## Polarization in hadronic $\Lambda$ hyperon production and chiral-odd twist-3 distribution

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Polarization of the  $\Lambda$  hyperon produced with a large transverse momentum in unpolarized nucleon-nucleon collisions is analyzed in the framework of QCD factorization. We focus on the mechanism in which the soft-gluon component of the chiral-odd spin-independent twist-3 quark distribution  $E_F(x,x)$  becomes a source of the polarized quark fragmenting into the polarized  $\Lambda$ . Our simple model estimate for this contribution indicates that it gives rise to a significant  $\Lambda$  polarization at large  $x_F$ . This is in parallel with the observation that the soft gluon pole mechanism gives rise to a large single transverse spin asymmetry in pion production at  $x_F \rightarrow 1$ .

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It is a well known experimental fact that the hyperons produced in unpolarized nucleon-nucleon collisions are polarized transversely to the production plane [1-3]. In this paper we focus on the polarization for  $\Lambda$  hyperon production with a large transverse momentum  $(l_T)$  in *pp* collisions:

$$N(P) + N'(P') \to \Lambda(l, \vec{S}_{\perp}) + X. \tag{1}$$

The ongoing experiment at the BNL Relativistic Heavy Ion Collider (RHIC) is expected to provide more data and shed light on the mechanism for polarization. As in the case of single transverse-spin asymmetries in direct photon or pion production,  $p^{\uparrow}+p \rightarrow \{\gamma,\pi\}+X$ , nonzero  $\Lambda$  polarization in the above process (1) indicates the effect of particular quarkgluon (higher twist) correlations and/or the transverse momentum of partons either in the unpolarized nucleon or in the fragmentation function for  $\Lambda$  [4–12]. According to the generalized QCD factorization theorem [13], the polarized cross section for this process consists of two types of twist-3 contributions:

(A) 
$$E_a(x_1, x_2) \otimes q_b(x') \otimes \delta D_{c \to \Lambda}(z) \otimes \hat{\sigma}_{ab \to c}$$
, (2)

(B) 
$$q_a(x) \otimes q_b(x') \otimes D^{(3)}_{c \to \Lambda}(z_1, z_2) \otimes \hat{\sigma}'_{ab \to c}.$$
 (3)

Here the functions  $E_a(x_1,x_2)$  and  $D_{c\to\Lambda}^{(3)}(z_1,z_2)$  are the twist-3 quantities representing, respectively, the unpolarized distribution and the fragmentation function for the transversely polarized  $\Lambda$  hyperon, and a, b, and c stand for the parton's species. Other functions are twist-2:  $q_b(x)$  the unpolarized distribution (quark or gluon) and  $\delta D_{c\to\Lambda}(z)$  the transversity fragmentation function for the  $\Lambda$ . The symbol  $\otimes$  denotes convolution.  $\hat{\sigma}_{ab\to c}$  and  $\hat{\sigma}'_{ab\to c}$  represent the partonic cross section for the process  $a+b\to c+anything$  that yields large transverse momentum of the parton c. Note that (A) contains two chiral-odd functions  $E_a$  and  $\delta D_{c\to\Lambda}$ , while (B) contains only chiral-even functions.

In this paper, we derive a QCD formula for the polarized cross section (1) from the (A) term which becomes dominant in the kinematic region  $x_F \rightarrow 1$ , using the valence quark-soft gluon approximation proposed by Qiu and Sterman [9]. Employing this approximation, they reproduced the E704 data for the single transverse-spin asymmetries in the pion pro-

duction at  $x_F \rightarrow 1$  reasonably well. The fact that the perturbative QCD description for the pion production is valid at the pion transverse momenta as low as a few GeV encouraged us to apply the method to the polarized  $\Lambda$  hyperon production (1) for which the data exist only in the same low  $l_T$  region. At large  $x_F > 0$ , where the large x and small x' region of the parton distributions is mainly probed, the cross section is dominated by the particular terms in (A) that contain the derivatives of the valence twist-3 distribution  $[\partial/\partial x)E_{Fa}(x,x)] \gg E_{Fa}(x,x)$  owing to the behavior of  $E_{Fa}(x,x) \sim (1-x)^{\beta}$  ( $\beta > 0$ ) at  $x \rightarrow 1$ . We thus keep only the terms with the derivative of  $E_{Fa}(x,x)$  for the valence quark (valence quark-soft gluon approximation).

The polarized cross section for Eq. (1) is a function of three independent variables,  $S = (P+P')^2 \simeq (2P)P'$ ,  $x_F = 2l_{\parallel}/\sqrt{S} [=(T-U)/S]$ , and  $x_T = 2l_T/\sqrt{S}$ .  $T = (P-l)^2 \simeq -(2P)l$  and  $U = (P'-l)^2 \simeq -2P'l$  are given in terms of these three variables by  $T = -S[\sqrt{x_F^2 + x_T^2} - x_F]/2$  and  $U = -S[\sqrt{x_F^2 + x_T^2} + x_F]/2$ . In this convention, production of  $\Lambda$  in the forward hemisphere in the direction of the incident nucleon [N(P)] corresponds to  $x_F > 0$ . Since  $-1 < x_F < 1$ ,  $0 < x_T < 1$ , and  $\sqrt{x_F^2 + x_T^2} < 1$ ,  $x_F \rightarrow 1$  corresponds to the region with  $-U \sim S$  and  $T \sim 0$ .

The cross section for Eq. (1) from the (A) term can be derived by the method described in Eqs. (8),(9),(11). In the valence quark-soft gluon approximation, it is obtained as

$$E_{l} \frac{d^{3} \Delta \sigma^{A}(S_{\perp})}{dl^{3}}$$

$$= \frac{\pi M \alpha_{s}^{2}}{S} \sum_{a,c} \int_{z_{min}}^{1} \frac{dz}{z^{3}} \delta D_{c \to \Lambda}(z) \int_{x_{min}}^{1} \frac{dx}{x} \frac{1}{xS + U/z}$$

$$\times \int_{0}^{1} \frac{dx'}{x'} \delta \left( x' + \frac{xT/z}{xS + U/z} \right) \varepsilon_{lS_{\perp}pn} \left( \frac{1}{-\hat{u}} \right)$$

$$\times \left[ -x \frac{\partial}{\partial x} E_{Fa}(x,x) \right] \left[ G(x') \delta \hat{\sigma}_{ag \to c} \right]$$

$$+ \sum_{b} q_{b}(x') \delta \hat{\sigma}_{ab \to c} \left] \delta_{ac}, \qquad (4)$$

where *p* and *n* are the two lightlike vectors defined from the momentum of the unpolarized nucleon (mass *M*) as  $P = p + M^2 n/2$ , pn = 1, and  $\varepsilon_{lS_\perp pn} = \varepsilon_{\mu\nu\lambda\sigma} l^{\mu}S_{\perp}^{\nu}p^{\lambda}n^{\sigma} \sim \sin\phi$  with  $\phi$  the azimuthal angle between the spin vector of the  $\Lambda$  hyperon and the production plane. The invariants in the parton level are defined as

$$\hat{s} = (p_a + p_b)^2 \simeq (xP + x'P')^2 \simeq xx'S,$$
  

$$\hat{t} = (p_a - p_c)^2 \simeq (xP - l/z)^2 \simeq xT/z,$$
  

$$\hat{u} = (p_b - p_c)^2 \simeq (x'P' - l/z)^2 \simeq x'U/z.$$
(5)

The lower limits for the integration variables are

$$z_{min} = \frac{-(T+U)}{S} = \sqrt{x_F^2 + x_T^2}, \quad x_{min} = \frac{-U/z}{S + T/z}.$$
 (6)

 $q_b(x')$  and G(x') are the unpolarized quark and gluon distributions, respectively.  $\delta \hat{\sigma}_{ag \to c}$  and  $\delta \hat{\sigma}_{ab \to c}$  are the partonic cross sections for the quark-gluon and quark-quark subprocesses, respectively.  $E_F(x,x)$  is the soft gluon component of the unpolarized twist-3 distribution defined as

$$E_{Fa}(x,y) = \frac{-i}{2M} \int \frac{d\lambda}{2\pi} e^{i\lambda x} \langle P | \bar{\psi}^a(0) h \gamma_{\perp \sigma} \left\{ \int \frac{d\mu}{2\pi} e^{i\mu(y-x)} \right. \\ \left. \times g F^{\sigma\beta}(\mu n) n_\beta \right\} \psi^a(\lambda n) | P \rangle.$$
(7)

 $\delta D_{c \to \Lambda}(z)$  is the twist-2 transversity fragmentation function for  $\Lambda$  defined by

$$\delta D_{c \to \Lambda}(z) = \sum_{X} \frac{z}{4} \int \frac{d\lambda}{2\pi} e^{-i\lambda/z} \langle 0 | \gamma_5 \$_{\perp} \hbar_{\Lambda} \psi^c(0) | \Lambda(lS_{\perp}) X \rangle$$
$$\times \langle \Lambda(lS_{\perp}) X | \overline{\psi}^c(\lambda n_{\Lambda}) | 0 \rangle, \qquad (8)$$

where the lightlike vector  $n_{\Lambda}$  is an analog of *n* for *l* ( $ln_{\Lambda}$ = 1). The summation for the flavor indices of  $E_{Fa}(x,x)$  is to be over *u*- and *d*-valence quarks, while that for the twist-2 distributions is over  $u, d, \overline{u}, \overline{d}, s, \overline{s}$ . The missing contributions in Eq. (4) are the soft gluon pole contribution proportional to  $E_{Fa}(x,x)$  itself (without derivative) and the soft Fermion pole contribution proportional to  $E_D(x,0)$  which does not appear with the derivative.  $E_D(x,y)$  is obtained by the replacement of the gluon field strength  $gF^{\mu\nu}n_{\nu}$  by the covariant derivative  $D^{\mu}$  in  $E_F(x,y)$  (see [11]). As stated above, at large  $x_F$  (i.e., large x)  $(d/dx)E_F(x,x)$  receives an enhancement, and Eq. (4) becomes the most dominant contribution. At large  $x_F$ , z is also large and we anticipate that the term proportional to  $(d/dz)D_{c\to\Lambda}^{(3)}(z,z)$  in the (B) contribution (3) also brings another large contribution. We left the analysis of this term for future study.

After the soft gluon poles are properly handled,  $\delta \hat{\sigma}_{ab \to c}$ and  $\delta \hat{\sigma}_{ag \to c}$  in Eq. (4) can be obtained from the  $2 \to 2$  cut diagrams shown in Figs. 1 and 2, respectively [8,9,11]. In these figures, the quark lines labeled as *a* come from



FIG. 1. Quark-quark  $2 \rightarrow 2$  scattering diagrams contributing to the hard cross section.

 $E_{Fa}(x,x)$  and the quark line labeled as *c* fragments into  $\Lambda$ . Because of the chiral-odd nature of  $E_{Fa}$  and  $\delta D_{c \to \Lambda}$  they have to appear in a pair along a fermion line, and hence only four diagrams in Fig. 1 contribute to  $\delta \hat{\sigma}_{ab \to c}$ . The result for the hard cross section reads

$$\delta\hat{\sigma}_{qq' \to q} = \left(\frac{\hat{s}\hat{u}}{\hat{t}^2}\right) \left[\frac{2}{9} + \frac{1}{9}\left(1 + \frac{\hat{u}}{\hat{t}}\right)\right]$$

[Fig. 1(a)],

$$\delta \hat{\sigma}_{q\bar{q}' \to q} = \left(\frac{\hat{s}\hat{u}}{\hat{t}^2}\right) \left[\frac{7}{9} + \frac{1}{9}\left(1 + \frac{\hat{u}}{\hat{t}}\right)\right]$$

[Fig. 1(b)],

$$\delta\hat{\sigma}_{qq \to q} = -\left(\frac{\hat{s}}{\hat{t}}\right) \left[\frac{10}{27} + \frac{1}{27}\left(1 + \frac{\hat{u}}{\hat{t}}\right)\right] \tag{9}$$

[Figs. 1(c) and 1(d)] for  $\delta \hat{\sigma}_{ab \to c}$ , and

$$\delta\hat{\sigma}_{qg \to q} = \frac{9}{8} \left( \frac{\hat{s}\hat{u}}{\hat{t}^2} \right) + \frac{9}{8} \left( \frac{\hat{u}}{\hat{t}} \right) + \frac{1}{8} + \left[ \frac{1}{4} \left( \frac{\hat{s}\hat{u}}{\hat{t}^2} \right) + \frac{1}{72} \right] \left( 1 + \frac{\hat{u}}{\hat{t}} \right), \tag{10}$$

for  $\delta \hat{\sigma}_{ag \to c}$ . We note that in the large  $x_F$  region  $\hat{t}$  becomes small compared to  $\hat{s}$  and  $\hat{u}$ . Therefore  $\delta \hat{\sigma}_{ab \to c}$  [from Figs. 1(a) and 1(b)] and  $\delta \hat{\sigma}_{ag \to c}$  (from Fig. 2) contribute to a large polarization as was the case for the single transverse-spin asymmetry for the pion production [9]. This is in strong contrast to the chiral-odd contribution for the single transversespin asymmetry for the pion production studied in [11]. In the contribution identified in [11], two chiral-odd distributions come from the initial nucleons and have to form a closed quark loop in a pair. Therefore there is no contribu-



FIG. 2. Quark-gluon  $2 \rightarrow 2$  scattering diagrams contributing to the hard cross section.

tion from Figs. 1(a) and 1(b) and Fig. 2, leading to the negligible asymmetry. (See Ref. [11](b).)

We now present a simple estimate of the  $\Lambda$  polarization  $P_{\Lambda}$ . To this end we employ a model for  $E_F(x,x)$  introduced in Ref. [11]. It is based on the comparison of the explicit form (7) with the transversity distribution

$$\delta q_a(x) = \frac{i}{2} \varepsilon_{S_\perp \sigma p n} \int \frac{d\lambda}{2\pi} e^{i\lambda x} \langle PS | \bar{\psi}^a(0) \not n \gamma_\perp^\sigma \psi^a(\lambda n) | PS \rangle,$$
(11)

where  $\varepsilon_{S_{\perp}\sigma pn} \equiv \varepsilon_{\mu\sigma\nu\lambda} S_{\perp}^{\mu} p^{\nu} n^{\lambda}$ . We make an ansatz

$$E_{Fa}(x,x) = K_a \delta q_a(x) \tag{12}$$

with a flavor-dependent parameter  $K_a$  which simulates the effect of the gluon field with zero momentum in  $E_F(x,x)$ . We note that even though  $E_F(x,x)$  is an unpolarized distribution, the quarks in  $E_F(x,x)$  are "transversely polarized" which eventually fragments into the transversely polarized  $\Lambda$ . The relation (12) is in parallel with the ansatz originally introduced in [9],

$$G_{Fa}(x,x) = K'_a q_a(x), \tag{13}$$

which is also motivated by the explicit forms for  $G_{Fa}(x,x)$ and  $q_a(x)$ :

$$G_{Fa}(x,x) = \frac{1}{M} \varepsilon_{S_{\perp} \sigma p n} \int \frac{d\lambda}{2\pi} e^{i\lambda x} \langle P | \bar{\psi}^{a}(0) \hbar \\ \times \left\{ \int \frac{d\mu}{2\pi} g F^{\sigma\beta}(\mu n) n_{\beta} \right\} \psi^{a}(\lambda n) | P \rangle, \quad (14)$$

$$q_a(x) = \frac{1}{2} \int \frac{d\lambda}{2\pi} e^{i\lambda x} \langle PS | \bar{\psi}^a(0) \hbar \psi^a(\lambda n) | PS \rangle.$$
(15)

From these relations, we expect that the signs of  $K_a$  and  $K'_a$  are the same and their magnitude is similar. Using the unpolarized parton distribution in [14] (scale  $\mu = 1.5$  GeV) and the fragmentation function for the pion in [15] (scale  $\mu = 2$  GeV),  $K'_{u,d}$  was determined to be  $K'_u = -K'_d = 0.06$  so that it approximately reproduces the Fermilab E704 data of the single transverse-spin asymmetry  $A_N$  in the pion production [16] at large  $x_F$ .

In calculating the  $\Lambda$  polarization, we use each distribution and fragmentation function at the scale 1.1 GeV which is equal to the average transverse momentum  $l_T$  of the produced  $\Lambda$  in CERN R608 data [2]. For the unpolarized distribution  $q_a(x)$  and G(x), we use the Glück-Reya-Vogt (GRV) leading-order (LO) distribution [14] (the same distribution to determine  $K'_{u,d}$  from  $A_N$ ). For the transversity distribution  $\delta q_a(x)$ , we use the Glück-Reya-Strectmann-Vogelgang (GRSV) helicity distribution  $\Delta q_a(x)$  (LO, standard scenario) [17] assuming  $\delta q_a(x) = \Delta q_a(x)$  at the scale  $\mu = 1.1$  GeV. Fragmentation functions of  $\Lambda$  are taken from Ref. [18]. For the transversity fragmentation function  $\delta D_{c \to \Lambda}(z)$  we use longitudinally polarized fragmentation  $\Delta D_{c \to \Lambda}(z)$  (three different scenarios) assuming  $\delta D_{c \to \Lambda}(z) = \Delta D_{c \to \Lambda}(z)$  at the



FIG. 3. The calculated  $\Lambda$  polarization  $P_{\Lambda}$  with three scenarios for  $\delta D_{c \to \Lambda}$  at  $\sqrt{S} = 62$  GeV and the transverse momentum of  $\Lambda$  at  $l_T = 1.1$  GeV together with the R608 data [2]. For the scenario 2, the result with  $K_{u,d} = K'_{u,d}$  and  $K_u = -K_d = 0.24$  are shown by dashed and solid lines, respectively. For the scenarios 1 and 3, only the result with  $K_{u,d} = K'_{u,d}$  is shown by dash-dotted and dashdouble-dotted lines, respectively.

scale  $\mu = 1.1$  GeV. We note that the  $\Lambda$  fragmentation functions given in [18] are for  $\Lambda + \overline{\Lambda}$ . But in the kinematic region of our interest (large  $x_F \sim$  large z),  $D_{u,d \to \Lambda + \overline{\Lambda}}(z)$  can be regarded as the one for  $\Lambda$  itself, so that we use  $D^{\Lambda}$  in [18] for the  $\Lambda$  fragmentation function. With these preparations, our first choice for  $K_{u,d}$  is to set  $K_{u,d} = K'_{u,d}$  as noted above. We also show the result for  $K_u = -K_d = 0.24$  which is determined to approximately reproduce the R608 data on  $P_{\Lambda}$  (see below).

The calculated  $\Lambda$  polarization  $P_{\Lambda}$  obtained with  $K_{u,d}$  $=K'_{u,d}$  is shown in Fig. 3 for the three scenarios of  $\delta D_{c \to \Lambda}(z)$  in [18] together with the CERN R608 data. Scenario 1 corresponds to the expectation from the naive nonrelativistic quark model, where only strange quarks can fragment into a polarized  $\Lambda$ . In our approximation,  $E_F(x,x) = 0$ for the s-quark and thus the polarization is zero in this scenario. Scenario 2 is based on the assumption that the flavor dependence of  $\Delta D_{c \to \Lambda}(z)$  is the same as that of the polarized quark distribution in  $\Lambda$  obtained by the SU(3) symmetry from  $g_1^p$  data,  $\delta D_{u \to \Lambda} = \delta D_{d \to \Lambda} = -0.2 \delta D_{s \to \Lambda}$ . In scenario 3, three flavors of quarks equally fragment into the polarized  $\Lambda$ ,  $\delta D_{u \to \Lambda} = \delta D_{d \to \Lambda} = \delta D_{s \to \Lambda}$ . From Fig. 3 one sees that scenarios 2 and 3 give rise to increasing polarization at large  $x_F$  as expected, the former giving the same sign of  $P_{\Lambda}$  as the data. To get negative  $P_{\Lambda}$ , negative  $\delta D_{u,d\to\Lambda}(z)$  is necessary if the (A) term turns out to be responsible for the hyperon polarization. To get an approximate fit to the data with scenario 2 for  $\delta D(z)$ , one needs  $K_u = -K_d = 0.24$ . This curve is also shown by the solid line in Fig. 3. At intermediate  $x_F$ , we expect a moderate contribution from the nonderivative term of the soft-gluon pole as well as the soft-fermion pole contributions, which may bring better agreement with the data.

The curves in Fig. 3 are obtained by the ansatz for the transversity distribution and fragmentation functions,  $\delta q(x) = \Delta q(x)$ ,  $\delta D(x) = \Delta D(x)$ . If we had used the saturation assumption in the inequality [19],  $2|\delta q(x)| \leq q(x) + \Delta q(x)$ ,



FIG. 4. The  $\Lambda$  polarization  $P_{\Lambda}$  at  $\sqrt{S} = 20$ , 40, 200 GeV and  $l_T = 1.1$  GeV. Scenario 2 is used for  $\delta D_{c \to \Lambda}$  and  $K_u = -K_d = 0.24$ .

 $2|\delta D(x)| \leq D(x) + \Delta D(x)$ , we would have gotten smaller  $K_{u,d}$  than estimated above.

Since we kept only the valence distribution for  $E_F(x,x)$ , polarization of  $\overline{\Lambda}$  becomes identically zero in all three scenarios, which is consistent with the experimental data. (For other antihyperons such as  $\overline{\Sigma}$  and  $\overline{\Xi}$ , however, nonzero polarization has been experimentally observed [3], so that the nonvalence component of the distribution function is certainly necessary for the complete description of the hyperon polarization.)

In Fig. 4 we plotted the  $\Lambda$  polarization for  $\sqrt{S} = 20$ , 40, 200 GeV at  $l_T = 1.1$  GeV with the scenario 2 for  $\delta D_{c \to \Lambda}$  and  $K_u = -K_d = 0.24$ . One sees that the result is almost independent of  $\sqrt{S}$ . This tendency is the same as the experimental data.

The hyperon polarization shown above shares common features with the single transverse spin asymmetry  $A_N$  in the pion production,  $p^{\uparrow}+p \rightarrow \pi(l_T)+X$ , studied in [9]. With our model assumption (12) for  $E_F(x,x)$ , the approximate formula for the  $\Lambda$  polarization at large  $x_F$  can be written as

$$P_{\Lambda} \sim \frac{K\pi M l_T}{(-U)} \left[ 1 + O\left(\frac{U}{T}\right) \right] \frac{1}{1 - x_F} \frac{\delta q(x) \otimes \delta D(z)}{q(x) \otimes D(z)}.$$
(16)

Here the factor  $l_T/(-U)$  comes from  $\varepsilon_{lS_+pn}/(-\hat{u})$  in Eq. (4), the O(U/T) term is from those with  $\hat{u}/\hat{t}$  in Eqs. (9) and (10) with the coefficients smaller than for the other terms and  $1/(1-x_F)$  dependence comes from the derivative of  $E_F(x,x)$ . The last factor represents the ratio of parton distribution and fragmentation functions between polarized and unpolarized production cross section, which is absent in the analogous formula for  $A_N$  [9].  $P_{\Lambda}$  obtained with the assumption  $K_{u,d} = K'_{u,d}$  in the scenario 2 is smaller than  $A_N$ , which is mainly due to the small ratio  $\delta D(z)/D(z)$ . At  $x_F \gg x_T$ ,  $U \rightarrow -x_F S$  and  $T \rightarrow -l_T^2/x_F$ . Accordingly,  $P_\Lambda$  has typically two contributions proportional to  $l_T/S$  and  $1/l_T$  corresponding to  $O(l_T/(-U))$  and  $O(l_T/(-T))$  terms in Eq. (16), respectively. Figure 5 shows the  $l_T$  dependence of the  $\Lambda$  polarization at  $x_F = 0.7$  for  $\sqrt{S} = 20$ , 40, 200 GeV. The polarization changes rapidly at  $l_T < 2$  GeV, which indicates the second term in Eq. (16) becomes dominant in this region. Experimentally,  $P_{\Lambda}$  tends to decrease as  $l_T$  increases in the



FIG. 5.  $l_T$ -dependence of  $P_{\Lambda}$  at  $\sqrt{S} = 20$ , 40, 200 GeV and  $x_F = 0.7$ . Scenario 2 is used for  $\delta D_{c \to \Lambda}$  and  $K_u = -K_d = 0.24$ .

region  $l_T \sim 1-3$  GeV at  $x_F > 0.6$ , while it shows flat  $l_T$  dependence in the smaller  $x_F$  region. At  $\sqrt{S} = 20$  GeV,  $P_{\Lambda}$  increases slightly as  $l_T$  increases toward the edge of the phase space ( $\sim 7$  GeV), as was seen in  $A_N$  [9].

We remark that the effect of the  $\Lambda$  mass  $M_{\Lambda}$  is completely ignored in our formula for  $P_{\Lambda}$ . The rapid increase of  $P_{\Lambda}$  at small  $l_T$  is due to our unjustified neglect of  $M_{\Lambda}$  compared with  $l_T$ . In particular,  $P_{\Lambda}$  should be zero at  $l_T=0$ . The comparison of our formula with experimental data should be more ideally done at larger  $l_T$  ( $l_T \gg M_{\Lambda}$ ) as well as large  $x_F$ in future collider experiments.

The complete formula for the hyperon polarization receives the (B) contribution in Eq. (3). Analysis of this term involves some complications due to the presence of two kinds of hadron matrix elements for the twist-3 fragmentation function. We hope to present the analysis of this term in a separate publication. At this stage we can only say that the (A) contribution in Eq. (2) brings a significant polarization of  $\Lambda$  at large  $x_F$  if the soft-gluon pole mechanism analyzed in [9] is a major source for the single transverse-spin asymmetry in the pion production.

A different approach to the hyperon polarization introduces the so-called T-odd distribution or fragmentation functions with the intrinsic transverse momentum [5,6,12,20] instead of twist-3 distributions introduced here. Similarly to (A) and (B), this approach starts from the factorization assumption for the two types of contributions to the polarization: (i)  $h_1^{\perp}(x, \mathbf{p}_{\perp}) \otimes q(x') \otimes \delta D(z) \otimes \hat{\sigma}$ , (ii)  $q(x) \otimes q(x')$  $\otimes D_{1T}^{\perp}(z, \mathbf{k}_{\perp}) \otimes \hat{\sigma}'$ , where  $h_1^{\perp}$  represents distribution of a transversely polarized quark with nonzero tranverse momentum inside the unpolarized nucleon, and  $D_{1T}^{\perp}$  represents a fragmentation function for an unpolarized quark fragmenting into a transversely polarized  $\Lambda$  with the transverse momentum ("polarizing fragmentation function"). Anselmino et al. fitted the experimental data for the  $\Lambda$  polarization assuming the above (ii) is the sole origin of the polarization [12]. Experimentally, however, no significant transverse polarization of  $\Lambda$  has been observed in  $e^+ + e^- \rightarrow \Lambda^{\uparrow} + X$  [21], in which origin of the  $\Lambda$  polarization resides only in the fragmentation process. These data suggest that the whole  $\Lambda$  polarization may not be ascribed to the  $D_{1T}^{\perp}$  effect in the hadronic production.<sup>1</sup> We also expect from the present study that the significant portion of the  $\Lambda$  polarization should be ascribed to the twist-3 distribution in the unpolarized nucleon and  $\delta D(z)$  which should be related to the above contribution (i). It is also interesting to explore the connection between the present approach and that in [12].

To summarize, we have derived a cross section formula for the polarized  $\Lambda$  production in the unpolarized nucleon-

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<sup>&</sup>lt;sup>1</sup>A recent paper [22] ascribed the smallness of the  $\Lambda$  polarization in [21] to the Sudakov suppression.