Standard model neutrinos as warm dark matter

Gian F. Giudice,[1] Edward W. Kolb,[2] Antonio Riotto,[3] Dmitry V. Semikoz,[4,5]
and Igor I. Tkachev[1,5]
[1]*CERN Theory Division, CH-1211 Geneva 23, Switzerland*
[2]*NASA/Fermilab Astrophysics Center, Fermi National Accelerator Laboratory, Batavia, Illinois 60510-0500*
and Department of Astronomy and Astrophysics, Enrico Fermi Institute, The University of Chicago, Chicago, Illinois 60637-1433
[3]*Scuola Normale Superiore, Piazza dei Cavalieri 7, I-56126 Pisa, Italy*
[4]*Max Planck Insitut für Physik, Föhringer Ring 6, 80805 München, Germany*
[5]*Institute for Nuclear Research of the Academy of Sciences of Russia, Moscow 117312, Russia*
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Standard model neutrinos are not usually considered plausible dark matter candidates because the usual treatment of their decoupling in the early universe implies that their mass must be sufficiently small to make them ''hot'' dark matter. In this paper we show that decoupling of standard model neutrinos in low reheat models may result in neutrino densities very much less than usually assumed, and thus their mass may be in the keV range. Standard model neutrinos may therefore be warm dark matter candidates.

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I. INTRODUCTION

Cosmological dark matter in the form of neutrinos with masses in the eV range is the quintessential example of hot dark matter (HDM) [1]. Indeed, for a standard relic that decouples from the surrounding thermal bath when still relativistic, the current abundance can be easily estimated to be [2]

$$\Omega_X h^2 \simeq 78\left(\frac{g_X}{g_*(T_D)}\right)\left(\frac{m_X}{\mathrm{keV}}\right), \quad (1)$$

where g_X is the number of degrees of freedom of the particle and $g_*(T_D)$ is the total effective number of relativistic degrees of freedom at the decoupling temperature T_D. Relativistic standard model neutrinos decouple from chemical equilibrium at a temperature of a few MeV, when $g_*(T_D) \simeq 10.75$. Since current observations indicate that the dark matter density amounts to approximately $\Omega_X \simeq 0.3$, one usually concludes that the mass of standard model neutrinos cannot be larger than about $30h^2$ eV [2,3].

Since neutrinos must be light in order to avoid overclosing the universe, they were moving at nearly the speed of light at redshift $z \sim 10^6$ when the cosmic horizon first encompassed $10^{12} M_\odot$, the amount of dark matter contained in the halo of a large galaxy like the Milky Way. This implies that the free streaming of light neutrinos destroyed any fluctuations smaller than that of a supercluster (about $10^{15} M_\odot$). Cosmological structure forms in a light-neutrino dominated universe in a top-down scenario, in which superclusters of galaxies form first, with galaxies and clusters forming through a process of fragmentation. However, in this scenario galaxies form too late and their distribution is much more inhomogeneous than observations indicate. An even more serious problem is that when normalized to the low amplitude of the cosmic microwave background fluctuations detected by the Cosmic Background Explorer (COBE) satellite [4], the HDM spectrum is only beginning to reach nonlinearity at the present epoch and the free-streaming wavelength cutoff in the amplitude of the spectrum should be considerably smaller to form any structure by the present. Because of these difficulties, the idea of standard model neutrinos as dark matter has been gradually abandoned in favor of scenarios where most of the mass in the universe is in the form of cold dark matter (CDM).

The purpose of this paper is to show that in low-reheat cosmologies, standard model neutrinos may play the role of warm dark matter (WDM), avoiding the pitfalls of HDM, and may be responsible for the cosmological structures we observe.

The key point of our observation is that the limit to the contribution of neutrinos to the present density need not constrain neutrino masses to be in the eV region. On the contrary, neutrino number densities in low-reheat models may be more than an order of magnitude lower than usually assumed, and their mass, m_ν, more than an order of magnitude higher, perhaps in the keV range. Fluctuations corresponding to sufficiently large galaxy halos with masses around $10^{11} M_\odot$ may survive free streaming and standard model neutrinos may act as WDM. Before launching into more details, let us see why standard model neutrinos may be heavier than usually thought.

II. ON THE NEUTRINO DENSITY

The traditional computation of the abundance of standard model neutrinos in the early universe is based on the simple, but *untested,* assumption that light neutrinos were in chemical equilibrium at temperatures larger than T_D. In other words, in the standard cosmological bound, $m_\nu \lesssim 30h^2$ eV, it is tacitly assumed that the universe had gone through a radiation-dominated phase with temperatures larger than about an MeV with active neutrinos in equilibrium. The assumption of an initial condition of neutrinos in thermal and chemical equilibrium in a radiation-dominated universe is then equivalent to the hypothesis that the maximum temperature obtained during the (last) radiation-dominated era, which we will refer to as the reheating temperature T_{RH}, is much larger than the decoupling temperature. The fact that we have no physical evidence of the radiation-dominated era well before the epoch of nucleosynthesis is a simple, but crucial, point. It was shown in Ref. [5] that light-element nucleosynthesis limits T_{RH} to be larger than 0.7 MeV. If the parent particle ϕ generating the reheating process has a nonvanishing branching ratio B_h into hadrons, the corresponding limit on T_{RH} can be tighter. For instance, for $B_h=10^{-2}$ and the mass of the ϕ-particle $m_\phi=10^2$ TeV, the limit is $T_{RH}>2.5$ MeV [5]. The limit on T_{RH} for nonvanishing B_h can be evaded if either $m_\phi<1$ GeV (since baryonic final states are not kinematically allowed) or if m_ϕ is somewhat larger than 10^4 TeV for $B_h=10^{-2}$. The latter condition guarantees that the ϕ-particle density right before the completion of reheating is sufficiently small $(n_\phi/s\simeq 0.3T_{RH}/m_\phi)$ not to affect the neutron-to-proton ratio. As a consequence, in the following we will consider T_{RH} as an unknown quantity that can take any value as low as $\sim$ MeV.

It is usually assumed that the radiation-dominated era commences after a period of inflation, and that the cold universe at the end of inflation becomes the hot universe of the radiation-dominated era in a process known as reheating. The reheating process need not be instantaneous. On the contrary, before the radiation-dominated phase there may have been a prolonged phase during which the energy density of the universe was dominated by some component other than radiation. This component is often represented by a coherent oscillating field such as the inflaton field, but one could just as easily imagine that the universe is dominated by some unstable massive particle species. During reheating there is a slow formation of a thermal bath of relativistic particles. The temperature of this thermal bath has a peculiar behavior [2]. It reaches a maximum temperature $T_{\max}\sim T_{RH}(H_I^2 M_{Pl}^2/T_{RH}^4)^{1/4}$ (H_I is the value of the Hubble rate at the beginning of the reheating process) and then has a less steep dependence on the scale factor a than in the radiation-dominated era, $T\sim a^{-3/8}$. During this phase entropy is continuously created (the universe is reheating!) and the Hubble rate scales like $H\sim[g_*(T)/g_*^{1/2}(T_{RH})](T^4/T_{RH}^2 M_{Pl})$.

At a given temperature, the expansion is faster for smaller reheat temperatures. When the temperature decreases to T_{RH}, the universe enters the radiation phase, and one recovers the more familiar Hubble law, $H\sim T^2/M_{Pl}$.

Let us now assume that the largest temperature of the universe during the radiation-dominated phase is very small, of the order of a few MeV. Since neutrinos have only weak interactions, it is very difficult for the thermal scatterings during the reheating stage to generate standard model neutrinos through processes like $e^+e^-\to\nu\bar{\nu}$ and to bring neutrinos into chemical equilibrium. Furthermore, decreasing the reheat temperature increases the rate of the expansion of the universe, making it more and more difficult for the weak interactions to bring the neutrinos to chemical equilibrium. Therefore, if the reheating temperature is small enough, standard model neutrinos produced during the reheating stage *never* go into chemical equilibrium. In this case, neutrinos are present in the thermal bath at the beginning of the radiation-dominated phase, but they have a number density n_ν that is *smaller* than the equilibrium number density [6]. This simple argument shows that the present abundance of neutrinos may be much smaller than predicted assuming that the largest temperature of the radiation-dominated universe was much larger than a few MeV. Thus, standard model neutrinos heavier than about $30h^2$ eV are perfectly compatible with cosmology.

The above expectation was confirmed in Ref. [6], where the effective number density of neutrinos was computed by solving the corresponding Boltzmann equation obtained under the assumptions of Maxwell–Boltzmann statistics and local thermodynamic equilibrium in the calculation of the thermal averaged cross section. This amounts to assuming that electrons have an equilibrium Boltzmann distribution function and to neglect all the Pauli blocking factors. In this paper we have performed a more refined and correct computation by solving directly the kinetic equations for the neutrino phase-space distribution $f_\nu(p,t)$.

We will use the numerical code developed in Ref. [7], where the collision integrals are analytically reduced to two dimensions [8]. The kinetic equations for neutrinos have the form

$$\frac{\partial f_i(p_1,t)}{\partial t}-H(t)p_1\frac{\partial f_i(p_1,t)}{\partial p_1}=I_{i,\mathrm{coll}}, \tag{2}$$

where the collision integral I_{coll} is dominated by two-body reactions $1+2\to3+4$, and is given by the expression

$$I_{\mathrm{coll}}=\frac{S}{2E_1}\sum\int\frac{d^3p_2}{2E_2(2\pi)^3}\frac{d^3p_3}{2E_3(2\pi)^3}\frac{d^3p_4}{2E_4(2\pi)^3}(2\pi)^4 \times\delta^{(4)}(p_1+p_2-p_3-p_4)F(f_1,f_2,f_3,f_4)|\mathcal{M}|^2_{12\to34}. \tag{3}$$

Here $F=f_3f_4(1-f_1)(1-f_2)-f_1f_2(1-f_3)(1-f_4)$, $|\mathcal{M}|^2$ is the square of the weak-interaction amplitude summed over spins of all particles except the first one, and S is the symmetrization factor which includes (1/2!) for each pair of identical particles in the initial and final states and a factor of 2 if there are two identical particles in the initial state. Finally, there is a summation over all possible sets of leptons 2, 3, and 4. Notice that a similar approach was taken in Ref. [5], even though there the interactions among neutrinos as well as the electron mass were neglected and electrons were assumed to have a Boltzmann distribution. (These approximations allow the reduction of the collision integrals in the kinetic equations to one-dimensional integrals, thus simplifying the numerical calculations.) Furthermore, in Ref. [5] the emphasis was on the impact of very low reheating temperatures on standard big-bang nucleosynthesis, and therefore neutrinos were taken to be massless.

We have assumed that reheating is due to the decay into light states of a particle ϕ, which might be the inflaton field, a modulus field, or any unstable particle which dominated the energy density of the universe before the radiation-dominated phase. The time evolution of the ρ_ϕ energy density is given by

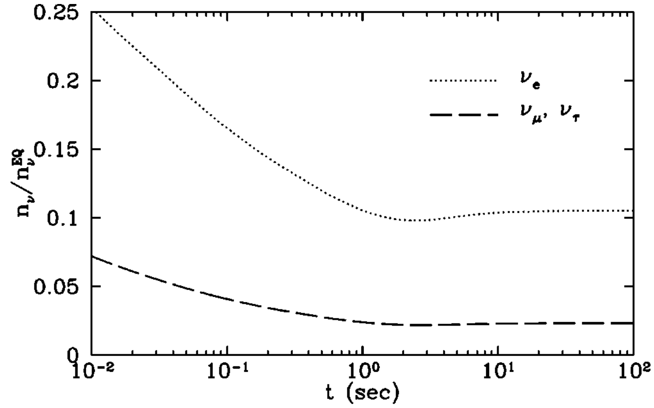

FIG. 1. The relative neutrino densities as functions of time for $T_{RH}=1$ MeV.

$$\frac{d\rho_\phi}{dt}=-\Gamma\rho_\phi-3H\rho_\phi, \tag{4}$$

where Γ is the decay rate of the ϕ field, which may be expressed in terms of the reheat temperature T_{RH} as $\Gamma=3H=3(T_{RH}^2/M_{Pl})[8\pi^3 g_*(T_{RH})/90]^{1/2}$ where $g_*(T_{RH})$ is the number of relativistic degrees of freedom at T_{RH}. Notice that in Refs. [5,6], the value of g_* in the $(\Gamma-T_{RH})$ relation was fixed to 10.75. This is not necessarily the actual value of $g_*(T_{RH})$; we will return to this point below. Also, in Ref. [6], the definition of T_{RH} is $\Gamma=H$, rather than $\Gamma=3H$.

We imposed the covariant energy conservation $\dot{\rho}(t)=-3H(\rho+P)$, where ρ as the total energy density is

$$\rho=\rho_\phi(t)+\frac{\pi^2T_\gamma^4}{15}+\frac{2}{\pi^2}\int dq q^2\frac{\sqrt{q^2+m_e^2}}{\exp(E/T_\gamma)+1}$$
$$+\frac{1}{\pi^2}\int dq q^3 f_{\nu_e}(q)+\frac{2}{\pi^2}\int dq q^3 f_{\nu_\mu}(q), \tag{5}$$

and a similar expression holds for the pressure P. The initial density of the scalar field ρ_ϕ is not relevant since it just defines the initial time of the evolution.

At the beginning of the evolution, the electromagnetic interactions are much faster than the neutrino interaction rates and therefore, before neutrinos are produced, the plasma reaches temperatures higher than the final reheating temperature T_{RH} [2,6]. Later, neutrinos ν_i of a given flavor start being produced by electron-positron annihilations $(e^+e^-\to\nu_i\bar{\nu}_i)$ and by neutrino-(anti)neutrino annihilations $(\nu_j\bar{\nu}_j\to\nu_i\bar{\nu}_i$ with $i\neq j)$. While tau- and muon-neutrinos are produced only by neutral current interactions, electron-neutrino production has a contribution from charged current interactions as well. However, since the present bound on m_{ν_e} is in the eV range, we will not be interested in ν_e's as dark matter. At late times, the neutrino distribution reaches some dynamical shape which differs from the equilibrium one. We have checked that the neutrino abundances follow the same evolution curve regardless of their initial abundances and the final neutrino distribution is insensitive to the

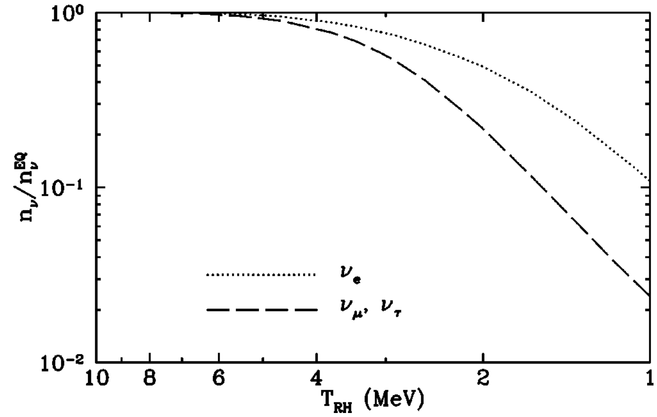

FIG. 2. The relative number density of different neutrino species to the equilibrium neutrino number density as a function of the reheating temperature.

initial conditions. The time dependence of neutrino density (at late times) is shown in Fig. 1.

The final tau- and muon-neutrino number density (they are the same) normalized to the neutrino number density in equilibrium n_ν/n_ν^{EQ} (where $n_\nu^{EQ}=2\xi(3)T_\nu^3/\pi^2$) is plotted in Fig. 2 as a function of T_{RH}. This ratio becomes smaller than unity for T_{RH} smaller than about 8 MeV, signaling a departure from equilibrium. The final momentum distribution function for $T_{RH}=1$ MeV is plotted in Fig. 3. For comparison, we have plotted the equilibrium neutrino distribution function taking into account the fact that the neutrino temperature is a factor 1.4 times smaller then the photon temperature T_γ due to $e^\pm$ annihilation after neutrino decoupling.

What is relevant is that the abundance of tau- and muon-neutrinos is about a factor of 2.7×10^{-2} smaller than the standard abundance for reheating temperatures around 1 MeV. This in turn implies that the standard upper bound of $30h^2$ eV on m_ν no longer applies and the abundance of tau- and muon-neutrinos for 1 MeV$\lesssim T_{RH}\lesssim 3$ MeV can be expressed as

$$\Omega_{\nu_\tau}h^2=\Omega_{\nu_\mu}h^2=\left(\frac{m_\nu}{4\ \text{keV}}\right)\left(\frac{T_{RH}}{1\text{MeV}}\right)^3, \tag{6}$$

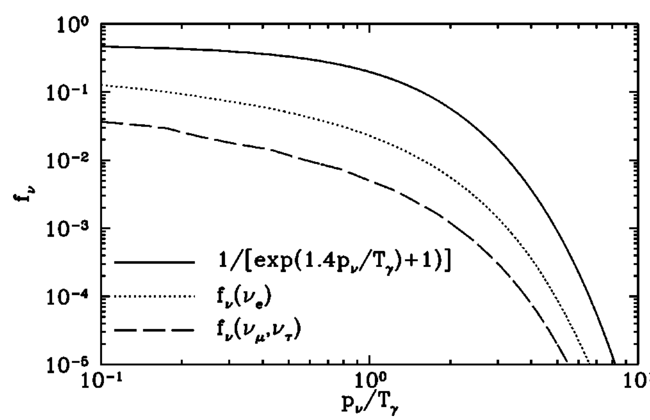

FIG. 3. The final distribution functions for ν_e, ν_μ and ν_τ for $T_{RH}=1$ MeV as function of the dimensionless momentum p_ν/T_γ. The dotted line corresponds to the standard model neutrino distribution function.

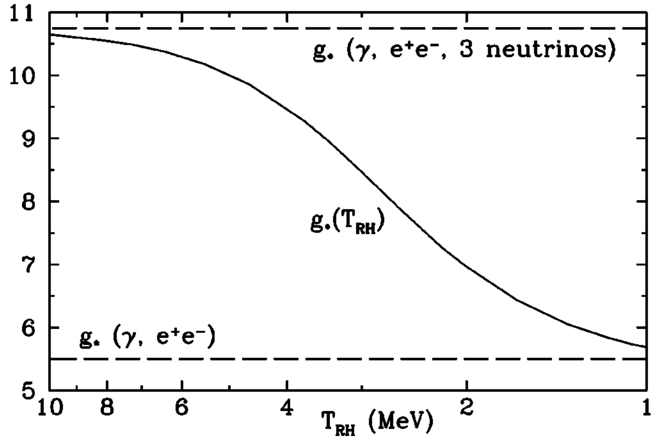

FIG. 4. The actual value of g_* at T_{RH} as a function of T_{RH}.

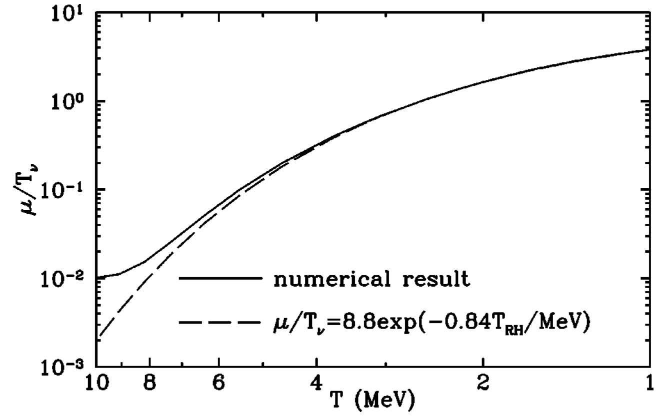

FIG. 5. The effective chemical potential as a function of reheat temperature.

where we used a power-law approximation $n_\nu/n_\nu^{\rm EQ} \approx 0.024(T_{RH}/1\ {\rm MeV})^3$. For larger reheat temperatures, $3\ {\rm MeV} \lesssim T_{RH} \lesssim 8\ {\rm MeV}$, one needs a slightly more complicated expression to fit n_ν: $n_\nu/n_\nu^{\rm EQ} \simeq 0.44 \tan^{-1}[(T_{RH}({\rm MeV})-2.6)/1.17]+0.43$.

As mentioned above, the value of g_* in the $(\Gamma - T_{RH})$ relation is actually a function of T_{RH}. The value of $g_*(T_{RH})$ provides an indication of the relative abundance of neutrinos in the thermal bath. The value of $g_*(T_{RH})$ as a function of T_{RH} is given in Fig. 4 and allows us to relate the decay rate to the reheat temperature, $\Gamma^{-1} = ({\rm MeV}/T_{RH})^2 g_*(T_{RH})^{-1/2} 1.6\ {\rm sec} \simeq 0.56({\rm MeV}/T_{RH})^{2.16}$ sec.

Our result in Eq. (6) is slightly smaller than the relic neutrino abundance computed in Ref. [5]. For instance, Fig. 2 of Ref. [5] shows a value of $\Omega_\nu h^2$ a factor of 1.6 larger than what is given by Eq. (6), for $T_{RH}=2$ MeV. The calculation in Ref. [5] makes the approximation of (i) neglecting the electron mass in the collision integrals, (ii) assuming a Boltzmann distribution for the electrons, (iii) neglecting $\nu-\nu$ interactions. We find that the first approximation is well justified, but the other two give errors of up to 30% (depending on the value of T_{RH}), which however are opposite in sign and roughly cancel each other. The numerical discrepancy is largely explained by the fact that the authors of Ref. [5] fix $g_*(T_{RH})=10.75$, and therefore their result should be multiplied by a correction factor $[10.75/g_*(T_{RH})]^{3/4}$. For the same reason, the limit on T_{RH} derived in Ref. [5] from nucleosynthesis should also be rescaled by a factor $[10.75/g_*(T_{RH})]^{1/4}$. With this procedure we find that nucleosynthesis gives slightly more stringent lower bounds on T_{RH}: $T_{RH}>1.2$ MeV (at 68% C.L.) and $T_{RH}>0.8$ MeV (at 95% C.L.).

The result of Eq. (6) is similar to that found in Ref. [6] (allowing for the differing definition of T_{RH} mentioned above). This small difference traces to the assumptions made in Ref. [6] of Maxwell-Boltzmann statistics and local thermodynamic equilibrium in the calculation of the thermal averaged cross section.

III. STANDARD MODEL NEUTRINOS AS WARM DARK MATTER

CDM reproduces the observable universe at large scales, but appears to be in conflict with observations on subgalactic scales. CDM produces too many dwarf galaxies and overdense galactic cores compared to observations. The resolution of this difficulty has been searched for along different routes. Many suggestions can be united under recipe of reducing the power on small scales. In the WDM scenario, this reduction occurs naturally via the mechanism of free streaming. The comoving smoothing scale can be estimated as the comoving horizon at matter-radiation equality times the *rms* velocity of dark matter particles at that time [9]

$$R \approx 0.2(\Omega_\nu h^2)^{1/3}({\rm keV}/m_X)^{4/3}{\rm Mpc}. \tag{7}$$

Interestingly, Ω_X also takes correct value within the same range of m_X if the reheating temperature is of order 1 MeV [see Eq. (6)]. This numerical coincidence deserves further study and detailed numerical investigation.

Encouraging preliminary results were obtained in Ref. [10]. To aid such studies, we parametrize our final distribution function of neutrinos as a thermal distribution with some effective "temperature" and a "chemical potential." The difference between the effective temperature and the standard-model neutrino temperature is insignificant (only a few percent) and the modification to the distribution function can be attributed to the chemical potential, which is shown in Fig. 5.

At large T_{RH} the chemical potential approaches the standard-model value $\mu=0.01T_\nu$. At small T_{RH} it can be fitted as $\mu/T_\nu = 8.8\exp(-0.84T_{RH}/{\rm MeV})$.

To conclude, the combination of a standard model neutrino of mass of a few keV and a reheat temperature of about an MeV will result in neutrinos as candidate WDM. Assuming $\Omega_\nu=0.3$ in one species of neutrino and $h=0.65$, then $R=0.4(T_{RH}/{\rm MeV})^4$ Mpc. As discussed above, any reheat temperature above about an MeV is consistent with BBN.

We want to stress that the hypothesis of neutrinos as WDM is not necessarily inconsistent at present with the recent data on atmospheric and solar neutrino anomalies. Atmospheric neutrino data could be accomodated by oscillations between quasi-degenerate ν_μ and ν_τ states and in such a case solar neutrino observations would require ν_e to oscillate into a sterile state. Alternatively, although this possibility is disfavored by Superkamiokande data, the atmospheric neutrino deficit might be explained by the conversion of ν_μ into a sterile state, and the solar neutrino data by conversion of ν_e into an active state.

ACKNOWLEDGMENTS

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