Muon anomalous magnetic dipole moment in supersymmetric theories

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We study the muon anomalous magnetic dipole moment in supersymmetric theories. The impact of the recent Brookhaven E821 experimental measurement on both model-independent and model-dependent supersymmetric parameter spaces is discussed in detail. We find that values of $\tan\beta$ as low as 3 can be obtained while remaining within the E821 one-sigma bound. This requires a light smuon; however, we show that, somewhat surprisingly, no model-independent bound can be placed on the mass of the lightest chargino for any $\tan\beta \geqslant 3$. We also show that the maximum contributions to the anomalous magnetic moment are insensitive to CP-violating phases. We provide analyses of the supersymmetric contribution to the muon anomalous magnetic moment in dilaton-dominated supergravity models and gauge-mediated supersymmetry-breaking models. Finally, we discuss how other phenomena, such as $B(b\to s\gamma)$, relic abundance of the lightest superpartner, and the Higgs boson mass may be correlated with the anomalous magnetic moment, but do not significantly impact the viability of a supersymmetric explanation, or the mass limits obtainable on smuons and charginos.

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I. THE MUON ANOMALOUS MAGNETIC DIPOLE MOMENT

A. Standard model prediction and experiment

The amplitude for the photon-muon-muon coupling in the limit of the photon momentum q going to zero can be written as

$$\text{Amplitude} = ie\,\overline{u}\left[\gamma^\lambda + a_\mu \frac{i\sigma^{\lambda\beta}q_\beta}{2m_\mu}\right]uA_\lambda, \tag{1.1}$$

where $e=\sqrt{4\pi\alpha_{\rm EM}}$. The second term comes from loop corrections, and is given to one-loop order in QED by $a_\mu = \alpha/2\pi$. Being a small correction to the tree-level magnetic moment of the muon, it is called the anomalous magnetic moment.

The state of the art calculation of a_μ within the standard model (SM) is [1]

$$a_\mu^{\rm SM} = 11\,659\,159.6(6.7)\times 10^{-10}. \tag{1.2}$$

The majority of the uncertainty comes from hadrons in the photon vacuum polarization diagram.

Recently the Brookhaven E821 experiment has released a new measurement of a_μ and found [2]

$$a_\mu^{\rm E821} = 11\,659\,202(14)(6)\times 10^{-10}. \tag{1.3}$$

From this one concludes [2]

$$\delta a_\mu = a_\mu^{\rm E821} - a_\mu^{\rm SM} = (43\pm 16)\times 10^{-10}. \tag{1.4}$$

This result indicates that the anomalous magnetic moment of the muon may need additional contributions beyond the SM to be consistent with the experimental measurement.

B. Supersymmetric contributions

There are many reasons to believe that the SM is an incomplete description of nature besides the present indications from a_μ. For example, the SM does not explain baryogenesis, dark matter, the ratios of fundamental scales, or the strengths of gauge and Yukawa interactions. Supersymmetry is an appealing theoretical framework that may answer many of the questions unanswerable within the SM [3,4].

The supersymmetry effects [5–20] on a_μ include loops with a chargino and a muon sneutrino, and loops with a neutralino and a smuon. Summations are performed over all such chargino, neutralino and smuon mass eigenstates. The one-loop superpartner contributions to a_μ, including the effects of possible complex phases, are

$$\delta a_\mu^{\chi^0} = \frac{m_\mu}{16\pi^2}\sum_{i,m}\left\{ -\frac{m_\mu}{12m^2_{\tilde\mu_m}}(|n^L_{im}|^2+|n^R_{im}|^2)F^N_1(x_{im}) + \frac{m_{\chi^0_i}}{3m^2_{\tilde\mu_m}}\mathrm{Re}[n^L_{im}n^R_{im}]F^N_2(x_{im})\right\} \tag{1.5}$$

$$\delta a_\mu^{\chi^\pm} = \frac{m_\mu}{16\pi^2}\sum_{k}\left\{ \frac{m_\mu}{12m^2_{\tilde\nu_\mu}}(|c^L_{k}|^2+|c^R_{k}|^2)F^C_1(x_{k}) + \frac{2m_{\chi^\pm_k}}{3m^2_{\tilde\nu_\mu}}\mathrm{Re}[c^L_{k}c^R_{k}]F^C_2(x_{k})\right\} \tag{1.6}$$

where $i=1,2,3,4$ and $m=1,2$ and $k=1,2$ are neutralino and smuon and chargino mass eigenstate labels respectively, and

$$n_{im}^{R}=\sqrt{2}g_1 N_{i1}X_{m2}+y_\mu N_{i3}X_{m1} \tag{1.7}$$

$$n_{im}^{L}=\frac{1}{\sqrt{2}}(g_2 N_{i2}+g_1 N_{i1})X_{m1}^{*}-y_\mu N_{i3}X_{m2}^{*} \tag{1.8}$$

$$c_k^R=y_\mu U_{k2} \tag{1.9}$$

$$c_k^L=-g_2 V_{k1} \tag{1.10}$$

and $y_\mu=g_2 m_\mu/\sqrt{2}m_W\cos\beta$ is the muon Yukawa coupling. The kinematic loop functions depend on the variables $x_{im}=m_{\chi_i^0}^2/m_{\tilde{\mu}_m}^2$, $x_k=m_{\chi_k^\pm}^2/m_{\tilde{\nu}_\mu}^2$ and are given by

$$F_1^N(x)=\frac{2}{(1-x)^4}[1-6x+3x^2+2x^3-6x^2\ln x], \tag{1.11}$$

$$F_2^N(x)=\frac{3}{(1-x)^3}[1-x^2+2x\ln x] \tag{1.12}$$

$$F_1^C(x)=\frac{2}{(1-x)^4}[2+3x-6x^2+x^3+6x\ln x] \tag{1.13}$$

$$F_2^C(x)=-\frac{3}{2(1-x)^3}[3-4x+x^2+2\ln x], \tag{1.14}$$

normalized so that $F_1^N(1)=F_2^N(1)=F_1^C(1)=F_2^C(1)=1$, corresponding to degenerate sparticles.

By definition $g_2\simeq 0.66$ and $g_1\simeq 0.36$ are the $SU(2)_L$ and $U(1)_Y$ gauge couplings. The phase convention for μ follows Refs. [3,4], so that the neutralino and chargino mass matrices are given by

$$M_{\chi^0}=\begin{pmatrix} M_1 & 0 & -c_\beta s_W m_Z & s_\beta s_W m_Z \\ 0 & M_2 & c_\beta c_W m_Z & -s_\beta c_W m_Z \\ -c_\beta s_W m_Z & c_\beta c_W m_Z & 0 & -\mu \\ s_\beta s_W m_Z & -s_\beta c_W m_Z & -\mu & 0 \end{pmatrix} \tag{1.15}$$

and

$$M_{\chi^\pm}=\begin{pmatrix} M_2 & \sqrt{2}s_\beta m_W \\ \sqrt{2}c_\beta m_W & \mu \end{pmatrix}. \tag{1.16}$$

Here we have used abbreviations $s_\beta=\sin\beta$, $c_\beta=\cos\beta$, $s_W=\sin\theta_W$, and $c_W=\cos\theta_W$. The neutralino mixing matrix N_{ij} and the chargino mixing matrices U_{kl} and V_{kl} are identical to those in Refs. [3,4]; they satisfy

$$N^{*}M_{\chi^0}N^{\dagger}=\mathrm{diag}(m_{\chi_1^0},m_{\chi_2^0},m_{\chi_3^0},m_{\chi_4^0}) \tag{1.17}$$

$$U^{*}M_{\chi^\pm}V^{\dagger}=\mathrm{diag}(m_{\chi_1^\pm},m_{\chi_2^\pm}). \tag{1.18}$$

In particular, the neutralino and chargino mass eigenvalues are always chosen to be real and positive, regardless of the complex phases of the underlying Lagrangian parameters; all non-trivial phases are contained in the unitary mixing matrices N, U, V. The smuon mass matrix, written in the $\{\tilde{\mu}_L,\tilde{\mu}_R\}$ basis is

$$M_{\tilde{\mu}}^2=\begin{pmatrix} m_L^2+\left(s_W^2-\frac{1}{2}\right)m_Z^2\cos 2\beta & m_\mu(A_{\tilde{\mu}}^{*}-\mu\tan\beta) \\ m_\mu(A_{\tilde{\mu}}-\mu^{*}\tan\beta) & m_R^2-s_W^2 m_Z^2\cos 2\beta \end{pmatrix}, \tag{1.19}$$

and the unitary matrix X_{mn} is defined by

$$XM_{\tilde{\mu}}^2X^{\dagger}=\mathrm{diag}(m_{\tilde{\mu}_1}^2,m_{\tilde{\mu}_2}^2). \tag{1.20}$$

The muon sneutrino mass is related to the left-handed smuon mass parameter by

$$m_{\tilde{\nu}}^2=m_L^2+\frac{1}{2}m_Z^2\cos 2\beta. \tag{1.21}$$

The simplest analytic result to obtain from supersymmetry is to assume that all superpartners have the same mass M_{SUSY}, which leads to

$$\begin{aligned}\delta a_\mu^{\mathrm{SUSY}}&=\frac{\tan\beta}{192\pi^2}\frac{m_\mu^2}{M_{\mathrm{SUSY}}^2}(5g_2^2+g_1^2)\\ &=14\tan\beta\left(\frac{100\ \mathrm{GeV}}{M_{\mathrm{SUSY}}}\right)^2 10^{-10}\end{aligned} \tag{1.22}$$

with the chargino contribution dominating the neutralino contribution [14]. The large $\tan\beta$ scaling is easy to understand, and is analogous to the large $\tan\beta$ enhancements of $B(b\to s\gamma)$ and Δm_b corrections. a_μ requires a muon chirality flip, which usually costs a m_μ suppression. However, the Higgsino-smuon-muon vertex coupling can perform the chirality flip with the muon Yukawa coupling y_μ, leading to an enhancement $y_\mu\propto m_\mu\tan\beta$ at large $\tan\beta$.

Another important limit which will play a role in the discussion of the next section is the case in which $M_1\ll M_2,\mu$, so that only loops containing a light B-ino and the smuons are important. In that limit, we find

$$\delta a_\mu^{\mathrm{light}\ B\text{-ino}}=\frac{g_1^2}{48\pi^2}\frac{m_\mu^2 M_1\mathrm{Re}[\mu\tan\beta-A_\mu^{*}]}{m_{\tilde{\mu}_2}^2-m_{\tilde{\mu}_1}^2}\times\left[\frac{F_2^N(x_{11})}{m_{\tilde{\mu}_1}^2}-\frac{F_2^N(x_{12})}{m_{\tilde{\mu}_2}^2}\right] \tag{1.23}$$

where $x_{1m}=M_1^2/m_{\tilde{\mu}_m}^2$. Note that Eq. (1.23) has a smooth limit as the sleptons become degenerate. This yields a quite sizeable contribution in the case that all neutralinos and charginos except the light B-ino become heavy. For example, in the case $m_{\tilde{\mu}_1}\approx m_{\tilde{\mu}_2}=2.0M_1$, Eq. (1.23) becomes

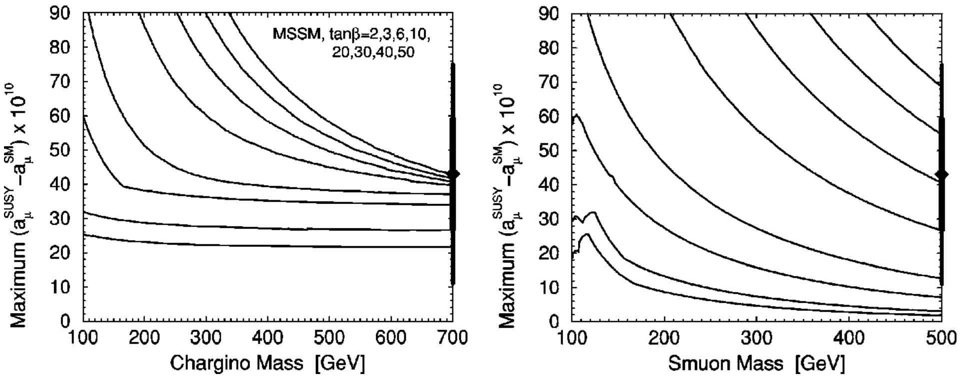

FIG. 1. The maximum possible values for the supersymmetric contribution to the muon anomalous magnetic moment, as a function of the lighter chargino mass and the lighter smuon mass. Gaugino mass unification conditions have not been imposed. All charged superpartners are required to be heavier than 100 GeV, and the lightest neutralino is required to be heavier than 50 GeV. The maximum allowed value for $|\mu|$ is taken to be 1000 GeV. The contours are shown from bottom to top for $\tan\beta=2$, 3, 6, 10, 20, 30, 40, 50. The black bars on the right vertical axes indicate the 1-σ and 2-σ allowed regions from the Brookhaven E821 experiment.

$$\delta a_\mu^{\text{light }B\text{-ino}}=18\tan\beta\left(\frac{100\ \text{GeV}}{m_{\tilde{\mu}}}\right)^3\left(\frac{\mu-A_\mu\cot\beta}{1000\ \ \text{GeV}}\right)10^{-10}. \tag{1.24}$$

(This formula will eventually fail to be accurate for extremely huge $\mu\tan\beta$, in accord with decoupling, since then $m_{\tilde{\mu}_1}\approx m_{\tilde{\mu}_2}$ must fail badly.) This situation is not quite in effect in the usual supergravity-inspired and gauge-mediated supersymmetry breaking scenarios, but is certainly obtainable within a model-independent framework, as we shall see. Furthermore, it could arise quite naturally in certain well-defined extensions of the minimal supersymmetric standard model (MSSM). For example, if supersymmetry breaking is manifested by an F-term vacuum expectation value (VEV) transforming in the adjoint **24** representation of a grand unified theory (GUT) $SU(5)$ gauge group, then the gaugino mass parameters are in the approximate ratio $M_1: M_2: M_3::$ $1:6:-12$ at the electroweak scale [21,22]. Another class of examples occurs in gauge-mediated supersymmetry breaking [23,24] (GMSB) models which have messengers that are not in complete $SU(5)$ multiplets, but rather in representations with more electroweak singlets than doublets. These models naturally predict a B-ino and right-handed sleptons which are much lighter than all electroweak doublet superpartners [25].

The leading log contribution from two-loop evaluation [26] yields a suppression

$$51.25a_{\mu,2\ \text{loop}}^{\text{SUSY}}=a_{\mu,1\ \text{loop}}^{\text{SUSY}}\left(1-\frac{4\alpha}{\pi}\ln\frac{M_{\text{SUSY}}}{m_\mu}\right) \tag{1.25}$$

where M_{SUSY} is a typical superpartner mass. This suppression factor varies between about 7% and 9% for the parameter space we consider. Although a complete next leading order (NLO) calculation has yet to be carried through in supersymmetry, we have imposed in all of our numerical results below a uniform 7% reduction from the 1-loop calculation based on this leading-log estimate.

II. RESULTS FOR GENERAL SUPERSYMMETRIC MODELS

A. General MSSM parameters

The full minimal supersymmetric standard model (MSSM) parameter space contains dozens of parameters. However, the supersymmetric contribution to the muon anomalous magnetic moment depends at the tree level only on the quantities M_1, M_2, μ, $\tan\beta$, m_L^2, m_R^2, and A_μ. Therefore it is possible to comprehend the impact of supersymmetry by using scans over parameter space which include experimental constraints. Several recent papers have examined the question of whether bounds can be put on superpartner masses and other parameters by taking the E821 results at face value. In this section, we remark on the possibility of extracting such bounds in a model-independent, and therefore maximally conservative, supersymmetric framework.

We have conducted an exhaustive examination of the relevant MSSM parameter space without imposing conditions that follow from model-building prejudice, in particular without requiring the usual gaugino mass unification condition between M_1 and M_2. In general, the supersymmetric contribution to a_μ can be made larger for larger values of $\tan\beta$ and smaller masses of the lighter chargino. However, in contrast to some recent reports, we find that it is quite possible to accommodate the E821 results even with rather low $\tan\beta$ and for arbitrarily heavy charginos. This can be seen directly from Eq. (1.23), by plugging in typical values. As long as M_1 and M_2 are not tied together by a unification condition, the charginos can become very heavy for very large M_2 and μ while still leaving behind a contribution which is large enough to fall within the E821 1-σ bounds, provided only that a smuon is light and $|\mu|$ is not too small. Even for $\tan\beta=2$, the contribution can be large enough to fall within the present 2-σ bounds.

The results for the maximum possible value of $a_\mu^{\text{SUSY}}-a_\mu^{\text{SM}}$ are shown in Fig. 1. Here we have chosen to present contours for different values of $\tan\beta=2$, 3, 6, 10, 20, 30, 40,

50. All of the other parameters are taken to be independent, subject to the constraint that all charged superpartners are heavier than 100 GeV and the lightest neutralino is heavier than 50 GeV (to provide approximate agreement with present and imminent bounds from the final results of the CERN e^+e^- collider LEP2 experiments). In order to avoid the possibility of charge- and color-breaking vacua, we have imposed a constraint $|A_\mu| \leq 3\,\mathrm{Min}[m_L, m_R]$. The precise value of this bound generally does not have a large effect on the contours shown. However, in these plots we have found it appropriate to make two significant concessions to model-dependent prejudice, as follows.

First, we have also included a 100 GeV lower bound on the lighter stau mass, by assuming universality in soft slepton parameters $A_{\tilde\tau} = A_{\tilde\mu}$ and $m^2_{L,\tilde\tau} = m^2_{L,\tilde\mu}$ and $m^2_{R,\tilde\tau} = m^2_{R,\tilde\mu}$, and then requiring $m_{\tilde\tau_1} > 100$ GeV. With these assumptions, requiring the staus to be heavier than 100 GeV imposes a stronger indirect constraint on the smuon, because of the mixings proportional to $m_\tau \mu \tan\beta$ for staus and $m_\mu \mu \tan\beta$ for smuons. Strictly speaking, this type of requirement does not correspond to a model-independent framework, where lepton flavor universality need not be imposed; all bounds from low-energy lepton-number violation can be evaded by simple alignment in lepton flavor space. However, perhaps the most natural way to satisfy these constraints is to impose lepton universality at high energies. This constraint is most significant for smaller smuon masses and smaller values of $\tan\beta$ (less than roughly 10 or so), where the chargino-sneutrino loops do not necessarily dominate in $a_\mu^{\rm SUSY} - a_\mu^{\rm SM}$. Since in most cases the bounds are saturated by large mixing in the slepton sectors arising dominantly from the effects of large μ, this requirement is not very sensitive to the precise values used for the soft parameters $A_{\tilde\tau}$, $m_{R,\tilde\tau}$ and $m_{L,\tilde\tau}$, which can be affected by renormalization group running from a high scale where universality is imposed. In any case, we emphasize that in principle *even larger* values of $a_\mu^{\rm SUSY} - a_\mu^{\rm SM}$ can be obtained than are presented here.

Second, we have imposed a maximum value of $|\mu| < 1000$ GeV. If one chooses to allow larger values of μ, then one can construct models with larger contributions to $a_\mu^{\rm SUSY}$, resulting from neutralino-smuon loops dominated by a light smuon with a large mixing angle due to the off-diagonal terms proportional to μ in the squared-mass matrix. The prospect of very large $|\mu|$ often causes discomfort since it requires fine-tuning in the Higgs potential in order to obtain electroweak symmetry breaking in accord with experiment. However, it should be noted that in general the upper bound on contributions to $a_\mu^{\rm SUSY}$ increases with the assumed maximum allowed $|\mu|$.

We have not imposed any requirement that the lightest supersymmetric particle (LSP) is a neutralino. While the existence of a neutralino LSP could make an attractive candidate for the cold dark matter, in a general model framework it is neither a necessary nor a sufficient condition for an acceptable cosmology. Furthermore, in models which saturate the maximum possible $a_\mu^{\rm SUSY}$, a neutralino is typically light, so that imposing such a constraint would generally not affect our results, except below when we impose gaugino mass unification on the parameter space.

Several features of Fig. 1 deserve comment. In the graph of maximum $a_\mu^{\rm SUSY} - a_\mu^{\rm SM}$ as a function of chargino mass, there are distinct regions separated by an "elbow" (which is most visibly pronounced in the case of $\tan\beta = 6$). For chargino masses to the right of the elbow in each case, the bound is saturated by models with the maximum allowed value of $|\mu|$ and small $|M_1|$, and in fact the graph is nearly flat as the dominant contribution comes mainly from neutralino-smuon loops, as in Eq. (1.23). These models also have smuon masses (and stau masses) near their lower bound. For chargino masses to the left of the elbow, the maximum of $a_\mu^{\rm SUSY} - a_\mu^{\rm SM}$ tends to be saturated for models with much smaller values of $|\mu|$, and the chargino loops play a more important role. As $\tan\beta$ is increased, the chargino loops become relatively more important, and the dependence on the chargino masses extends out to much larger values before they decouple.

In the graph of maximum $a_\mu^{\rm SUSY} - a_\mu^{\rm SM}$ as a function of smuon mass in Fig. 1, the cases with smaller $\tan\beta$ exhibit some structure. For smuon masses just above 100 GeV, the models that saturate the bound have $|\mu|$ of order 200 GeV; much larger values of $|\mu|$ which could otherwise increase $a_\mu^{\rm SUSY}$ would conflict with our assumptions stated above regarding the limit on the lighter stau mass. In an intermediate region for the lightest smuon mass, the models that saturate the bound are the ones with the maximum allowed value of $|\mu|$ and small $|M_1|$, as suggested by Eq. (1.23). This leads to a bump in the maximum $a_\mu^{\rm SUSY} - a_\mu^{\rm SM}$; this is prominent for $\tan\beta = 2,3$, is just barely visible for $\tan\beta = 6$ (near smuon mass of 145 GeV) and disappears entirely for larger values of $\tan\beta$. For larger $\tan\beta$ or larger smuon masses, the models that saturate the bound again have much smaller $|\mu|$ (of order 200 GeV).

The effect of varying the maximum allowed value for $|\mu|$ is illustrated in Fig. 2 for $\tan\beta = 3$, using $|\mu| < 500, 1000$, and 2000 GeV. The graph shows that for a given chargino mass, the upper bound on $a_\mu^{\rm SUSY} - a_\mu^{\rm SM}$ is usually obtained for the maximum allowed $|\mu|$. However, as a function of the lighter smuon mass, the upper bound on $a_\mu^{\rm SUSY} - a_\mu^{\rm SM}$ is saturated for large $|\mu|$ only in a finite range of the smuon mass. Again, this is because for smuon masses very close to the experimental limit, the effects of large $|\mu|$ are limited by our requirement that the stau is not too light, while for sufficiently large smuon masses the chargino-sneutrino loops become more important.

B. Gaugino mass unification

It is also interesting to see how our results would change if one restricts to a class of models that make the usual assumption of gaugino mass unification predicted by supergravity-inspired models with unification of gauge couplings and universal soft-supersymmetry breaking couplings, namely

$$M_1 = \frac{5}{3}\tan^2\theta_W M_2 \simeq 0.5 M_2. \tag{2.1}$$

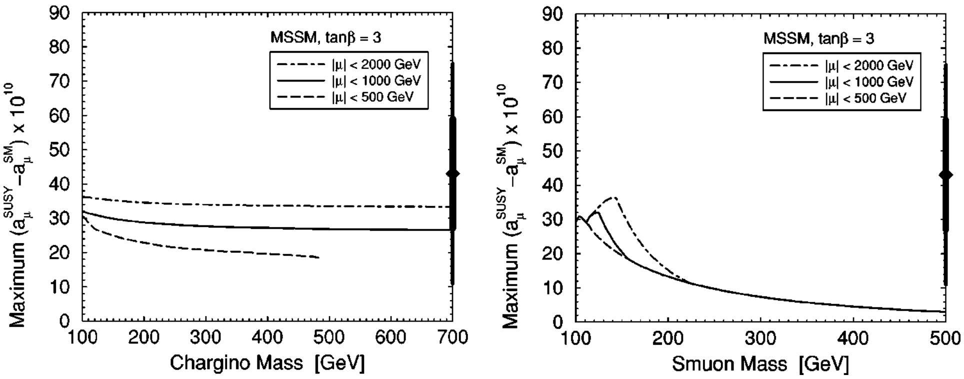

FIG. 2. Effects of larger allowed $|\mu|$: the maximum possible values for the supersymmetric contribution to the muon anomalous magnetic moment, as a function of the lighter chargino mass and the lighter smuon mass, for $\tan\beta = 3$ and different values (2000 GeV, 1000 GeV, 500 GeV from top to bottom) of the allowed maximum $|\mu|$. Gaugino mass unification conditions have not been imposed. All charged superpartners are required to be heavier than 100 GeV, and the lightest neutralino is required to be heavier than 50 GeV. The black bars on the right vertical axes indicate the 1-σ and 2-σ allowed regions from the Brookhaven E821 experiment.

It is plausible from a model-building perspective that slepton and Higgs boson soft squared masses can be affected by unknown D-term contributions [27–31] and other sources of non-universality. This supports the idea of an unrestricted parameter space for m_L^2, m_R^2, μ, and A_μ, while still maintaining the condition Eq. (2.1). Therefore we show the effects of imposing this assumption on the parameter space in Fig. 3. This graph shows that requiring gaugino mass unification does significantly impact the maximum obtainable $a_\mu^{\rm SUSY} - a_\mu^{\rm SM}$ for larger values of the chargino mass. This is clearly because if gaugino mass unification is imposed, heavy charginos necessarily means that the neutralino-smuon loop also decouples. Without the gaugino mass unification requirement, a significant contribution from the lightest smuon and B-ino–like neutralino loop can be independent of the chargino masses. However, the results for the maximum $a_\mu^{\rm SUSY} - a_\mu^{\rm SM}$ as a function of the lighter smuon mass are essentially unaffected by the requirement of gaugino mass unification, since the bounds in that case are saturated by models with lighter charginos anyway.

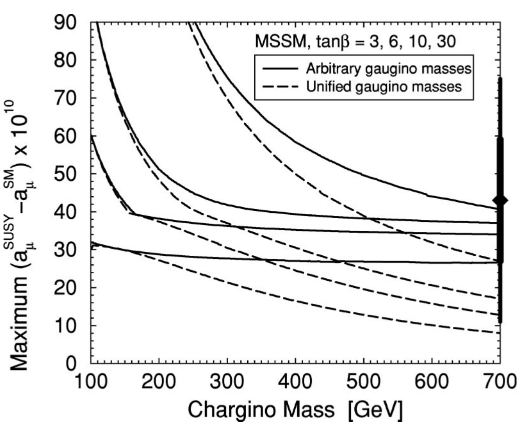

FIG. 3. Effects of a gaugino mass unification requirement: the maximum possible values for the supersymmetric contribution to the muon anomalous magnetic moment, as a function of the lighter chargino mass. The solid lines are the general results as before for $\tan\beta = 3, 6, 10, 30$, while the dashed lines are obtained with the additional condition $M_1 = (5/3)\tan^2\theta_W M_2$ imposed. The maximum allowed value of $|\mu|$ is 1000 GeV. All charged superpartners are required to be heavier than 100 GeV, and the lightest neutralino is required to be heavier than 50 GeV. The black bars on the right vertical axis indicate the 1-σ and 2-σ allowed regions from the Brookhaven E821 experiment. (The corresponding plot as a function of the lighter smuon mass is essentially unaffected by the gaugino mass unification condition.)

C. Constraints on the effects of complex phases

The above results were obtained for general values of all phases of M_1, M_2, μ, and A_μ. It is useful to remark that the maximum values of $a_\mu^{\rm SUSY} - a_\mu^{\rm SM}$, for fixed magnitudes of the parameters, is generally obtained when they are all real. For example, this is illustrated for a particular choice of parameters (close to a dilaton-dominated supergravity model) in Fig. 4. In the left graph, we show $a_\mu^{\rm SUSY} - a_\mu^{\rm SM}$ as a function of chargino mass, with fixed $\tan\beta = 30$, $|M_1| = 140$ GeV, $|M_2| = 280$ GeV, $|\mu| = 500$ GeV, and slepton parameters $m_L = 300$ GeV, $m_R = 225$ GeV, and $|A_\mu| = 320$ GeV. The range of values for $\delta a_\mu^{\rm SUSY}$, obtained by varying over all possible phase values, fills out the region enclosed in the solid lines, while the circles at the corners denote the points obtained when all parameters are real. In the right-hand graph, the same thing is done for the same model, but with $|\mu| = 280$ GeV (equal to M_2) so that chargino mixing and neutralino mixing effects are larger.

This illustrates that while the dependence on the phases is quite strong as has been noted in Refs. [16,17], the maximal contribution to the muon anomalous magnetic moment occurs for real parameters; in particular it usually occurs for positive real μ, if M_1 and M_2 are both positive and real. This result is not surprising. Unlike chiral violating interactions, CP violation breaks no symmetry critical to a_μ and so its introduction cannot overwhelm the calculation. The extremes of constructive and destructive interference naturally occur for $e^{i\phi_k} = \pm 1$ (i.e., $\phi_k = 0$ or π). Therefore, imposing bounds from CP-violation experiments has no effect on the results shown in Figs. 1–3.

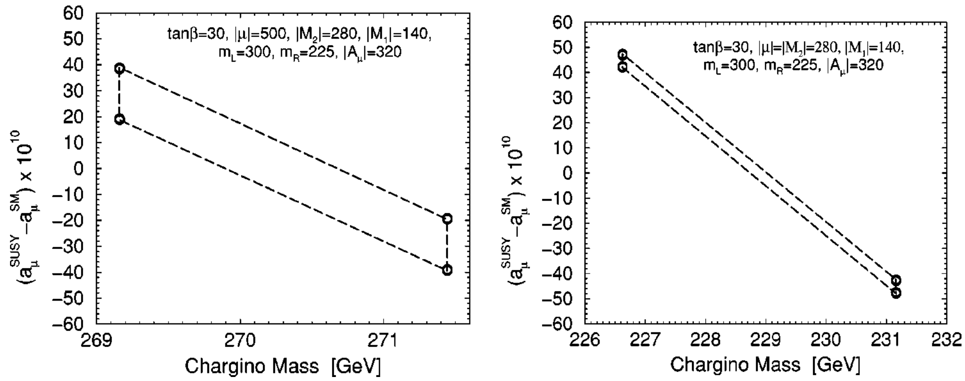

FIG. 4. Effects of phases: the contribution to the muon anomalous magnetic moment as a function of the lighter chargino mass, with magnitudes of all parameters held fixed. The dashed lines enclose the region obtained as the phases of all parameters are varied over all possible values. The circles at the corners of the regions are obtained when all parameters are required to be real. The graph on the left is obtained for a model close to a dilaton-dominated supergravity model, while the graph on the right is the same but with $|\mu|$ adjusted to equal $|M_2|$.

III. EXPECTATIONS IN MINIMAL SUPERSYMMETRY MODELS

The full supersymmetry parameter space, including all supersymmetric masses and mixing angles, contains well over 100 free parameters. The vast majority of this parameter space is ruled out by experimental measurements of proton lifetime, flavor changing neutral currents, and CP-violating observables. Ideas to solve these problems in supersymmetry are varied. However, there exists two baseline, or minimal models, that are largely immune from all past experimental constraints, and are often employed to estimate accessibility of supersymmetry in new experiments. These two models are called ''minimal supergravity'' (SUGRA) and ''minimal gauge mediation'' (GMSB).

One advantage of having minimal models as baselines for comparing expectations of supersymmetry is that they existed and were well-motivated before anomalies were seen by experiment. They therefore provide a more restrictive but still dispassionate view of how easily supersymmetry can accommodate non-SM effects, complementary to that obtained from the fully model-independent framework described in the previous section. When the measurement of $R_b=\Gamma(Z\to b\overline{b})/\Gamma(Z\to \text{had})$ appeared to have a 3σ deviation from the SM prediction, it was shown that supergravity (SUGRA) could not accommodate it [32]. One could attain R_b^{expt} in supersymmetry only by entertaining unusual corners of parameter space. It might be accurate to say that the SUGRA analysis of R_b has turned out to be the most enlightening one.

We perform the SUGRA analysis here for similar dispassionate reasons. One expects a large class of viable supersymmetric theories to be in the neighborhood of SUGRA, especially for the subset of MSSM parameters that enter into the a_μ calculation. We also do an analysis for GMSB since that constitutes a separate, equally interesting minimal model positioned in a different large neighborhood of viable supersymmetric theories. For the reader's ability to reproduce our results, we define our models by feeding SUGRA and GMSB spectra from the ISAJET sugrun code [33] into the a_μ formulas presented above.

A. Minimal supergravity

SUGRA simplifies the derivation of the superpartner spectrum by assuming that all gauginos unify at the grand unified (GUT) scale with mass $m_{1/2}$, and all scalars unify at the GUT scale with mass m_0. Additional free parameters are $\tan\beta$ (the ratio of Higgs vacuum expectation values), A_0 (common trilinear scalar coupling at the GUT scale), and the sign of μ (the superpotential Higgs boson mixing mass parameter with sign convention of Refs. [3,4]). For a more thorough description of SUGRA and this parametrization, see [34,22].

We will illustrate the generic effects that SUGRA has on a_μ by initially restricting ourselves to the so-called dilaton dominated scenario where

$$m_{1/2}=-A_0=\sqrt{3}m_0 \quad (\text{dilaton dominated}). \tag{3.1}$$

In Fig. 5 we have plotted $\delta a_\mu^{\text{SUSY}}$ vs superpartner mass (chargino and lightest smuon) for various $\tan\beta$. The dashed lines mean $m_h<114$ GeV, in apparent conflict with LEP2 bounds on the Higgs boson [35]. If $m_h\simeq 114$ GeV turns out to be the actual Higgs boson mass, as some tantalizing data seem to suggest, then one can spot the prediction for $\delta a_\mu^{\text{SUSY}}$ by focusing on the interface between the dashed lines and the solid lines.

Going from right to left, some of the lines terminate abruptly. The reason for this is that we have required all lines, dashed or solid, to be consistent with $m_{\tilde{\tau}_1}>100$ GeV, which is our conservative cut based on anticipated limits from the final LEP2 analyses. Since the stau mass matrix in most models, including this one, is correlated closely with the smuon mass matrix, we can test unambiguously if $m_{\tilde{\tau}_1}<100$ GeV. When $\tan\beta$ is large, $m_{\tilde{\tau}_1}<m_{\tilde{\mu}_1}$ because the off-diagonal part of the mixing matrix, $-m_{\mu,\tau}\mu\tan\beta$, is larger for the $\tilde{\tau}$ than $\tilde{\mu}$, and level repulsion of mass eigenstates will

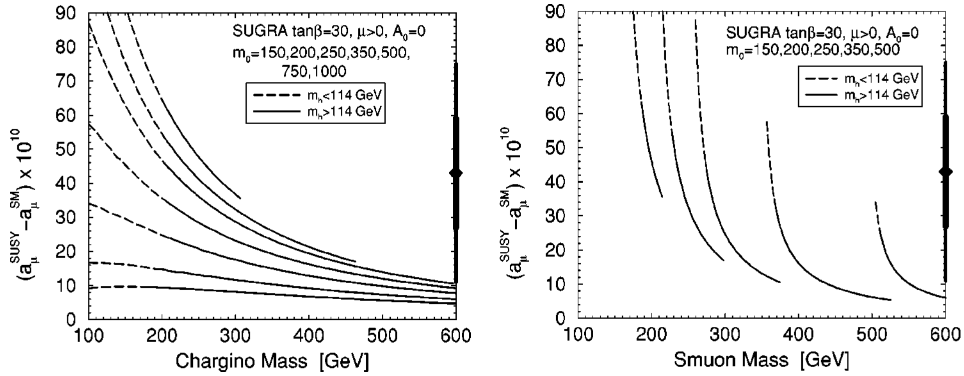

FIG. 5. Predictions for $a_\mu^{\rm SUSY}-a_\mu^{\rm SM}$ in dilaton-dominated supergravity models with various $\tan\beta = 2, 3, 6, 10, 20, 30, 40$ (from bottom to top), as a function of the lighter chargino mass and the lighter smuon mass. All charged superpartners are required to have mass above 100 GeV. The solid lines indicate where the lightest Higgs scalar boson mass m_h exceeds its approximate LEP2 bound of 114 GeV, while the dashed lines indicate where $m_h < 114$ GeV. The black bars on the right vertical axes indicate the 1-σ and 2-σ allowed regions from the Brookhaven E821 experiment.

push $m_{\tilde{\tau}_1}$ lower than $m_{\tilde{\mu}_1}$. The available smuon masses are also constrained by $m_{\chi_1^\pm} > 100$ GeV for $\tan\beta = 2,3,6$. For these reasons, Fig. 5 has some lines ending within the plots.

As expected, the higher values of $\tan\beta$ have higher $\delta a_\mu^{\rm SUSY}$ contributions, have less problem with the $m_h > 114$ GeV constraint, and have more problem with the $m_{\tilde{\tau}_1} > 100$ GeV constraint. The Higgs boson and $\tilde{\tau}_1$ mass constraints are competing effects in the drive to get high $\delta a_\mu^{\rm SUSY}$. In the end, large $\tan\beta$ still wins out and we can easily get within the 1σ allowed region by requiring $\tan\beta \gtrsim 20$, $m_{\chi_1^\pm} \lesssim 260$ GeV and $m_{\tilde{\mu}_1} \lesssim 230$ GeV; or $\tan\beta \gtrsim 30$, $m_{\chi_1^\pm} \lesssim 325$ GeV and $m_{\tilde{\mu}_1} \lesssim 280$ GeV.

Since $\delta a_\mu^{\rm SUSY}$ scales as $\tan\beta$ for large $\tan\beta$ we now suppress discussion of this known behavior by fixing $\tan\beta = 30$ and vary m_0 within the SUGRA framework. We are comfortable with this larger $\tan\beta$ choice for another reason. Namely, $t-b-\tau$ Yukawa coupling unification is most easily satisfied for larger $\tan\beta$ theories [36]. This tri-unification of Yukawa couplings is preferred in minimal version of $SO(10)$ grand unification. Figure 6 plots the prediction of $\delta a_\mu^{\rm SUSY}$ vs chargino mass and lightest slepton mass for various values of m_0. Again, the dashed lines indicate $m_h < 114$ GeV. The dashed lines terminate on the left where $m_{\chi_1^\pm} < 100$ GeV. Going from left to right, the solid lines terminate because $\tilde{\tau}_1$ becomes the LSP. There are two problems with this. First, charged LSPs are cosmologically disfavored [37]. And second, even if one assumes R-parity violation will decay away the dangerous charged relics, we would have to give up on the very attractive neutralino LSP of SUGRA. For this reason we have terminated the lines when $m_{\tilde{\tau}_1} < m_{\chi_1^0}$, although it is easy enough to visually follow where the lines would have extended in the higher chargino mass region.

From Fig. 6 we learn that for a large value of $\tan\beta$, such as the choice here of 30, large contributions are possible for $\delta a_\mu^{\rm SUSY}$, but the superpartner effects decouple rapidly. For $\tan\beta = 30$, one requires $m_{\chi_1^\pm} \lesssim 350$ GeV and $m_{\tilde{\mu}_1} \lesssim 500$ GeV to be within 1σ of the measured value. Both these masses increase to approximately 600 GeV to find oneself within 2σ of the measured value.

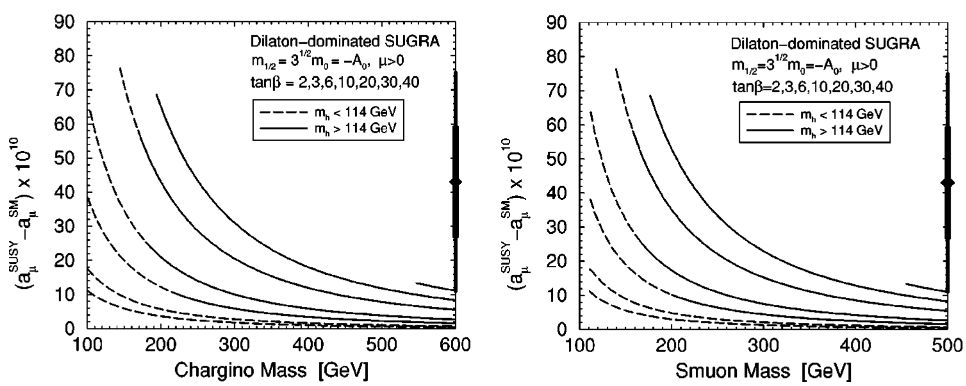

FIG. 6. Predictions for $a_\mu^{\rm SUSY}-a_\mu^{\rm SM}$ in minimal supergravity models with various $m_0 = 150, 200, 250, 350, 500, 750, 1000$ GeV, from top to bottom as a function of the lighter chargino mass and from left to right as a function of the lighter smuon mass. All charged superpartners are required to have mass above 100 GeV. The solid lines indicate where the lightest Higgs scalar boson mass m_h exceeds its approximate LEP2 bound of 114 GeV, while the dashed lines indicate where $m_h < 114$ GeV. The black bars on the right vertical axes indicate the 1-σ and 2-σ allowed regions from the Brookhaven E821 experiment.

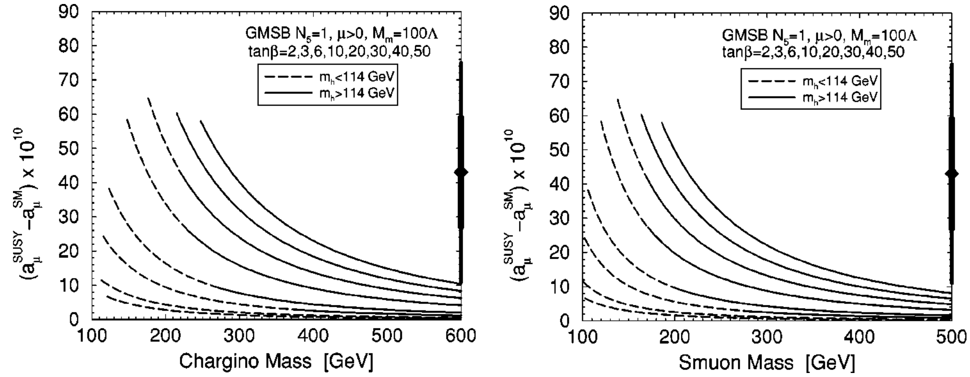

FIG. 7. Predictions for $a_\mu^{\rm SUSY} - a_\mu^{\rm SM}$ in minimal GMSB models for $N_{5+\bar{5}}=1$ and various $\tan\beta = 2$, 3, 6, 10, 20, 30, 40 and 50 (from bottom to top), as a function of the lighter chargino mass and the lighter smuon mass. All charged superpartners are required to have mass above 100 GeV. The solid lines indicate where the lightest Higgs scalar boson mass m_h exceeds its approximate LEP2 bound of 114 GeV, while the dashed lines indicate where $m_h < 114$ GeV. The black bars on the right vertical axes indicate the 1-σ and 2-σ allowed regions from the Brookhaven E821 experiment.

In short, the SUGRA model with large $\tan\beta$ generically gives large values of $\delta a_\mu^{\rm SUSY}$ for superpartners with mass at least as high as three times the current experimental limits. Therefore, SUGRA or some approximate to it would not be a surprising solution to the measured non-SM contribution of the muon anomalous magnetic moment.

B. Minimal gauge mediation

GMSB organizes the superpartner spectrum in an entirely different way, but with equal simplicity, by assuming that all superpartners get their masses by interacting through ordinary gauge bosons with messenger fields that feel supersymmetry breaking. In the minimal model the messenger fields are assumed to be equivalent to an integer number (N_5) of complete multiplets of $\mathbf{5}+\overline{\mathbf{5}}$ fields of $SU(5)$. Along with N_5, other free parameters are the supersymmetry breaking scale $\sqrt{F}$, the messenger mass scale M_m, and $\tan\beta$. For simplicity in this analysis we assume the reasonable relation $M_m = 100\Lambda$, where $\Lambda = F/M_m$ sets the scale of the MSSM sparticle masses. For a more thorough description of GMSB and this parametrization, see [23,24].

Our first illustration of the GMSB predictions will be for the most minimal model of one messenger $\mathbf{5}+\overline{\mathbf{5}}$, i.e., $N_5 = 1$. In Fig. 7 we plot $\delta a_\mu^{\rm SUSY}$ vs lightest chargino mass and lightest smuon mass for various $\tan\beta$. Again, the dashed lines represent $m_h < 114$ GeV for comparison with LEP2 searches, and the lines terminate to the left because $m_{\tilde{\tau}_1} < 100$ GeV.

We witness from Fig. 8 yet another example of how large $\tan\beta$ enhances the value of $\delta a_\mu^{\rm SUSY}$. For large but reasonable values of $\tan\beta$, $\delta a_\mu^{\rm SUSY}$ is within 1σ of the measured value. Again, masses can be several times heavier than the current limits to accomplish this, and no additional constraints such as m_h or $m_{\tilde{\tau}_1}$ limits disturb the result. An intriguing feature of this plot is the near-equal predictions of SUGRA dilaton dominated scenario and $N_5 = 1$ GMSB for fixed chargino mass. This only means that in both these minimal models the relative masses of the charginos and smuons are close for the same values of $\tan\beta$.

Variations in the spectrum occur for different values of N_5. In Fig. 8, we fix $\tan\beta = 30$ and plot $\delta a_\mu^{\rm SUSY}$ for various N_5. The higher the number of $\mathbf{5}+\overline{\mathbf{5}}$ representations the higher

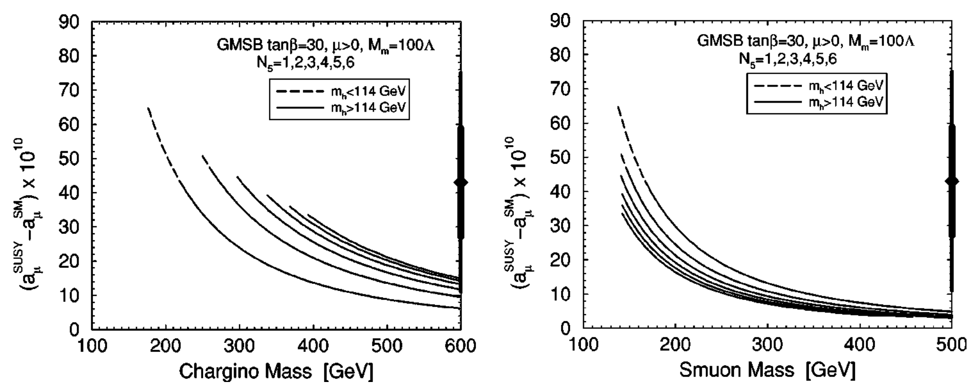

FIG. 8. Predictions for $a_\mu^{\rm SUSY} - a_\mu^{\rm SM}$ in minimal GMSB models for $\tan\beta = 30$ and various $N_5 = 1$, 2, 3, 4, 5, 6 from bottom to top as a function of the lighter chargino mass, and from top to bottom as a function of the lighter smuon mass. All charged superpartners are required to have mass above 100 GeV. The solid lines indicate where the lightest Higgs scalar boson mass m_h exceeds its approximate LEP2 bound of 114 GeV, while the dashed lines indicate where $m_h < 114$ GeV. The black bars on the right vertical axes indicate the 1-σ and 2-σ allowed regions from the Brookhaven E821 experiment.

$\delta a_\mu^{\rm SUSY}$ for a given chargino mass. This is simply because $m_{\tilde{\tau}_1}/m_{\chi_1^\pm}\propto 1/\sqrt{N_5}$. Of course, if $m_{\tilde{\tau}_1}$ dips below 100 GeV the line is not extended, which explains the curious result in Fig. 8 that the highest allowed $\delta a_\mu^{\rm SUSY}$ for a fixed $\tan\beta$ comes from *lower* N_5.

In short, the simplest GMSB models have similar predictions as SUGRA for $\delta a_\mu^{\rm SUSY}$, and can naturally produce a result within 1σ of the measured value for reasonable superpartner masses well above direct experimental mass limits. We find the results for SUGRA and GMSB encouraging for the supersymmetric interpretation of a_μ.

IV. DISCUSSION OF CORRELATING PHENOMENA

The anomalous magnetic moment of the muon is just one observable out of many that supersymmetry can affect. Using just this one quantity to divine predictions for other observables is difficult for the obvious reason that each observable requires a different set of supersymmetry masses and mixing angles. Furthermore, even within a narrowly defined version of supersymmetry, such as SUGRA, a single value of $\delta a_\mu^{\rm SUSY}$ maps to a vast parameter space within the model.

A. Superpartners at colliders

With the above caveats we make a few general comments on expected correlating phenomena. All remarks are based on one generally drawn conclusion: the large a_μ measurement at E821 likes a supersymmetric interpretation with larger $\tan\beta$ and lighter superpartners. The exact values of $\tan\beta$ and superpartner masses are model-dependent. In the previous sections, we have shown that in the most general MSSM no meaningful bounds can be placed on the chargino mass, and weak bounds can be placed on the lightest smuon mass. For example, if $\tan\beta<20$ then $m_{\tilde{\mu}_1}\lesssim 500$ GeV. However, we readily admit that one does not find *generically* in MSSM parameter space that the lightest smuon mass can be above several hundred GeV and $\delta a_\mu^{\rm SUSY}$ within 1σ. This assessment is made by analyzing minimal models and making agenda-less tours in supersymmetry parameter space. Therefore, we would *cautiously* agree [38–40] that the first statement to make about correlating phenomena is that smuons should be light. Smuons are notoriously difficult objects to discover at hadron colliders [41]. They are relatively easy to find at e^+e^- machines, but of course the center of mass energy must be sufficient to produce them.

B. Higgs boson mass

Other conclusions are a bit more subtle. For example, large $\tan\beta$ is also preferred by Higgs boson search results at LEP2. Higgs bosons greater than (or equal to) 114 GeV have put strain on low $\tan\beta$ models (see, e.g. [42,43]). This can be seen most readily by the prediction for the lightest supersymmetric scalar Higgs boson mass eigenstate in the $m_Z^2/m_A^2\ll 1$ limit:

$$m_h^2=m_Z^2\cos^2 2\beta+\frac{3g_2^2m_t^4}{8\pi^2m_W^2}\ln\frac{\Delta^2}{m_t^2}. \tag{4.1}$$

This formula is exact if one is willing to tolerate an extremely complicated form for Δ^2, otherwise it can be interpreted as approximate [44,45] with $\Delta^2\simeq m_{\tilde{t}_1}m_{\tilde{t}_2}$. The first term representing the tree-level mass prediction grows larger with $\tan\beta$. Furthermore, although not obvious from the form of Eq. (4.1), the radiative corrections to the Higgs boson mass can increase with larger $\tan\beta$ also (i.e., Δ depends mildly on $\tan\beta$). As a simple illustration, for $\Delta=1$ TeV ($=500$ GeV) we need $\tan\beta\geqslant 2.8$ ($\geqslant 12$) to ensure $m_h>114$ GeV. This tendency for higher $\tan\beta$ with lower superpartner masses to satisfy the Higgs boson mass bound is in the same direction as the requirements of $\delta a_\mu^{\rm SUSY}$.

We remark that we would not be surprised if the Higgs sector were different than the simple two Higgs doublets of the MSSM. For example, an additional singlet field with superpotential term λSH_uH_d may even be more preferred since it can lead to spontaneous generation of the $\mu=\lambda\langle S\rangle$ term, among other advantages. The lightest Higgs boson in this case would then get a contribution to its mass proportional to λ^2v^2, potentially making $\tan\beta$ limits from Higgs boson mass in the MSSM irrelevant, depending on the size of the Yukawa coupling λ.

C. Neutralino dark matter

Another potentially important correlation is in dark matter relic abundance and dark matter detection. Several authors [46–49] have noted that within some specific frameworks, e.g. SUGRA, the dark matter detection rate prediction is large when $\delta a_\mu^{\rm SUSY}$ is large. This is partly because coherent scattering of dark matter off nuclei similarly requires a chirality flip and so is enhanced by larger $\tan\beta$.

Recently, DAMA has claimed a signal in the annual modulation of weakly interacting massive particle (WIMP) nuclei scattering [50]. The supersymmetric interpretation implies a large spin-independent coherent scattering cross section, which is easier to attain at large $\tan\beta$ [51]. The DAMA signal may or may not be real, but the correlation remains: large $\tan\beta$ implied by $\delta a_\mu^{\rm SUSY}$ generally implies larger scattering cross sections for dark matter detectors. Of course, it is possible that supersymmetry has nothing to do with dark matter because *R*-parity is not conserved, or some other reason, in which case these issues become irrelevant.

As for relic abundance of the lightest neutralino, light sleptons could create a problem for the supersymmetric interpretation of dark matter since they induce an efficient *t*-channel annihilation channel in $\chi_1^0\chi_1^0\to l^-l^+$. The larger the annihilation channel the smaller the relic abundance ($\Omega\sim 1/\sigma v$). However, there is a large region of parameter space for light sleptons [52–55] (but heavier than 100 GeV) that is consistent with adequate thermal relic abundance to be cosmologically interesting, $0.1\lesssim\Omega h^2\lesssim 0.4$. Furthermore, in the regions where there is small thermal relic abundance from light sleptons or large coannihilation effects [56], there are non-thermal sources [57–59] of the LSPs that could regenerate them as dark matter. Therefore, we do not think relic abundance considerations add significantly to the dialog on $\delta a_\mu^{\rm SUSY}$ at this point.

D. $B(b\rightarrow s\gamma)$ constraint

Lastly, we remark on $B(b\rightarrow s\gamma)$. There is a close similarity between a_μ and $B(b\rightarrow s\gamma)$ in that both get large $\tan\beta$ enhancements from a Higgsino-sfermion-fermion interaction vertex with a down-fermion Yukawa coupling. If the E821 experiment had measured $\delta a_\mu^{\rm SUSY}\simeq -43\times 10^{-10}$ instead of $+43\times 10^{-10}$, the measurement of $B(b\rightarrow s\gamma)$ would have disfavored many supersymmetric interpretations. However, it happens that $\delta a_\mu^{\rm SUSY}$ prefers $\mu>0$ (for real positive gaugino masses) and large $\tan\beta$, and $B(b\rightarrow s\gamma)$ severely restricts $\mu<0$ and large $\tan\beta$, but does not significantly restrict $\mu>0$.

It is well known that $M_3\mu>0$ is not as restricted by B($b\rightarrow s\gamma$) [60,61,55] as $M_3\mu<0$, since the signs of the amplitudes in this circumstance imply partial cancellations. Recently, this conclusion was strengthened even more by the evaluation of higher-order calculations to $B(b\rightarrow s\gamma)$. At higher order one must self-consistently take into account the finite b-quark mass corrections which are enhanced dramatically at large $\tan\beta$. These corrections imply smaller b-quark Yukawa coupling and therefore smaller magnitude for the Higgsino-squark-quark chirality flip. From Fig. 2 of [62] one can see the reduction in the supersymmetric prediction for large $\tan\beta$ with $M_3\mu>0$, rendering $B(b\rightarrow s\gamma)$ unable to significantly constrain large *positive* $\delta a_\mu^{\rm SUSY}$ scenarios.

The above discussion is mostly based on SUGRA-like relations among superpartner masses. A similar conclusion can be inferred from Ref. [63], wherein $B(b\rightarrow s\gamma)$ has little impact on the viability of the CMSSM to explain $\delta a_\mu^{\rm SUSY}$. The same discussion holds for GMSB since the squarks are even heavier in that model, and $B(b\rightarrow s\gamma)$ was never much of a serious constraint [64–66] when all the uncertainties are accounted for. Other theories of supersymmetry breaking such as anomaly mediated supersymmetry breaking (AMSB) [67,68] appear to have difficulty accomodating $\delta a_\mu^{\rm SUSY}$ [39,69]. This difficulty arises because $M_3<0$ and the lightest gauginos are W-inos with small positive M_2, leading to a severe constraint on $\mu>0$ parameter space from $B(b\rightarrow s\gamma)$. The $SU(5)$ model with a supersymmetry breaking F-term in the **24** representation discussed at the end of Sec. I B also has $M_3<0$. However, in this case the ratios $|M_2/M_1|\simeq 6$ and $|M_3/M_1|\simeq 12$ imply that the lightest neutralino and smuon entering the $\delta a_\mu^{\rm SUSY}$ loop corrections would be significantly lighter than the squarks and charginos that affect $B(b\rightarrow s\gamma)$. Even though these mass hierarchies make $B(b\rightarrow s\gamma)$ less important of a constraint than it is in AMSB, careful evaluation of the next-to-leading order $B(b\rightarrow s\gamma)$ prediction would need to be compared with experiment to ultimately judge the viability of this model to explain $\delta a_\mu^{\rm SUSY}$.

One also must approach the $B(b\rightarrow s\gamma)$ observable with a bit of caution when trying to rule out parameter space consistent with $\delta a_\mu^{\rm SUSY}$. Most analyses implicitly assume that the theory prediction is precise, and it need only fit into the range obtained from experimental measurement, often quoted to be between $(2-4)\times 10^{-4}$. The SM theory prediction [72] is

$$(3.29\pm 0.33)\times 10^{-4}\quad (\text{SM theory}). \tag{4.2}$$

This 10% error, whether one interprets it as a 1σ error or 95% C.L., clearly implies that there should be comparable error in the theory prediction of any theory evaluated at the same NLO rigor. Supersymmetry, it should be noted, has not been calculated fully to NLO. Therefore it is safe to presume that the supersymmetry prediction will be at least 10% uncertain, and it must be taken into account in any careful analysis.

Equally important as the accuracy of the theory prediction is the fact that the experimental measurement of $B(b\rightarrow s\gamma)$ is not a pure observable in that a severe cut on the photon energy is needed to reduce charm backgrounds in the analysis at CLEO. This introduces theoretical uncertainties [71,72] in addition to the obvious ones, such as imprecise knowledge of the b-quark mass, α_s and the not-completely-known contributions scaling as $m_c^2/\Lambda_{\rm QCD}^2$. Therefore, the CLEO measurement is expressed as [70]

$$(3.15\pm 0.35\pm 0.32\pm 0.26)\times 10^{-4}\quad (\text{CLEO}) \tag{4.3}$$

where the errors are statistical, systematic, and model dependence, respectively. These kinds of varied errors should give pause when advocating a hard cut on $B(b\rightarrow s\gamma)$ in supersymmetry, and one should be wary about deleting any part of parameter space based on an apparent incompatibility with the $B(b\rightarrow s\gamma)$ constraint.

In our more general MSSM discussion of Sec. II, the $B(b\rightarrow s\gamma)$ constraint does not even need to be discussed since no values of the squarks masses enter. For this more general model, we can simply claim that the squark masses are sufficiently massive as to contribute little to $B(b\rightarrow s\gamma)$. From the discussions above, we conclude that $B(b\rightarrow s\gamma)$, as with all other observables, usually adds no significant burden in a quest to find a supersymmetric explanation for $\delta a_\mu^{\rm SUSY}$.

ACKNOWLEDGMENTS

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