

Localization of Kaluza-Klein gauge fields on a brane

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(Received 7 March 2001; published 27 July 2001)

In phenomenological models with extra dimensions there is a natural symmetry group associated with a brane universe—the group of rotations of the normal bundle of the brane. We consider Kaluza-Klein gauge fields corresponding to this group and show that they can be localized on the brane in models with warped extra dimensions. These gauge fields are coupled to matter fields which have a nonzero rotation moment around the brane. In a particular example of a four-dimensional brane embedded into six-dimensional asymptotically anti-de Sitter space, we calculate the effective four-dimensional coupling constant between the localized fermion zero modes and the Kaluza-Klein gauge field.

DOI: 10.1103/PhysRevD.64.044018

PACS number(s): 04.50.+h, 11.10.Kk, 11.15.-q

I. INTRODUCTION

The possibility to treat observable gauge fields as arising from dimensional reduction of higher-dimensional general relativity has a long history [1]. If we suppose that the space-time is a direct product of four-dimensional Minkowski space R^4 with coordinates x^μ , $\mu=0, \dots, 3$ on an n -dimensional “internal space” K^n parametrized by coordinates y^a , $a=1, \dots, n$, the four-dimensional gauge fields A_μ^a can be described by the μa components of the higher-dimensional metric

$$g_{AB} = \begin{pmatrix} g_{\mu\nu} + g_{ab} A_\mu^a A_\nu^b & g_{ab} A_\mu^b \\ g_{ab} A_\mu^b & g_{ab} \end{pmatrix}. \quad (1.1)$$

In this simple form such an approach to the gauge interactions faces certain problems such as, for example, the problem of obtaining realistic pattern of four-dimensional fermions with different charges with respect to A_μ^a after dimensional reduction of higher-dimensional fermion fields [2]. The problem of chiral fermions can be solved in models with additional fundamental higher-dimensional gauge fields in topologically nontrivial configurations [3], or in models with noncompact internal spaces [4].

If we deal with noncompact internal spaces, we face a problem how to explain the observable four-dimensional structure of the universe. One is forced to consider the observable space-time as a surface (brane) embedded in a higher-dimensional manifold [5,6]. The idea of the “brane world” received considerable attention recently due to new developments in the string theory. A new impulse to higher-dimensional model building was given by Refs. [7,8] where the possibility of having “large” or infinite extra dimensions was considered in relation to the hierarchy problem of particle physics. The observational consequences of the brane world picture for the accelerator physics [9], astrophysics [10], and cosmology [11–14] were extensively studied recently. The possibility of solving the fermion mass hierarchy

[15], cosmological constant [16–18], supersymmetry breaking [19] problems with large extra dimensions was also discussed.

An important problem which must be addressed in phenomenological models with a brane universe is how the observable matter fields of the standard model of particle physics are localized on the brane. The mechanism of localization of fermions is known for a long time [20] while mechanisms of localization of gauge fields [21–24] were only found recently.

In this paper we discuss the question whether it is possible to localize the Kaluza-Klein fields A_μ^a on the brane and to treat them as observable gauge fields. If the brane is embedded in a space-time with $n \geq 2$ extra dimensions there is a “natural” symmetry group associated to the brane: the group of rotations of its normal bundle. In the case of $n=2$ extra dimensions the brane can be treated as a stringlike defect in a higher-dimensional space-time. The symmetry group of the normal bundle is $U(1)$ and corresponds to rotations of normal vectors around the brane. We analyze the gauge field A_μ which corresponds to this symmetry in Kaluza-Klein ansatz (1.1). We show that A_μ can be localized on a four-dimensional string even if the extra dimensions are noncompact and have infinite volume, as it is, for example, in the space-times considered in Ref. [6]. We consider coupling of the field A_μ to the localized fermion fields in a particular model where a four-dimensional brane is embedded into a six-dimensional (asymptotically anti-de Sitter) space-time [25–30]. In this space-time both the Kaluza-Klein field A_μ which corresponds to the $U(1)$ symmetry of rotations of the normal bundle and the fermion zero modes which have nonzero momentum of rotation around the string are localized. We calculate the effective four-dimensional fine structure constant α in this model and find that it is related to the curvature radius of anti-de Sitter space-time and to the thickness of the brane.

II. GAUGE FIELDS ASSOCIATED TO THE GROUP OF ROTATIONS OF NORMAL BUNDLE

In the models with large or infinite extra dimensions the gauge fields A_μ^a which appear by dimensional reduction mechanism (1.1) are not considered as observable gauge

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fields of the standard model because of the following reasons. The Kaluza-Klein fields A_μ^a (1.1) are, in general, not confined to the brane and can propagate in the bulk. The tower of Kaluza-Klein excitations of these fields starts from very low masses. For example, in the model with submillimeter extra dimensions [7] the mass scale of Kaluza-Klein excitations is of order of 10^{-3} eV which means that there must be corrections to Coulomb law at distances of order of 1 mm. In the case of noncompact extra dimensions one encounters another difficulty: some of the Kaluza-Klein gauge fields can be completely reshuffled into other components of metric perturbations. Consider the case of single extra dimension. A four-dimensional brane M^4 is embedded in higher dimensional bulk \mathcal{M}^5 as a level surface of some function $F(x^A)$, $A=0, \dots, 5$

$$M^4: \{F(x^A) = 0\}. \quad (2.1)$$

If we take coordinates x^μ on the brane and the function F as a coordinate system in the vicinity of the brane, the metric on \mathcal{M}^5 takes the form

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu + g_{44} dF^2 \quad (2.2)$$

since vector $\vec{N}_A = F_{,A}$ is a normal to the surfaces $F(x^A) = \text{const}$. Comparing Eq. (2.2) with Eq. (1.1) we find that it is always possible to find a coordinate system in the neighborhood of the brane in which

$$A_\mu(x^A) \equiv 0. \quad (2.3)$$

If the extra dimension is noncompact, such a coordinate system can be chosen globally and A_μ can be always removed by a gauge transformation of five-dimensional theory.

The situation changes if the number of extra dimensions is $n \geq 2$. In this case we have $4+n$ coordinate transformations

$$x^A \rightarrow x^A + \xi^A(x) \quad (2.4)$$

at our disposal. Therefore we can impose $4+n$ gauge conditions on the metric. For example, $g_{4\mu} = 0$, $g_{44} = 1$, $g_{4a} = 0$, $a = 1, \dots, (n-1)$. After such a gauge fixing the metric becomes

$$g_{AB} = \begin{pmatrix} g_{\mu\nu} + g_{ab} A_\mu^a A_\nu^b & 0 & g_{ab} A_\mu^b \\ 0 & 1 & 0 \\ g_{ab} A_\mu^b & 0 & g_{ab} \end{pmatrix}. \quad (2.5)$$

The fields A_μ^a , $a = 1, \dots, (n-1)$ cannot be removed by a coordinate transformation. These fields have a clear geometrical meaning. Let the coordinate $r = x^4$ be a distance from the brane M^4 placed at $r = 0$. Then the fields A_μ^a are the gauge fields associated to the symmetry group G of rotations of the normal bundle of the brane. In the simplest case of a thin four-dimensional brane M^4 embedded into a higher-dimensional manifold \mathcal{M}^{4+n} the coordinates θ^a , $a = 1, \dots, (n-1)$ parametrize a small $(n-1)$ -dimensional sphere around the point $r = 0$ of location of the brane and the group of the normal bundle is $SO(n)$. The group G may be

more involved if we consider branes of nonzero thickness. The stress-energy tensor of the brane is different from zero in a region $M^4 \times K^n = \{r \leq \epsilon\}$ for some small ϵ . The surface $r = \epsilon$ is a $3+n$ -dimensional boundary $M^4 \times \partial K^n$ of the thick brane. The group G is the group of isometries of ∂K^n . It can be quite arbitrary. For example, in the model considered in Ref. [18], this group is $SU(2) \times U(1)$.

If we introduce a coordinate system (x^μ, θ^a) on $M^4 \times \partial K^n$, the background metric (with $A_\mu^a = 0$) in the vicinity of the brane can be written in the form

$$ds^2 = e^{\nu(r)} \eta_{\mu\nu} dx^\mu dx^\nu + e^{\lambda(r)} dr^2 + \varphi(r) g_{ab}(\theta) d\theta^a d\theta^b, \quad (2.6)$$

where $g_{ab}(\theta)$ is a G -symmetric metric on ∂K^n . The functions $\nu(r), \lambda(r)$ and $\varphi(r)$ are found from the $(4+n)$ -dimensional Einstein equations

$$R_{AB} - \frac{1}{2} g_{AB} R + \Lambda g_{AB} = 8\pi G_{4+n} T_{AB}, \quad (2.7)$$

where G_{4+n} , Λ are $(4+n)$ -dimensional gravitational and cosmological constants and T_{AB} is stress-energy tensor generated by the brane.

In what follows we will analyze in detail the case of two extra dimensions when there is single angular coordinate θ . We concentrate our attention only on $g_{\mu\theta}$ components of metric perturbations and leave aside a more complicated question of self-consistent analysis of interaction of six-dimensional gravitational perturbations with matter localized on the brane.

III. EQUATIONS OF MOTION FOR A_μ

The Ricci tensor for the metric (2.6) perturbed by the fields A_μ (2.5) in the linear in A_μ approximation has the form

$$R_{\mu\nu} = -e^{\nu-\lambda} \eta_{\mu\nu} \left(\frac{\nu''}{2} - \frac{\nu' \lambda'}{4} + \nu'^2 + \frac{\nu' \varphi'}{4\varphi} \right) + \frac{1}{2} (\dot{A}_{\mu,\nu} + \dot{A}_{\nu,\mu}), \quad (3.1)$$

$$R_{\mu\theta} = \frac{e^{-\nu} \varphi}{2} (A_{\nu,\mu\nu} - A_{\nu,\mu\mu}) + \frac{\varphi e^{-\lambda}}{2} \left[-A_\mu'' + \left(\frac{\lambda'}{2} - \nu' - \frac{5\varphi'}{2} \right) A_\mu' + \left(\frac{\varphi'^2}{2\varphi^2} - \frac{\varphi''}{\varphi} + \frac{\varphi' \lambda'}{2\varphi} - \frac{\varphi' \nu'}{\varphi} \right) A_\mu \right], \quad (3.2)$$

$$R_{\mu r} = \frac{1}{2\varphi} (\varphi \dot{A}_\mu)' - \frac{\nu'}{2} \dot{A}_\mu, \quad (3.3)$$

$$R_{rr} = -2\nu'' - \nu'^2 + \nu' \lambda' + \frac{1}{2} \left(\frac{\varphi'^2}{2\varphi^2} + \frac{\varphi' \lambda'}{2\varphi} - \frac{\varphi''}{\varphi} \right), \quad (3.4)$$

$$R_{r\theta} = \frac{\varphi e^{-\nu}}{2} A'_{\mu,\mu}, \quad (3.5)$$

$$R_{\theta\theta} = \varphi e^{-\nu} \dot{A}_{\mu,\mu} + e^{-\lambda} \left(\frac{\varphi' \lambda'}{4} - \frac{\varphi'^2}{4\varphi} - \frac{\varphi''}{2} - \varphi' \nu' \right). \quad (3.6)$$

When $A_\mu = 0$ we get from Eq. (2.7), the expression of $\nu(r)$, $\lambda(r)$, and $\varphi(r)$ through the brane stress-energy tensor T_{AB} . In the presence of A_μ the $\mu\nu$, μr , $r\theta$, and $\theta\theta$ components of Einstein equations (2.7) become

$$\dot{A}_{(\mu,\nu)} = 0, \quad (3.7)$$

$$(e^{-\nu} \varphi \dot{A}_\mu)' = 0, \quad (3.8)$$

$$\dot{A}_{\mu,\mu} = A'_{\mu,\mu} = 0. \quad (3.9)$$

Here a prime denotes the derivative with respect to r and dot denotes the θ derivative. The only nontrivial equation is the $\mu\theta$ component of Einstein equations. In the most simple case of flat extra dimensions when the background metric is

$$ds^2 = \eta_{\mu\nu} dx^\mu dx^\nu + dr^2 + r^2 d\theta^2 \quad (3.10)$$

it reduces to

$$A''_\mu + \frac{5}{r} A'_\mu + F_{\mu\nu,\nu} = 0, \quad (3.11)$$

where we have denoted

$$F_{\mu\nu} = A_{\mu,\nu} - A_{\nu,\mu}. \quad (3.12)$$

First of all we can see that the ‘‘zero mode’’ $A_\mu(x^\nu)$ which does not depend on r, θ and satisfies

$$F_{\mu\nu,\nu} = 0 \quad (3.13)$$

is a solution of the six-dimensional Einstein equations. The last system of equations is just the four-dimensional Maxwell equations on electromagnetic field $A_\mu(x)$. The $U(1)$ gauge group of electromagnetism is identified with the group $SO(2)$ of rotations of the normal bundle of the brane. Indeed, let us make a coordinate transformation

$$\tilde{\theta} = \theta + \nu(x^\mu) \quad (3.14)$$

which rotates normal vectors to the brane at the point x^μ on a small angle $\nu(x^\mu)$. The background metric (3.10) in new coordinates takes the form

$$ds^2 = (\eta_{\mu\nu} + r^2 \nu_{,\mu} \nu_{,\nu}) dx^\mu dx^\nu + dr^2 + r^2 d\tilde{\theta}^2 - 2r^2 \nu_{,\mu} dx^\mu d\tilde{\theta} \quad (3.15)$$

from where we see that in new coordinates

$$A_\mu = -\nu_{,\mu}. \quad (3.16)$$

Since the metric (3.15) differs from Eq. (3.10) on coordinate transformation, it is again a solution of the Einstein equations.

Obviously, by the same way of reasoning, the pure gauge configuration (3.16) must be a solution of the perturbed Einstein equations on a general background of the form (2.6). The $\mu\theta$ component of the Einstein equations in the general case is

$$(e^{\nu-\lambda/2} \varphi^{5/2} A'_\mu)' + e^{\lambda/2} \varphi^{5/2} F_{\mu\nu,\nu} = 0. \quad (3.17)$$

Taking the θ derivative of Eq. (3.17) and combining it with Eq. (3.8) we find that A_μ does not depend on θ . Making a coordinate transformation (3.14) we can always fix the coordinate system in such a way that the condition

$$A_{\mu,\mu} = 0 \quad (3.18)$$

is satisfied on a particular surface $r=r_0$. Then, from Eq. (3.9) we conclude that the condition (3.18) remains valid for all r . Equation (3.17) in the gauge (3.18) reduces after the Fourier transform $\tilde{A}_\mu(p,r) = \int d^4x e^{-ip_\nu x^\nu} A_\mu(x,r)$ to

$$(e^{\nu-\lambda/2} \varphi^{5/2} \tilde{A}'_\mu)' + e^{\lambda/2} \varphi^{5/2} m^2 \tilde{A}_\mu = 0, \quad (3.19)$$

where we have denoted $m^2 = -p_\mu p^\mu$. This equation can be solved, for example, in the case of flat extra dimensions (3.10), $\lambda = \nu = 0$, $\varphi = r^2$. The modes with $m^2 \neq 0$ which are regular at $r=0$ have the form

$$A_\mu = \frac{C}{r^2} J_2(mr), \quad (3.20)$$

where $J_2(z)$ is the Bessel function and C is an arbitrary constant.

Although the zero mode $A_\mu(x)$ is always a solution of Einstein equations locally, it can fail to be a global solution if the rotation invariance (3.14) is spontaneously broken. A simple example of this is when the flat extra dimensions (3.10) are compactified on two-dimensional torus T^2 .

IV. LOCALIZATION OF A_μ ON THE BRANE

If the rotation symmetry (3.14) is not broken, the zero mode of the field A_μ is a global solution of higher-dimensional Einstein equations which, from the four-dimensional point of view, describes massless gauge field propagating along the brane. Equation (3.19) is a Sturm-Liouville equation and the zero mode $A_\mu(x^\nu)$ is its normalizable solution in a space-time (2.6) which satisfies the condition

$$N = \int dr e^{\lambda/2} \varphi^{5/2} < \infty. \quad (4.1)$$

In such a space-time the massless field $A_\mu(x^\nu)$ mediates a Coulomb-like interaction between particles localized on the brane. For example, if we take a space-time with compact extra dimensions with topology of a sphere S^2 , the bulk metric is

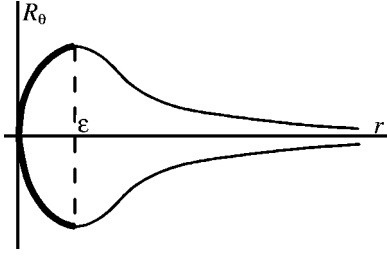


FIG. 1. The geometry extra dimensions.

$$ds^2 = \eta_{\mu\nu} dx^\mu dx^\nu + \frac{1}{(1+r^2/a^2)^2} dr^2 + \frac{r^2}{(1+r^2/a^2)^2} d\theta^2, \quad (4.2)$$

where a is the radius of the sphere. This corresponds to

$$\begin{aligned} \nu &= 0, \\ \lambda &= -2 \log(1+r^2/a^2), \\ \varphi &= r^2/(1+r^2/a^2) \end{aligned} \quad (4.3)$$

and the integral N (4.1), obviously, converges.

Zero mode of the field A_μ can be normalizable also in space-times with noncompact extra dimensions. Let us take, for example, a space-time considered in Ref. [6]. This space-time is a solution of six-dimensional Einstein equations coupled to cylindrically symmetric magnetic field

$$f_{r\theta} = B_0 \sqrt{\varphi} e^{\lambda/2 - 2\nu}, \quad (4.4)$$

where B_0 is an arbitrary magnetic field strength. In the case when the cosmological constant in the bulk and on the brane are zero, the space-time metric is

$$ds^2 = \left(1 + \frac{r^2}{a^2}\right)^{2/3} (\eta_{\mu\nu} dx^\mu dx^\nu + dr^2) + \frac{r^2 d\theta^2}{(1+r^2/a^2)^2}, \quad (4.5)$$

where the parameter a is related to the field strength (4.4)

$$a^2 = \frac{2}{3\pi G_6 B_0^2}. \quad (4.6)$$

The geometry of extra dimensions is schematically presented on Fig. 1. The size R_θ of the circle parametrized by θ shrinks to zero as $r \rightarrow \infty$.

The integral N (4.1) converges

$$N = \int_0^\infty \frac{r^5 dr}{(1+r^2/a^2)^{14/3}} = \frac{27a^6}{440} \quad (4.7)$$

and the zero mode $A_\mu(x^\nu)$ is a normalizable solution of Eq. (3.19). Note, that the volume of extra dimensions

$$V = \int \sqrt{-g} dr d\theta = \int \left(1 + \frac{r^2}{a^2}\right)^{2/3} r dr d\theta \quad (4.8)$$

is infinite so that the metric (4.5) does not localize gravity.

It is interesting to consider situation when both gravity and the gauge field are localized on the brane. This can be achieved in a six-dimensional generalization of Randall-Sundrum model which was considered in Refs. [26,27,23]. The space-time is a solution of Einstein equations (2.7) with negative cosmological constant Λ . A string of finite size ϵ is placed at $r=0$ so that the stress-energy tensor T_{AB} is different from zero in a small region $0 \leq r < \epsilon$ around $r=0$. Outside the string in the region $\epsilon < r < \infty$ the space-time metric is the six-dimensional anti-de Sitter metric

$$ds^2 = e^{-cr} \eta_{\mu\nu} dx^\mu dx^\nu + dr^2 + e^{-cr} R^2 d\theta^2. \quad (4.9)$$

Here R is an arbitrary constant which sets the size of the circle S^1 parametrized by θ . Parameter c is related to the bulk cosmological constant (2.7) as

$$c = \sqrt{-\frac{2\Lambda}{5}}. \quad (4.10)$$

The geometry of extra dimensions in this case is similar to that of space-time (4.5): the size of the circle S^1 parametrized by θ goes to zero as $r \rightarrow \infty$ (see Fig. 1). The metric inside the brane is determined by the detailed structure of the brane stress-energy tensor $T_{AB}(r)$.

Equation (3.19) on A_μ reduces for the metric (4.9) to

$$(e^{-7cr/2} \tilde{A}'_\mu)' + e^{-5cr/2} m^2 \tilde{A}_\mu = 0. \quad (4.11)$$

Its general solution in the gauge (3.19) is

$$\tilde{A}_\mu^0 = e^{7r/4} \left[C_1 J_{7/2} \left(\frac{2m}{c} e^{cr/2} \right) + C_2 Y_{7/2} \left(\frac{2m}{c} e^{cr/2} \right) \right] \quad (4.12)$$

The zero mode solution $A_\mu(x)$ of Eq. (4.11) is a normalizable solution of Eq. (4.11) since the integral N (4.1) is converging

$$N = \frac{2R^5}{5c}. \quad (4.13)$$

Note that the zero mode of A_μ is normalizable even in a more general space-time of ‘‘global string,’’ considered in Refs. [23,31]

$$ds^2 = e^{-c_1 r} \eta_{\mu\nu} dx^\mu dx^\nu + dr^2 + e^{-c_2 r} d\theta^2. \quad (4.14)$$

Here c_1 and c_2 are arbitrary positive constants related to the stress-energy tensor $T_{AB}(r)$ of the string which is nonzero even when $r \rightarrow \infty$.

V. MATTER FIELDS BOUND TO THE BRANE

The field A_μ interacts with matter fields bound to the brane. As an example we consider fermion fields propagating

on the background (2.6). Consider a six-dimensional Dirac spinor Ψ which satisfies the equation

$$\Gamma^A D_A \Psi = 0. \quad (5.1)$$

The six-dimensional gamma matrices Γ^A are defined with the help of vielbein $E_{\hat{B}}^A$ and flat space gamma matrices $\Gamma^{\hat{A}}$

$$\Gamma^A = E_{\hat{B}}^A \Gamma^{\hat{B}} \quad (5.2)$$

(the indexes with hat are six-dimensional Lorentz indexes). The covariant derivative is defined as

$$D_A \Psi = \Psi_{,A} + \frac{1}{2} \omega_A^{\hat{A}\hat{B}} \sigma_{\hat{A}\hat{B}} \Psi, \quad (5.3)$$

where $\omega_A^{\hat{B}\hat{C}}$ is the spin connection expressed through vielbein $E_{\hat{B}}^A$ and $\sigma_{\hat{B}\hat{C}} = \frac{1}{4} [\Gamma_{\hat{B}} \Gamma_{\hat{C}}]$. Taking the coordinate vielbein for the metric (2.6)

$$\begin{aligned} E_{\hat{\mu}}^{\hat{\alpha}} &= e^{\nu/2} \delta_{\hat{\mu}}^{\hat{\alpha}}, \\ E_{\hat{r}}^{\hat{r}} &= e^{\lambda/2}, \\ E_{\hat{\theta}}^{\hat{\theta}} &= \sqrt{\varphi} \end{aligned} \quad (5.4)$$

we find

$$\begin{aligned} \omega_{\hat{\mu}}^{\hat{A}\hat{B}} \sigma_{\hat{A}\hat{B}} &= \frac{\nu'}{2} e^{(\nu-\lambda)/2} \Gamma_{\hat{\mu}} \Gamma_{\hat{r}}, \\ \omega_{\hat{\theta}}^{\hat{A}\hat{B}} \sigma_{\hat{A}\hat{B}} &= \frac{\varphi'}{2\sqrt{\varphi}} e^{-\lambda/2} \Gamma_{\hat{\theta}} \Gamma_{\hat{r}}. \end{aligned} \quad (5.5)$$

Let us consider the solutions of Eq. (5.1) which satisfy the condition

$$\Gamma_{\hat{0}} \cdots \Gamma_{\hat{3}} \Gamma_{\hat{r}} \Gamma_{\hat{\theta}} \Psi = \Psi \quad (5.6)$$

and can be presented in the form

$$\Psi = e^{iq\theta} \chi(r) \psi(x^\mu), \quad (5.7)$$

where q is an integer. The two-component spinor $\chi(r)$ satisfies

$$i\Gamma_{\hat{r}} \Gamma_{\hat{\theta}} \chi = -\chi \quad (5.8)$$

while the four-component spinor $\psi(x)$ is chiral in four-dimensional sense

$$i\Gamma_{\hat{0}} \cdots \Gamma_{\hat{3}} \psi = \psi \quad (5.9)$$

and satisfies the four-dimensional massless Dirac equation

$$\Gamma^{\hat{\mu}} \partial_{\hat{\mu}} \psi = 0. \quad (5.10)$$

From Eq. (5.1) we derive an equation on χ

$$\chi' + \left(\nu' + \frac{\varphi'}{4\varphi} + \frac{e^{\lambda/2}}{\sqrt{\varphi}} q \right) \chi = 0 \quad (5.11)$$

which is readily integrated

$$\chi(r) = \text{const } \varphi^{-1/4} \exp \left\{ -\nu - q \int_{r_0}^r dr' \frac{e^{\lambda/2}}{\sqrt{\varphi}} \right\}. \quad (5.12)$$

The solutions of Eq. (5.1) must be normalizable with respect to the norm

$$\begin{aligned} \langle \Psi, \Psi \rangle &= \int d^4x dr d\theta \sqrt{-g} \bar{\Psi} \Gamma^0 \Psi \\ &= \int dr e^{3\nu/2 + \lambda/2} \sqrt{\varphi} |\chi|^2. \end{aligned} \quad (5.13)$$

In the case of space-time (4.9) the normalized solutions of Eq. (5.11) are given by

$$\chi_q(r) = \frac{q^{1/2}}{\pi^{1/2} R} e^{5cr/4} \exp \left\{ \frac{2q}{cR} (1 - e^{cr/2}) \right\}. \quad (5.14)$$

The solution with $q=0$ with trivial dependence on θ grows at large r and is not localized on the brane. But the solutions with $q>0$ decrease far from the brane. Here we have neglected a contribution from the region $0 \leq r < \epsilon$ inside the brane into the integral (5.13). In principle, the requirement of convergence of Eq. (5.13) in the limit $r \rightarrow 0$ will impose restrictions on the number of normalizable zero mode solutions of Eq. (5.1).

VI. COUPLING OF A_μ TO MATTER FIELDS

The fermion field Ψ (5.1) produces a stress-energy tensor

$$T_{AB}^\Psi = \frac{i}{2} (\bar{\Psi} \Gamma_{(A} D_{B)} \Psi - D_{(A} \bar{\Psi} \Gamma_{B)} \Psi). \quad (6.1)$$

The $\mu\theta$ component of T_{AB}^Ψ is

$$\begin{aligned} T_{\mu\theta}^\Psi &= \frac{i}{4} e^{\nu/2} (\bar{\Psi} \Gamma_{\hat{\mu}} \Psi_{,\theta} - \bar{\Psi}_{,\theta} \Gamma_{\hat{\mu}} \Psi) + \frac{i\sqrt{\varphi}}{4} (\bar{\Psi} \Gamma_{\hat{\theta}} \Psi_{,\mu} \\ &\quad - \bar{\Psi}_{,\mu} \Gamma_{\hat{\theta}} \Psi) + \frac{i\sqrt{\varphi}}{4} e^{(\nu-\lambda)/2} \left(\nu' - \frac{\varphi'}{\varphi} \right) \bar{\Psi} \Gamma_{\hat{\mu}} \Gamma_{\hat{r}} \Gamma_{\hat{\theta}} \Psi. \end{aligned} \quad (6.2)$$

If we restrict our attention to the Ψ configurations of the form (5.7), (5.8), (5.9), then Eq. (6.2) reduces to

$$T_{\mu\theta}^\Psi = \frac{e^{\nu/2} |\chi(r)|^2}{2} \left[q - e^{-\lambda/2} \frac{\sqrt{\varphi}}{2} \left(\nu' - \frac{\varphi'}{\varphi} \right) \right] \bar{\psi} \Gamma_{\hat{\mu}} \psi. \quad (6.3)$$

We see that $T_{\mu\theta}^\Psi$ is proportional to the four-dimensional current

$$j_\mu = \bar{\psi} \Gamma_{\hat{\mu}} \psi. \quad (6.4)$$

Substituting the stress-energy tensor (6.3) into the Einstein equations (2.7) we find that Eq. (3.19) on A_μ is modified

$$(e^{\nu-\lambda/2}\varphi^{5/2}\tilde{A}'_\mu)' + e^{\lambda/2}\varphi^{5/2}m^2\tilde{A}_\mu = -8\pi G_6 e^{2\nu+\lambda/2}\varphi^{3/2}|\chi|^2 Q\tilde{J}_\mu, \quad (6.5)$$

where \tilde{J}_μ is the Fourier transform $\tilde{J}_\mu(p, r) = \int d^4p e^{-ip_\mu x^\mu} j_\mu(x, r)$ of the four-dimensional current. We also have denoted

$$Q = q - e^{-\lambda/2} \frac{\sqrt{\varphi}}{2} \left(\nu' - \frac{\varphi'}{\varphi} \right). \quad (6.6)$$

We can write a solution of this equation in terms of a Green's function $\tilde{\Delta}(p, r, r')$

$$\tilde{A}_\mu = -8\pi G_6 \tilde{J}_\mu \int dr' \tilde{\Delta}(p, r, r') Q e^{2\nu+\lambda/2} \varphi^{3/2} |\chi|^2. \quad (6.7)$$

The Green's function of Eq. (6.5) is expressed in terms of normalized solutions $A_m^0(r)$ of homogeneous equation (3.19) for a fixed m^2

$$\tilde{\Delta}(p, r, r') = \frac{1}{Np^2} + \sum_{m^2 > 0} \frac{A_m^0(r)A_m^0(r')}{p^2 + m^2}, \quad (6.8)$$

where N is the normalization constant (4.1). If we are interested in the behavior of A_μ at large distances from the source, we can restrict attention to the limit $p^2 \rightarrow 0$ in which zero mode solution of Eq. (3.19) gives the leading contribution into the Green's function $\tilde{\Delta}$. In this limit

$$\tilde{A}_\mu \approx \frac{8\pi G_6 \tilde{J}_\mu(p)}{N p^2} \int dr' Q \varphi^{3/2} e^{2\nu+\lambda/2} |\chi(r')|^2. \quad (6.9)$$

In the case of asymptotically anti-de Sitter space-time (4.9) the normalization constant N is given by Eq. (4.13) while the charged zero modes of Dirac field have the radial profile $\chi(r)$ given by Eq. (5.14). In the limit

$$cR \ll 1. \quad (6.10)$$

when the curvature radius $\rho = c^{-1}$ of anti-de Sitter space is much larger than the thickness of the brane R , the expression (6.9) for \tilde{A}_μ reduces to

$$\tilde{A}_\mu = \frac{10cG_6}{R^3} \frac{(q\tilde{J}_\mu)}{p^2}. \quad (6.11)$$

Comparing the last equation to the standard expression of the four-dimensional Maxwell theory we find the effective fine structure constant

$$\alpha = \frac{5cG_6}{2\pi R^3}. \quad (6.12)$$

From Eq. (5.14) we can see that the fermion zero modes with nonzero charge q are localized in a region with a size of order of the brane thickness R . If R is small enough we can approximate the profile $\chi(r)$ by delta function

$$|\chi|^2 \approx \frac{1}{R} \delta(r). \quad (6.13)$$

If we are interested in interactions of particles localized on the brane we need only the expression for the Green's function $\tilde{\Delta}$ at $r, r' = 0$. The exact expression for $\tilde{\Delta}$ can be found in a way similar to the one used in Ref. [32] for the calculation of the Green's function for gravitational perturbations of Randall-Sundrum model. For $r, r' = 0$ we get

$$\tilde{\Delta}(p) = \frac{5c}{2k^2} - \frac{1}{k} \frac{H_{3/2}^{(1)}(2k/c)}{H_{5/2}^{(1)}(2k/c)}, \quad (6.14)$$

where $k^2 = -p^2$. Expressing the Hankel functions $H_{3/2}^{(1)}(z)$ and $H_{5/2}^{(1)}(z)$ through the elementary functions we get for A_μ

$$\tilde{A}_\mu = \frac{4G_6}{R^3} \left(\frac{5c}{2k^2} + \frac{2(c-2ik)}{4k^2 + 6ick - 3c^2} \right) q\tilde{J}_\mu \quad (6.15)$$

In the case of static configurations $p_0 = 0$ the first term in the last equation gives the conventional Coulomb law at large distances from a static source. The second term of Eq. (6.15) results in corrections of order of $O(1/L^3)$ to Coulomb law at distances $L \sim c^{-1}$ along the brane.

VII. CONCLUSION

We have considered models in which a brane universe M^4 is embedded into a space-time \mathcal{M}^6 with $n=2$ warped infinite extra dimensions. The Kaluza-Klein field A_μ associated to the $U(1)$ group of rotations of the normal bundle of M^4 is localized on the brane if the background metric (2.6) is such that the integral N (4.1) is converging. In the case of asymptotically anti-de Sitter metric in six-dimensional bulk the fermion zero modes which possess nonzero rotation moment q around the brane are localized on M^4 [see Eq. (5.14)]. They are charged with respect to A_μ and A_μ mediates Coulomb-like interaction between these zero modes. We have calculated the effective four-dimensional fine structure constant α (6.12) in such a model. It is expressed through the inverse curvature radius c of the bulk anti-de Sitter space and the brane thickness R . The presence of infinite extra dimensions results in modification of the four-dimensional photon propagator and in power-law corrections to the Coulomb law at the distances of order of the inverse curvature radius c of anti-de Sitter space (6.15).

ACKNOWLEDGMENTS

I am grateful to A.O. Barvinsky, V.F. Mukhanov, V.A. Rubakov, I. Sachs, and S. Solodukhin for fruitful discussions of the subject of the paper. This work was supported by Grant No. SFB 375 der Deutschen Forschungsgemeinschaft.

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