Possible color octet quark-antiquark condensate in the instanton model

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Inspired by a recent proposal for a Higgs description of QCD we study the possible formation of a color- and flavor-octet quark-antiquark condensate in the instanton liquid model. For this purpose we calculate two-point correlation functions of color-singlet and octet quark-antiquark operators. We find long range order in the standard $\langle\bar{\psi}\psi\rangle$ channel, but not in the color-octet channel. We emphasize that similar calculations in lattice QCD can check whether or not a color-flavor locked Higgs phase is realized in QCD at zero temperature and baryon density.

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It was recently argued that confinement in QCD can be understood in terms of an effective Higgs description where $SU(3)$ gauge invariance is spontaneously broken by a color-octet quark-antiquark condensate [1–4] (see [5,6] for earlier speculations in this direction)

$$\langle\bar{\psi}(\lambda^A)_C(\lambda^A)_F^T\psi\rangle = 2\left(\delta_i^a\delta_j^b - \frac{1}{3}\delta^{ab}\delta_{ij}\right)\langle\bar{\psi}_i^a\psi_j^b\rangle. \quad (1)$$

Here, $(\lambda^A)_C$ $(A=1,\ldots,8)$ is a color $SU(3)$ matrix and $(\lambda^A)_F$ is a flavor $SU(3)$ matrix. Color indices in the fundamental representation of $SU(3)$ are denoted by a,b and flavor indices by i,j. The condensate (1) breaks the local gauge symmetry as well as the $SU(3)_L\times SU(3)_R$ chiral symmetry but leaves a diagonal $SU(3)$ unbroken. This symmetry acts as the physical flavor symmetry. The Higgs field (1) allows for a remarkably simple description of the gross features of the QCD spectrum. Gluons acquire a mass via the Higgs mechanism and carry the flavor quantum numbers and charges of the vector meson octet. The quark degrees of freedom transform as an octet and a heavier singlet and carry the quantum numbers of the baryon octet and singlet.

This type of Higgs description was originally proposed as a complementary picture of QCD at large baryon chemical potential [7,8]. In that case, the primary Higgs field is given by the color-flavor locked diquark condensate

$$\langle\psi_i^a C\gamma_5\psi_j^b\rangle = \phi(\delta_i^a\delta_j^b - \delta_i^b\delta_j^a). \quad (2)$$

This condensate has the same residual color-flavor symmetry as Eq. (1) but also breaks the $U(1)$ of baryon number. At large baryon density this is not a concern and simply corresponds to baryon superfluidity. The diquark condensate is dynamically generated by attractive interactions in the color anti-triplet quark-quark channel. At very large chemical potential this attraction is generated by one-gluon exchange [9] while at intermediate density instanton induced interactions are likely to play a role [10,11]. Because the color-octet quark-anti-quark condensate (1) is consistent with the symmetries of the color-flavor locked diquark phase we expect this operator to have a non-zero expectation value in the high density phase [12–14]. This expectation is borne out by explicit calculations [12,13] but the value of the color-octet condensate is strongly suppressed with respect to the primary Higgs field, $\langle\bar{\psi}(\lambda^A)_C(\lambda^A)_F^T\psi\rangle \simeq \langle\bar{\psi}\psi\rangle \ll \langle\psi C\gamma_5(\lambda^A)_C(\lambda^A)_F\psi\rangle$. The suppression is due to an approximate Z_2 symmetry in the high density phase.

At zero baryon density the dynamical origin of a possible color-octet quark-antiquark is unclear. Both one-gluon exchange and instantons are repulsive in this channel. Nevertheless, given the attractiveness of the scenario outlined in [1–4] it seems worthwhile to investigate this question beyond perturbation theory. In this Brief Report, we report on a calculation using the instanton liquid model [15,16]. This model correctly accounts for chiral symmetry breaking at zero baryon density and the formation of a diquark condensate at high density. The color-octet quark-antiquark condensate at high density is also induced by instantons. Furthermore, Wetterich suggested that an instability in the instanton induced effective potential in three flavor QCD might be the dynamical origin of the color-octet quark-antiquark condensate [3].

First we have to discuss how to identify the Higgs phase characterized by Eq. (1). The color-octet condensate (1) is not a gauge invariant operator and therefore it cannot be used as an order parameter. The simplest alternative is the square of the octet condensate. This is similar to studying the vacuum expectation value (VEV) of the square of the $SU(2)$ Higgs field in the standard model. In the present case, however, this is not very useful. The square of the octet condensate receives large contributions from fluctuations associated with ordinary chiral symmetry breaking. This is suggested by the factorization approximation [17] which gives

$$\langle(\bar{\psi}(\lambda^A)_C(\lambda^A)_F^T\psi)^2\rangle \simeq \frac{8}{81}\langle\bar{\psi}\psi\rangle^2. \quad (3)$$

In the instanton model $\langle(\bar{\psi}(\lambda^A)_C(\lambda^A)_F^T\psi)^2\rangle$ is about 2–3 times larger than this estimate, but even larger deviations from factorization are observed in other Lorentz-scalar four-quark operators. Instead of the square of the octet condensate we propose to study the color-octet quark-antiquark correlation function. If QCD can be described in terms of the Higgs picture advocated in [1] we expect to observe long range order in this correlation function. We should note that the color-octet correlation function is also not a gauge invariant object. This problem can be addressed by calculating the correlator in some fixed gauge or by inserting two gauge strings. One might worry that if the gauge strings are included the correlator will no longer approach a constant even if there is long range order. Nevertheless, if the Higgs description makes sense there has to be a clear difference between the color-octet correlator and a generic gauge invariant correlation function without long range order.

We have calculated the correlation functions in the color- and flavor-singlet, color-singlet and flavor-octet, and color- and flavor-octet channel. The correlation functions are defined by

$$\Pi_{[1][1]}(x,y) = -\langle \mathrm{Tr}[S^{ab}(x,y)S^{ba}(y,x)]\rangle + \langle \mathrm{Tr}[S^{aa}(x,x)]\mathrm{Tr}[S^{bb}(y,y)]\rangle, \tag{4}$$

$$\Pi_{[1][8]}(x,y) = -\langle \mathrm{Tr}[S^{ab}(x,y)S^{ba}(y,x)]\rangle, \tag{5}$$

$$\Pi_{[8][8]}(x,y) = -4\langle \mathrm{Tr}[S^{aa}(x,y)S^{bb}(y,x)]\rangle + \frac{4}{3}\langle \mathrm{Tr}[S^{ab}(x,y)S^{ba}(y,x)]\rangle, \tag{6}$$

where $S^{ab}(x,y)$ is the quark propagator with color indices a,b and the traces are taken over Dirac indices. We have assumed exact flavor symmetry. The flavor-singlet correlation function $\Pi_{[1][1]}$ is strongly attractive and expected to approach $\langle \bar{\psi}\psi\rangle^2$ as $x-y$ tends to infinity. The flavor-octet correlator $\Pi_{[1][8]}$ is expected to decay exponentially with a characteristic mass $m_{[1][8]} \simeq m_{a_0} \simeq 1$ GeV. Depending on whether the Higgs description is valid we expect the color-flavor octet correlator $\Pi_{[8][8]}$ to behave similar to $\Pi_{[1][1]}$ or to $\Pi_{[1][8]}$.

Figures 1 and 2 show the three correlation functions (4)–(6) measured in quenched and unquenched instanton liquid simulation. In both cases we observe long range order in the $\bar{\psi}\psi$ channel, $\Pi_{[1][1]}(x) \to \mathrm{const}$. From the asymptotic value of the correlation function we find $\langle \bar{\psi}\psi\rangle \simeq -(250\ \mathrm{MeV})^3$ in the quenched case and $-(220\ \mathrm{MeV})^3$ in the unquenched case. The flavor-octet $\Pi_{[1][8]}$ correlation function decays exponentially. In the quenched calculation the correlator becomes unphysical for $x > 0.5$ fm. In the unquenched calculation the screening mass is consistent with 1 GeV.

We observe that the color-flavor octet correlator $\Pi_{[8][8]}$ also decays exponentially. There is no sign of long range order. In the unquenched calculation the screening mass is 700 MeV, while it is even larger in the quenched calculation. We have also calculated the correlator with the gauge links included. We observe no qualitative difference. In fact, the screening mass is slightly increased.

Finally, we would like to address another potential concern regarding our method for studying the scenario proposed in [1]. The color-octet condensate (1) breaks no physical symmetries except for the chiral $SU(3)_L \times SU(3)_R$ symmetry. It does, however, break the original flavor symmetry of the theory. The unbroken $SU(3)_V$ is a linear combination of the original $SU(3)_V$ and $SU(3)_C$ symmetries.

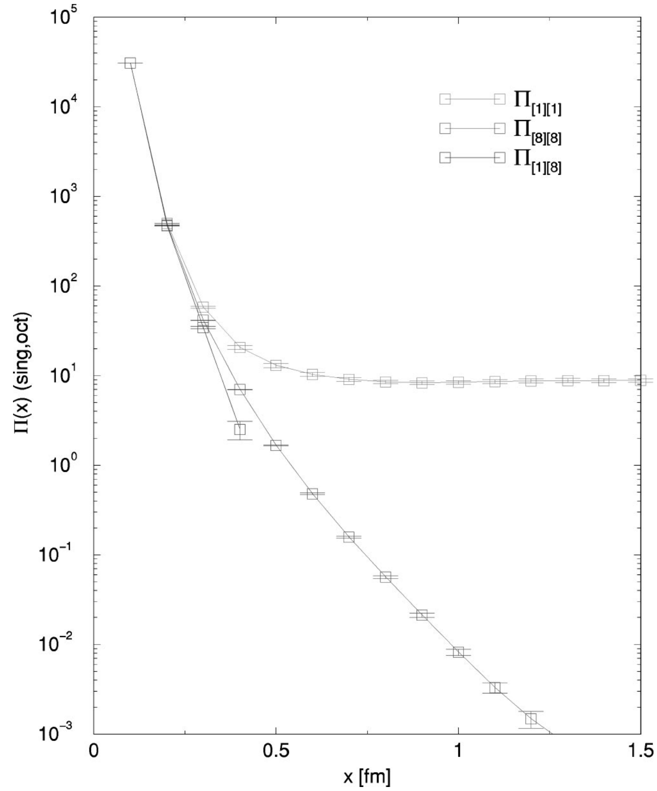

FIG. 1. Correlation functions in the color- and flavor-singlet ([1][1]), color- and flavor-octet ([8][8]) and color-singlet and flavor-octet ([1][8]) channel. The correlators were calculated in a quenched instanton ensemble. The function $\Pi_{[8][8]}$ was multiplied by a factor 3/32 in order to normalize the short distance behavior of all correlation functions in the same way.

This might imply that the color-octet condensate (1) cannot be observed in a finite volume unless a flavor-symmetry breaking source term is added. The proper way to extract the color-octet condensate is then to take the thermodynamic limit first and set the source term to zero afterwards. In order to investigate this possibility we have calculated the color- and flavor-octet correlation function in the case of unequal quark masses m_u, m_d, m_s. This means that the flavor symmetry is completely broken. The color- and flavor-octet correlation function is given by

$$\Pi_{[8][8]}(x,y) = 4\Bigg\{ \sum_{f,g} \Big\langle \Big(\mathrm{Tr}[S_f^{ff}(x,x)] - \frac{1}{3}\mathrm{Tr}[S_f^{aa}(x,x)] \Big) \times \Big(\mathrm{Tr}[S_g^{gg}(y,y)] - \frac{1}{3}\mathrm{Tr}[S_g^{aa}(y,y)] \Big) \Big\rangle - \sum_{f,g} \langle \mathrm{Tr}[S_f^{ff}(x,y)S_g^{gg}(y,x)]\rangle + \frac{2}{3}\sum_f \langle \mathrm{Tr}[S_f^{fa}(x,y)S_f^{af}(y,x)]\rangle - \frac{1}{9}\sum_f \langle \mathrm{Tr}[S_f^{ab}(x,y)S_f^{ba}(y,x)]\rangle \Bigg\}. \tag{7}$$

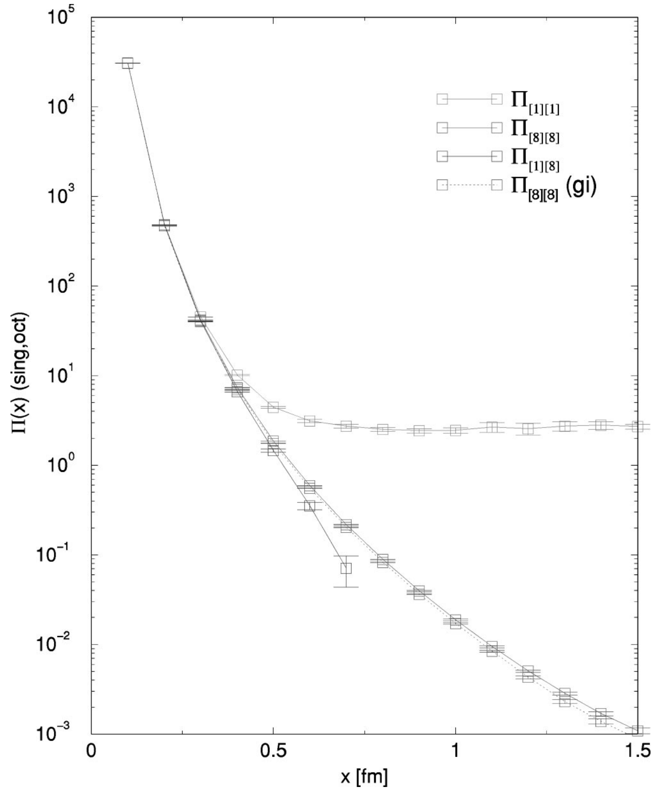

FIG. 2. Same correlators as in Fig. 1 calculated in an unquenched instanton ensemble. $\Pi_{[8][8]}(\mathrm{gi})$ denotes the color-octet correlator with the gauge links included.

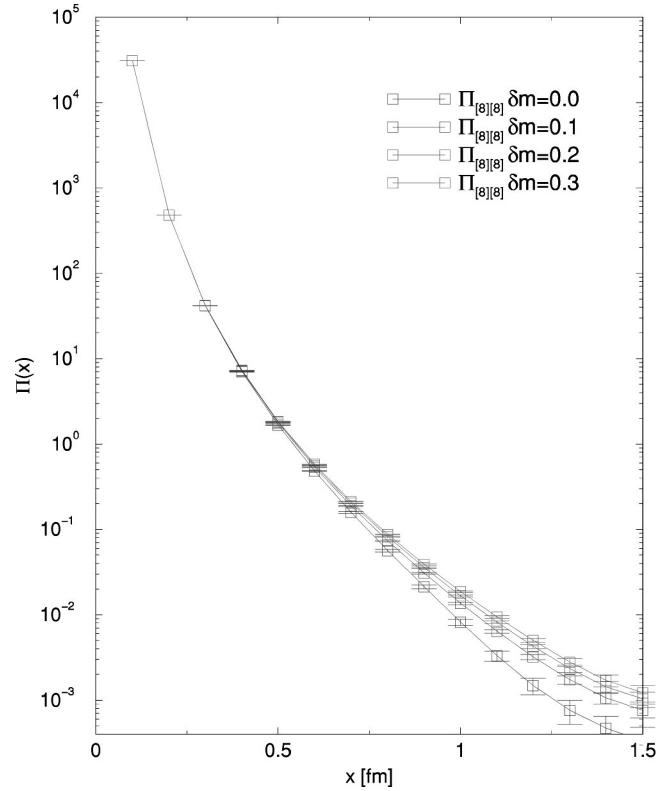

FIG. 3. Correlation functions in the color- and flavor-octet ([8][8]) channel for different values of the explicit flavor symmetry breaking parameter $\delta m = 0.0,0.1,0.2,0.3\ \mathrm{fm}^{-1}$. The correlators were calculated in a quenched instanton ensemble.

Here, f,g are flavor indices, $S_f^{ab} = \{S_u^{ab}, S_d^{ab}, S_s^{ab}\}$, $S_f^{fa} = \{S_u^{1a}, S_d^{2a}, S_s^{3a}\}$ and $S_f^{ff} = \{S_u^{11}, S_d^{22}, S_s^{33}\}$. It is easy to check that in the case of exact flavor symmetry, $S_u^{ab} = S_d^{ab} = S_s^{ab}$, expression (7) reduces to our earlier result (6). We note that the correlation function (7), unlike the correlator in the flavor-symmetric case (6), contains disconnected contributions. This makes it more likely to observe long range order. In Fig. 3 we show the color- and flavor-octet correlation function for different values of the flavor-symmetry breaking parameter δm. This parameter is related to the masses of the up, down and strange quarks by $m_u = m_0$, $m_d = m_0 + \delta m$ and $m_s = m_0 + 2\,\delta m$. Results are shown for $\delta m = 0.0,0.1,0.2,0.3\ \mathrm{fm}^{-1}$ where $m_0 = 0.1\ \mathrm{fm}^{-1}$ is kept fixed. We observe that the color- and flavor-octet correlation function increases with δm, but again there is no evidence for long range order. There is also no clear evidence for a non-analytic dependence on δm.

We should note that a similar analysis can be carried out for the color-singlet and flavor-octet correlation function $\Pi_{[1][8]}$. In this case the appearance of long range order for $\delta m \neq 0$ would correspond to spontaneous flavor symmetry breaking and $\langle \bar{u}u - \bar{d}d \rangle \neq 0$. This possibility is excluded by the Witten-Vafa theorem. In agreement with this theorem, no spontaneous flavor symmetry breaking is observed in the instanton model.

In summary we observe no evidence for long range order in the color-flavor octet channel. We suggest that lattice calculations of the correlation functions (4)–(7) can provide a definitive answer to the question whether the Higgs picture suggested in [1] is realized in nature.

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