Bounds for lepton flavor violation and the pseudoscalar Higgs boson in the general two Higgs doublet model using the $g-2$ muon factor

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Current experimental data from the $g-2$ muon factor seem to show the necessity of physics beyond the standard model (SM), since the difference between SM and experimental predictions is approximately 2.6σ. In the framework of the general two Higgs doublet model, we calculate the muon anomalous magnetic moment to get lower and upper bounds for the flavor changing Yukawa couplings in the leptonic sector. We also obtain lower bounds for the mass of the Higgs pseudoscalar (m_{A^0}) as a function of the parameters of the model.

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Current muon anomalous magnetic moment a_μ data have challenged the standard model (SM) and seem to open a window for new physics. Because of the high precision in the a_μ value, it gives very restrictive bounds on physics beyond the SM. Although the a_e measurement is about 350 more precise [1], a_μ is much more sensitive to new physics because contributions to a_l are usually proportional to m_l^2.

The most accurate measurement of a_μ hitherto has been provided by the Brookhaven Alternating Gradient Syncrotron (AGS) [2]. Their data have an error that is one-third that of the combined previous data [3]; Ref. [2] reports

$$a_{\mu^+} = 11659202(14)(6)\times 10^{-10}. \tag{1}$$

On the other hand, SM predictions for a_μ have been estimated taking into account the contributions from QED, hadronic loops, and electroweak corrections. The final current result is [1,2]

$$a_\mu^{SM} = 11\,659\,159.7(6.7)\times 10^{-10}. \tag{2}$$

Taking into account Eq. (2) we obtain

$$\Delta a_\mu^{NP} = a_\mu^{\text{exp}} - a_\mu^{SM} = 42.6(16.5)\times 10^{-10}, \tag{3}$$

where a_μ^{exp} is the world average experimental value [1]. Consequently, at 90% C.L.,

$$21.5\times 10^{-10} \leqslant \Delta a_\mu^{NP} \leqslant 63.7\times 10^{-10}. \tag{4}$$

Δa_μ^{NP} gives the room available for new physics, so a_μ^{exp} differs from a_μ^{SM} approximately in 2.6σ. Therefore physics beyond the SM is needed to achieve an acceptable theoretical experimental agreement. The most studied contributions to a_μ has been carried out in the framework of radiative muon mass models as well as the minimal supersymmetric standard model (MSSM), E_6 string-inspired models, and extensions of MSSM with an extra singlet [4].

Moreover, a very interesting suggestion to conciliate the new experimental data with theoretical predictions is to consider models that include flavor-changing neutral currents (FCNC) at the tree level. Interactions involving FCNC are forbidden at the tree level in the SM, but could be present at the one loop level as in the case of $b\to s\gamma$ [5], $K^0\to\mu^+\mu^-$ [6], $K^0-\overline{K}^0$ [7], $t\to c\gamma$ [8], etc. Many extensions of the SM permit FCNC at the tree level. For example, the introduction of new representations of fermions different from doublets produce them by means of the Z coupling [9]. Additionally, they are generated at the tree level by adding a second doublet to the SM [10], such couplings can be gotten as well in supersymmetry (SUSY) theories without R parity. The first one loop electroweak corrections for Δa_μ^{NP} were calculated in Refs. [11], and calculations coming from one loop Higgs boson contributions were carried out in Refs. [12].

Some other important new sources for FCNC might be provided by a muon collider, as the processes $\mu\mu\to\mu\tau(e\tau)$ mediated by Higgs exchange [13,14], which produce lepton flavor violation (LFV).

However, there are several mechanisms to avoid FCNC at the tree level. Glashow and Weinberg [15] proposed a discrete symmetry to supress them in the two Higgs doublet model (2HDM) which is the simplest one that exhibits these rare processes at the tree level. There are two kinds of models which are phenomenologically plausible with the discrete symmetry imposed. In the model type I, one Higgs doublet provides masses to the up-type and down-type quarks, simultaneously. In the model type II, one Higgs doublet gives masses to the up-type quarks and the other one to the down-type quarks. But the discrete symmetry [15] is not compulsory and both doublets may generate the masses of the quarks of up-type and down-type simultaneously; in such case we are in the model type III [16]. It has been used to search for FCNC at the tree level [17,18].

Recently, the 2HDM type III has been discussed and classified [19], depending on the way in which the flavor mixing matrices are rotated, showing that there are two types of rotations which generate two Lagrangians in the leptonic sector. The well known 2HDM types I and II could be gen-

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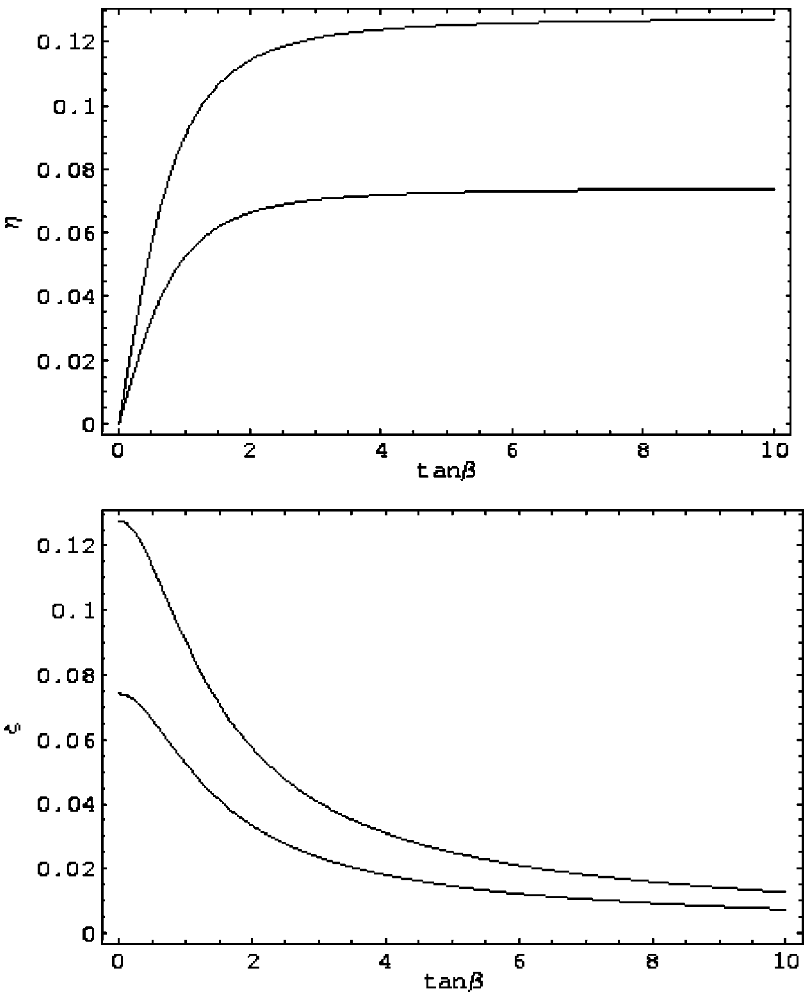

FIG. 1. Lower and upper bounds for $\eta_{\mu\tau}(\xi_{\mu\tau})$ vs $\tan\beta$ for rotations I and II using $m_{h^0}=m_{H^0}=150$ GeV and $m_{A^0}\rightarrow\infty$.

erated from them in the limit in which the FC vertices vanish. It has been pointed out that the bounds for the free parameters of the 2HDM type III depend on the rotation used for the mixing matrices.

In this paper, we calculate the contributions to Δa_μ^{NP} coming from the 2HDM, which includes FCNC at the tree level. We will constrain the FC vertex involving the second and third charged leptonic sector by using the result for Δa_μ^{NP}, Eq. (4). Additionally, we get lower bounds on the Higgs Pseudoscalar mass by taking into account the lower experimental value of Δa_μ^{NP} at 90% C.L. and making reasonable assumptions on the FC vertex.

The Yukawa's Lagrangian for the 2HDM type III, is as follows:

$$-\pounds_Y=\eta_{ij}^U\overline{Q}_{iL}\widetilde{\Phi}_1U_{jR}+\eta_{ij}^D\overline{Q}_{iL}\Phi_1D_{jR}+\eta_{ij}^E\overline{l}_{iL}\Phi_1E_{jR}+\xi_{ij}^U\overline{Q}_{iL}\widetilde{\Phi}_2U_{jR}+\xi_{ij}^D\overline{Q}_{iL}\Phi_2D_{jR}+\xi_{ij}^E\overline{l}_{iL}\Phi_2E_{jR}+\text{H.c.} \tag{5}$$

where $\Phi_{1,2}$ are the Higgs doublets, η_{ij} and ξ_{ij} are nondiagonal 3×3 matrices and *i*, *j* are family indices. In this work, we are interested only in neutral currents in the leptonic sector. We consider a CP-conserving model with a parametrization in which both Higgs doublets acquire a vacuum expectation value (VEV):

$$\langle\Phi_1\rangle_0=\begin{pmatrix}0\\ v_1/\sqrt{2}\end{pmatrix},\quad \langle\Phi_2\rangle_0=\begin{pmatrix}0\\ v_2/\sqrt{2}\end{pmatrix}. \tag{6}$$

The neutral mass eigenstates are given by [20]

$$\begin{pmatrix}G_Z^0\\ A^0\end{pmatrix}=\begin{pmatrix}\cos\beta & \sin\beta\\ -\sin\beta & \cos\beta\end{pmatrix}\begin{pmatrix}\sqrt{2}\,\mathrm{Im}\phi_1^0\\ \sqrt{2}\,\mathrm{Im}\phi_2^0\end{pmatrix},$$

$$\begin{pmatrix}H^0\\ h^0\end{pmatrix}=\begin{pmatrix}\cos\alpha & \sin\alpha\\ -\sin\alpha & \cos\alpha\end{pmatrix}\begin{pmatrix}\sqrt{2}\,\mathrm{Re}\,\phi_1^0-v_1\\ \sqrt{2}\,\mathrm{Re}\,\phi_2^0-v_2\end{pmatrix}, \tag{7}$$

where $\tan\beta=v_2/v_1$ and α is the mixing angle of the CP-even neutral Higgs sector. G_Z is the would-be Goldstone boson of Z and A^0 is the CP-odd neutral Higgs boson.

Now, to convert the Lagrangian (5) into mass eigenstates we make the unitary transformations

$$E_{L,R}=(V_{L,R})E_{L,R}^0 \tag{8}$$

from which we obtain the mass matrix

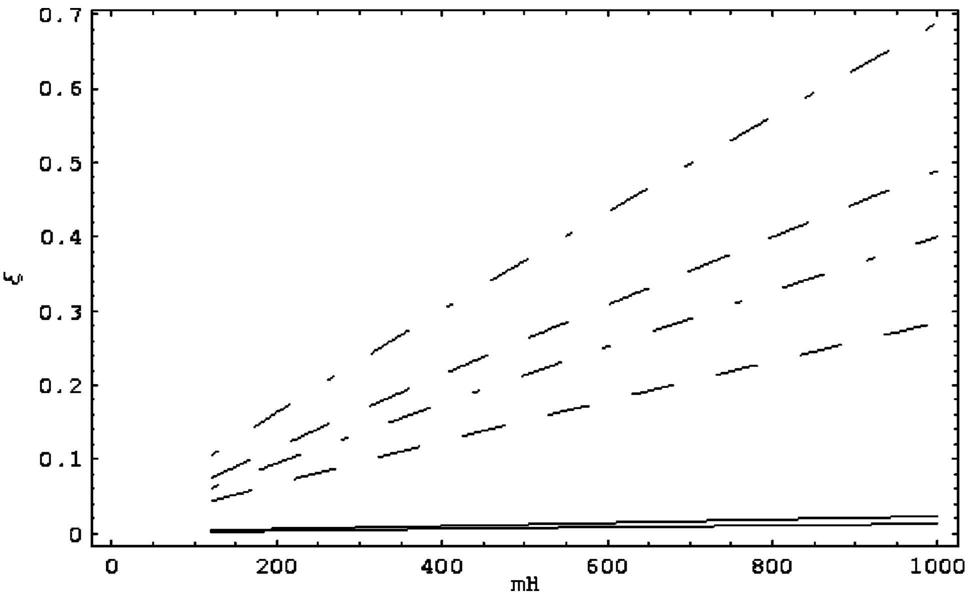

FIG. 2. Lower and upper bounds for $\xi_{\mu\tau}$ vs m_{H^0}, for rotation of type II, taking $m_{h^0}=m_{H^0}$ and $m_{A^0}\to\infty$; the pair of dash-dotted lines correspond to $\tan\beta=0.1$, the dashed lines are for $\tan\beta=1$, and the solid lines are for $\tan\beta=30$.

$$M_E^{diag}=V_L\left[\frac{v_1}{\sqrt{2}}\eta^{E,0}+\frac{v_2}{\sqrt{2}}\xi^{E,0}\right]V_R^{\dagger}, \tag{9}$$

where M_E^{diag} is the diagonal mass matrix for the three lepton families. From Eq. (9) we can solve for $\xi^{E,0}$ obtaining

$$\xi^{E,0}=\frac{\sqrt{2}}{v_2}V_L^{\dagger}M_E^{diag}V_R-\frac{v_1}{v_2}\eta^{E,0} \tag{10}$$

which we call a rotation of type I. Replacing it into Eq. (5), the expanded Lagrangian for the neutral leptonic sector is

$$\begin{aligned}-\pounds_{Y(E)}^{(I)}=&\frac{g}{2M_W\sin\beta}\bar{E}M_E^{diag}E(\sin\alpha H^0+\cos\alpha h^0)\\&+\frac{ig}{2M_W}\bar{E}M_E^{diag}\gamma_5 EG^0+\frac{ig\cot\beta}{2M_W}\bar{E}M_E^{diag}\gamma_5 EA^0\\&-\frac{1}{\sqrt{2}\sin\beta}\bar{E}\eta^E E[\sin(\alpha-\beta)H^0+\cos(\alpha-\beta)h^0]\\&-\frac{i}{\sqrt{2}\sin\beta}\bar{E}\eta^E\gamma_5 EA^0+\text{H.c.},\end{aligned} \tag{11}$$

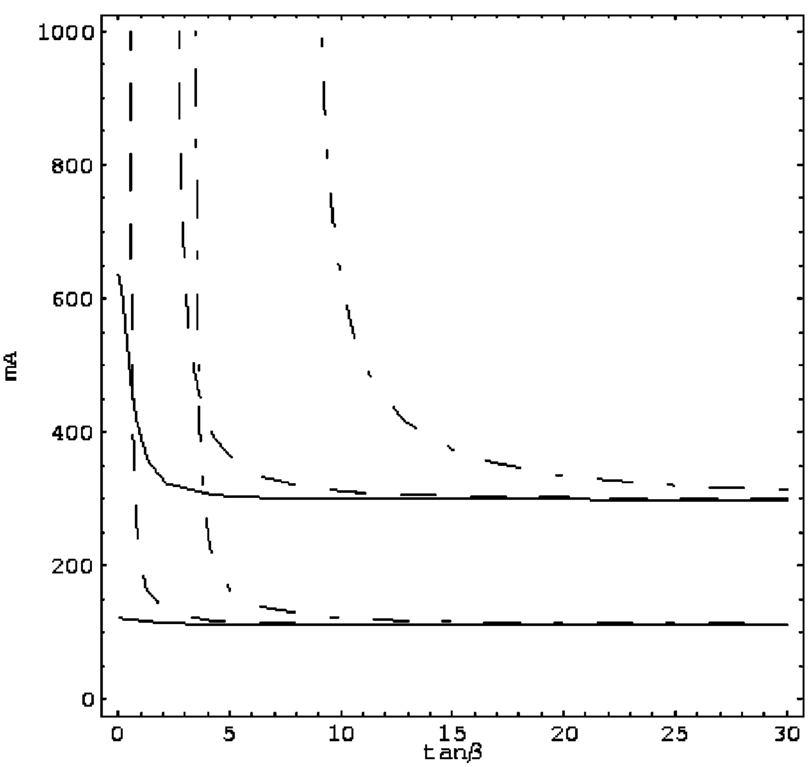

FIG. 3. Contour plot of m_{A^0} vs $\tan\beta$ using rotation type II and assuming $m_{h^0}=m_{H^0}$. Dash-dotted lines correspond to $\xi_{\mu\tau}=2.5\times10^{-4}$ for $m_{H^0}=110$ GeV (below) and $m_{H^0}=300$ GeV (above). Dashed lines correspond to $\xi_{\mu\tau}=2.5\times10^{-3}$ for $m_{H^0}=110$ GeV (below) and $m_{H^0}=300$ GeV (above). Finally, solid lines correspond to $\xi_{\mu\tau}=2.5\times10^{-2}$ for $m_{H^0}=110$ GeV (below) and $m_{H^0}=300$ GeV (above).

where the superindex (I) refers to the rotation type I. It is easy to check that Lagrangian (11) is just the one in the 2HDM type I [20], plus some FC interactions. Therefore we obtain the Lagrangian of the 2HDM type I from Eq. (11) by setting $\eta^E=0$. In this case it is clear that when $\tan\beta\to 0$ then η^E should go to zero, in order to have a finite contribution for FCNC at the tree level.

On the other hand, from Eq. (9) we can also solve for $\eta^{E,0}$ instead of $\xi^{E,0}$, to get

$$\eta^{E,0}=\frac{\sqrt{2}}{v_1}V_L^{\dagger}M_E^{diag}V_R-\frac{v_2}{v_1}\xi^{E,0} \tag{12}$$

which we call a rotation of type II. Replacing it into Eq. (5) the expanded Lagrangian for the neutral leptonic sector is

$$\begin{aligned}-\pounds_{Y(E)}^{(II)}=&\frac{g}{2M_W\cos\beta}\bar{E}M_E^{diag}E(\cos\alpha H^0-\sin\alpha h^0)\\&+\frac{ig}{2M_W}\bar{E}M_E^{diag}\gamma_5 EG^0-\frac{ig\tan\beta}{2M_W}\bar{E}M_E^{diag}\gamma_5 EA^0\\&+\frac{1}{\sqrt{2}\cos\beta}\bar{E}\xi^E E[\sin(\alpha-\beta)H^0+\cos(\alpha-\beta)h^0]\\&+\frac{i}{\sqrt{2}\cos\beta}\bar{E}\xi^E\gamma_5 EA^0+\text{H.c.}\end{aligned} \tag{13}$$

The Lagrangian (13) coincides with the one of the 2HDM type II [20], plus some FC interactions. So, the Lagrangian of the 2HDM type II is obtained setting $\xi^E=0$. In this case it is clear that when $\tan\beta\to\infty$ then ξ^E should go to zero, in order to have a finite contribution for FCNC at the tree level.

In the present report, we calculate Δa_μ^{NP} in the 2HDM with FC interactions. If we assume that $m_\mu^2\ll m_\tau^2$ and $m_\mu^2\ll m_{h^0,H^0,A^0}^2$ in the calculation of the Feynman integrals, the contribution at the one loop level from all neutral Higgs bosons is given by

$$\begin{aligned}\Delta a_\mu=&\frac{m_\mu m_\tau}{16\pi^2}\sum_i b_i^2\left[F(m_{H_i})+\frac{m_\mu}{3m_\tau}G(m_{H_i})\right]+a_i^2\left[F(m_{H_i})\right.\\&\left.-\frac{m_\mu}{3m_\tau}G(m_{H_i})\right],\end{aligned} \tag{14}$$

where

$$\begin{aligned}G(m_{H_i})&\equiv\left[\frac{2+3\hat{m}_{H_i}^2+6\hat{m}_{H_i}^2\ln(\hat{m}_{H_i}^2)-6\hat{m}_{H_i}^4+\hat{m}_{H_i}^6}{m_{H_i}^2(1-\hat{m}_{H_i}^2)^4}\right]\\F(m_{H_i})&=\frac{[3+\hat{m}_{H_i}^2(\hat{m}_{H_i}^2-4)+2\ln\hat{m}_{H_i}^2]\hat{m}_{H_i}}{m_{H_i}(1-\hat{m}_{H_i}^2)^3}\end{aligned} \tag{15}$$

with $\hat{m}_{H_i}=m_\tau/m_{H_i}$. The sum is over the index $i=m_{h^0},m_{H^0},m_{A^0}$. a_i,b_i are the coefficients of the Feynman rules for scalar and pseudoscalar Higgs bosons, respectively. We have neglected the contribution of the charged Higgs boson, because of two reasons: on one side, the contribution involves the neutrino mass and on the other hand, the CERN e^+e^- collider LEP bound on its mass is $m_{H^\pm}\geqslant 80.5$ GeV. Additionally, we can notice that $m_\mu/3m_\tau G(m_{H_i})\ll F(m_{H_i})$ and its contribution is negligible. However, we include it for completeness.

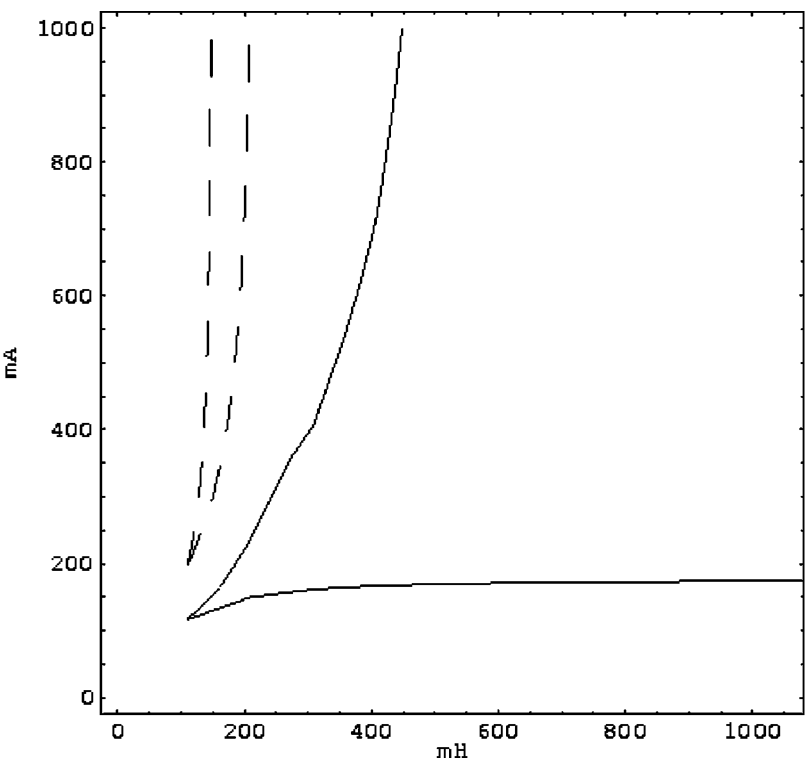

FIG. 4. Contour plot of m_{A^0} vs m_{H^0} setting $\tan\beta=1$, dashed lines correspond to $\xi_{\mu\tau}=2.5\times10^{-3}$ for $m_{h^0}=110$ GeV (right) and $m_{h^0}=m_{H^0}$ (left). Solid lines correspond to $\xi_{\mu\tau}=2.5\times10^{-2}$ for $m_{h^0}=110$ GeV (below) and $m_{h^0}=m_{H^0}$ (above).

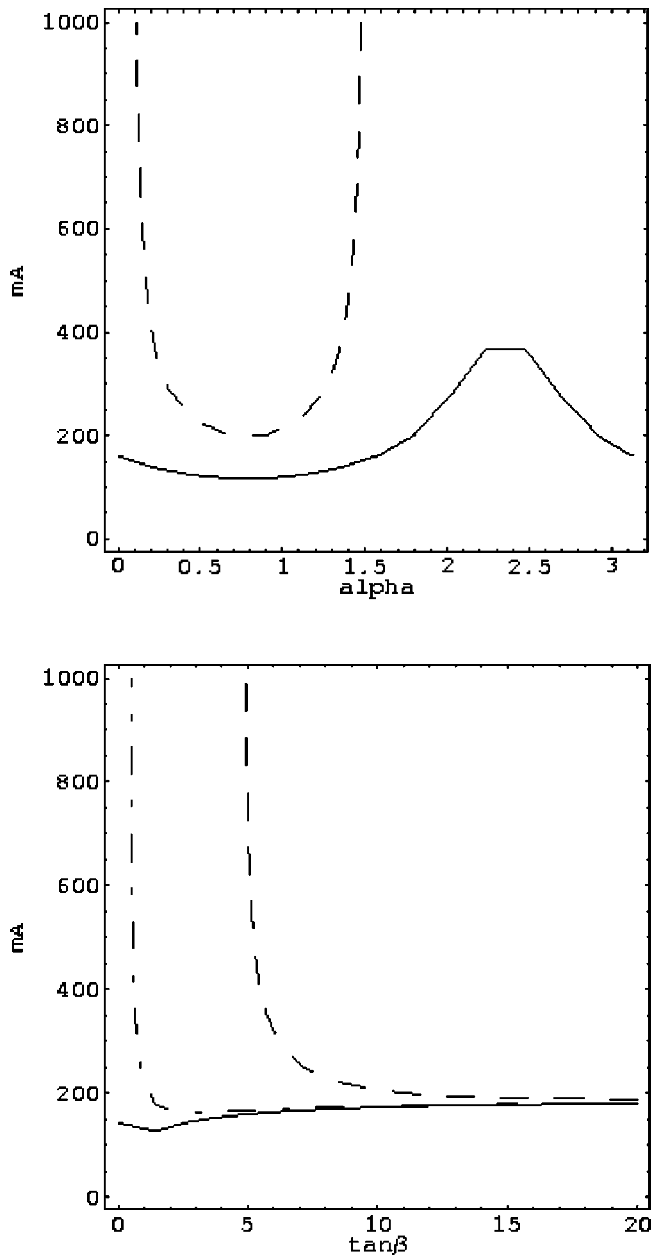

FIG. 5. (Top) Contour plot of m_{A^0} vs α, for rotation type II, with $m_{h^0}=110$ GeV, $m_{H^0}=300$ GeV and $\tan\beta=1$. Dashed line corresponds to $\xi_{\mu\tau}=2.5\times10^{-3}$, solid line corresponds to $\xi_{\mu\tau}=2.5\times10^{-2}$. (Bottom) Contour plot of m_{A^0} vs $\tan\beta$ for $m_{h^0}=110$ GeV, $m_{H^0}=300$ GeV, $\alpha=\pi/6$, and for rotation type II. Dash-dotted line corresponds to $\xi_{\mu\tau}=2.5\times10^{-4}$, dashed line corresponds to $\xi_{\mu\tau}=2.5\times10^{-3}$, and solid line corresponds to $\xi_{\mu\tau}=2.5\times10^{-2}$.

If we take into account the experimental data (4), we get some lower and upper bounds on the mixing vertex $\eta(\xi)_{\mu\tau}$ for the rotations of type I (II). In Fig. 1, we display lower and upper bounds for the FC vertices as a function of $\tan\beta$ for both types of rotations with $m_{h^0}=m_{H^0}=150$ GeV and $m_{A^0}\to\infty$. In the first case, rotation type I, the allowed region for $\eta_{\mu\tau}$ is $0.07\lesssim\eta_{\mu\tau}\lesssim0.13$ for large values of $\tan\beta$. Meanwhile, for rotation type II, the allowed region for small $\tan\beta$ is the same. From Lagrangian (11), which describes rotation type I, we can see that when $\tan\beta\to0$, $\eta_{\mu\tau}$ should go to zero as well to maintain a finite contribution to Δa_μ. This behavior can be seen from Fig. 1. For rotation type II occurs the same but in the limit $\tan\beta\to\infty$.

In Fig. 2, we show lower and upper bounds for the FC vertex as a function of m_{H^0} for rotation of type II when $m_{h^0}=m_{H^0}$ and $m_{A^0}\to\infty$. We see that the larger value for $\tan\beta$ the smaller value of $\xi_{\mu\tau}$. We only consider the case of rotation type II because there is a complementary behavior between both rotations as could be seen in Fig. 1. In particular, for $\tan\beta=1$, the behavior of the bounds for both rotations is the same.

Observe that according to the Feynman rules from Eqs. (11) and (13), the scalar (pseudoscalar) contribution to Δa_μ^{NP}, Eqs. (14) and (15), is positive (negative). Such fact permits us to impose lower bounds on the pseudoscalar Higgs mass, by using the lower limit in Eq. (4). According to this equation the room for new physics from $g-2$ muon factor is positive definite, and it is a new feature from most updated results [2].

Now, to take into account the experimental value (4), we should make a supposition about the value of the FC vertex. A reasonable assumption consists of taking the geometric average of the Yukawa couplings [21] i.e., $\eta(\xi)_{\mu\tau}\approx2.5\times10^{-3}$. Additionally, we shall use also the values $\eta(\xi)_{\mu\tau}\approx2.5\times10^{-2}$ and $\eta(\xi)_{\mu\tau}\approx2.5\times10^{-4}$ which are one order of magnitude larger and smaller than the former. Using these suppositions and the experimental value (4) we get lower bounds for m_{A^0} and they are plotted in Figs. 3–5.

Figure 3 displays m_{A^0} vs $\tan\beta$ using rotation type II with the three values of $\xi_{\mu\tau}$ mentioned above and setting $m_{h^0}=m_{H^0}$ with $m_{h^0}=110$, 300 GeV. It could be seen that in the limit of large $\tan\beta$, the lower limit reduces to $m_{A^0}\approx m_{h^0}$. The same behavior can be seen in rotation type I but the bound $m_{A^0}\approx m_{h^0}$ is gotten in the limit of small $\tan\beta$. We also see that the smaller value of $\xi_{\mu\tau}$ the stronger lower limit for m_{A^0}.

Figure 4 shows m_{A^0} vs m_{H^0} with $\xi_{\mu\tau}=2.5\times10^{-3}$, 2.5×10^{-2} and $\tan\beta=1$, using $m_{h^0}=m_{H^0}$ and $m_{h^0}=110$ GeV. With this settings, the value $\xi_{\mu\tau}=2.5\times10^{-4}$ is excluded. Using such specific arrangements, the bounds are identical in both types of rotations.

In Fig. 5 we suppose that $m_{h^0}=110$ GeV, $m_{H^0}=300$ GeV. The top figure shows the sensitivity of lower bounds on m_{A^0} with the mixing angle α, for rotation type II, taking $\tan\beta=1$. The value $\xi_{\mu\tau}=2.5\times10^{-4}$ is excluded again. The constraints are very sensitive to the α mixing angle for $\xi_{\mu\tau}=2.5\times10^{-3}$ but less sensitive for $\xi_{\mu\tau}=2.5\times10^{-2}$. The bottom figure shows m_{A^0} vs $\tan\beta$ for $m_{h^0}=110$ GeV, $m_{H^0}=300$ GeV, $\alpha=\pi/6$, for rotation type II and considering the same three values of $\xi_{\mu\tau}$. The m_{A^0} lower asymptotic limit for large $\tan\beta$ is approximately m_{h^0}.

In conclusion, we have found lower and upper bounds for the FC vertex $\eta(\xi)_{\mu\tau}$ in the context of the general 2HDM by using the allowed range for Δa_μ^{NP} at 90% CL and utilizing several sets of values for the parameters of the model. Additionally, in the limit $m_{A^0}\to\infty$, we get that for small (large) values of $\tan\beta$ the allowed range for the FC vertex $\eta_{\mu\tau}(\xi_{\mu\tau})$

becomes narrower, and both upper and lower bounds go to zero in the rotation of type I (II).

On the other hand, we have gotten lower bounds on the pseudoscalar Higgs mass of the 2HDM coming from the $g-2$ muon factor, by using the experimental value of Δa_μ^{NP} and making reasonable assumptions on the FC vertex $\eta(\xi)_{\mu\tau}$. Specifically, we have taken for $\eta(\xi)_{\mu\tau}$ the geometric average of the Yukawa couplings, and we also utilized values one order of magnitude larger and one order of magnitude smaller. Taking these three values for the FC vertex we find that smaller values for $\eta(\xi)_{\mu\tau}$ imply more stringent lower bounds for m_{A^0}. Additionally, assuming $m_{H^0}=m_{h^0}$, we show that in the limit of small (large) $\tan\beta$ the lower bound of m_{A^0} becomes merely $m_{A^0}\approx m_{h^0}$ for rotation of type I (II). In the case of different scalar masses, there is still a lower asymptotic limit for m_{A^0}. Notwithstanding, these lower constraints on m_{A^0} should be considered carefully, since for $\eta(\xi)_{\mu\tau}$ we can only make reasonable estimations, but they are unknown so far.

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