots M_s}$ with respect to the vector ε_1^M

$$\begin{aligned}(\mathcal{L}_{\varepsilon_1} G_s)^{M_1 M_2 \cdots M_s} = \varepsilon_1 \cdot \partial G_s^{M_1 M_2 \cdots M_s} &- \partial_K \varepsilon_1^{M_1} G_s^{K M_2 \cdots M_s}\\ &- \cdots - \partial_K \varepsilon_1^{M_s} G_s^{M_1 M_2 \cdots K}.\end{aligned} \tag{39}$$

In the other terms, $\varepsilon_k \cdot \partial G_l$ (similarly $G_k \cdot \partial \varepsilon_l$) is the tensor

$$\varepsilon_k \cdot \partial G_l = \frac{k!\,l!}{(k+l-1)!} \varepsilon_k^{M_1 (M_2 \cdots M_k)} \partial_{M_1} G_l^{(M_{k+1}, \cdots M_{k+l})}, \tag{40}$$

where all unsummed upper indices ($k+l-1$ of them) are symmetrized, and $\varepsilon_k F G_l$ is the tensor

$$\varepsilon_k F G_l = \frac{k!\,l!}{(k+l-2)!} \varepsilon_k^{M_1 (M_2 \cdots M_k} G_l^{M_{k+1} \cdots) M_{k+l}} F_{M_1 M_{k+l}}, \tag{41}$$

where all unsummed upper indices ($k+l-2$ of them) are symmetrized. Finally $\tilde{\varepsilon}_0$ which appears in δA is defined by $\tilde{\varepsilon}_0 = \varepsilon_0 + \varepsilon_1 \cdot A$.

From δA_M it is evident that $\tilde{\varepsilon}_0(X)$ is a Yang-Mills type gauge parameter, and from $\mathcal{L}_{\varepsilon_1} G_s$ it is clear that $\varepsilon_1^M(X)$ is the parameter of general coordinate transformations in position space. The remaining parameters $\varepsilon_{s\geqslant 2}(X)$ are gauge parameters for high spin fields (note that the derivative of the ε_s appear in the transformation rules). From the transformation laws for $\delta A, \delta G_s$ we find the transformation law for $\delta V_s^{M_1 M_2 \cdots M_s}$ by contracting both sides of the equation above with $\partial_M W(X)$. After using the subgroup condition (27) and the definition (31) we find

$$\delta V_0 = -\mathcal{L}_{\varepsilon_1} V_0, \tag{42}$$

$$\delta V_1 = -\mathcal{L}_{\varepsilon_1} V_1 - (\varepsilon_2 \cdot \partial V_0 + \varepsilon_2 F V_1) \tag{43}$$

$$\begin{aligned}\delta V_{s\geqslant 2} = -\mathcal{L}_{\varepsilon_1} V_s &- (\varepsilon_2 \cdot \partial V_{s-1} - V_{s-1} \cdot \partial \varepsilon_2 + \varepsilon_2 F V_s)\\ &- \cdots - (\varepsilon_s \cdot \partial V_1 - V_1 \cdot \partial \varepsilon_s + \varepsilon_s F V_2)\\ &- (\varepsilon_{s+1} \cdot \partial V_0 + \varepsilon_{s+1} F V_1).\end{aligned} \tag{44}$$

The form of δV_k is similar to the form of δG_k as might be expected, since it can also be obtained from $\delta Q_{12} = \{\varepsilon, Q_{12}\}$, but we have derived it by taking into account the restriction (31) and the subgroup condition (27).

For V_s of the form (32) the subgroup parameters are sufficient to transform to a frame where $V_{s\geqslant 2}=0$. Therefore, we may always start from a frame of the form

$$\tilde{Q}_{11}(X,P) = W(X); \quad V_1 \cdot \partial W = 2W, \tag{45}$$

$$\tilde{Q}_{12}(X,P) = V_0 + V_1^M (P_M + A_M); \quad V_0 = \frac{1}{4} \partial_N W G_1^N,$$

$$V_1^M = \frac{1}{2} \partial_N W G_2^{MN}, \tag{46}$$

$$\begin{aligned}\tilde{Q}_{22}(X,P) = \sum_{s=0}^{\infty} G_s^{M_1 M_2 \cdots M_s} &(P_{M_1} + A_{M_1})\\ &\times (P_{M_2} + A_{M_2}) \cdots (P_{M_s} + A_{M_s})\end{aligned} \tag{47}$$

and transform to the most general solution via

$$Q_{ij}(X,P) = \lim_{\hbar \to 0} e^{i\varepsilon/\hbar} \star \tilde{Q}_{ij}(X,P) \star e^{-i\varepsilon/\hbar}. \tag{48}$$

In $\tilde{Q}_{22}$ the term G_1^M may be set equal to zero by shifting $A_M \to A_M - \frac{1}{2}(G_2)_{MN} G_1^N + \cdots$, and then redefining all other background fields. Here we have assumed that the tensor G_2^{MN} has an inverse $(G_2)_{MN}$; in fact, as we will see soon, it will have the meaning of a metric. Therefore, we will assume $G_1^M = 0$ without any loss of generality. In that case we see from Eq. (46) that we must also have $V_0 = 0$.

It suffices to impose the remaining relations of the Sp(2) algebra in this frame. By comparing the coefficients of every power of $(P+A)$ in the condition $\{\tilde{Q}_{12}, \tilde{Q}_{22}\} = 2\tilde{Q}_{22}$ we derive the following equations:

$$V_1^M F_{MN} = 0, \quad \mathcal{L}_{V_1} G_s = -2 G_s, \tag{49}$$

where $\mathcal{L}_{V_1} G_s$ is the Lie derivative with respect to the vector V_1 [see Eq. (39)]. These, together with

$$V_1^M = \frac{1}{2} \partial_N W G_2^{MN}, \quad V_1 \cdot \partial W = 2W, \quad V_0 = 0,$$

$$G_1 = 0, \quad \partial W \cdot G_{s\geqslant 3} = 0, \tag{50}$$

that we used before, provide the complete set of equations that must be satisfied to have a closure of the Sp(2) algebra. These background fields, together with the background fields provided by the general $\varepsilon(X,P)$ through Eq. (48), generalize the results of Ref. [4], where only A_M, G_0, and G_2^{MN} had been included.

There still is remaining canonical symmetry that keeps the form of the above $\tilde{Q}_{ij}$ unchanged. This is given by the subgroup of symmetries associated with $\tilde{\varepsilon}_0(X),\varepsilon_1^M(X)$ which have the meaning of local parameters for Yang-Mills and general coordinate transformations, and also the higher-spin symmetries that satisfy

$$\partial W\cdot\varepsilon_{s\geqslant1}=0,\ \ \mathcal{L}_{V_1}\varepsilon_{s\geqslant1}=0,\ \ \partial G_0\cdot\varepsilon_2=0. \tag{51}$$

The conditions in Eq. (51) are obtained after setting $G_1=V_{s\geqslant2}=0$ and $\delta G_1=\delta V_{s\geqslant2}=0$, as well as using Eq. (27).

It is possible to go further in using the remaining $\varepsilon_s(X)$ transformations, but this will not be necessary since the physical content of the worldline system will be more transparent by using the background fields G_s and A_M identified up to this stage. However, we will return to the remaining symmetry at a later stage to clarify its action on the fields, and thus discover that there are two disconnected branches.

IV. CHOOSING COORDINATES AND $W(X)$

As mentioned in the beginning of the previous section the original $\varepsilon(X,P)$ transformations permits a choice for the function $W(X)$, while the surviving $\varepsilon_1^M(X)$ which is equivalent to general coordinate transformations further permits a choice for the vector $V_1^M(X)$, as long as it is consistent with the differential conditions given above. Given this freedom we will explore two choices for $W(X)$ and $V_1^M(X)$ in this section.

A. SO$(d,2)$ covariant $W(X)=X^2$

We choose $W(X)$ and $V_1^M(X)$ as follows:

$$W(X)=X^2=X^MX^N\eta_{MN},\ \ V_1^M(X)=X^M, \tag{52}$$

where η_{MN} is the metric for SO$(d,2)$. These coincide with part of the simplest Sp(2) system (7). We cannot choose any other signature η_{MN} since we already know that the constraints $Q_{ij}(X,P)=0$ have solutions only when the signature includes two timelike dimensions.

Using Eqs. (49) and (50), the metric $G_2^{MN}(X)$ takes the form

$$G_2^{MN}=\eta^{MN}+h_2^{MN}(X),\ \ X\cdot\partial h_2^{MN}=0,\ \ h_2^{MN}X_N=0. \tag{53}$$

G_2^{MN} is an invertible metric. The fluctuation $h_2^{MN}(X)$ is any homogeneous function of degree zero and it is orthogonal to X_N.

Using the $\varepsilon_0(X)$ gauge degree of freedom we work in the axial gauge $X\cdot A=0$, then the condition $X^MF_{MN}=0$ reduces to

$$(X\cdot\partial+1)A_M=0,\ \ X\cdot A=0. \tag{54}$$

Therefore $A_M(X)$ is any homogeneous vector of degree (-1) and it is orthogonal to X_M. There still is remaining gauge symmetry $\delta A_M=\partial_M\varepsilon_0$ provided $\varepsilon_0(X)$ is a homogeneous function of degree zero

$$X\cdot\partial\varepsilon_0=0. \tag{55}$$

Similarly, the higher-spin fields in Eqs. (49),(50) satisfy

$$(X\cdot\partial-s+2)G_{s\geqslant3}^{M_1M_2\cdots M_s}=0,\ \ X_{M_1}G_{s\geqslant3}^{M_1M_2\cdots M_s}=0. \tag{56}$$

These equations are easily solved by homogeneous tensors of degree $s-2$ that are orthogonal to X_N.

The $\tilde{Q}_{ij}$ now take the SO$(d,2)$ covariant form

$$\tilde{Q}_{11}=X^2,\ \ \tilde{Q}_{12}=X\cdot P, \tag{57}$$

$$\tilde{Q}_{22}=G_0+\sum_{s=2}^{\infty}G_s^{M_1\cdots M_s}(P+A)_{M_1}\cdots(P+A)_{M_s}. \tag{58}$$

Thus, Q_{11} and Q_{12} are reduced to the form of the simplest 2T physics system (7), while Q_{22} contains the nontrivial background fields. The remaining symmetry of Eq. (51) is given by

$$\partial G_0\cdot\varepsilon_2=0;\ \ (X\cdot\partial-s)\varepsilon_{s\geqslant0}^{M_1M_2\cdots M_s}=0,\ \ X_{M_1}\varepsilon_{s\geqslant1}^{M_1M_2\cdots M_s}=0, \tag{59}$$

where all dot products involve the metric η_{MN} of SO$(d,2)$. Hence the frame is SO$(d,2)$ covariant, and this will be reflected in any of the gauge fixed versions of the theory. As before, $\varepsilon_0(X)$ is the (homogeneous) Yang-Mills-type gauge parameter and the $\varepsilon_{s\geqslant1}$ play the role of gauge parameters for higher-spin fields as in Eq. (37).

To solve the constraints $Q_{ij}=0$ we can choose various Sp(2) gauges that produce the 2T to 1T holographic reduction. This identifies some combination of the $X^M(\tau)$ with the τ parameter, thus reducing the 2T physics description to the 1T physics description. Depending on the choice made, the 1T dynamics of the resulting holographic picture in d dimensions appears different from the point of view of one-time. This produces various holographic pictures in an analogous way to the free case discussed previously in Ref. [1]. We plan to discuss several examples of holographic pictures in the presence of background fields in a future publication.

B. Lightcone type $W(X)=-2\kappa w$

There are coordinate choices that provide a shortcut to some of the holographic pictures, although they do not illustrate the magical unification of various 1T dynamics into a single 2T dynamics as clearly as the SO$(d,2)$ formalism of the previous section. Nevertheless, since such coordinate systems can be useful, we analyze one that is closely related to the relativistic particle dynamics in d dimensions.[1] Following Ref. [4] we consider a coordinate system $X^M = (\kappa, w, x^\mu)$ and use the symmetries to choose $V_1^M = (\kappa, w, 0)$ and $W = -2w\kappa$. Then, as in Ref. [4] the solution for the gauge field, the spin-2 gravity field G_2^{MN} and the scalar field G_0 are

$$A_\kappa = -\frac{w}{2\kappa^2} B\left(\frac{w}{\kappa}, x\right), \quad A_w = \frac{1}{2\kappa} B\left(\frac{w}{\kappa}, x\right), \quad A_\mu = A_\mu\left(\frac{w}{\kappa}, x\right), \tag{60}$$

$$G_2^{MN} = \begin{pmatrix} \frac{\kappa}{w}(\gamma-1) & -\gamma & \frac{1}{\kappa} W^\nu \\ -\gamma & \frac{w}{\kappa}(\gamma-1) & -\frac{w}{\kappa^2} W^\nu \\ \frac{1}{\kappa} W^\mu & -\frac{w}{\kappa^2} W^\mu & \frac{g^{\mu\nu}}{\kappa^2} \end{pmatrix}, \tag{61}$$

$$G_0 = \frac{1}{\kappa^2} u\left(x, \frac{w}{\kappa}\right), \tag{62}$$

where the functions $A_\mu(w/\kappa, x)$, $B(w/\kappa, x)$, $\gamma(x, w/\kappa)$, $W^\mu(x, w/\kappa)$, $g^{\mu\nu}(x, w/\kappa)$, $u(x, w/\kappa)$ are arbitrary functions of only x^μ and the ratio w/κ.

We now extend this analysis to the higher-spin fields. The equation

$$G_{s\geq 3}^{M_1 M_2 \cdots M_s} \cdot \partial_{M_s} W = 0 \tag{63}$$

becomes

$$w G_{s\geq 3}^{M_1 \cdots M_{s-1}\kappa} = -\kappa G_{s\geq 3}^{M_1 \cdots M_{s-1} w}. \tag{64}$$

This shows that not all the components of $G_{s\geq 3}^{M_1 M_2 \cdots M_s}$ are independent. The condition

$$\mathcal{L}_{V_1} G_s = -2 G_s \tag{65}$$

becomes

$$(\kappa\partial_\kappa + w\partial_w) G_s^{M_1 \cdots M_s} - \sum_{n=1}^{s} \delta_\kappa^{M_n} G_s^{M_1 \cdots M_{n-1}\kappa M_{n+1} \cdots M_s} - \sum_{n=1}^{s} \delta_w^{M_n} G_s^{M_1 \cdots M_{n-1} w M_{n+1} \cdots M_s} = -2 G_s^{M_1 \cdots M_s}. \tag{66}$$

Specializing the indices for independent components and also using the relation (64) between the components of $G_{s\geq 3}^{M_1 M_2 \cdots M_s}$ we get the solution for all components of $G_{s\geq 3}^{M_1 M_2 \cdots M_s}$ as

$$G_s^{\overbrace{\kappa\cdots\kappa}^{m}\overbrace{w\cdots w}^{n}\mu_1\cdots\mu_{s-n-m}} = \kappa^{m-2}(-w)^n g_{s,(s-n-m)}^{\mu_1\cdots\mu_{s-n-m}}, \tag{67}$$

where $g_{s,k}^{\mu_1\mu_2\cdots\mu_k}(x, w/\kappa)$, where $k = 1, \ldots, s$, are arbitrary functions and independent of each other.

For this solution, the generators of Sp(2,R) in Eqs. (45)–(47) become

$$\tilde{Q}_{11} = -2\kappa w, \tag{68}$$

$$\tilde{Q}_{12} = \kappa p_\kappa + w p_w, \tag{69}$$

$$\tilde{Q}_{22} = -\frac{1}{\kappa w}\left[\left(\kappa p_\kappa - \frac{wB}{2\kappa}\right)^2 + \left(w p_w + \frac{wB}{2\kappa}\right)^2\right] + \frac{H + H'}{\kappa^2}, \tag{70}$$

where H, H', which contain the background fields, are defined by

$$H = u + g^{\mu\nu}(p_\mu + A_\mu)(p_\nu + A_\nu) + \sum_{s=3}^{\infty} g_{s,s}^{\mu_1\cdots\mu_s} \times (p_{\mu_1} + A_{\mu_1}) \cdots (p_{\mu_s} + A_{\mu_s}), \tag{71}$$

$$H' = \sum_{s=2}^{\infty} \sum_{k=0}^{s-1} g_{s,k}^{\mu_1\cdots\mu_k}\left(\kappa p_\kappa - w p_w - \frac{wB}{\kappa}\right)^{s-k} \times (p_{\mu_1} + A_{\mu_1}) \cdots (p_{\mu_k} + A_{\mu_k}). \tag{72}$$

H contains only the highest spin components $g_{s,s}^{\mu_1\mu_2\cdots\mu_s}$ that emerge from $G_{s\geq 2}^{M_1 M_2 \cdots M_s}$. Here we have defined the metric $g^{\mu\nu} = g_{2,2}^{\mu\nu}$ as in Eq. (61). All the remaining lower spin components $g_{s,k}^{\mu_1\mu_2\cdots\mu_k}$ with $k \leq s-1$ are included in H'. In the $s=2$ term of H' we have defined $g_{2,0} \equiv \gamma\kappa/w$ and $g_{2,1}^\mu \equiv W^\mu$ in comparison to Eq. (61). It can be easily verified that these $\tilde{Q}_{ij}$ obey the Sp(2,R) algebra for any background fields u, $g_{\mu\nu}$, A_μ, B and $g_{s,k}^{\mu_1\mu_2\cdots\mu_k}$ $(k = 0, \ldots, s)$ that are *arbitrary functions of* $(x^\mu, w/\kappa)$.

We next can choose some Sp(2,R) gauges to solve the Sp(2,R) constraints $\tilde{Q}_{ij} = 0$ and reduce to a one-time theory containing the higher-spin fields. As in the low spin-1 and spin-2 cases of Ref. [4], we choose $\kappa(\tau) = 1$ and $p_w(\tau) = 0$,

[1] We call the coordinate system in this section ‘‘lightcone type’’ because, in the Sp(2) gauge $\kappa = 1$, it can be related to a lightcone type Sp(2) gauge ($X^{+'} = 1$) in the SO(d,2) covariant formalism of the previous section. Once the gauge is fixed from either point of view, the 1T holographic picture describes the massless relativistic particle (see, e.g., Ref. [1]) including its interactions with background fields.

and solve $\tilde{Q}_{11}=\tilde{Q}_{12}=0$ in the form $w(\tau)=p_\kappa(\tau)=0$. We also use the canonical freedom ε_0 to work in a gauge that insures $wB/\kappa\to 0$, as $w/\kappa\to 0$. Then the $\tilde{Q}_{ij}$ simplify to

$$\tilde{Q}_{11}=\tilde{Q}_{12}=0,\ \ \tilde{Q}_{22}=H. \tag{73}$$

At this point, the two-time $(d+2)$-dimensional theory described by the original action (1) reduces to a one-time theory in d dimensions

$$S=\int d\tau\left(\partial_\tau x^\mu p_\mu-\frac{1}{2}A^{22}H\right). \tag{74}$$

This is a particular 2T to 1T holographic picture of the higher-dimensional theory obtained in a specific gauge. There remains unfixed one gauge subgroup of Sp(2,R) which corresponds to τ reparametrization, and the corresponding Hamiltonian constraint is $H\sim 0$. There is also remaining canonical freedom which we will discuss below. Here, in addition to the usual background fields $g_{\mu\nu}(x)$, $A_\mu(x)$, $u(x)$, the Hamiltonian includes the higher-spin fields $g_{s,s}^{\mu_1\mu_2\cdots\mu_s}$ that now are functions of only the d dimensional coordinates x^μ, since $w/\kappa=0$. Similar to γ and W^μ in the gravity case, the nonleading $g_{s,k}^{\mu_1\mu_2\cdots\mu_k}$ for $k<s$ decuple from the dynamics that govern the time development of $x^\mu(\tau)$ in this Sp(2) gauge.

A similar conclusion is obtained if we use the SO(d,2) covariant formalism of the previous section when we choose the Sp(2) gauges $X^{+'}=1$, and $P^{+'}=-P_{-'}=0$. The algebra for arriving at the final conclusion (74) is simpler in the coordinate frame of the present section,[2] and this was the reason for introducing the "lightcone type" $W=-2\kappa w$. However, from the SO(d,2) covariant formalism we learn that there is a hidden SO(d,2) in the d-dimensional action Eq. (74). This can be explored by examining the SO(d,2) transformations produced by $\varepsilon_1^M=\omega^{MN}X_N$, obeying Eq. (59), on all the fields through the Lie derivative $\delta A_M=\mathcal{L}_{\varepsilon_1}A_M$, $\delta G_s=-\mathcal{L}_{\varepsilon_1}G_s$, but this will not be further pursued here.

In the present Sp(2) gauge we find a link to Ref. [11] where the action (74) was discussed. The symmetries inherited from our $(d+2)$-dimensional approach (discussed below) have some overlap with those discussed in Ref. [11]. It was shown in Ref. [11] that the first order action (74) improves and completes the second order action discussed in Ref. [13]. Also, the incomplete local invariance discussed in Ref. [13] is now completed by the inclusion of the higher powers of velocity which were unknown in Ref. [13]. In the second order formalism one verifies once more that the action describes a particle moving in the background of arbitrary electromagnetic, gravitational and higher-spin fields in the remaining d dimensional spacetime.

[2] The (κ,w,x^μ) coordinate system can be related to the one in the previous section by a change of variables as follows. Starting from the previous section define a light cone type basis $X^{\pm'}=(X^{0'}\pm X^{1'})/\sqrt{2}$, and then make the change of variables $X^{+'}=\kappa$, $X^\mu=\kappa x^\mu$, $X^{-'}=w+\kappa x^2/2$. Then $W=X\cdot X=-2X^{+'}X^{-'}+X^\mu X_\mu=-2\kappa w$. The momenta (with lower indices) are transformed as follows: $P_{+'}=p_\kappa+p_w x^2/2-x\cdot p/\kappa$, $P_{-'}=p_w$, and $P_\mu=p_\mu/\kappa-p_w x_\mu$. One can varify that $\dot{X}\cdot P=\dot{X}^{+'}P_{+'}+\dot{X}^{-'}P_{-'}+\dot{X}^\mu P_\mu=\dot{\kappa}p_\kappa+\dot{w}p_w+\dot{x}\cdot p$. In this coordinate basis $X\cdot P=\kappa p_\kappa+wp_w$ and the dimension operator $X\cdot\partial$ takes the form $X\cdot\partial=\kappa\partial_\kappa+w\partial_w$. This shows that all the results obtained with the lightcone type $W=-2\kappa w$ can also be recovered from the covariant $W(X)=X^2$, and vice versa.

C. Surviving canonical symmetry in d dimensions

Let us now analyze the form of the d dimensional canonical symmetry inherited from our $(d+2)$ dimensional approach. Recall that the infinite dimensional canonical symmetry $\varepsilon(X,P)$ is not a symmetry of the action, it is only a symmetry if the fields are permitted to transform in the space of all possible worldline actions. What we wish to determine here is: what is the subset of d-dimensional actions that are related to each other by the surviving canonical symmetry in the remaining d dimensions. As we will see, there are disconnected branches, one for low spin backgrounds and one for high spin backgrounds. These branches may correspond to independent theories, or to different phases or limits of the same theory. Interestingly, string theory seems to offer a possibility of making a connection to these branches in the zero and infinite tension limits. Furthermore, we will show that the noncommutative field theory constructed in Ref. [9], which includes interactions, contains precisely the same branches in the free limit.

As shown at the end of Sec. III, a subgroup of the higher-spin symmetries that keeps the form of Q_{ij} unchanged satisfy

$$\partial W\cdot\varepsilon_{s\geq 1}=0,\ \ \mathcal{L}_{V_1}\varepsilon_{s\geq 1}=0,\ \ \partial G_0\cdot\varepsilon_2=0. \tag{75}$$

We will solve these equations explicitly and identify the unconstrained remaining symmetry parameters. We will discuss the case for $W=-2\kappa w$ and $V_1^M=(\kappa,w,0)$ of the previous subsection. The first equation becomes

$$w\varepsilon_{s\geq 1}^{M_1\cdots M_{s-1}\kappa}=-\kappa\varepsilon_{s\geq 1}^{M_1\cdots M_{s-1}w} \tag{76}$$

and the second equation becomes

$$(\kappa\partial_\kappa+w\partial_w)\varepsilon_{s\geq 1}^{M_1\cdots M_s}-\sum_{n=1}^{s}\delta_\kappa^{M_n}\varepsilon_{s\geq 1}^{M_1\cdots M_{n-1}\kappa M_{n+1}\cdots M_s}-\sum_{n=1}^{s}\delta_w^{M_n}\varepsilon_{s\geq 1}^{M_1\cdots M_{n-1}wM_{n+1}\cdots M_s}=0. \tag{77}$$

Specializing the indices for independent components and also using Eq. (76) we get the solution for all components of the higher-spin symmetry parameters, that obey the subgroup conditions, as

$$\varepsilon_{s\geq 1}^{\overbrace{\kappa\cdots\kappa}^{m}\overbrace{w\cdots w}^{n}\mu_1\cdots\mu_{s-n-m}}=(-1)^n\kappa^m w^n\varepsilon_{s,(s-n-m)}^{\mu_1\cdots\mu_{s-n-m}}, \tag{78}$$

where $\varepsilon_{s,k}^{\mu_1\cdots\mu_k}(x,w/\kappa)$, with $k=0,1,\ldots,s$, are arbitrary parameters and independent of each other. Therefore the form of $\varepsilon(X,P)$ that satisfies all the conditions for the remaining symmetry takes the form

$$\varepsilon_{\text{remain}}(X,P)=\sum_{s=0}^{\infty}\sum_{k=0}^{s}\varepsilon_{s,k}^{\mu_1\cdots\mu_k}\left(\kappa p_\kappa-wp_w-\frac{w}{\kappa}B\right)^{s-k}\times(p_{\mu_1}+A_{\mu_1})\cdots(p_{\mu_k}+A_{\mu_k}). \tag{79}$$

This identifies $\varepsilon_{s,k}^{\mu_1\cdots\mu_k}(x,w/\kappa)$, with $k=0,1,\ldots,s$, as the unconstrained remaining canonical transformation parameters.

For notational purposes we are going to use the symbol ε_s^k for $\varepsilon_{s,k}^{\mu_1\cdots\mu_k}$ from now on. We will also indicate the highest-spin fields $g_{s,s}^{\mu_1\mu_2\cdots\mu_s}$ in d dimensions as simply g_s. The third condition in Eq. (51) gives some extra constraint on ε_2^{MN} which will not be needed here, so we are going to ignore that condition in the rest of this discussion.

Let us now consider the gauge $\kappa(\tau)=1$ and $p_\kappa(\tau)=0$, $B=0$, and the physical sector that satisfies $\tilde{Q}_{11}=\tilde{Q}_{12}=0$ [or $w(\tau)=p_w(\tau)=0$] as described by the d-dimensional holographic picture whose action is Eq. (74). We discuss the role of the remaining canonical symmetry in this gauge. The transformation laws for the relevant high-spin fields g_s, computed from Eq. (33) through $\{\varepsilon_{\text{remain}},\tilde{Q}_{22}\}$, come only from the terms $k=(s-1),s$ in Eq. (79) since we set $w=p_w=p_\kappa=0$ and $\kappa=1$ after performing the differentiation in the Poisson bracket $\{\varepsilon_{\text{remain}},\tilde{Q}_{22}\}$. Equivalently, one may obtain the transformation laws in this gauge by specializing the indices in Eq. (37). The result is

$$\delta g_s=(2\varepsilon_1^0 g_s-\mathcal{L}_{\varepsilon_1^1}g_s)+\sum_{n=2}^{s-1}(2\varepsilon_n^{n-1}g_{s-n+1}-\varepsilon_n^n\cdot\partial g_{s-n+1}+g_{s-n+1}\cdot\partial\varepsilon_n^n-\varepsilon_n^nFg_{s-n+2})-\varepsilon_s^sFg_2+2(s+1)\varepsilon_{s+1}^s u-\varepsilon_{s+1}^{s+1}\cdot\partial u. \tag{80}$$

Each higher-spin field g_s is transformed by lower-rank transformation parameters, ε_n^{n-1} and ε_n^n $(n=1,\ldots,s-1)$, and also by ε_s^s, ε_{s+1}^s and ε_{s+1}^{s+1}. In passing we note that these transformations inherited from $d+2$ dimensions are somewhat different than those considered in Ref. [11] although there is some overlap.

If we specialize to $s=2$, we get

$$\delta g_2^{\mu\nu}=2\varepsilon_1^0 g_2^{\mu\nu}-\mathcal{L}_{\varepsilon_1^1}g_2^{\mu\nu}-2\varepsilon_2^{\rho(\mu}F_{\rho\sigma}g_2^{\nu)\sigma}+6\varepsilon_3^{\mu\nu}u-3\varepsilon_3^{\mu\nu\rho}\partial_\rho u. \tag{81}$$

Other than the usual general coordinate transformations associated with ε_1^1 and the Weyl dilatations associated with ε_1^0, it contains second rank $\varepsilon_2^{\rho\mu},\varepsilon_3^{\mu\nu}$ and third rank $\varepsilon_3^{\mu\nu\rho}$ transformation parameters. The latter unusual transformations mix the gravitational field with the gauge field $F_{\rho\sigma}$ and with the scalar field u. Under such transformations, if a field theory with such local symmetry could exist, one could remove the gravitational field completely. In fact the same remark applies to all g_s. If these could be true gauge symmetries, all worldline theories would be canonically transformed to trivial backgrounds. However, there are no known field theories that realize this local symmetry, and therefore it does not make sense to interpret them as symmetries in the larger space of d-dimensional worldline theories. This was of concern in Ref. [11]. Fortunately there is a legitimate resolution by realizing that there are two branches of worldline theories, one for low spin $(s\leqslant 2)$ and one for high spin $(s\geqslant 2)$, that form consistent sets under the transformations as follows.

The first branch is associated with familiar field theories for the low spin sector including $u,A_\mu,g_{\mu\nu}$. The corresponding set of worldline actions $S(u,A,g_2)$, in which all background fields $g_{s\geqslant 3}$ vanish, are transformed into each other under gauge transformations $\varepsilon_0(x)$, dilatations ε_1^0 and general coordinate transformations ε_1^1. Since $g_{s\geqslant 3}=0$, all $\varepsilon_{s\geqslant 2}$ must be set to zero, and then the low spin parameters $\varepsilon_0,\varepsilon_1^0,\varepsilon_1^1$ form a closed group of local transformations realized on only $u,A_\mu,g_{\mu\nu}$, as seen from the transformation laws given above. This defines a branch of worldline theories for low spins that are connected to each other by the low spin canonical transformations. This is the usual set of familiar symmetries and actions.

A second branch of worldline theories exists when the background fields u,A_μ vanish. In this high spin branch only $g_{s\geqslant 2}$ occurs and therefore, according to the transformations given above they form a basis for a representation including only the lower rank gauge parameters ε_k^{k-1} and ε_k^k $(k=1,\ldots,s-1)$. Then the transformation rule for the higher-spin fields in d dimensions becomes

$$\delta g_{s\geqslant 2}=\sum_{n=1}^{s-1}(2\varepsilon_n^{n-1}g_{s-n+1}-\varepsilon_n^n\cdot\partial g_{s-n+1}+g_{s-n+1}\cdot\partial\varepsilon_n^n) \tag{82}$$

$$=(2\varepsilon_1^0 g_s-\mathcal{L}_{\varepsilon_1^1}g_s)+(2\varepsilon_2^1 g_{s-1}-\varepsilon_2^2\cdot\partial g_{s-1}+g_{s-1}\cdot\partial\varepsilon_2^2)+\cdots+(2\varepsilon_{s-1}^{s-2}g_2-\varepsilon_{s-1}^{s-1}\cdot\partial g_2+g_2\cdot\partial\varepsilon_{s-1}^{s-1}). \tag{83}$$

We note that the very last term contains $g_2^{\mu\nu}$, which is the d dimensional metric that can be used to raise indices

$$\delta g_s^{\mu_1\mu_2\cdots\mu_s}=\cdots+(2g_2^{(\mu_1\mu_2}\varepsilon_{s-1,s-2}^{\mu_3\cdots\mu_s)}-\varepsilon_{s-1,s-1}^{\mu(\mu_3\cdots\mu_s)}\partial_\mu g_2^{(\mu_1\mu_2)}+\partial^{(\mu_1}\varepsilon_{s-1,s-1}^{\mu_2\cdots\mu_s)}). \tag{84}$$

The very last term contains the usual derivative term expected in the gauge transformation laws of a high spin gauge field in d dimensions.

Not all components of the remaining g_s can be removed with these gauge transformations; therefore physical components survive in this high spin branch. In particular, there is enough remaining freedom to make further gauge choices such that $g_s^{\mu_1\mu_2\cdots\mu_s}$ is double traceless [i.e.,

$g_{s\geq 4}^{\mu_1\mu_2\cdots\mu_s}(g_2)_{\mu_1\mu_2}(g_2)_{\mu_3\mu_4}=0$], as needed for a correct description of high spin fields [14]. The high-spin background fields defined in this way belong to a unitary theory. It is known that with the double traceless condition on g_s, and the gauge symmetry generated by traceless $\varepsilon_{s-1,s-1}$ (which is a subgroup of our case), the correct kinetic terms for high spin fields are written uniquely in a field theory approach. Thus, the worldline theory constructed with the double traceless g_s makes sense physically. We would not be allowed to make canonical transformations to further simplify the worldline theory if we assume that it corresponds to a more complete theory in which the extra transformations could not be implemented.

Having clarified this point, we may still analyze the fate of the canonical symmetry left over after the double traceless condition. The remaining gauge parameters must satisfy the conditions that follow from the double tracelessness of δg_s:

$$(\cdots+2g_2^{(\mu_1\mu_2}\varepsilon_{s-1,s-2}^{\mu_3\cdots\mu_s)}-\varepsilon_{s-1,s-1}^{\mu(\mu_3\cdots\mu_s}\partial_\mu g_2^{\mu_1\mu_2)} + \partial^{(\mu_1}\varepsilon_{s-1,s-1}^{\mu_2\cdots\mu_s)})(g_2)_{\mu_1\mu_2}(g_2)_{\mu_3\mu_4}=0. \tag{85}$$

If not prevented by some mechanism in a complete theory, this remaining symmetry is sufficiently strong to make the $g_s^{\mu_1\mu_2\cdots\mu_s}$ not just double traceless, but also traceless. In this case, the resulting gravity theory would be conformal gravity, which is naively nonunitary. However, there are ways of curing the problem in a conformal gravitational field theory setting. One approach is to include "compensator" fields that absorb the extra gauge symmetry, thus leaving behind only the correct amount of symmetry as described in the previous paragraph. The possibility for such a mechanism appears to be present in the local Sp$(2,R)$ noncommutative field theory formalism of Ref. [9] that includes interactions, and in which $\varepsilon(X,P)$ plays the role of gauge symmetry parameters. Indeed, the background field configurations described so far in the worldline formalism also emerge in the solution of the noncommutative field equations of this theory, in the free limit, as described in the following section.

It is also interesting to note that string theory seems to be compatible with our results. String theory contains two branches of massless states in two extreme limits, that is, when the string tension vanishes or goes to infinity, as outlined in the introduction. To better understand this possible relation to string theory we would have to construct transformation rules for the extremes of string theory, which are not presently known in the literature. Hence, the proposed connection to string theory is a conjecture at this stage. If this connection is verified, it is interesting to speculate that the high energy, fixed angle, string scattering amplitudes, computed by Gross and Mende [15], may describe the scattering of a particle in the type of background fields we find in this paper. Note that an appropriate infinite slope limit $\alpha'\to\infty$ can be imitated by the limit $s,t,u\to\infty$ (at fixed angle) used by Gross and Mende, since α' multiplies these quantities in string amplitudes.

We also find a connection between our transformation rules inherited from $d+2$ dimensions, and the transformation rules in $\mathcal{W}$ geometry analyzed by Hull [17] in the special cases of $d=1,2$. The $\mathcal{W}$ geometry or generalized Riemannian geometry, which is the background geometry in W gravity theories [16], is defined by a generalized metric function, on the tangent bundle TM of the target manifold M, which defines the square of the length of a tangent vector $y^\mu\in T_xM$ at $x\in M$. The inverse metric is also generalized by introducing a co-metric function $F(x,y)$ on the cotangent bundle, which is expanded in y as in [17]

$$F(x,y)=\sum_s \frac{1}{s}g_s^{\mu_1\cdots\mu_s}(x)y_{\mu_1}\cdots y_{\mu_s}, \tag{86}$$

where the coefficients $g_s^{\mu_1\cdots\mu_s}(x)$ are contravariant tensors on M. It is observed in Ref. [17] that the coefficients $g_s^{\mu_1\cdots\mu_s}(x)$ in cometric function can be associated to higher-spin gauge fields on M only if the cometric function is invariant under symplectic diffeomorphism group of the cotangent bundle of M in $d=1$ and under a subgroup of it in $d=2$. This leads to a natural set of transformation rules for the gauge fields $g_s^{\mu_1\cdots\mu_s}(x)$ in dimensions $d=1$ and 2. The transformation rules that are given in Ref. [17] for $g_s^{\mu_1\cdots\mu_s}(x)$ in $d=1$ and $d=2$ exactly matches the transformation rules (82) that we found in any dimension by using the 2T physics techniques. In the language of Ref. [17] the first term in Eq. (82) is the $\mathcal{W}$-Weyl transformation, and the second and the third terms combined are the action of some subgroup of the symplectic diffeomorhisms of the cotangent bundle of space-time. We emphasize that our results are valid in any dimension.

V. SOLUTION OF NCFT EQUATION TO ALL ORDERS IN $\hbar$

One may ask the question: which field theory could one write down, such that its equations of motion, after ignoring field interactions, reproduce the first quantized version of the physics described by our worldline theory. That is, we wish to construct the analog of the Klein-Gordon equation reproducing the first quantization of the relativistic particle. Then in the form of field theory interactions are included. A noncommutative field theory (NCFT) formulation of 2T physics which addresses and solves this question is introduced in Ref. [9]. The basic ingredient is the local Sp(2) symmetry, but now in a NC field theoretic setting. The NCFT equations have a special solution described by the NC field equations (10),(11). We would like to find all $Q_{ij}(X,P)$ that satisfies these equations to all orders of $\hbar$ which appears in the star products.

It is clear that the classical solution for the background fields discussed up to now is a solution in the $\hbar\to 0$ limit, since then the star commutator reduces to the classical Poisson bracket. However, surprisingly, by using an appropriate set of coordinates, the classical solution is also an exact quantum solution. These magical coordinates occur whenever $W(X)$ is at the most quadratic in X^M and $V_1^M(X)$ is at the most linear in X^M Thus both of the cases $W=X^2$ and

$W=-2\kappa w$ discussed in the previous section provide exact quantum solutions, and similarly others can be constructed as well.

To understand this assertion let us examine the transformation rules given in Sec. II, but now for general $\hbar$ using the full star product. Evidently, the classical transformations get modified by all higher orders in $\hbar$. These are the local Sp(2) gauge transformation rules of the Q_{ij} in the NCFT where $\varepsilon(X,P)$ is the local gauge parameter [9]. With these rules we can still map $Q_{11}=W(X)$ as in Eq. (25). However, if we proceed in the same manner as in Sec. II, since the Poisson bracket would be replaced by the star commutator everywhere, we are bound to find higher order $\hbar$ corrections in all the expressions. However, consider the star commutator of $W(X)$ with any other quantity $[W(X),\dots]_\star$. This is a power series containing only odd powers of $\hbar$. If $W(X)$ is at the most quadratic function of X^M, the expression contains only the first power of $\hbar$. Hence for quadratic $W(X)=X^2$ or $W=-2\kappa w$ the star commutator is effectively replaced by the Poisson brackets, and all expressions involving such $W(X)$ produce the same results as the classical analysis.

Similarly, we can argue that, despite the complications of the star product, we can use the remaining gauge freedom to fix $V_{s\geqslant 2}=0$, $V_0=0$, $G_1=0$, and $V_1^M(X)$ linear in X^M. Again, with linear $V_1^M(X)$ all of its star commutators are replaced by Poisson brackets.

Then, the classical analysis of the background fields, and their transformation rules, apply intact in the solution of the NCFT field equations (10). The conclusion, again, is that there are two disconnected branches, one for low spins $s\leqslant 2$ and one for high spins $s\geqslant 2$, that seem to have an analog in string theory at the extreme tension limits.

The NCFT of Ref. [9] allows more general field configurations in which the higher-spin fields interact with each other and with matter to all orders of $\hbar$ and with higher derivatives, consistently with the gauge symmetries. In the full theory, the type of field that appears in Eq. (11) can play the role of the "compensators" alluded to in the previous section. This would provide an example of an interacting field theory for higher-spin fields.

VI. CONCLUSIONS AND REMARKS

In this paper it was demonstrated that in a worldline formalism, all the usual d-dimensional Yang-Mills, gravitational and scalar interactions experienced by a particle, plus interactions with higher-spin fields, can be embedded in $(d+2)$-dimensional 2T physics as a natural solution of the two-time background field equations (49), taken in a fixed Sp$(2,R)$ gauge. Since 2T physics provides many d dimensional holographic images that appear as different 1T dynamics, a new level of higher-dimensional unification is achieved by the realization that a family of d-dimensional dynamical systems (with background fields) are unified as a single $(d+2)$-dimensional theory.

It is also argued that the same perspective is true in field theory provided we use the NCFT approach to 2T physics proposed recently in Ref. [9] which, beyond the worldline theory, provides a coupling of all these gauge fields to each other and to matter. In the NCFT counterpart the same picture emerges for a special solution of the NC field equations. Furthermore, the classical solution that determines the phase space configuration of the background fields is also a special exact solution of the NCFT equations to all orders of $\hbar$ when, by using gauge freedom, $W(X)$ is chosen as any quadratic function of X^M (equivalently, V_1^M taken a linear function of X^M). In the present paper we gave two illustrations by taking $W=X^2$ and $W=-2w\kappa$. For nonquadratic forms of $W(X)$ there would be higher powers of $\hbar$ in the solutions of the NCFT equations.

By considering the canonical transformations in phase space in the worldline formalism (or the gauge symmetry in NCFT formalism) it is argued that a given solution for a fixed set of background fields can be transformed into new solutions for other sets of background fields. The physical interpretation of this larger set of solutions could be very rich, but it is not investigated in this paper.

The holographic image of the $(d+2)$-dimensional theory, in the massless particle gauge, makes connections with other formalisms for higher-spin fields. In particular in one gauge our $(d+2)$-dimensional approach yields the d-dimensional action discussed in Ref. [11]. As it is shown there, the first order action (in phase space) is a completed version of an action originally proposed by de Wit and Freedman [13] in position-velocity space. The completion consists of including all powers of the velocities that couple to the higher-spin fields, and their effect in the complete form of transformation rules. Some problems pointed out in Ref. [11] can be resolved by three observations: first, there are different branches of solutions, one for the low spin sector, and one for the high spin sector starting with spin 2; second, a worldline theory with the correct unitary high spin fields certainly is permitted as one of the holographic pictures of the $d+2$ theory; and third, the stronger canonical gauge symmetries that could lead to nonunitary conformal gravity need not exist in a complete interacting theory.

Our description of higher-spin fields appears to be consistent in the worldline formalism, while the noncommutative field theory approach of Ref. [9] provides a field theoretic action for them, with interactions. In this paper we touched upon this aspect only superficially. This is an old problem [12] that deserves further careful study. Furthermore, our solution may correspond to self-consistent subsectors of string theory at extreme limits of the tension. It would also be very interesting to further study the holographic aspects of the 2T physics theory.

ACKNOWLEDGMENTS

I.B. would like to thank Edward Witten, Misha Vasiliev, and Djordje Minic for discussions. This research was in part supported by the U.S. Department of Energy under Grant No. DE-FG03-84ER40168 and by the CIT-USC Center of Theoretical Physics.

[1] For the most recent review see I. Bars, ''Survey of two-time physics,'' hep-th/0008164.
[2] I. Bars, C. Deliduman, and O. Andreev, Phys. Rev. D **58**, 066004 (1998); I. Bars, *ibid.* **58**, 066006 (1998); ''Two-Time Physics,'' hep-th/9809034; Phys. Rev. D **59**, 045019 (1999); I. Bars and C. Deliduman, *ibid.* **58**, 106004 (1998); S. Vongehr, ''Examples of Black Holes in Two-Time Physics,'' hep-th/9907077.
[3] I. Bars, C. Deliduman, and D. Minic, Phys. Rev. D **59**, 125004 (1999); Phys. Lett. B **457**, 275 (1999); **466**, 135 (1999).
[4] I. Bars, Phys. Rev. D **62**, 085015 (2000).
[5] I. Bars, Phys. Rev. D **62**, 046007 (2000).
[6] I. Bars, Phys. Lett. B **483**, 248 (2000).
[7] I. Bars, ''$AdS_5 \times S^5$ supersymmetric Kaluza-Klein towers as a 12-dimensional theory in 2T-physics'' (in preparation).
[8] I. Bars, ''A toy-M-model'' (in preparation).
[9] I. Bars and S.J. Rey, ''Noncommutative Sp(2,R) Gauge Theories: A Field Theory Approach to Two-Time Physics,'' hep-th/0104135.
[10] P.A.M. Dirac, Ann. Math. **37**, 429 (1936); H.A. Kastrup, Phys. Rev. **150**, 1183 (1966); G. Mack and A. Salam, Ann. Phys. (N.Y.) **53**, 174 (1969); C.R. Preitschopf and M.A. Vasiliev, Nucl. Phys. **B549**, 450 (1999).
[11] A.Yu. Segal, ''Point particle in general background fields and generalized equivalence principle,'' hep-th/0008105.
[12] M.A. Vasiliev, Int. J. Mod. Phys. D **5**, 763 (1996); ''Higher spin gauge theories, star products and AdS space,'' hep-th/9910096.
[13] B. de Wit and D.Z. Freedman, Phys. Rev. D **21**, 358 (1980).
[14] C. Fronsdal, Phys. Rev. D **18**, 3624 (1978); **20**, 848 (1979); J. Fang and C. Fronsdal, *ibid.* **18**, 3630 (1978); **22**, 1361 (1980).
[15] D.J. Gross and P. Mende, Phys. Lett. B **197**, 129 (1987); Nucl. Phys. **B303**, 407 (1988).
[16] C.M. Hull, Phys. Lett. B **240**, 110 (1989); K. Schoutens, A. Sevrin, and P. van Nieuwenhuizen, *ibid.* **243**, 245 (1990); E. Bergshoeff, C.N. Pope, L.J. Romans, E. Sezgin, X. Shen, and K.S. Stelle, *ibid.* **243**, 350 (1990); M. Awada and Z. Qiu, *ibid.* **245**, 85 (1990); **245**, 359 (1990).
[17] C.M. Hull, Phys. Lett. B **269**, 257 (1991); Commun. Math. Phys. **156**, 245 (1993).