Nonfactorizable contributions to the decay mode $D^0 \to K^0 \overline{K^0}$

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We point out that the decay mode $D^0 \to K^0 \overline{K^0}$ has no factorizable contribution. In chiral perturbation language, treating D^0 as heavy, the $\mathcal{O}(p)$ contribution is zero. We calculate the nonfactorizable chiral loop contributions of order $\mathcal{O}(p^3)$. Then, we use a heavy-light type chiral quark model to calculate nonfactorizable tree level terms, also of order $\mathcal{O}(p^3)$, proportional to the gluon condensate. *A priori*, chiral loops are not expected to give good precision because the energy release in this decay is almost 800 MeV. Still, we find that both the chiral loops and the gluon condensate contributions are of the same order of magnitude as the experimental amplitude.

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I. INTRODUCTION

The decay mechanism of the weak nonleptonic D^0 decays has motivated numerous studies [1–10]. For nonleptonic decays of D mesons, as well as for K's and B's, the so-called *factorization* hypothesis has been commonly used. For nonleptonic decays, the effective Lagrangian at the quark level has the form

$$\mathcal{L}_W = \sum_i C_i Q_i, \tag{1}$$

where the coefficients C_i contain all the short distance electroweak and QCD effects to a certain order in perturbation theory, and the Q_i's are quark operators. Typically, these quark operators are products of (pseudo)scalar or vector currents $Q = j(1)j(2)$. Then, for a nonleptonic decay $M \to M_1 + M_2$, the factorization hypothesis (also called vacuum saturation approximation) gives prescriptions of the form

$$\langle M_1 M_2 | Q | M \rangle \to \langle M_1 | j(1) | 0 \rangle \langle M_2 | j(2) | M \rangle. \tag{2}$$

The factorization hypothesis are known to fail badly for nonleptonic K decays [11–13]. On the other hand, there are certain heavy hadron weak decays where factorization might apply. Recently, the understanding of factorization for exclusive nonleptonic decays of B mesons in terms of QCD in the heavy quark limit has been considerably improved [14]. In this paper we will discuss nonfactorizable terms for D decays, in particular for the decay mode $D^0 \to K^0 \overline{K^0}$. Additional motivation to consider this decay mode comes from the very recent experimental result [15] for the CP violating asymmetry in $D^0 \to K_S K_S$.

Even though the factorization hypothesis might work reasonably well if one is interested in an order of magnitude estimate, it does not reproduce experimental data completely. For example, a naive application of factorization in charm decays leads to rates for the $D^0 \to \pi^0 \overline{K^0}$, $D^0 \to \pi^0 \pi^0$, $D^0 \to K^+ K^-$, $D^0 \to \pi^+ \pi^-$ decays which are too strongly suppressed. Moreover, and this is the important point of this paper, in $D^0 \to K^0 \overline{K^0}$, the factorization misses completely, predicting a vanishing branching ratio, in contrast with the experimental situation.

To see this, note that at tree level the $D^0 \to K^0 \overline{K^0}$ decay might occur due to two annihilation diagrams [1] which could potentially create the $K^0 \overline{K^0}$ state. However, they cancel each other by the Glashow-Iliopoulos-Maiani (GIM) mechanism. Moreover, in the factorization limit, the amplitude is proportional to

$$\langle K^0 \overline{K^0} | V_\mu | 0 \rangle \langle 0 | A^\mu | D^0 \rangle \simeq (p_{K^0} - p_{\overline{K^0}})_\mu f_D p_D^\mu = 0. \tag{3}$$

In many of the studies (e.g., [2–5,7]) this decay has been understood as a result of final state interactions (FSI). In the analysis of Ref. [2] the rescattering mechanism included K^+K^- and $\pi^+\pi^-$ states leading to a branching ratio $B(D^0 \to K^0 \overline{K^0}) = \frac{1}{2} B(D^0 \to K^+ K^-)$. Experimental data on the other hand are [16] $B(D^0 \to K^0 \overline{K^0}) = (6.5 \pm 1.8) \times 10^{-4}$ and $B(D^0 \to K^+ K^-) = (4.25 \pm 0.16) \times 10^{-3}$. A recent investigation of the $D^0 \to K^0 \overline{K^0}$ decay mode performed in [3] has focused on the s channel and the t channel one particle exchange contributions. The s channel contribution has been taken into account through the poorly known scalar meson $f_0(1710)$ and was found to be very small, while the one particle t exchanges yielded higher contributions, with the pion exchange being the highest. In the approach of [8] the $D^0 \to K^0 \overline{K^0}$ was realized through the scalar glueball or gluerich scalar meson.

The second instructive case concerning the factorization hypothesis is offered by the analyses of nonleptonic K meson decays. Namely, it is well known that factorization does not work in nonleptonic K decays. Among many approaches the chiral quark model (χQM) [17] was shown to be able to accommodate the intriguing $\Delta I = 1/2$ rule in $K \to \pi\pi$ decays, as well as CP violating parameters, by systematic involvement of the soft gluon emission forming gluon condensates and chiral loops at $\mathcal{O}(p^4)$ order [12]. In the χQM [18], the light quarks (u,d,s) couple to the would-be Goldstone octet mesons (K,π,η) in a chiral invariant way, such that all effects are, in principle, calculable in terms of physical quantities and a few model dependent parameters, namely the quark condensate, the gluon condensate, and the constituent quark mass [12,17,19]. Also in "generalized factorization," it was shown [13] that the inclusion of gluon condensates is important in order to understand the $\Delta I = 1/2$ rule for $K \to 2\pi$ decays.

As the χQM approach successfully indicated the main mechanisms in $K \to \pi\pi$ decays, it seems worthwhile to investigate decays of charm mesons within a similar framework. In doing this, we should strongly emphasize that one cannot *a priori* expect chiral perturbation theory (χPT) to work to a good precision in the process $D \to K\bar{K}$ because the energy release is almost 800 MeV. However, the leading contributions that we consider in this paper, do turn out to describe data reasonably well. Moreover, in the case of D meson decays one has to extend the ideas of the χQM to the sector involving a heavy quark (c) using the chiral symmetry of light degrees of freedom as well as heavy quark symmetry and heavy quark effective field theory (HQEFT). Such ideas have already been presented in previous papers [20–22] and lead to the formulation of heavy-light chiral quark models (HLχQM). In our formulation of the HLχQM Lagrangian, an unknown coupling constant appears in the term that couples the heavy meson to a heavy and a light quark. Our strategy is to relate expressions involving this coupling to physical quantities, as it is done within the χQM [12]. We perform the bosonization by integrating out the light and heavy quarks and obtain a heavy quark symmetric chiral Lagrangian involving light and heavy mesons [23,24].

Because the $\mathcal{O}(p)$ (factorizable) contribution is zero as seen in Eq. (3), we will try in this paper to approach to the $D^0 \to K^0 \bar{K}^0$ decay systematically to $\mathcal{O}(p^3)$. We do this by including first the nonfactorizable contributions coming from the chiral loops. These are based on the weak Lagrangian corresponding to the factorizable $\mathcal{O}(p)$ terms for $D^0 \to \pi^+\pi^-$ and $D^0 \to K^+K^-$. [Note that the velocity $v^\mu = p_D^\mu / m_D$ is considered to be $\mathcal{O}(p^0)$ in the chiral counting]. Second, we consider the gluon condensate contributions, also of $\mathcal{O}(p^3)$ within the χQM and HLχQM framework. As already pointed out, the energy release in $D \to K\bar{K}$ is $p = 788$ MeV and hence p/Λ_χ (for $\Lambda_\chi \geqslant 1$ GeV) is close to unity. The next to leading $\mathcal{O}(p^5)$ terms might be almost of the same order of magnitude compared to our $\mathcal{O}(p^3)$ terms. However, we expect a weak suppression of the order p^2/Λ_χ^2. On the other hand, the inclusion of $\mathcal{O}(p^5)$ order in this framework is not straightforward. Before doing loop calculations at that order, one has to find a reliable framework to include light resonances like ρ, K^*, $a_0(980)$, $f_0(975)$, etc. In the present approaches, which combine heavy quark effective theory and chiral Lagrangian, the light resonances are treated within hidden gauge symmetry (see, e.g., [23]), which is not compatible with chiral perturbation theory. Even if the light resonances were included in the HLχQM Lagrangian, one would face the problem of determining their couplings to the rest of heavy and light states. The poorly known scalar resonances would introduce a rather large uncertainty [3]. Right now, the consistent calculation of this or higher orders does not seem to be possible. Still, the amplitude of $D^0 \to K^0 \bar{K}^0$ calculated within the framework of $\mathcal{O}(p^3)$ turns out to be of the same order of magnitude as the experimental result. Note that we have omitted $1/m_Q$ terms in the framework of HQEFT.

Our paper is organized as follows: In Sec. II we write down the basic Lagrangians including the weak Lagrangian at quark level (with special emphasis on the terms giving rise to the nonfactorizable gluon condensate contributions) as well as the standard strong chiral Lagrangians for the light and heavy meson sectors. The chiral loop contributions to the decay amplitudes are presented in Sec. III. The details of the heavy-light chiral quark model (HLχQM) are presented in Sec. IV, while the bosonization of the weak quark currents is given in Sec. V. The results are given in Sec. VI. Appendix A contains some details from the chiral loop integrals, Appendix B some details about the D meson decay constant, while Appendix C contains some loop integrals within the HLχQM.

II. BASIC LAGRANGIANS

The effective weak Lagrangian at quark level relevant for $D \to \pi\pi, K\bar{K}$ is

$$\mathcal{L}_W = \widetilde{G}[c_A(Q_A - Q_C) + c_B(Q_B^{(s)} - Q_B^{(d)})], \tag{4}$$

where $\widetilde{G} = -2\sqrt{2} G_F V_{us} V_{cs}^*$, and

$$Q_A = (\overline{s_L}\gamma^\mu c_L)(\overline{u_L}\gamma_\mu s_L), \quad Q_C = (\overline{d_L}\gamma^\mu c_L)(\overline{u_L}\gamma_\mu d_L),$$

$$Q_B^{(q)} = (\overline{u_L}\gamma^\mu c_L)(\overline{q_L}\gamma_\mu q_L), \quad (q = s,d), \tag{5}$$

are quark operators.

Using Fierz transformations and the following relation between the generators of $SU(3)_c$ (i,j,l,n are color indices running from 1 to 3 and a is an index running over the eight gluon charges):

$$\delta_{ij}\delta_{ln} = \frac{1}{N_c}\delta_{in}\delta_{lj} + 2 t_{in}^a t_{lj}^a, \tag{6}$$

one obtains

$$Q_A = \frac{1}{N_c} Q_B^{(s)} + R_B^{(s)}, \quad Q_C = \frac{1}{N_c} Q_B^{(d)} + R_B^{(d)},$$

$$Q_B^{(s)} = \frac{1}{N_c} Q_A + R_A, \quad Q_B^{(d)} = \frac{1}{N_c} Q_C + R_C, \tag{7}$$

where the R's correspond to a color exchange between two currents and is genuinely nonfactorizable,

$$R_A = 2(\overline{s_L}\gamma^\mu t^a c_L)(\overline{u_L} t^a \gamma_\mu s_L),$$

$$R_C = 2(\overline{d_L}\gamma^\mu t^a c_L)(\overline{u_L} t^a \gamma_\mu d_L),$$

$$R_B^{(q)} = 2(\overline{u_L}\gamma^\mu t^a c_L)(\overline{q_L} t^a \gamma_\mu q_L), \quad (q=s,d). \tag{8}$$

The operators can be written in terms of currents: for instance,

$$Q_B^{(s)} - Q_B^{(d)} = J_\mu^Y j_X^\mu, \quad R_B^{(s)} - R_B^{(d)} = 2J_\mu^{Y,a} j_X^{\mu,a}, \tag{9}$$

where

$$J_\mu^{Y,a} \equiv \overline{u_L}\gamma^\mu t^a c_L, \quad j_X^{\mu,a} \equiv \overline{s_L} t^a \gamma^\mu s_L - [s \to d]. \tag{10}$$

The currents without color index are given by the corresponding expressions by removing the color matrix.

The factorization approach amounts to writing the currents J^Y_μ, j^X_μ in terms of hadron (in our case meson) fields only, so that the operator $Q_B^{(s)} - Q_B^{(d)}$ in Eq. (9) is equal to the product of two meson currents. The color currents in Eq. (9) are then zero if hadronized (mesons are color singlet objects). There is also a replacement of the Wilson coefficients in the hadronized effective weak Lagrangian $c_{A,B} \to c_{A,B}(1 + 1/N_c)$. Combining heavy quark symmetry and chiral symmetry of the light sector, we can obtain the weak chiral Lagrangian for nonleptonic D meson decays due to factorizable terms. Then we can first use this to calculate nonfactorizable contributions due to chiral loops. Second, we can calculate the color currents' contribution using the gluon condensate within the framework of the HLχQM.

Treating the light pseudoscalar mesons as pseudo-Goldstone bosons one obtains the usual $\mathcal{O}(p^2)$ chiral Lagrangian

$$\mathcal{L}_{str}^{(2)} = \frac{f^2}{8}\mathrm{tr}(\partial^\mu \Sigma \partial_\mu \Sigma^\dagger) + \frac{f^2 B_0}{4}\mathrm{tr}(\mathcal{M}_q \Sigma + \mathcal{M}_q \Sigma^\dagger), \tag{11}$$

where $\Sigma = \exp(2i\Phi/f)$ with $\Phi = \Sigma_j \lambda^j \pi^j/\sqrt{2}$ containing the Goldstone bosons π, K, η, while the trace (tr) runs over flavor indices and $\mathcal{M}_q = \mathrm{diag}(m_u, m_d, m_s)$ is the current quark mass matrix. From this Lagrangian, we can deduce the light weak current to $\mathcal{O}(p)$:

$$j_\mu^X = -i\frac{f^2}{4}\mathrm{tr}(\Sigma \partial_\mu \Sigma^\dagger \lambda^X), \tag{12}$$

corresponding to the quark current $j_\mu^X = \overline{q_L}\gamma_\mu \lambda^X q_L$. [$\lambda^X$ is a SU(3) flavor matrix.]

In the heavy meson sector interacting with light mesons we have the following lowest order $\mathcal{O}(p)$ chiral Lagrangian

$$\mathcal{L}_{str}^{(1)} = -\mathrm{Tr}(\overline{H}_{va} iv\cdot D_{ab} H_{vb}) - g\mathrm{Tr}(\overline{H}_{va} H_{vb}\gamma_\mu \mathcal{A}_{ba}^\mu \gamma_5), \tag{13}$$

where $iD^\mu_{ab}H_b = i\partial^\mu H_a - H_b \mathcal{V}^\mu_{ba}$, the trace (Tr) runs over Dirac indices. Note that in Eq. (13) and the rest of this section a and b are *flavor* indices.

The vector and axial vector fields $\mathcal{V}_\mu$ and $\mathcal{A}_\mu$ in Eq. (13) are given by

$$\mathcal{V}_\mu \equiv \frac{i}{2}(\xi \partial_\mu \xi^\dagger + \xi^\dagger \partial_\mu \xi), \quad \mathcal{A}_\mu \equiv \frac{i}{2}(\xi \partial_\mu \xi^\dagger - \xi^\dagger \partial_\mu \xi), \tag{14}$$

where $\xi = \exp(i\Phi/f)$. The heavy meson field H_{va} contains a spin zero and spin one boson,

$$H_{va} \equiv P_+(P_{\mu a}\gamma^\mu - iP_{5a}\gamma_5), \tag{15}$$

$$\overline{H}_{va} = \gamma^0 (H_{va})^\dagger \gamma^0 = [P^\dagger_{\mu a}\gamma^\mu - iP^\dagger_{5a}\gamma_5]P_+, \tag{16}$$

with $P_\pm = (1 \pm \gamma^\mu v_\mu)/2$ being the projection operators. The field $P_5(P_5^\dagger)$ annihilates (creates) a pseudoscalar meson with a heavy quark having velocity v and similar for spin one mesons.

For a decaying heavy quark, the weak current is given by

$$J_a^\lambda = \overline{q_a}\gamma^\lambda L Q, \tag{17}$$

where $L = (1-\gamma_5)/2$ and Q is the heavy quark field in the full theory, in our case a c quark, and q is the light quark field. [For flavor $a=u$, this is the current J_μ^Y in Eq. (9).]

From symmetry grounds, the heavy-light weak current is to $\mathcal{O}(p^0)$ bosonized in the following way, [24],

$$J_a^\lambda = \tfrac{1}{2}\alpha_H \mathrm{Tr}[\gamma^\lambda L H_{vb}\xi^\dagger_{ba}], \tag{18}$$

where α_H is related to the physical decay constant f_D through the well known matrix element

$$\langle 0|\overline{u}\gamma^\lambda \gamma_5 c|D^0\rangle = -2\langle 0|J_a^\lambda|D^0\rangle = i m_D v^\lambda f_D. \tag{19}$$

Note that the current (18) is $\mathcal{O}(p^0)$ in the chiral counting.

III. CHIRAL LOOP CONTRIBUTIONS

In the factorization limit there are no contributions to $D^0 \to K^0 \overline{K^0}$ at tree level. The observation of a partial decay width $B(D \to K^0 \overline{K^0}) = (6.5 \pm 1.8) \times 10^{-4}$ on the other hand implies that we can expect sizable contributions at the one loop level. Calculations to one loop in the framework of combined chiral perturbation theory and HQEFT involves a construction of the most general effective Lagrangian that has the correct symmetry properties in order to make the renormalization work. We discuss constructions of counterterms in the end of Sec. V.

We work strictly in the modified minimal subtraction $(\overline{\mathrm{MS}})$ renormalization scheme, where we put $\overline{\Delta} = 2/\epsilon - \gamma + \ln(4\pi) + 1$ equal to one in the loop calculations. This choice, $\overline{\Delta} = 1$, determines the appropriate renormalization of couplings in the $\mathcal{O}(p^3)$ effective Lagrangian and is the same as made by Stewart in [25], while it differs from the one used

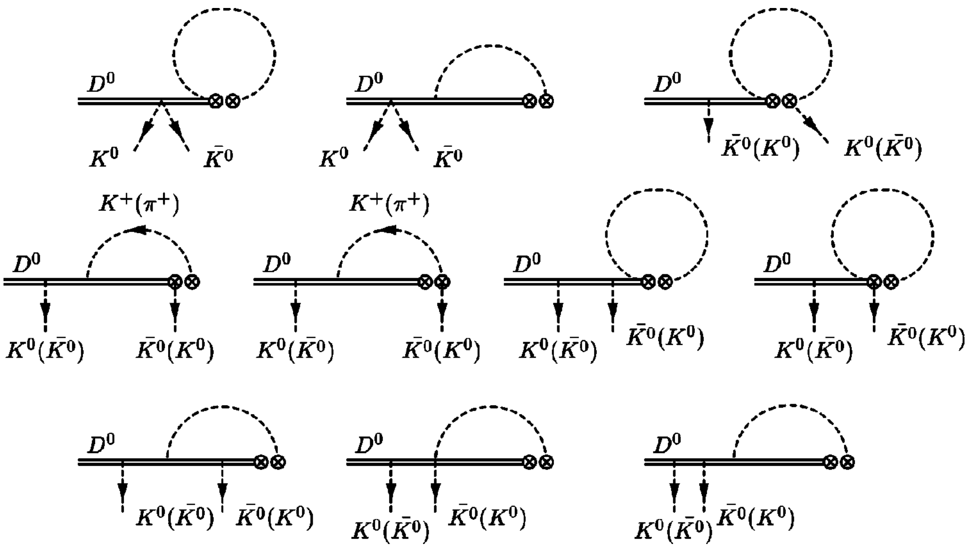

FIG. 1. Diagrams that give zero contribution since the relevant vertices appearing in the heavy meson chiral Lagrangian (13) are zero. The double line represents heavy meson D or D^*, while dashed lines denote pseudo Goldstone bosons.

by authors of Ref. [23], who use $\bar{\Delta}=0$. We consider only contributions coming from the c_A part of the weak Lagrangian as c_B is suppressed compared to c_A [26].

Writing down the most general one loop graphs with two outgoing Goldstone bosons (K^0 and $\bar{K}^0$) one arrives at 26 Feynman diagrams. A number of these give zero contributions and are shown on Figs. 1–3, while the graphs that do contribute to $D^0\to K^0\bar{K}^0$ decay are shown on Fig. 4. Note that factorizable loops which renormalize vertices are omitted (they do appear, however, in the loop determination of the α_H coupling related to f_D. See Appendix B.)

To shorten the notation, the common factors in the S matrix have been organized such that the amplitude is written

$$\mathcal{M}(D^0\to K^0\bar{K}^0)=-\frac{G_F}{\sqrt{2}}c_A V_{us}V_{cs}^*\frac{F}{8\pi^2}\sqrt{m_D}, \tag{20}$$

where $F=\Sigma_n F_n$ is the sum of the amplitudes corresponding to the graphs on Figs. 1–4. The partial decay width for the decay $D^0\to K^0\overline{K^0}$ is then

$$\Gamma_{D^0\to K^0\bar{K}^0}=\frac{1}{2\pi}\frac{G_F^2}{8m_D}c_A^2|V_{us}V_{cs}^*|^2\frac{|F|^2}{(8\pi^2)^2}p, \tag{21}$$

where p is the $K^0(\bar{K}^0)$ three-momentum in the D^0 rest frame

$$p=\tfrac{1}{2}\sqrt{m_D^2-4m_K^2}. \tag{22}$$

The nonzero amplitudes corresponding to the graphs in Fig. 4 are

$$F_1+F_2+F_3=-\frac{g\alpha_H}{f^2}\frac{13}{4}[\Delta_d^* J_1(m_\pi,\Delta_d^*) - \Delta_s^* J_1(m_K,\Delta_s^*)], \tag{23}$$

$$\begin{aligned}F_4=-\frac{\alpha_H}{3f^2}\frac{m_D}{2}\{&(m_D^2-2m_K^2)[N_0(m_\pi,m_D^2) - N_0(m_K,m_D^2)]+m_D^2[N_2(m_\pi,m_D^2)\\&-N_2(m_K,m_D^2)]+[N_3(m_\pi,m_D^2)\\&-N_3(m_K,m_D^2)]-(m_\pi^2-m_K^2)N_0(m_\pi,m_D^2)\},\end{aligned} \tag{24}$$

$$F_5+F_6=\frac{\alpha_H m_D}{f^2}\frac{7}{24}[I_1(m_\pi)-I_1(m_K)], \tag{25}$$

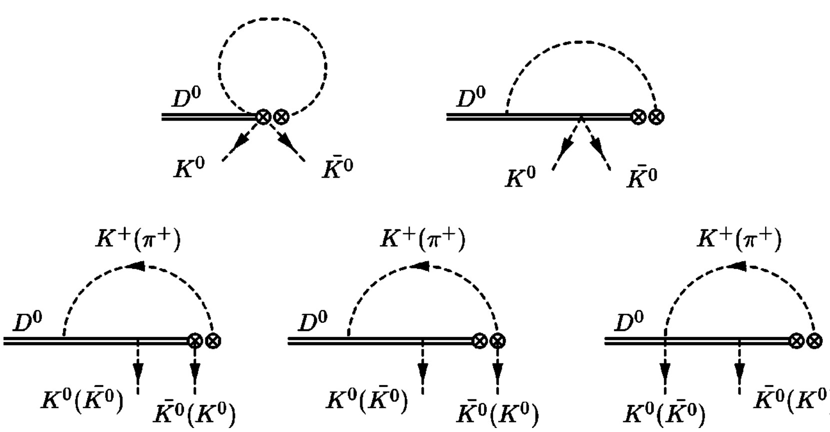

FIG. 2. Diagrams that give zero contributions since the loop integrals are zero. The double line represents heavy meson D or D^*, while dashed lines denote pseudo Goldstone bosons.

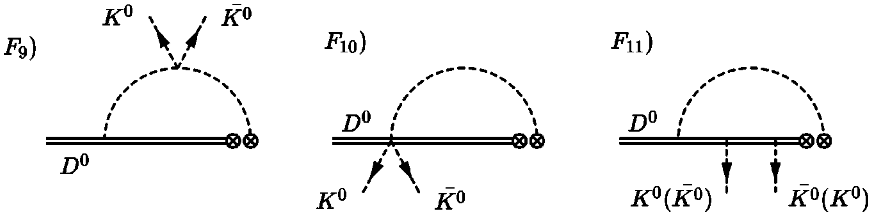

FIG. 3. Power suppressed diagrams (neglected in the calculation).

$$
\begin{aligned}
F_7+F_8=\frac{\alpha_H}{4f^2}\Bigg\{ & \tilde{\Delta}_d[J_1(m_K,\tilde{\Delta}_d)+J_2(m_K,\tilde{\Delta}_d)] \\
& -\tilde{\Delta}_s[J_1(m_\pi,\tilde{\Delta}_s)+J_2(m_\pi,\tilde{\Delta}_s)] \\
& +m_D\frac{\Delta_d}{\tilde{\Delta}_d}I_2(m_K,\tilde{\Delta}_d)-m_D\frac{\Delta_s}{\tilde{\Delta}_s}I_2(m_\pi,\tilde{\Delta}_s) \\
& +\frac{m_D}{2\tilde{\Delta}_d}I_1(m_K)-\frac{m_D}{2\tilde{\Delta}_s}I_1(m_\pi).\Bigg\},
\end{aligned} \tag{26}
$$

where $\Delta_q^{(*)}=m_{D_q^{(*)}}-m_{D^0}$ and $\tilde{\Delta}_q=m_D/2+\Delta_q$ for $q=d,s$. Note that $\tilde{\Delta}_q$ are of the order $m_D/2$, a consequence of relatively high momenta flowing in the loops of graphs F_7,F_8. The functions $I_1(m)$, $I_2(m,\Delta)$, $J_1(m,\Delta)$, $J_2(m,\Delta)$, $N_0(m,k^2)$, $N_2(m,k^2)$, $N_3(m,k^2)$ appearing in the amplitudes (24–27) can be found in Appendix A.

It should be noted that in Eqs. (23)–(26) all the expressions vanish in the exact SU(3) limit, where $m_K\to m_\pi$ and $\Delta_s\to\Delta_d$, $\tilde{\Delta}_s\to\tilde{\Delta}_d$. This shows explicitly that the $D^0\to K^0\overline{K}^0$ decay mode is a manifestation of SU(3) breaking effects [as already noted by H. Lipkin [4], if U symmetry is exact, then $\Gamma(D^0\to K^0\overline{K}^0)=0$].

The amplitudes shown on Figs. 1–3 are either exactly zero or are suppressed by powers of $1/m_D$ and g (we present results for the value $g=0.27$ [25] and the value $g=0.57$ preferred by a recent determination of D^{*+} decay width [27]). The amplitudes corresponding to diagrams on Figs. 1,2 are zero due to symmetry reasons (because there are no such couplings in the heavy sector chiral Lagrangian (13), or because of Lorentz covariance), while the amplitudes F_9, F_{10}, and F_{11} shown on Fig. 3 are power suppressed. An analysis of the loop integrals leads to the conclusion that $F_9\sim g(\tilde{q}/m_D)^2F_4$, $F_{10}\sim g(\tilde{q}/m_D)F_4$, and $F_{11}\sim g^3(\tilde{q}/m_D)F_4$, where $\tilde{q}$ is a typical loop momentum less than $m_D/2$, so the suppression need not be substantial. However, a direct evaluation of the amplitude F_{10} shows that it is about 20 (10) times smaller than F_4 if one uses $g=0.27$ ($g=0.57$). Therefore, in our numerical calculation we neglect contributions of F_9, F_{10}, and F_{11}. In Fig. 5 we show the graphs for F_9 and F_4 with momentum routing.

IV. A HEAVY-LIGHT CHIRAL QUARK MODEL (HLχQM)

The nonfactorizable contributions to $D^0\to K^0\overline{K}^0$ coming from the chiral loop correction at the meson level obtained in the previous section are not the only contributions to $\mathcal{O}(p^3)$. In the effective weak Lagrangian (4) there are, after Fierz transformations, terms that involve color currents [see Eqs. (9),(10)]. As mesons are color singlet objects, the product of color currents does not contribute at meson level in the factorization limit. However, at quark level they do contribute through the gluon condensate as will be shown in the next section. In order to estimate this contribution we have to

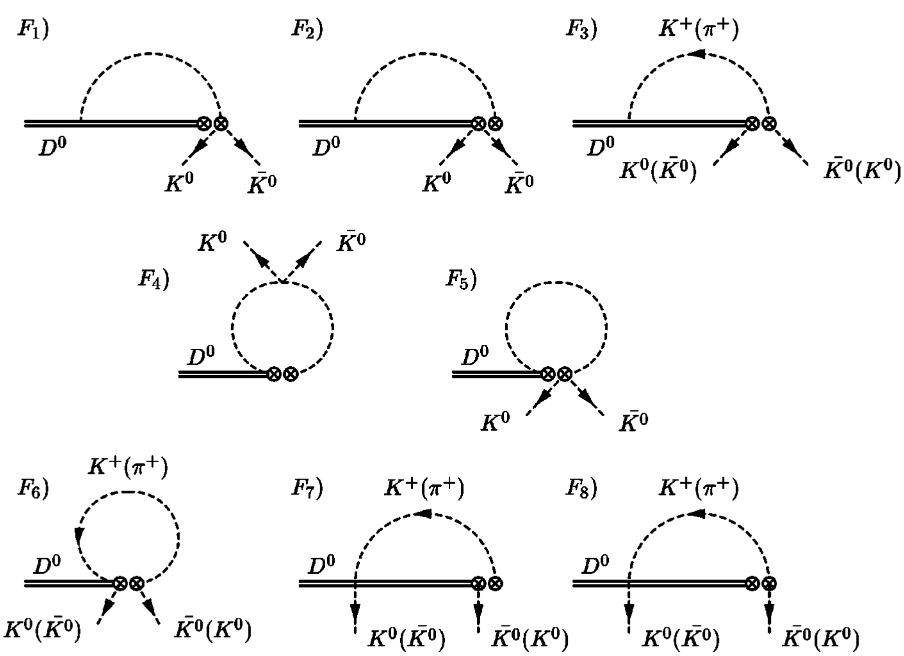

FIG. 4. The nonzero diagrams in $D^0\to K^0\overline{K}^0$ decay.

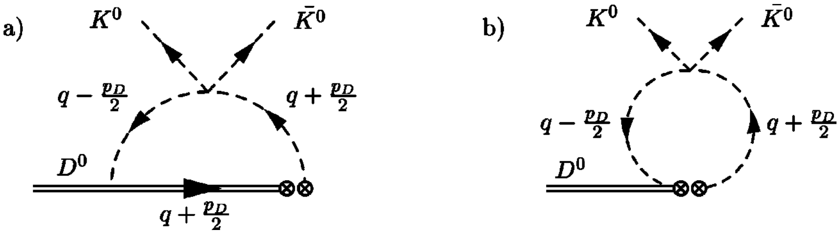

FIG. 5. The momenta flowing in the graphs corresponding to (a) power suppressed F_9 amplitude and (b) the leading contribution F_4 amplitude.

establish the connection between the underlying quark-gluon dynamics and the meson level picture. This is done through the use of a heavy-light chiral quark model.

Our starting point is the following Lagrangian containing both quark and meson fields:

$$\mathcal{L}=\mathcal{L}_{HQ}+\mathcal{L}_{\chi QM}+\mathcal{L}_{Int}, \tag{27}$$

where

$$\mathcal{L}_{HQ}=\overline{Q_v}iv\cdot DQ_v+\mathcal{O}(m_Q^{-1}) \tag{28}$$

is the Lagrangian for heavy quark effective field theory [28]. The heavy quark field Q_v annihilates a heavy quark with velocity v and mass m_Q. D_μ is the covariant derivative containing the gluon field. The light quark sector is described by the chiral quark model (χQM),

$$\mathcal{L}_{\chi QM}=\overline{q}(i\gamma^\mu D_\mu-\mathcal{M}_q)q-m_\chi(\overline{q_R}\Sigma^\dagger q_L+\overline{q_L}\Sigma q_R), \tag{29}$$

where $q=(u,d,s)$ are the light quark fields. The left- and right-handed projections q_L and q_R are transforming under $SU(3)_L$ and $SU(3)_R$, respectively. $\mathcal{M}_q$ is the current quark mass matrix, and Σ is a 3 by 3 matrix containing the (would be) Goldstone octet (π,K,η), appearing already in Eq. (11). The quantity m_χ is interpreted as the [SU(3)-invariant] constituent quark mass for light quarks, expected to appear due to the chiral symmetry breakdown at a scale $\Lambda_\chi\sim 1$ GeV.

The χQM has a "rotated version" with flavor rotated quark fields χ given by

$$\chi_L=\xi^\dagger q_L, \quad \chi_R=\xi q_R, \quad \xi\cdot\xi=\Sigma. \tag{30}$$

In the rotated version, the chiral interactions are rotated into the kinetic term while the interaction term [proportional to m_χ in Eq. (29) and responsible for the π-quark couplings] becomes a pure (constituent) mass term

$$\mathcal{L}_{\chi QM}=\overline{\chi}[\gamma^\mu(iD_\mu+\mathcal{V}_\mu+\gamma_5\mathcal{A}_\mu)-m_\chi]\chi-\overline{\chi}\widetilde{M}_q\chi, \tag{31}$$

and $\widetilde{M}_q$ defines the rotated version of the current mass term

$$\widetilde{M}_q\equiv\xi^\dagger\mathcal{M}_q\xi^\dagger R+\xi\mathcal{M}_q^\dagger\xi L\equiv\widetilde{M}_q^R R+\widetilde{M}_q^L L\equiv\widetilde{M}_q^V+\widetilde{M}_q^A\gamma_5, \tag{32}$$

where $L=(1-\gamma_5)/2$ is the left-handed projector in Dirac space, and R is the corresponding right handed projector. The Lagrangian (31) is manifestly invariant under the unbroken symmetry $SU(3)_V$ (if $\mathcal{M}_q$ is formally chosen to transform as Σ). In the light sector, the various pieces of the strong chiral Lagrangian (11) can be obtained by integrating out the constituent quark fields χ. This is the *bosonization* to be discussed in more detail in the next section.

Similarly, a left-handed current can be written [λ^X is a SU(3) flavor matrix]

$$\overline{q_L}\gamma^\mu\lambda^X q_L=\overline{\chi_L}\gamma^\mu\Lambda^X\chi_L, \quad \Lambda^X\equiv\xi^\dagger\lambda^X\xi. \tag{33}$$

By coupling the fields $\mathcal{A}_\mu,\widetilde{M}_q^{V,A},\Lambda^X$ to quark loops, the chiral Lagrangians of the weak sector can be obtained.

In the heavy-light case, the generalization of the meson-quark interactions in the pure light sector χQM is given by the following $SU(3)_V$ invariant Lagrangian [20–22,29]:

$$\mathcal{L}_{Int}=-G_H[\overline{\chi_f}\overline{H_{vf}}Q_v+\overline{Q_v}H_{vf}\chi_f], \tag{34}$$

where G_H is a coupling constant which is related through bosonizations to physical quantities like α_H and g appearing in Eqs. (13) and (18), as well as f_π and m_χ. (See Appendix C.)

Within HQEFT the heavy-light weak current in Eq. (17) will, below the renormalization scale $\mu=m_c$, be modified in the following way [28]:

$$J_a^\lambda=C_\gamma(\mu)\overline{\chi_b}\xi_{ba}^\dagger\gamma^\lambda LQ_v+C_v(\mu)\overline{\chi_b}\xi_{ba}^\dagger v^\lambda LQ_v, \tag{35}$$

where the coefficients $C_{\gamma,v}$ are determined by QCD renormalization for $\mu<m_c$. However, for $\mu\simeq\Lambda_\chi$, $C_\gamma\simeq 1$ and $C_v\simeq 0$. The bosonization of Eq. (35) will lead to Eq. (18) by using Eq. (34).

V. BOSONIZATION

The Lagrangian (27) from the previous section can now be used for bosonization, i.e., we integrate out the quark fields. This can be done in the path integral formalism, or as we do here, by expanding in terms of Feynman diagrams. For instance, the lowest order (kinetic) chiral Lagrangian (11) in the light sector (involving π,K,η's) can be obtained by coupling two axial fields to a quark loop using the Lagrangian in Eq. (31),

$$i\mathcal{L}_{str}^{(2)}(\pi,K,\eta)=-N_c\int\frac{d^dp}{(2\pi)^d}\mathrm{Tr}[(\gamma_\sigma\gamma_5\mathcal{A}^\sigma)S(p)\times(\gamma_\rho\gamma_5\mathcal{A}^\rho)S(p)]\sim\mathrm{tr}[\mathcal{A}_\mu\mathcal{A}^\mu], \tag{36}$$

where $S(p)=(\gamma\cdot p-m_\chi)^{-1}$, and the trace is both in flavor and Dirac spaces. This is the standard form of the lowest order chiral Lagrangian (11), which can easily be seen by using the relations

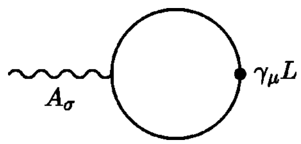

FIG. 6. Feynman diagram for bosonization of the left-handed current to order $\mathcal{O}(p)$.

$$\mathcal{A}_\mu = -\frac{1}{2i}\xi(\partial_\mu\Sigma^\dagger)\xi = \frac{1}{2i}\xi^\dagger(\partial_\mu\Sigma)\xi^\dagger. \tag{37}$$

Similarly one obtains the lowest order $\mathcal{O}(p)$ strong chiral Lagrangian (13) in the heavy sector .

Let us now consider the bosonization of the pure light weak current. The lowest order term $\mathcal{O}(p)$ is obtained when the vertex Λ^X from Eq. (33) and axial vertex ($\sim\mathcal{A}_\mu$) from Eq. (31) are combined with quark loops (see Fig. 6),

$$j_\mu^X(\mathcal{A}) = -iN_c\int\frac{d^dp}{(2\pi)^d}\mathrm{Tr}[(\gamma_\mu L\Lambda^X)S(p)(\gamma_\sigma\gamma_5\mathcal{A}^\sigma)S(p)]$$
$$\sim \mathrm{tr}[\Lambda^X\mathcal{A}_\mu]. \tag{38}$$

This coincides with Eq. (12) when Eq. (37) is used.

To obtain a nonzero nonfactorizable contribution to $D^0\to K^0\overline{K^0}$ at tree level, we have to consider the color current $j_\mu^{X,a}$ to $\mathcal{O}(p^3)$, involving insertions of the "mass fields" $\widetilde{M}_q$ in Eq. (32). From Fig. 7, one obtains the contribution

$$j_\mu^{X,a}(G^b,\mathcal{A},\widetilde{M}_q|\mathrm{Fig.\ 7}) = i\int\frac{d^dp}{(2\pi)^d}\mathrm{Tr}[(\gamma_\mu L\Lambda^X)S(p)$$
$$\times(\gamma_\sigma\gamma_5\mathcal{A}^\sigma)S(p)\widetilde{M}_qS_1(p,G^b)], \tag{39}$$

where [30,12],

$$S_1(p,G^b) = -\frac{g_s}{4}G^b_{\alpha\beta}t^b[\sigma^{\alpha\beta}(\gamma\cdot p+m_\chi)+(\gamma\cdot p+m_\chi)\sigma^{\alpha\beta}]$$
$$\times(p^2-m_\chi^2)^{-2}, \tag{40}$$

is the light quark propagator in a gluonic background (to first order in the gluon field) and g_s is the strong coupling constant. Moreover, a,b are color octet indices. Summing all six diagrams with permutated vertices compared to the one in Fig. 7 we obtain in total

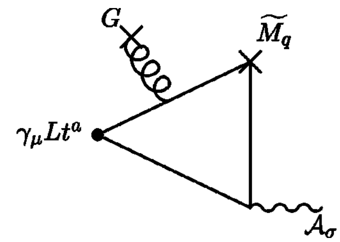

FIG. 7. Diagram for bosonization of the color current to $\mathcal{O}(p^3)$.

$$j_\mu^{X,a}(G^b,\mathcal{A},\widetilde{M}_q) = \frac{g_s}{12m_\chi}\frac{1}{16\pi^2}G^{a,\kappa\lambda}[i\varepsilon_{\mu\rho\kappa\lambda}T_\varepsilon^{X,\rho}+(\eta_{\mu\kappa}\eta_{\rho\lambda}$$
$$-\eta_{\mu\lambda}\eta_{\rho\kappa})T_g^{X,\rho}], \tag{41}$$

where (we have used the analytical computer program FORM [31]),

$$T_\varepsilon^{X,\rho} = 4S_\rho^K - 3(S_\rho^L+S_\rho^R), T_g^{X,\rho} = S_\rho^L - S_\rho^R. \tag{42}$$

The S's are chiral Lagrangian terms:

$$S_\rho^L \equiv \mathrm{tr}[\Lambda^X\mathcal{A}_\rho\widetilde{M}_q^L] = \frac{1}{2i}\mathrm{tr}[\lambda^X(D_\rho\Sigma)\mathcal{M}_q^\dagger],$$

$$S_\rho^R \equiv \mathrm{tr}[\Lambda^X\widetilde{M}_q^R\mathcal{A}^\rho] = \frac{-1}{2i}\mathrm{tr}[\lambda^X\mathcal{M}_q(D_\rho\Sigma^\dagger)],$$

$$S_\rho^K \equiv \frac{1}{2}\mathrm{tr}[\Lambda^X(\mathcal{A}^\rho\widetilde{M}_q^R+\widetilde{M}_q^L\mathcal{A}^\rho)]$$
$$= \frac{1}{4i}\mathrm{tr}[\lambda^X((D_\rho\Sigma)\Sigma^\dagger\mathcal{M}_q\Sigma^\dagger$$
$$-\Sigma\mathcal{M}_q^\dagger\Sigma(D_\rho\Sigma^\dagger))]. \tag{43}$$

Within the heavy-light sector, the weak current can be bosonized to lowest order $[\mathcal{O}(p^0)]$ by calculating the Feynman diagram shown in Fig. 8, left. The obtained result is Eq. (18) with α_H related to G_H (see Appendix C).

The bosonization of the color current given by Eq. (35) with an extra color matrix t^a inserted and with an extra gluon emitted is given by the following loop integral (Fig. 8, right):

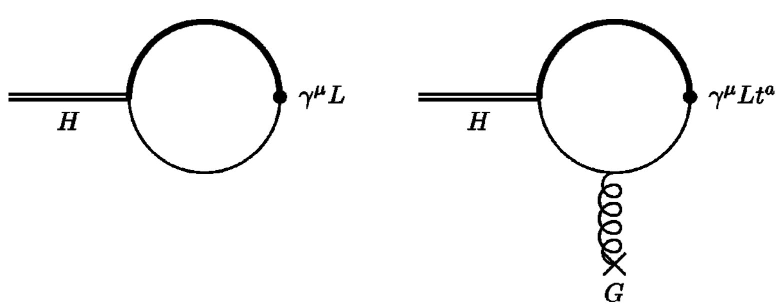

FIG. 8. Diagrams representing bosonization of heavy-light weak current. The boldface line represents the heavy quark, the solid line the light quark.

$$J^{\sigma}(H_v G^a)^f = -\int \frac{d^d k}{(2\pi)^d} \mathrm{Tr}[(-iG_H H_v \xi^\dagger)^f (iS_1(k,G^b)) \times (\gamma^\sigma L t^a)(i\Delta_v(k))], \tag{44}$$

where $\Delta_v(k) = P_+ / k\cdot v$ is the heavy quark propagator. Notice that emission of a gluon from the heavy quark is suppressed by $1/m_Q$ and omitted. The result can be written

$$J^{\sigma}(H_v G^a)^f = G_H g_s G^a_{\alpha\beta} \mathrm{Tr}\{\gamma_\sigma L (H_v \xi^\dagger)^f [I_{G1}\sigma^{\alpha\beta} - iI_{G2}(\gamma^\alpha v^\beta - \gamma^\beta v^\alpha)]\}, \tag{45}$$

where I_{G1} and I_{G2} are loop integrals given in Appendix C. Keeping only the pseudoscalar field P_5 representing D^0, we find

$$J^{Y,a}_{\mu}(P_5, G^b) = \frac{g_s G_H}{16\pi^2}(P_5 \xi^\dagger)^Y G^{a,\alpha\beta}[iB_\varepsilon \varepsilon_{\mu\sigma\alpha\beta} v^\sigma + B_g(\eta_{\mu\alpha} v_\beta - \eta_{\mu\beta} v_\alpha)], \tag{46}$$

where $B_{\varepsilon,g}$ are obtained from loop integrals in Eq. (45). Then we find the nonfactorizable (gluon condensate) contribution,

$$\mathcal{L}_{\mathrm{eff}}(D^0 \mathrm{decay})_{\langle G^2\rangle} = 2\tilde{G} c_A \left(\frac{g_s G_H}{16\pi^2}\right)\left(\frac{g_s}{12 m_\chi}\frac{1}{16\pi^2}\right) \times \langle G^2\rangle v_\rho [B_\varepsilon T^{X,\rho}_\varepsilon + B_g T^{X,\rho}_g](P_5 \xi^\dagger)^Y, \tag{47}$$

where $\langle G^2\rangle$ is the gluon condensate, obtained by the prescription

$$G^a_{\mu\nu} G^a_{\alpha\beta} \to \tfrac{1}{12}(\eta_{\mu\alpha}\eta_{\nu\beta} - \eta_{\mu\beta}\eta_{\nu\alpha})\langle G^2\rangle. \tag{48}$$

In order to make predictions, we have to relate G_H in Eq. (47) and the various loop integrals to physical quantities like m_χ, f_π, and $\alpha_H \simeq f_D \sqrt{m_D}$.

It should be noted that there are *a priori* other terms than the one in Eq. (47). There is one possible term where the field $\tilde{M}_q$ occurring in Fig. 7 may instead be attached to the light quark line in the diagram in Fig. 8 (right). However, this term will not give contributions to $D^0 \to K^0 \overline{K^0}$. Moreover, there is *a priori* a term where the field $\mathcal{A}_\sigma$ attached in Fig. 7 is instead attached to the light quark line in Fig. 8 (right). This term is identically zero.

In the language of chiral perturbation theory, the term (47) can be interpreted as a counterterm. To be more specific, the (divergent part of the) counterterm has the Lorentz and flavor structure of the second line of Eq. (47) and is multiplied with a (divergent) coefficient adjusted to cancel the loop divergences obtained in Sec. III.

VI. RESULTS

In numerical calculation we use the values of α_H, g, and f obtained within the same framework [23,25,32-34]. The coupling g is extracted from existing experimental data on $D^* \to D\pi$ and $D^* \to D\gamma$ decays. The analysis in [25] includes chiral corrections at one loop order and yields $g = 0.27^{+0.04+0.05}_{-0.02-0.02}$, leaving the sign undetermined. Recently the CLEO Collaboration has announced the first measurement of D^{*+} decay width $\Gamma(D^{*+}) = 96 \pm 4 \pm 22$ keV [27]. Using this value together with the data on branching ratios $Br(D^{*+} \to D^0\pi^+) = (67.7 \pm 0.5)\%$ and $Br(D^{*+} \to D^+\pi^0) = (30.7 \pm 0.5)\%$ [16] one immediately finds at tree level $g = 0.57 \pm 0.08$. We present results for $g = 0.27$ and $g = 0.57$. The larger value seems to be in better agreement with the results coming from different approaches listed in [23]. The one loop chiral corrections also reduce the bare pion decay constant from $f_\pi = 0.132$ GeV to $f = 0.120$ GeV [25], which is the value we use. In order to obtain the α_H coupling, we use present experimental data on D_s leptonic decays. Namely, at the tree level there is a relation $f_D = f_{D_s} = \alpha_H / \sqrt{m_D}$. This relation receives 10–20% chiral corrections [23,32]. From the experimental branching ratio $D_s \to \mu\nu_\mu$ and the D_s decay width [16] one gets $f_{D_s} = 0.23 \pm 0.05$ GeV and taking into account chiral loop contributions, we find $\alpha_H = 0.23 \pm 0.04$ GeV$^{3/2}$ using $g = 0.27$ or $\alpha_H = 0.21 \pm 0.04$ GeV$^{3/2}$ using $g = 0.57$ (for details see Appendix B). Note that in [23] the $\overline{\Delta} = 0$ has been used, while we use the strict $\overline{\mathrm{MS}}$ prescription $\overline{\Delta} = 1$ as in [25]. We put everywhere $\mu = 1$ GeV$\simeq \Lambda_\chi$.

For the Wilson coefficients $c_{A,B}$ of Eq. (4) we use $c_A = 1.10 \pm 0.05$ and $c_B = -0.06 \pm 0.12$ [26], calculated at the scale $\mu = 1$ GeV with the number of colors $N_c = 3$. Within the framework of ''new'' or ''generalized'' factorization, where nonfactorizable effects are taken into account in a phenomenological way, one uses the ''effective values'' $c_A^{\mathrm{eff}} = 1.26$ and $c_B^{\mathrm{eff}} = -0.47$. However, in this paper we calculate nonfactorizable effects in terms of chiral loops and gluon condensates and therefore we use the values of [26]. Due to the suppression of c_B in comparison with c_A, we do not include terms proportional to c_B.[1] We present our numerical results for the nonzero one chiral loop amplitudes in Table I.

The imaginary part of the amplitude comes from the F_4 graph, when the π's or the K's in the loops are on shell. All other graphs contribute only to the real part of the amplitude. The imaginary part of the amplitude is scale and scheme independent within chiral perturbation theory. This amplitude is also obtained from unitarity, and is valid beyond the chiral loop expansion. We also mention that the rescattering contribution, considered in [2,10] is the same contribution as the one we calculate from graphs on Fig. 5.

In order to cancel divergences one has to construct counterterms. In our case, this is described at the end of Sec. V. Generally, one can do that by using the symmetry arguments as has been done in [34,25] for the semileptonic decays of heavy mesons and D^* decays. In the case of D^* [25] it was

[1]Even if the ''new factorization'' values had been used, the c_B part of the weak interaction would be suppressed by 1/3 compared to the c_A one.

TABLE I. Table of the one chiral loop amplitudes (see Fig. 5), where $\mathcal{M}=\Sigma_n\mathcal{M}_n$ is defined in Eq. (20). The second column shows the amplitudes calculated using $g=0.27$ and $\alpha_H=0.23$ GeV$^{3/2}$, while the third column amplitudes have been calculated using $g=0.57$ and $\alpha_H=0.21$ GeV$^{3/2}$. In the last line the sum of all amplitudes is presented. It can be compared with the experimental result $|\mathcal{M}_{\rm Exp}|=3.80\times10^{-7}$GeV.

−	$\mathcal{M}_i[\times10^{-7}\mathrm{GeV}]$ $(g=0.27)$	$\mathcal{M}_i[\times10^{-7}\mathrm{GeV}]$ $(g=0.57)$
$\mathcal{M}_1$	-0.42	-0.82
$\mathcal{M}_2$	-0.31	-0.62
$\mathcal{M}_3$	-0.62	-1.23
$\mathcal{M}_4$	$0.75-2.54i$	$0.70-2.37i$
$\mathcal{M}_5$	-0.81	-0.76
$\mathcal{M}_6$	-0.61	-0.57
$\mathcal{M}_7$	-0.99	-0.92
$\mathcal{M}_8$	0.91	0.85
$\Sigma_i\mathcal{M}_i$	$-2.11-2.54i$	$-3.37-2.37i$

estimated that the contribution of counterterms is not substantial.

To obtain the $D^0\to\overline{K}^0K^0$ amplitude due to the gluon condensate we have to know the coupling G_H. In addition, we have to find the tensors S in Eqs. (42), (43) (and thereby the T's) for $K^0\overline{K^0}$ in the final state. We find to lowest order for the parts of $T^{X,\rho}_{g,\varepsilon}$

$$S^L_\mu=-S^R_\mu=-\frac{1}{f^2}(m_s-m_d)(p+\overline{p})_\mu\,,$$

$$S^K_\mu=\frac{2}{f^2}(m_s+m_d)(p-\overline{p})_\mu\,, \tag{49}$$

where p and $\overline{p}$ are the momenta of K^0 and $\overline{K}^0$, respectively. From Eq. (46) we see that the momenta will be contracted with $v^\mu=p^\mu_D/M_D$; where $p_D=p+\overline{p}$. It is important that S^L_μ and S^R_μ have a different momentum structure than $\langle K^0\overline{K}^0|V_\mu|0\rangle$ in Eq. (3), and they will give a nonfactorizable contribution to $D^0\to K^0\overline{K}^0$ proportional to $\langle G^2\rangle$ while S^K_μ does not. Note that $T^{X,\rho}_\varepsilon$ of Eqs. (41),(42),(47) do not contribute. We find the gluon condensate contribution:

$$\mathcal{M}(D^0\to K^0\overline{K^0})_{\langle G^2\rangle}=-c_A(\widetilde{G}m_D^2)\frac{(m_s-m_d)}{m_\chi}\,\frac{\beta\delta_G}{12N_c}B_gf_D\,, \tag{50}$$

where

$$\delta_G\equiv N_c\frac{\langle\alpha_sG^2/\pi\rangle}{8\pi^2f^4},\qquad \beta\equiv\frac{f^2G_H}{2\alpha_H},$$

$$B_g=16i\pi^2(I_{G1}-I_{G2})=\frac{\pi}{4}. \tag{51}$$

When we take into account the various relations between the loop integrals (I's) and G_H, we find that $\beta\simeq1/4$. Using the values [12] $\langle\alpha_sG^2/\pi\rangle\simeq(334\ \mathrm{MeV})^4$, $m_\chi=200$ MeV, and $m_s\simeq$ 150 MeV, we obtain the numerical value

$$\mathcal{M}(D^0\to K^0\overline{K^0})_{\langle G^2\rangle}\simeq0.43\times10^{-7}\ \ \mathrm{GeV}, \tag{52}$$

which is also of the same order of magnitude as the chiral loop contributions in Table I.

Adding both the chiral loops and the gluon condensate (52) contributions, we obtain the total amplitude to $\mathcal{O}(p^3)$

$$g=0.27,\quad \mathcal{M}_{\rm Th}=(-1.68-2.54i)\times10^{-7}\ \ \mathrm{GeV},$$

$$g=0.57,\quad \mathcal{M}_{\rm Th}=(-2.94-2.37i)\times10^{-7}\ \ \mathrm{GeV}. \tag{53}$$

or in terms of branching ratio

$$g=0.27,\quad B(D^0\to K^0\overline{K}^0)_{\rm Th}=(4.2\pm1.4)\times10^{-4},$$

$$g=0.57,\quad B(D^0\to K^0\overline{K}^0)_{\rm Th}=(6.5\pm1.7)\times10^{-4}, \tag{54}$$

where the estimated uncertainties reflect the uncertainties in the rest of the input parameters. These results should be compared with experimental data [16] $B(D^0\to K^0\overline{K}^0)=(6.5\pm1.8)\times10^{-4}$.

Around the charm mesons mass region there are many resonances. One might think that their contribution will appear in this decay mode, either as scalar resonance exchange like in [3] or as K^* exchanges [3,7,10]. Within our framework they would appear as the next order contribution ($\mathcal{O}(p^5)$) in the chiral expansion. This is, however, beyond the present scope of our investigations. It is interesting to point out that the effects we calculate, both from chiral loops and from the gluon condensate, are results of the SU(3) flavor symmetry breaking. In the limit of exact symmetry both contributions will disappear.

We can summarize that we indicate the leading nonfactorizable contributions to $D^0\to K^0\overline{K}^0$. Even though the use of chiral perturbation theory in this decay mode could be questioned, the calculated chiral loops can be considered as part of the final state interactions. In the treatment of the final state interactions the light pseudoscalar meson exchanges have to be present due to unitarity. Although the next to leading $\mathcal{O}(p^5)$ order terms might give sizable contributions to this decay, we have demonstrated that contributions due to the chiral loops and gluon condensates are of the same order of magnitude as the amplitude extracted from the experimental result.

APPENDIX A: LIST OF INTEGRALS FROM CHIRAL LOOPS

Here we list the dimensionally regularized integrals needed for evaluation of χPT and HQEFT one-loop graphs shown in Fig. 4,

$$i\mu^{\epsilon}\int\frac{d^{4-\epsilon}q}{(2\pi)^{4-\epsilon}}\frac{1}{q^2-m^2}=\frac{1}{16\pi^2}I_1(m), \tag{A1}$$

$$i\mu^{\epsilon}\int\frac{d^{4-\epsilon}q}{(2\pi)^{4-\epsilon}}\frac{1}{(q^2-m^2)(q\cdot v-\Delta)}=\frac{1}{16\pi^2}\frac{1}{\Delta}I_2(m,\Delta), \tag{A2}$$

with

$$I_1(m)=m^2\ln\left(\frac{m^2}{\mu^2}\right)-m^2\bar{\Delta}, \tag{A3}$$

$$I_2(m,\Delta)=-2\Delta^2\ln\left(\frac{m^2}{\mu^2}\right)-4\Delta^2F\left(\frac{m}{\Delta}\right)+2\Delta^2(1+\bar{\Delta}), \tag{A4}$$

where $\bar{\Delta}=2/\epsilon-\gamma+\ln(4\pi)+1$ (in calculation $\bar{\Delta}=1$), while $F(x)$ is the function calculated by Stewart in [25], valid for negative and positive values of the argument

$$F\left(\frac{1}{x}\right)=\begin{cases}-\dfrac{\sqrt{1-x^2}}{x}\left[\dfrac{\pi}{2}-\tan^{-1}\left(\dfrac{x}{\sqrt{1-x^2}}\right)\right], & |x|\leqslant 1,\\ \dfrac{\sqrt{x^2-1}}{x}\ln(x+\sqrt{x^2-1}), & |x|\geqslant 1.\end{cases} \tag{A5}$$

The other integrals needed are

$$i\mu^{\epsilon}\int\frac{d^{4-\epsilon}q}{(2\pi)^{4-\epsilon}}\frac{q^{\mu}}{(q^2-m^2)(q\cdot v-\Delta)}=\frac{v^{\mu}}{16\pi^2}[I_2(m,\Delta)+I_1(m)], \tag{A6}$$

$$i\mu^{\epsilon}\int\frac{d^{4-\epsilon}q}{(2\pi)^{4-\epsilon}}\frac{q^{\mu}q^{\nu}}{(q^2-m^2)(q\cdot v-\Delta)}=\frac{1}{16\pi^2}\Delta[J_1(m,\Delta)\,\eta^{\mu\nu}+J_2(m,\Delta)v^{\mu}v^{\nu}], \tag{A7}$$

with

$$J_1(m,\Delta)=\left(-m^2+\frac{2}{3}\Delta^2\right)\ln\left(\frac{m^2}{\mu^2}\right)+\frac{4}{3}(\Delta^2-m^2)F\left(\frac{m}{\Delta}\right)-\frac{2}{3}\Delta^2(1+\bar{\Delta})+\frac{1}{3}m^2(2+3\bar{\Delta})+\frac{2}{3}m^2-\frac{4}{9}\Delta^2, \tag{A8a}$$

$$J_2(m,\Delta)=\left(2m^2-\frac{8}{3}\Delta^2\right)\ln\left(\frac{m^2}{\mu^2}\right)-\frac{4}{3}(4\Delta^2-m^2)F\left(\frac{m}{\Delta}\right)+\frac{8}{3}\Delta^2(1+\bar{\Delta})-\frac{2}{3}m^2(1+3\bar{\Delta})-\frac{2}{3}m^2+\frac{4}{9}\Delta^2. \tag{A8b}$$

The functions $J_1(m,\Delta),J_2(m,\Delta)$ differ from the ones in the Boyd-Grinstein list of integrals [34] by the last two terms in Eq. (A8) that are of the order of $\mathcal{O}(m^2,\Delta^2)$. These additional finite terms originate from the fact that $\eta^{\mu\nu}$ is $4-\epsilon$ dimensional metric tensor.

The chiral loop integrals needed are

$$i\mu^{\epsilon}\int\frac{d^{4-\epsilon}q}{(2\pi)^{4-\epsilon}}\frac{1}{[(q+k)^2-m^2](q^2-m^2)}=\frac{1}{16\pi^2}N_0(m,k^2), \tag{A9}$$

$$i\mu^{\epsilon}\int\frac{d^{4-\epsilon}q}{(2\pi)^{4-\epsilon}}\frac{q^{\mu}}{[(q+k)^2-m^2](q^2-m^2)}=\frac{k^{\mu}}{16\pi^2}N_1(m,k^2)=-\frac{1}{2}\frac{k^{\mu}}{16\pi^2}N_0(m,k^2), \tag{A10}$$

$$i\mu^{\epsilon}\int\frac{d^{4-\epsilon}q}{(2\pi)^{4-\epsilon}}\frac{q^{\mu}q^{\nu}}{[(q+k)^2-m^2](q^2-m^2)}=-\frac{k^{\mu}k^{\nu}}{16\pi^2}N_2\times(m,k^2)-\frac{\eta^{\mu\nu}}{16\pi^2}N_3(m,k^2), \tag{A11}$$

where

$$N_0(m,k^2)=-\bar{\Delta}+1-H\left(\frac{k^2}{m^2}\right)+\ln\left|\frac{m^2}{\mu^2}\right|-i\pi\Theta\left(-\frac{m^2}{\mu^2}\right)\mathrm{sign}(\mu^2), \tag{A12}$$

$$N_2(m,k^2)=\frac{1}{3}\left\{\bar{\Delta}+\frac{7}{6}-2\frac{m^2}{k^2}+2\left(\frac{m^2}{k^2}-1\right)\times\left[1-\frac{1}{2}H\left(\frac{k^2}{m^2}\right)\right]-\ln\left(\frac{m^2}{\mu^2}\right)+i\pi\Theta\left(-\frac{m^2}{\mu^2}\right)\mathrm{sign}(\mu^2)\right\}, \tag{A13}$$

$$N_3(m,k^2)=\frac{1}{2}\left(m^2-\frac{k^2}{6}\right)\bar{\Delta}-\frac{1}{2}\left\{\frac{1}{3}(4m^2-k^2)\left[1-\frac{1}{2}H\left(\frac{k^2}{m^2}\right)\right]-\frac{4}{3}m^2+\frac{5}{18}k^2+\left(m^2-\frac{k^2}{6}\right)\times\left[\ln\left|\frac{m^2}{\mu^2}\right|-i\pi\Theta\left(-\frac{m^2}{\mu^2}\right)\mathrm{sign}(\mu^2)\right]\right\}, \tag{A14}$$

and

$$H(a)=\begin{cases}2\left[1-\sqrt{4/a-1}\arctan\left(\frac{1}{\sqrt{4/a-1}}\right)\right], & 0<a<4,\\ 2\left(1-\frac{1}{2}\sqrt{1-4/a}\left[\ln\left|\frac{\sqrt{1-4/a}+1}{\sqrt{1-4/a}-1}\right|-i\pi\Theta(a-4)\right]\right), & \text{otherwise}\end{cases} \tag{A15}$$

while m^2 is assumed to be positive.

APPENDIX B: *D* MESON DECAY CONSTANT

Here we list results for one-loop chiral corrections to D meson decay constants and use them to obtain coupling α_H from experimental data. The one-loop chiral corrections have been calculated in [23,34] using $\bar{\Delta}=0$, while the leading logs have been obtained already in [32,35],

$$f_D=\frac{\alpha_H}{\sqrt{m_D}}\left\{1+\frac{3g^2}{32\pi^2f^2}\left[\frac{3}{2}C(\Delta_{D^*D},m_\pi)+C(\Delta_{D_s^*D},m_K)+\frac{1}{6}C(\Delta_{D^*D},m_\eta)\right]-\frac{1}{32\pi^2f^2}\left[\frac{3}{2}I_1(m_\pi)+I_1(m_K)+\frac{1}{6}I_1(m_\eta)\right]\right\}, \tag{B1a}$$

$$f_{D_s}=\frac{\alpha_H}{\sqrt{m_D}}\left\{1+\frac{3g^2}{32\pi^2f^2}\left[2C(\Delta_{D^*D_s},m_K)+\frac{2}{3}C(\Delta_{D_s^*D_s},m_\eta)\right]-\frac{1}{32\pi^2f^2}\left[2I_1(m_K)+\frac{2}{3}I_1(m_\eta)\right]\right\}, \tag{B1b}$$

where $C(\Delta,m)=J_1(m,\Delta)+\Delta(\partial/\partial\Delta)J_1(m,\Delta)$, while $J_1(m,\Delta)$ and $I_1(m)$ can be found in Appendix A. These formulas are valid at the leading order in $1/m_Q$ [23,34]. Using $f=120$ MeV, $\mu=1$ GeV, and $\bar{\Delta}=1$ one gets the numerical values

$$f_D=\frac{\alpha_H}{\sqrt{m_D}}(1+0.18-0.37g^2), \tag{B2a}$$

$$f_{D_s}=\frac{\alpha_H}{\sqrt{m_D}}(1+0.35+0.38g^2). \tag{B2b}$$

To obtain the α_H coupling we use experimental data on decays of D mesons into leptons. From the experimental value for branching ratio $B(D_s\to\mu\nu_\mu)=(4.6\pm1.9)\times10^{-3}$ and the D_s decay time $\tau_{D_s}=(0.496^{+0.010}_{-0.009})\times10^{-12}s$, one gets $f_{D_s}=0.23\pm0.05$ GeV. Using this value and $g=0.27$ [25] in Eq. (B2b) we get $\alpha_H=0.23\pm0.04$ GeV$^{3/2}$, while for $g=0.57$ we obtain $\alpha_H=0.21\pm0.04$ GeV$^{3/2}$.

From Eq. (B2a) one can also calculate $f_D=0.194\pm0.045$ GeV ($f_D=0.17\pm0.04$ GeV), using $\alpha_H=0.23\pm0.04$ GeV$^{3/2}$ ($\alpha_H=0.21\pm0.04$ GeV$^{3/2}$) and $g=0.27$ ($g=0.57$). These values are in fair agreement with the recent lattice results [36].

APPENDIX C: HEAVY-LIGHT QUARK LOOP INTEGRALS

The integrals entering heavy quark loops like the ones in Fig. 8 are of the form

$$R_{p,q}\equiv\int\frac{d^dk}{(2\pi)^d}\frac{1}{(v\cdot k)^p}\frac{1}{(k^2-m^2)^q}. \tag{C1}$$

Performing a shift of momentum integration combined with Feynman parametrization, we obtain

$$R_{p,q}=2^p\frac{\Gamma(p+q)}{\Gamma(p)\Gamma(q)}K(p+q,p-1), \tag{C2}$$

where

$$K(n,r)\equiv\int_0^\infty d\lambda\int\frac{d^dl}{(2\pi)^d}\frac{\lambda^r}{(l^2-m^2-\lambda^2)^n}. \tag{C3}$$

One should notice that to obtain the result in Eq. (13), we have to do the identification

$$8iN_cG_H^2I_{HH}=1, \tag{C4}$$

where I_{HH} is a logarithmically divergent loop integral given below. (There is also a similar relation for g.) One should notice that some authors use an extra factor m_H, the mass of the heavy meson, in front of the right hand side of Eq. (13). Choosing the normalization in Eq. (13), it means that a factor $\sqrt{m_H}$ is included in the heavy meson field H_v. For the left-handed current in Eqs. (18) and (19) we find that we have to identify

$$\alpha_H=-4iN_cG_HI_{HW}, \tag{C5}$$

where I_{HW} is a quadratically divergent loop integral. The loop integral's (the I's) relevant for us are

$$I_{HH} \equiv mK(3,1) + K(3,2), \tag{C6}$$

$$I_{HW} \equiv K(2,1) + mK(2,0), \tag{C7}$$

$$I_{G1} \equiv K(3,1) + mK(3,0), \tag{C8}$$

$$I_{G2} \equiv K(3,1). \tag{C9}$$

The regularization can be done in various ways (various cut-off prescriptions or by $\overline{\mathrm{MS}}$) and each regularization correspond to slightly different versions of this type model [12,19–22,29].

Within dimensional regularization, the expressions for some values of n and r are listed below,

$$K(2,1) = \frac{i}{2(4\pi)^{d/2}} \frac{\Gamma(1-d/2)}{(m^2)^{1-d/2}}, \tag{C10}$$

$$K(3,1) = -\frac{i}{4(4\pi)^{d/2}} \frac{\Gamma(2-d/2)}{(m^2)^{2-d/2}}, \tag{C11}$$

$$K(3,2) = -\frac{i}{16(4\pi)^{d/2-1/2}} \frac{\Gamma(3/2-d/2)}{(m^2)^{3/2-d/2}}. \tag{C12}$$

where $m = m_\chi$. From the properties of the Γ function it is easy to see that

$$K(2,0) = -4K(3,2), \quad m^2 K(3,0) = (3-d)K(3,2). \tag{C13}$$

Comparing with a cut-off regularization, we see that $K(2,1)$ is quadratically and $K(3,1)$ is logarithmically divergent. In a primitive cutoff regularization $K(2,0)$ and $K(3,2)$ appear as linearly divergent [20], while they appear as finite in dimensional regularization.

Note also that some of the integrals C3 can be obtained as the limits of integrals listed in Appendix A if one lets $\Delta \to 0$. Thus one has the relations

$$K(2,0) = -\frac{i}{32\pi^2} \lim_{\Delta\to 0} \frac{1}{\Delta} I_2(m,\Delta), \tag{C14}$$

$$K(2,1) = \frac{i}{32\pi^2} \lim_{\Delta\to 0} [I_2(m,\Delta) + I_1(m)]$$
$$= \frac{i}{32\pi^2} I_1(m), \tag{C15}$$

$$K(2,2) = -\frac{i}{32\pi^2} \lim_{\Delta\to 0} \Delta J_2(m,\Delta). \tag{C16}$$

When soft gluon emission is included in Eq. (45) above, gluon condensate contributions should also be included in loop integrals I_{HH} and I_{HW}, as it is for f_π in the light sector [12,17]. However, we will not go into these details here. In order to produce a leading order estimate for the coupling G_H, we identify the logarithmically divergent integral contained in I_{HH} with f_π obtained as within the χQM. Then we obtain from Eq. (C4) the expression

$$G_H \simeq \frac{2\sqrt{m_\chi}}{f_\pi}. \tag{C17}$$

Furthermore, we identify the quadratic divergence contained in the loop integral I_{HW} gotten from the diagram in Fig. 8 (left) with the quark condensate of the light quark, which is also quadratically divergent. Then, similar to Eq. (C17), we obtain from Eq. (C5) to leading order

$$G_H \simeq -2\frac{m_\chi \alpha_H}{\langle \bar{q}q \rangle}. \tag{C18}$$

Combining Eqs. (C17) and (C18) we obtain

$$\alpha_H \simeq -\frac{\langle \bar{q}q \rangle}{f_\pi \sqrt{m_\chi}}, \tag{C19}$$

which for the values $m_\chi = 200$ MeV, $f_\pi = 131$ MeV, and $\langle \bar{q}q \rangle = (-240 \text{ MeV})^3$ reproduces the value for α_H cited in Appendix B. The fact that Eq. (C19) works well numerically gives some support to the leading order estimates in Eqs. (C17) and (C18). These relations will be slightly modified when the details are elaborated [29], but here we will be satisfied with the simple relations (C17)–(C19). Using Eqs. (C17) and (C18) we obtain

$$\beta \simeq -\frac{m_\chi f_\pi^2}{\langle \bar{q}q \rangle} \simeq \frac{1}{4}, \tag{C20}$$

to be used in Eqs. (50) and (51) .

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