Locally localized gravity models in higher dimensions

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We explore the possibility of generalizing the locally localized gravity model in five space-time dimensions to arbitrary higher dimensions. In a space-time with a negative cosmological constant, there are essentially two kinds of higher-dimensional cousins which not only take an analytic form but also are free from the naked curvature singularity in a whole bulk space-time. One cousin is a trivial extension of five-dimensional model, while the other one is in essence in higher dimensions. One interesting observation is that in the latter model, only the anti–de Sitter (AdS_p) brane is physically meaningful, whereas de Sitter (dS_p) and Minkowski (M_p) branes are dismissed. Moreover, for the AdS_p brane in the latter model, we study the property of localization of various bulk fields on a single brane. In particular, it is shown that the presence of the brane cosmological constant enables a bulk gauge field and massless fermions to confine to the brane only by a gravitational interaction. We find a novel relation between the mass of the brane gauge field and the brane cosmological constant.

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I. INTRODUCTION

In recent years, an idea that our world is a 3-brane embedded in a higher-dimensional space-time, with some of the extra dimensions being macroscopically large, has attracted a lot of attention as a resolution of the hierarchy problem, supersymmetry breaking, the cosmological constant problem, and so on. In particular, Randall and Sundrum have found a solution to the five-dimensional Einstein equations with a Minkowski flat 3-brane in AdS_5 and have shown that the effects of four-dimensional gravity on the brane are reproduced without the need to compactify the fifth dimension [1,2]. (This model was generalized to the case of many branes in Refs. [3,4].)

One disadvantage of the Randall-Sundrum model [1] is the presence of a brane with negative tension. Although this brane is located at a fixed point of S^1/Z_2 orbifold in such a way that the fluctuation modes associated with the brane, which are necessarily physical ghost modes, do not appear, the existence of the negative tension brane violates the weak energy theorem in the bulk [5].

Another disadvantage in the Randall-Sundrum model is related to the localization of bulk fields on a brane [6]. In the conventional brane world scenario, the standard model gauge and matter fields are assumed to be localized on our brane, whereas gravity freely propagates in a bulk space-time. But this assumption is quite unnatural since we tacitly discriminate gravity from the other fields. Since the graviton corresponds to the fluctuation mode of the space-time geometry, it automatically sees the whole structure of the space-time and consequently lives in the bulk space-time. The physically plausible setup is then to treat the standard model gauge and matter fields on an equal footing with gravity and consider all the local fields as the fields living in the bulk space-time. From this context, we can regard the Randall-Sundrum model [2] as a successful model for the localization of gravity on a brane. However, it is well known that in the original Randall-Sundrum model it is very difficult to localize the gauge fields [7–10] and the massless fermions, those are, spin-1/2 massless Dirac spinor [11–14] and spin-3/2 massless gravitino [15], on a brane by a gravitational interaction.

Recently, there has been an interesting development which circumvents simultaneously the two disadvantages mentioned above [16–21]. The models deal with a single or two positive tension anti–de Sitter AdS_4 brane(s) in a five-dimensional anti–de Sitter space-time AdS_5, where four-dimensional gravity is induced on the AdS_4 brane owing to the localization of a massive and normalizable bound state [17]. Moreover, it was shown that all the standard model particles are localized on the AdS_4 brane only through the gravitational interaction [20]. For instance, the appearance of zero-mode with dependence of a fifth dimension supplies us with a novel mechanism for the localization of the bulk gauge field on the brane. Even if nature seems to favor a Minkowski brane M_4 with zero cosmological constant rather than an AdS_4 with negative cosmological constant, it is impossible to rule out the possibility that our world might have a very tiny negative cosmological constant which is consistent with the present observations. Interestingly enough, in the AdS_4 brane model the existence of a *massless* ''photon'' on a brane demands that the brane cosmological constant must be small in Planck units enough not to violate experiment [20].

The aim of this paper is to generalize this interesting model to higher dimensions. Such a generalization is of course of importance from the viewpoint of an underlying fundamental theory in higher dimensions such as ten-dimensional superstring theory.

We will regard the branes as *global* defects with the number p of longitudinal dimensions in a higher-dimensional space-time with D bulk dimensions and n extra transverse ones (so the equality $D=p+n$ holds). A set of n scalar fields with the Higgs potential, thereby breaking the *global* SO(n) symmetry to SO($n-1$) symmetry, are utilized to generate the *global* defects [22,23]. A topological argument

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$\Pi_{n-1}[\mathrm{SO}(n-1)]=Z$, which expresses the fact that a mapping of configuration space at spatial infinity to a vacuum manifold is topologically nontrivial, guarantees the stability of the defects under deformations.

The plan of the paper is as follows. In the next section we review the model setup and then in Sec. III we look for solutions to Einstein's equations. In Sec. IV we study the metric fluctuations and derive a Schrödinger-like equation. Because of a complicated form of the solution and the lack of the knowledge inside the core, it is difficult to understand an exact formula of Newton's potential so we shall be contented with some qualitative understanding of the solution. In Sec. V, we show that the zero-mode of bulk gauge field is normalizable owing to the presence of the cosmological constant, thereby leading to the localization of gauge field on a brane. But it is shown that the localization is not so sharp on the brane and spreads rather widely in a bulk. Section VI is devoted to the treatment of fermionic fields. Discussions and future works are summarized in Sec. VII.

II. MODEL SETUP

In this section, we shall review the construction of ''global'' topological defect model in higher dimensions [22,23]. The solutions to Einstein's equations which we shall derive below can be found in essence in the article of Olasagasti and Vilenkin [22,23], but we shall not only derive the solutions in a more unified metric ansatz but also examine their physical properties in detail from a different viewpoint. In this paper, we shall follow the notations and the conventions in our previous papers [24].

The action with which we start is that of gravity in general D dimensions, with the conventional Einstein-Hilbert action and some matter action which will be specified later

$$S=\frac{1}{2\kappa_D^2}\int d^Dx\sqrt{-g}(R-2\Lambda)+\int d^Dx\sqrt{-g}L_m. \quad (1)$$

Taking the variation of the action (1) with respect to the D-dimensional metric tensor g_{MN} we obtain Einstein's equations in D dimensions

$$R_{MN}-\frac{1}{2}g_{MN}R=-\Lambda g_{MN}+\kappa_D^2T_{MN}, \quad (2)$$

where the energy-momentum tensor is defined as

$$T_{MN}=-\frac{2}{\sqrt{-g}}\frac{\delta}{\delta g^{MN}}\int d^Dx\sqrt{-g}L_m. \quad (3)$$

To find the spherically symmetric solutions in the bulk, we shall adopt the following metric ansatz

$$\begin{aligned} ds^2&=g_{MN}dx^Mdx^N\\ &=g_{\mu\nu}dx^\mu dx^\nu+dr^2+g_{mn}dy^mdy^n\\ &=e^{-A(r)}\hat{g}_{\mu\nu}dx^\mu dx^\nu+dr^2+e^{-B(r)}d\Omega_{n-1}^2, \end{aligned} \quad (4)$$

where $M,N,\ldots,$ denote D-dimensional space-time indices, $\mu,\nu,\ldots,$ p-dimensional brane ones, and $m,n,\ldots,$ $(n-1)$-dimensional extra spatial ones, so the equality $D=p+n$ holds. (We assume $p\geq 4$.) We sometimes denote $g_{mn}=e^{-B(r)}\tilde{g}_{mn}(x^l)$. Note that the reason why we take account of this metric ansatz comes from the holographic principle where the ''radial'' coordinate r plays the role of scale of the AdS renormalization group, so it is straightforward to extend various results of AdS conformal field theory (CFT) correspondence such as ''c-theorem'' to the present case. Moreover, we shall take an ansatz for the energy-momentum tensor respecting the spherical symmetry:

$$\begin{aligned} &T^\mu_\nu=\delta^\mu_\nu t_0(r),\\ &T^r_r=t_r(r),\\ &T^{\theta_2}_{\theta_2}=T^{\theta_3}_{\theta_3}=\cdots=T^{\theta_n}_{\theta_n}=t_\theta(r), \end{aligned} \quad (5)$$

where $t_i(i=0,r,\theta)$ are functions of only the radial coordinate r.

Under these ansatz, after a straightforward calculation, Einstein's equations reduce to the forms

$$\begin{aligned} &e^A\hat{R}-\frac{p(n-1)}{2}A'B'-\frac{p(p-1)}{4}(A')^2\\ &-\frac{(n-1)(n-2)}{4}(B')^2+(n-1)(n-2)e^B-2\Lambda\\ &+2\kappa_D^2t_r=0, \end{aligned} \quad (6)$$

$$\begin{aligned} &e^A\hat{R}+(n-2)B''-\frac{p(n-2)}{2}A'B'-\frac{(n-1)(n-2)}{4}(B')^2\\ &+(n-2)(n-3)e^B+pA''-\frac{p(p+1)}{4}(A')^2\\ &-2\Lambda+2\kappa_D^2t_\theta=0, \end{aligned} \quad (7)$$

$$\begin{aligned} &\frac{p-2}{p}e^A\hat{R}+(p-1)\left(A''-\frac{n-1}{2}A'B'\right)-\frac{p(p-1)}{4}(A')^2\\ &+(n-1)\left[B''-\frac{n}{4}(B')^2+(n-2)e^B\right]-2\Lambda+2\kappa_D^2t_0=0, \end{aligned} \quad (8)$$

where the prime denotes the differentiation with respect to r, and $\hat{R}$ is the scalar curvature associated with the brane metric $\hat{g}_{\mu\nu}$. Here we define the cosmological constant on the $(p-1)$-brane Λ_p by the equation

$$\hat{R}_{\mu\nu}-\frac{1}{2}\hat{g}_{\mu\nu}\hat{R}=-\Lambda_p\hat{g}_{\mu\nu}. \quad (9)$$

In addition, the conservation law for the energy-momentum tensor $\nabla^MT_{MN}=0$ takes the form

$$t_r' = \frac{p}{2}A'(t_r - t_0) + \frac{n-1}{2}B'(t_r - t_\theta). \tag{10}$$

The formulation reviewed thus far [24] is rather general in that we have assumed only the metric ansatz (4). Here let us specify the model by fixing the matter action. Following Refs. [22,23], we shall take a multiplet of n scalar fields Φ^a with the Higgs potential

$$L_m = -\frac{1}{2}g^{MN}\partial_M\Phi^a\partial_N\Phi^a + \frac{\lambda}{4}(\Phi^a\Phi^a - \eta^2)^2, \tag{11}$$

from which the energy-momentum tensor takes the form

$$T_{MN} = \partial_M\Phi^a\partial_N\Phi^a - \frac{1}{2}g_{MN}\partial_P\Phi^a\partial^P\Phi^a + g_{MN}\frac{\lambda}{4}(\Phi^a\Phi^a - \eta^2)^2. \tag{12}$$

Then, the familiar ''hedgehog'' ansatz leads to a global defect

$$\Phi^a = f(r)\hat{r}^a, \tag{13}$$

where $\hat{r}^a$ is the unit vector on the $(n-1)$-sphere and the function $f(r)$ takes the form

$$f(0) = 0, \quad \lim_{r\to\infty} f(r) = \eta. \tag{14}$$

Namely, it is considered that the defect has $\Phi^a = 0$ at the center of the core and approaches the radial ''hedgehog'' configuration $\Phi^a = \eta\hat{r}^a$ outside the core. In this paper, we limit ourselves to the exterior solutions, where the configuration is given by $\Phi^a = \eta\hat{r}^a$. Note that this configuration becomes an accurate approximation as the coupling constant λ gets large. A big question about ''global'' defects is whether there could be a stable localized core or not.[1] This problem is closely connected with physics inside the core so now we cannot answer this important problem.

III. SOLUTIONS

In this section, we solve a set of Einstein's equations (6)–(8) derived in the previous section. In this paper, we pay attention to only the case of the bulk cosmological constant being negative, $\Lambda < 0$ in order to search higher dimensional analogs corresponding to an AdS_4 brane solution in AdS_5 [17].

First of all, let us notice that with the ansatz $\Phi^a = \eta\hat{r}^a$ which holds only outside the defect core, the energy-momentum tensor takes the forms

$$t_0 = t_r = -\frac{1}{2}(n-1)\eta^2 e^{B(r)}, \quad t_\theta = -\frac{1}{2}(n-3)\eta^2 e^{B(r)}, \tag{15}$$

which obviously satisfy the conservation law (10).

Next, to find analytic solutions we need to set up a more specific metric ansatz, for which we shall take the form

$$B(r) = cA(r) + d, \quad R_0^2 \equiv e^{-d}, \tag{16}$$

where c and d (or R_0) are constants, which will be later fixed by Einstein's equations. Then it is straightforward to solve Einstein's equations (6)–(8) whose solutions can be divided into two kinds of cousins. One cousin, being a trivial extension of branes in AdS_5, belongs to a class having $c = 0$. According to the signature of the brane cosmological constant, let us separate this class of solutions to three branes' solutions, de Sitter brane dS_p, Minkowski brane M_p, and anti–de Sitter brane AdS_p.[2]

(i) dS_p brane:

$$ds^2 = \sinh^2\omega r\, d\hat{s}_+^2 + dr^2 + R_0^2 d\Omega_{n-1}^2,$$

$$\hat{R} = -2\Lambda\frac{p-1}{n+p-2} > 0, \quad \Lambda_{dS} = -\Lambda\frac{(p-1)(p-2)}{p(n+p-2)} > 0. \tag{17}$$

(ii) M_p brane:

$$ds^2 = e^{\mp 2\omega r} d\hat{s}_0^2 + dr^2 + R_0^2 d\Omega_{n-1}^2,$$

$$\hat{R} = \Lambda_M = 0. \tag{18}$$

(iii) AdS_p brane:

$$ds^2 = \cosh^2\omega r\, d\hat{s}_-^2 + dr^2 + R_0^2 d\Omega_{n-1}^2$$

$$\hat{R} = 2\Lambda\frac{p-1}{n+p-2} < 0, \quad \Lambda_{AdS} = \Lambda\frac{(p-1)(p-2)}{p(n+p-2)} < 0. \tag{19}$$

Here ω, R_0^2 are, respectively, given by

$$\omega = \sqrt{\frac{-2\Lambda}{p(n+p-2)}}, \quad R_0^2 = \frac{1}{2\Lambda}(n+p-2)(n-2-\kappa_D^2\eta^2), \tag{20}$$

where $R_0^2 > 0$ requires $n-2-\kappa_D^2\eta^2 < 0$. This class of solutions has been first derived in Ref. [22]. The common feature in this class is that the $(n-1)$ sphere has a constant radius R_0, because of which we have called it a *trivial* extension of branes in AdS_5 in the above. Indeed, it is easy to show that this class of solutions shares the same properties such as the corrections to Newton's law as for corresponding

[1] In the absence of gravity, in other words, in Minkowski space, Virial theorem tells us that for $D \geqslant 3$ there are no such static solutions. This theorem is circumvented when gravity switches on and there is a negative cosmological constant as in the case at hand.

[2] In this paper, we consider only the maximally symmetric solutions on a brane.

five-dimensional cases. Thus we shall not consider this class of solutions anymore in this paper.

A different class of solutions are provided when $c=1$. Again we shall present the solutions below by following the signature of the brane cosmological constant.

(i) dS_p brane:

$$ds^2=\sinh^2 \omega r d\hat{s}_+^2+dr^2+R_0^2 \sinh^2 \omega r d\Omega_{n-1}^2,$$

$$\hat{R}=-2\Lambda \frac{p}{n+p-1}>0,$$

$$\Lambda_{\rm dS}=-\Lambda \frac{p-2}{n+p-1}>0, \quad n-2-\kappa_D^2\eta^2>0. \tag{21}$$

(ii) M_p brane:

$$ds^2=e^{\mp 2\omega r}d\hat{s}_0^2+dr^2+R_0^2 e^{\mp 2\omega r}d\Omega_{n-1}^2$$

$$\hat{R}=\Lambda_M=0, \quad n-2-\kappa_D^2\eta^2=0. \tag{22}$$

(iii) AdS_p brane:

$$ds^2=\cosh^2 \omega r d\hat{s}_-^2+dr^2+R_0^2 \cosh^2 \omega r d\Omega_{n-1}^2,$$

$$\hat{R}=2\Lambda \frac{p}{n+p-1}<0,$$

$$\Lambda_{\rm AdS}=\Lambda \frac{p-2}{n+p-1}<0, \quad n-2-\kappa_D^2\eta^2<0. \tag{23}$$

Here ω, R_0^2 are, respectively, given by

$$\omega=\sqrt{\frac{-2\Lambda}{(n+p-2)(n+p-1)}},$$

$$R_0^2=-\frac{1}{2\Lambda}(n+p-2)|n-2-\kappa_D^2\eta^2|, \tag{24}$$

but in the case of the Minkowski brane M_p, R_0 is a free parameter. This class of solutions has been also in essence derived in Ref. [22] but with a different metric ansatz from ours. Note that one advantage of our metric ansatz (16) over the ones in Ref. [22] is that we have derived two classes of solutions in a unified way, while the authors in Ref. (16) have set up different metric ansatze and needed the change of variables to reach the forms listed in the above.

Now let us attempt to understand the solutions (21)–(23) in more detail. To do so, let us calculate the D-dimensional scalar curvature under the ansatz (4) whose result is given by

$$\begin{aligned} R&=g^{MN}R_{MN}\\ &=e^A\hat{R}+pA''+(n-1)B''-\frac{p(p+2)}{4}(A')^2\\ &\quad-\frac{p(n-1)}{2}A'B'-\frac{n(n-1)}{4}(B')^2+(n-1)(n-2)e^B. \end{aligned} \tag{25}$$

In particular, the last term in R reveals that the cases of $n=1,2$ are qualitatively different from higher-dimensional cases $n\geqslant 3$. The reason is that for $n=1$ (domain wall) the extra space is flat and for $n=2$ (stringlike defect) the extra space is still conformally flat, while for $n\geqslant 3$ the extra space is essentially curved [25]. The presence of this term makes many solutions to Einstein's equations in higher dimensions physically uninteresting owing to the appearance of the naked curvature singularity in the bulk space-time. Some people do not regard the appearance of the naked curvature singularity as a sick property of solutions by taking the optimistic attitude that such a singularity would be smoothed by quantum effects or string theory corrections. In contrast, we consider the naked curvature singularity to be a serious problem of solutions and impose a strict criterion that classical solutions to Einstein's equations should be free from the naked curvature singularity.[3]

Imposing the singularity-free condition as the physical requirement, for $n\geqslant 3$ the dS_p brane in Eq. (21) must be dismissed from physical solutions. Note that in this case, the real problem is that the line element is singular at $r=0$ even in the absence of a defect ($\eta=0$) [22]. For $n=2$, dS_p brane is not the solution owing to the relation $n-2-\kappa_D^2\eta^2>0$ when there is no defect ($\eta=0$), so we also dismiss this case. Of course, for $n=1$, dS_p brane is physical and corresponds to the dS domain wall solution.

Next, in M_p brane Eq. (22), for $n\geqslant 3$, the solution with the upper sign has the naked curvature singularity at the spatial infinity, so we dismiss this solution. On the other hand, the solution with the lower sign is free from the curvature singularity, but it turns out that the solution cannot localize gravity on a defect, so we also dismiss this case. The remaining possibilities are when $n=1,2$. For $n=2$, the solution corresponds to Gregory's solution [27,28,24] and as seen from the relation $n-2-\kappa_D^2\eta^2=0$ this solution describes a *local* stringlike defect so we also dismiss this solution from our present consideration. The solution in the case of $n=1$ is nothing but the Randall-Sundrum solution [1,2] (when $p=4$).

We are ready to analyze AdS_p brane Eq. (23) in a similar manner. For $n=1$, the solution obviously corresponds to an AdS_p brane in AdS_{p+1} [17]. Note that for $n\geqslant 2$ the solution is completely free from the curvature singularity and constitutes a higher-dimensional nontrivial extension of an

[3]The existence of the curvature singularity at the origin $r=0$ might be admissible since in some cases this singularity could be identified with the core of the brane [26].

AdS_p brane in AdS_{p+1}. Remarkably, as shown later, this solution localizes all local bulk fields on a defect only through the gravitational interaction.

Before closing this section, let us summarize the results obtained. We have derived two classes of classical solutions to Einstein's equations in higher dimensions. One class of solutions is a trivial extension of the domain wall solutions. The other class of solutions is a nontrivial extension, but almost all solutions except AdS_p brane are unphysical because of the existence of the naked curvature singularity and the nonlocalization of gravity on a defect. It is rather surprising that in higher dimensions ($n \geqslant 2$) only the AdS_p brane solution is selected as a physical solution, while in the case of $n=1$ domain wall three types of brane, dS_p, M_p, and AdS_p, are permissible.

IV. GRAVITATIONAL FLUCTUATIONS

In the following sections, we shall turn our attention to the properties of an AdS_p brane solution (23) in higher-dimensional space-time. The aim of this section is to study the gravitational fluctuations around the background (23).

First, let us rewrite the metric (23) in terms of the conformal coordinates

$$\begin{aligned} ds^2 &= \cosh^2 \omega r \hat{g}_{\mu\nu} dx^\mu dx^\nu + dr^2 + R_0^2 \cosh^2 \omega r d\Omega_{n-1}^2 \\ &= e^{-A(z)} (\hat{g}_{\mu\nu} dx^\mu dx^\nu + dz^2 + R_0^2 d\Omega_{n-1}^2), \end{aligned} \tag{26}$$

where $e^{-A(z)}$ and the relation between two coordinate systems are, respectively, given by

$$e^{-A(z)} = \frac{1}{\sin^2 \omega z},$$

$$e^{\omega r} = \tan \frac{1}{2} \omega z. \tag{27}$$

Since the ''radial'' coordinate r runs from 0 to ∞, this relation yields the range of z, which is $\pi/2\omega \leqslant z \leqslant \pi/\omega$.

We will only consider the transverse, traceless fluctuations around the background metric (26) in the conformal z coordinates

$$ds^2 = e^{-A(z)} \{ [\hat{g}_{\mu\nu} + h_{\mu\nu}(x^M)] dx^\mu dx^\nu + dz^2 + R_0^2 d\Omega_{n-1}^2 \}, \tag{28}$$

where $\nabla^\mu h_{\mu\nu} = g^{\mu\nu} h_{\mu\nu} = 0$. Then, it is straightforward to show that Einstein's equations reduce to the form of the linearized equations

$$\frac{1}{\sqrt{-g}} \partial_M (\sqrt{-g} g^{MN} \partial_N h_{\mu\nu}) - 2\Lambda h_{\mu\nu} = 0. \tag{29}$$

Given the symmetries of the background metric, we separate variables as

$$h_{\mu\nu}(x^M) = \phi_{\mu\nu}(x^\mu) \check{Z}_{lm}(z) Y_{lm_i}(\Omega), \tag{30}$$

where $Y_{lm_i}(\Omega)$ are the spherical harmonics for the $(n-1)$-sphere with eigenvalue $\Delta_l = l(l+n-2)$. And $\phi_{\mu\nu}(x^\mu)$ satisfy the equations of motion $(\hat{\Box} - 2\Lambda e^{-A}) \phi_{\mu\nu} = m_0^2 \phi_{\mu\nu}$ with the definition of $\hat{\Box} \equiv (1/\sqrt{-\hat{g}}) \partial_\mu (\sqrt{-\hat{g}} \hat{g}^{\mu\nu} \partial_\nu)$. Equations (29) then reduce to

$$e^{[(D-2)/2]A} \partial_z (e^{[(D-2)/2]A} \partial_z \check{Z}_{lm}) + m^2 \check{Z}_{lm} = 0, \tag{31}$$

where $m^2 = m_0^2 + \Delta_l / R_0^2$. After changing to a new function

$$\check{Z}_{lm} = e^{[(D-2)/4]A} Z_{lm}, \tag{32}$$

we find a Schrödinger-like equation for Z_{lm}:

$$[-\partial_z^2 + V(z)] Z_{lm}(z) = m^2 Z_{lm}(z), \tag{33}$$

where the potential is of the form

$$V(z) = \frac{(D-2)^2}{16} (A')^2 - \frac{D-2}{4} A'', \tag{34}$$

with the prime denoting the differentiation with respect to z. If we introduce a new variable $w \equiv \omega z$, taking the range $\pi/2 \leqslant w \leqslant \pi$, instead of z and make use of the concrete expression of $A(z)$ in Eq. (27), we finally arrive at the equation

$$[-\partial_w^2 + U(w)] Z(w) = E Z(w), \tag{35}$$

where we have omitted to write the indices l,m on $Z(w)$ explicitly, and $U(w)$ and E are, respectively, defined as

$$U(w) = -\frac{(D-2)^2}{4} + \frac{D(D-2)}{4} \frac{1}{\sin^2 w},$$

$$E = \frac{m^2}{\omega^2}. \tag{36}$$

To have the second-rank linear differential equation well defined, we need to impose boundary conditions at $w = \pi/2, \pi$. The boundary condition at $w = \pi$ is the Dirichlet condition, $Z(\pi) = 0$, since the potential $U(w)$ becomes an infinity there. The delicate problem is what boundary condition we have to impose at $w = \pi/2$ where there is the core of a topological defect. We have only solved Einstein's equations in the exterior region outside the core so that in principle we have no knowledge about physics inside the core, which makes it difficult to set up the boundary condition at $w = \pi/2$. However, the condition that the differential operator should be self-adjoint, which is necessary for Z to have a complete basis, requires us to choose a homogeneous boundary condition at $w = \pi/2$:

$$\xi_1 Z'\left(\frac{\pi}{2}\right) + \xi_2 Z\left(\frac{\pi}{2}\right) = 0, \tag{37}$$

where ξ_1, ξ_2 are constants and the prime now denotes the differentiation with respect to w. Thus physics inside the core of a defect should satisfy this boundary condition to have a smooth continuity between inside and outside the core of a defect. (Here for simplicity we have neglected the core size.)

With these results in hand, we are ready to study a solution to Eq. (35). After some elementary manipulation [16–19], a solution is given by a linear combination of Gauss's hypergeometric function F:

$$Z=\frac{A_1}{(\sin w)^{(D-2)/2}}F\left(-\frac{D-2}{4}+\frac{\sqrt{4E+(D-2)^2}}{4},-\frac{D-2}{4}-\frac{\sqrt{4E+(D-2)^2}}{4},\frac{1}{2};\cos^2 w\right)$$
$$+\frac{A_2\cos w}{(\sin w)^{(D-2)/2}}F\left(-\frac{D-4}{4}+\frac{\sqrt{4E+(D-2)^2}}{4},-\frac{D-4}{4}-\frac{\sqrt{4E+(D-2)^2}}{4},\frac{3}{2};\cos^2 w\right), \tag{38}$$

where A_1,A_2 are integration constants. At this stage, given the boundary condition $Z(\pi)=0$, we find an equation

$$A_2=2A_1\frac{\Gamma\left(\frac{D+2}{4}-\frac{\sqrt{4E+(D-2)^2}}{4}\right)\Gamma\left(\frac{D+2}{4}+\frac{\sqrt{4E+(D-2)^2}}{4}\right)}{\Gamma\left(\frac{D}{4}-\frac{\sqrt{4E+(D-2)^2}}{4}\right)\Gamma\left(\frac{D}{4}+\frac{\sqrt{4E+(D-2)^2}}{4}\right)}. \tag{39}$$

The remaining work is to impose the boundary condition (37) to fix the integration constants A_1,A_2, but it is a very delicate problem because of a complicated structure of Gauss's hypergeometric function and in consequence only the numerical analysis is available. We leave this numerical analysis for a future work. Here we shall investigate only a set of massive excited states which are supported by $U(w)$. Then the natural choice of the boundary condition at $w=\pi/2$ is the Neumann boundary condition $Z'(\pi/2)=0$. Together with this boundary condition and Eq. (39), we obtain $A_2=0$ and eigenvalues of Eq. (35)

$$E_k=k(k+D-2), \tag{40}$$

where $k=1,2,\ldots$. This equation then gives the natural higher dimensional generalization of mass formula of Kaluza-Klein (KK) states in the AdS_4 brane [17]:

$$m_k^2=-\frac{2}{(p-2)(n+p-2)}k(k+D-2)\Lambda_{\mathrm{AdS}}, \tag{41}$$

where we have used Eqs. (23), (24), and (36). Hence, in the $\Lambda_{\mathrm{AdS}}\to 0$ limit, these states become massless degenerate states, thereby giving rise to corrections to Newton's law whose size is of the order $\mathcal{O}(\sqrt{\Lambda_{\mathrm{AdS}}})$.

On the other hand, a massive bound state, which is trapped and generates gravity on an AdS brane, is supported by the attractive potential around the core. In the model at hand, the information about this attractive potential is implicitly included in the boundary condition (37). Recall that the boundary condition of AdS_4 brane in AdS_5 certainly satisfies this equation. Anyway in order to understand this problem completely, it would be necessary to construct a physically plausible core model.

V. LOCALIZATION OF GAUGE FIELDS

In this section we are willing to consider the localization of spin-1 U(1) vector field on an AdS_p brane (23). Incidentally the generalization to the non-Abelian gauge fields is straightforward. As mentioned in Sec. I, it is well known that in the original Randall-Sundrum model the gauge fields cannot be localized on a domain wall by the gravitational interaction [7–10]. Since we have various gauge fields in our world, the impossibility of confining gauge fields to a brane imposes a serious drawback on the scenario of brane world. Of course, there might be some ingenious mechanism for the localization of gauge fields by invoking additional interactions except the gravitational one [7], but we believe that such a mechanism is artificial and the universal interaction, that is, the gravitational interaction should provide us with the localization mechanism for the whole local fields including gauge fields. From this context, it is of interest to ask whether or not AdS_p brane (23) presents the localization mechanism for the gauge fields.

Let us start with the familiar action of U(1) gauge field

$$S_1=-\frac{1}{4}\int d^Dx\sqrt{-g}g^{MN}g^{RS}F_{MR}F_{NS}, \tag{42}$$

where $F_{MN}=\partial_M A_N-\partial_N A_M$. The equations of motion become $\partial_M(\sqrt{-g}g^{MN}g^{RS}F_{NS})=0$. To study the localization of the gauge field, it is convenient to use the conformal z coordinates (26). In the coordinates, for simplicity, we shall focus on only the brane gauge field $A_\mu(x^M)$ and set $A_z=A_{\theta_i}=0$. Then, we look for a solution with the form of $A_\mu(x^M)=a_\mu(x^\lambda)u(z)\chi(y^m)$, where y^m denote the angular coordinates. Here we assume the following equations of motion $\hat{\nabla}^\mu a_\mu=\partial^\mu f_{\mu\nu}=\partial_m(\sqrt{\tilde{g}}\tilde{g}^{mn}\partial_n\chi)=0$ where $f_{\mu\nu}=\partial_\mu a_\nu$

$-\partial_\nu a_\mu$. With these ansatze, the equations of motion reduce to a single differential equation

$$\partial_z[e^{(-D/2+2)A(z)}\partial_z u(z)]=0. \tag{43}$$

In the case of a Minkowski brane, we have selected a constant zero-mode solution $u(z)=\text{const}$, which leads to nonlocalization of the vector fields [10]. On the other hand, in the case of an AdS brane, a new solution is available, which is given by $e^{(-D/2+2)A(z)}\partial_z u(z)=\text{const}\neq 0$. (Note that this solution is not localized on a Minkowski brane, either.) As a result, we obtain the following solution to Eq. (43). For $D=2k+3$ $(k=1,2,3,\dots)$,

$$u(z)=\frac{\alpha}{\omega}\frac{(-1)^k}{2^{2(k-1)}}\sum_{l=0}^{k-1}(-1)^l\binom{2k-1}{l}\frac{\cos[(2k-2l-1)\omega z]}{2k-2l-1}+\beta, \tag{44}$$

and for $D=2k+4$ $(k=1,2,3,\dots)$,

$$u(z)=\frac{\alpha}{\omega}\frac{(-1)^k}{2^{2k}}\left\{\sum_{l=0}^{k-1}(-1)^l\binom{2k}{l}\frac{\sin[2(k-l)\omega z]}{k-l}+(-1)^k\binom{2k}{k}\omega z\right\}+\beta, \tag{45}$$

where α,β are integration constants.

We would like to investigate whether this solution is localized on an AdS_p brane or not. The substitution of this solution into the action leads to

$$\begin{aligned} S_1^{(0)}&=-\frac{1}{4}\int d^Dx\sqrt{-g}g^{MN}g^{RS}F_{MR}^{(0)}F_{NS}^{(0)}\\ &=-\frac{1}{4}\int d^pxdzd^{n-1}y\sqrt{-\hat{g}}\sqrt{\tilde{g}}e^{(-D/2+2)A(z)}\\ &\quad\times[u^2\chi^2\hat{g}^{\mu\nu}\hat{g}^{\rho\sigma}f_{\mu\rho}f_{\nu\sigma}+2(\partial_z u)^2\chi^2\hat{g}^{\mu\nu}a_\mu a_\nu\\ &\quad+2u^2\tilde{g}^{mn}\partial_m\chi\partial_n\chi\hat{g}^{\mu\nu}a_\mu a_\nu]. \end{aligned} \tag{46}$$

Here we have carefully kept the KK mass term since we wish to examine later whether this solution leads to *massless* ‘‘photon’’ on a brane. The localization condition of this mode on a brane requires the integral over z in front of the kinetic term to be finite since the integral over the angular variables $\int d^{n-1}y\sqrt{\tilde{g}}\chi(y)^2$ is in general finite. Thus let us consider this integral first:

$$\begin{aligned} I_1&=\int dz e^{(-D/2+2)A(z)}u^2(z)\\ &=\int_{\pi/2\omega}^{\pi/\omega}dz\frac{1}{(\sin\omega z)^{D-4}}u^2(z). \end{aligned} \tag{47}$$

The expressions (44), (45) for $u(z)$ become more complicated as the number of space-time dimensions gets larger, so below we shall present explicitly only the results of the two simplest cases $D=5,6$, belonging to each branch of solutions although we have examined some remaining lower-dimensional cases and found similar results and repeated pattern depending on $D=2k+3$ or $D=2k+4$.

In the case of $D=5,p=4,n=1$, that is, an AdS_4 brane in AdS_5 [17], from Eq. (47) the integral $I_1^{D=5}$ reads

$$I_1^{D=5}=\int_{\pi/2\omega}^{\pi/\omega}dz\frac{1}{\sin\omega z}\left(-\frac{\alpha}{\omega}\cos\omega z+\beta\right)^2, \tag{48}$$

which is in general divergent, but only when the equality $\beta=-\alpha/\omega$ holds, does it become finite. Henceforth, we shall consider this specific case. Then, it is straightforward to calculate the above integral as well as the second integral over z in Eq. (46) associated with the KK mass term. [Note that in $D=5$ there does not exist the third term in the right-hand side in Eq. (46).] The result is given by

$$S_1^{(0)}=-\frac{1}{4}\int d^4x\sqrt{-\hat{g}}\times\left[\frac{\alpha^2}{\omega^3}(-1+2\log 2)\hat{g}^{\mu\nu}\hat{g}^{\rho\sigma}f_{\mu\rho}f_{\nu\sigma}+\frac{2\alpha^2}{\omega}\hat{g}^{\mu\nu}a_\mu a_\nu\right]. \tag{49}$$

The quantities in front of the kinetic and the mass terms are obviously finite, so the gauge field is localized on an AdS_4 brane, which is to be contrasted with the case of a Minkowski brane [6].

At this stage, it is worthwhile to examine the mass of the brane gauge field. Provided that we redefine the brane gauge field a_μ as

$$\frac{\alpha}{\omega^{3/2}}\sqrt{-1+2\log 2}\,a_\mu\to a_\mu, \tag{50}$$

Eq. (49) reads

$$S_1^{(0)}=-\frac{1}{4}\int d^4x\sqrt{-\hat{g}}\left[\hat{g}^{\mu\nu}\hat{g}^{\rho\sigma}f_{\mu\rho}f_{\nu\sigma}+\frac{2\omega^2}{-1+2\log 2}\hat{g}^{\mu\nu}a_\mu a_\nu\right]. \tag{51}$$

From this equation, we can read off the mass of the brane gauge field, which is expressed in terms of the brane cosmological constant by using Eqs. (23),(24) as

$$m^2=\frac{\omega^2}{-1+2\log 2}=-\frac{1}{3}\frac{1}{-1+2\log 2}\Lambda_{\text{AdS}}. \tag{52}$$

The physical condition that the U(1) gauge field a_μ must be *massless* ‘‘photon’’ on an AdS_4 brane requires that the brane cosmological constant is small enough. It is very intriguing that in the present brane model the smallness of the brane cosmological constant is directly connected with the smallness of mass of the brane gauge field, which, we think, is a miracle in the brane world scenario. From the current experimental data, the bound on the photon mass is $m < 2\times 10^{-16}$ eV so our relation (52) yields a weaker constraint on the four-dimensional cosmological constant although it does not conflict with the current experimental data.

As a remark, let us notice that when the equality $\beta = -\alpha/\omega$ holds, our solution reduces to the form

$$u(z) = -\frac{2\alpha}{\omega}\cos^2\frac{\omega z}{2}. \tag{53}$$

Similar to the graviton considered in the previous section, this solution satisfies the Dirichlet condition at $z=\pi/\omega$, where $u(z)=0$. It is quite of interest that the requirement of the localization for the gauge field naturally leads to the same boundary condition as the other bosonic fields. (We can show that scalar field also satisfies the same boundary condition $z=\pi/\omega$.)

As another remark, let us check if the bulk gauge field is sharply localized on a brane or spreads rather widely in a bulk. For this, it is useful to change the z coordinates to the radial coordinate whose relation can be found in Eq. (27) and then examine the normalized zero mode in an AdS_4. In the radial coordinates, the normalized zero mode in an AdS_4 has the form

$$\hat{u}(r) = \frac{1}{\sqrt{I_1^{D=5}}}u(r) = -2\sqrt{\frac{\omega}{-1+2\log 2}}\frac{1}{1+e^{2\omega r}}. \tag{54}$$

The present observations require $\omega\sqrt{G_N}\ll 1$ where G_N is the four-dimensional Newton's constant, so it turns out that this zero mode is not sharply localized but spreads rather widely in a bulk.

Next, let us consider $D=6, p=4, n=2$ case, that is, a stringlike defect model in six space-time dimensions. In this case, we also follow a similar path of arguments to the case of $D=5$ domain wall. The concrete expression for $u(z)$ is different between $D=2k+3$ and $D=2k+4$, so it is valuable to investigate this simplest case in the branch of $D=2k+4$.

In the case of $D=6, p=4, n=2$, from Eq. (45) the integral $I_1^{D=6}$ takes the form

$$I_1^{D=6} = \int_{\pi/2\omega}^{\pi/\omega} dz\,\frac{1}{\sin^2\omega z}\left(-\frac{\alpha}{4\omega}\sin 2\omega z + \frac{1}{2}\alpha z + \beta\right)^2, \tag{55}$$

which is also generally divergent, but only when the equality $\beta = -\alpha\pi/2\omega$ holds, it also becomes strictly finite. Again, from now on we shall confine ourselves to this specific case. Then, after a bit calculations the integral $I_1^{D=6}$ reduces to

$$I_1^{D=6} = \frac{\alpha^2}{16\omega^3}\left(1 + 4\pi\log 2 - 8\int_0^{\pi/2} d\zeta\,\zeta\cot\zeta\right), \tag{56}$$

which is finite since the last integral is known to be finite. Thus, the gauge field is also localized on an AdS_4 brane in a six-dimensional space-time with negative cosmological constant.

Evaluating the integrals over z, the classical action is of form

$$S_1^{(0)} = -\frac{1}{4}\int d^4x\sqrt{-\hat{g}}\int d\theta R_0\Bigg\{ I_1\chi^2\hat{g}^{\mu\nu}\hat{g}^{\rho\sigma}f_{\mu\rho}f_{\nu\sigma} + \left[\frac{\pi\alpha^2}{2\omega}\chi^2 + \frac{2I_1}{R_0^2}(\partial_\theta\chi)^2\right]\hat{g}^{\mu\nu}a_\mu a_\nu\Bigg\}. \tag{57}$$

With respect to integrations over θ, as mentioned before, they are always finite, which fact can be shown as follows. As seen in the derivation from Eqs. (42) and (43), $\chi(y^m)$ must satisfy the equation of motion $\partial_m(\sqrt{\tilde{g}}\tilde{g}^{mn}\partial_n\chi)=0$, which now reduces to $\partial_\theta^2\chi=0$, so a general solution to this equation is given by $\chi(\theta)=\chi_1+\theta\chi_2$ where χ_1, χ_2 are integration constants. Thus the integrals over θ appearing in Eq. (57), $\int_0^{2\pi}d\theta\chi^2, \int_0^{2\pi}d\theta(\partial_\theta\chi)^2$ are finite quantities as long as the integration constants are finite. We are now ready to examine the mass of the brane gauge field. To make the kinetic term take a canonical form, let us redefine the brane gauge field a_μ as follows:

$$\sqrt{R_0 I_1\int d\theta\chi^2(\theta)}\,a_\mu \to a_\mu. \tag{58}$$

As a result, it turns out that Eq. (57) becomes

$$S_1^{(0)} = -\frac{1}{4}\int d^4x\sqrt{-\hat{g}}\Bigg[\hat{g}^{\mu\nu}\hat{g}^{\rho\sigma}f_{\mu\rho}f_{\nu\sigma} + \frac{8\pi\omega^2\int d\theta\chi^2 + (2/R_0^2)K\int d\theta(\partial_\theta\chi)^2}{K\int d\theta\chi^2}\hat{g}^{\mu\nu}a_\mu a_\nu\Bigg], \tag{59}$$

where $K \equiv 1 + 4\pi\log 2 - 8\int_0^{\pi/2}d\zeta\,\zeta\cot\zeta$. Since it is reasonable to regard value of the integrals over θ as being of the order 1, the smallness of mass of the brane gauge field requires $\omega^2\approx 0, 1/R_0^2\approx 0$, both of which imply that the brane cosmological constant is extremely tiny as desired.

As a final check, let us study the zero-mode $u(z)$. When the equality $\beta = -\alpha\pi/2\omega$ holds, our solution reduces to the form

$$u(z) = -\frac{\alpha}{4\omega}\sin 2\omega z + \frac{\alpha}{2\omega}(\omega z - \pi). \tag{60}$$

This solution also satisfies the Dirichlet boundary condition at $z=\pi/\omega$, where $u(z)=0$. For the investigation of the localization of this mode on a brane, we also use the radial coordinates instead of the z coordinates. In the radial coordinates, after some calculations, the normalized zero-mode in an AdS_4 is proportional to

$$\hat{u}(r) \propto \sqrt{\omega}\left[\frac{1}{e^{\omega r}+e^{-\omega r}}\tanh(\omega r)-\tan^{-1}(e^{-\omega r})\right]. \quad (61)$$

Again, for $\omega\sqrt{G_N}\ll 1$, it turns out that the brane gauge field is not sharply localized on an AdS_4 brane.

VI. LOCALIZATION OF FERMIONIC FIELDS

Next let us turn to fermionic fields, those are, spin-1/2 spinor field and spin-3/2 gravitino field. First, let us consider spin-1/2 spinor field. The starting action is the conventional Dirac action with a mass term in D dimensions:

$$S_{1/2}=\int d^D x\sqrt{-g}\,\bar{\Psi} i[\Gamma^M D_M+m\varepsilon(z)]\Psi, \quad (62)$$

where the covariant derivative is defined as $D_M\Psi=(\partial_M+\frac{1}{4}\omega_M^{AB}\gamma_{AB})\Psi$ with the definition of $\gamma_{AB}=\frac{1}{2}[\gamma_A,\gamma_B]$, and $\varepsilon(z)$ is $\varepsilon(z)\equiv z/|z|$ and $\varepsilon(0)\equiv 0$. Here the indices A,B are the ones of the local Lorentz frame and the gamma matrices Γ^M and γ^A are related by the vielbeins e_A^M through the usual relations $\Gamma^M=e_A^M\gamma^A$, where $\{\Gamma^M,\Gamma^N\}=2g^{MN}$ and $\{\gamma^A,\gamma^B\}=2\eta^{AB}$. A feature of the action is the existence of a mass term with a ''kink'' profile. We have just introduced this type of mass term in the action since the existence has played a critical role in the localization of fermionic fields on a Minkowski brane in an arbitrary dimension [6].

In this section, we shall consider a more general metric ansatz than Eq. (4) in the ''radial'' coordinates. The metric ansatz we take into consideration is the following one:

$$\begin{aligned} ds^2&=g_{MN}dx^M dx^N\\ &=e^{-A(r)}\hat{g}_{\mu\nu}(x^\lambda)dx^\mu dx^\nu+dr^2+e^{-B(r)}\tilde{g}_{mn}(y^l)dy^m dy^n, \end{aligned} \quad (63)$$

where we have replaced a metric on S^{n-1} in Eq. (4) with a general curved metric $\tilde{g}_{mn}(y^l)$ depending only on extra dimensions y^l except r. In this background metric, the torsion-free conditions yield an explicit expression of the spin connections

$$\omega_\mu=\frac{1}{4}A'(r)\Gamma_r\Gamma_\mu+\hat{\omega}_\mu(\hat{e}), \quad \omega_r=0,$$

$$\omega_m=\frac{1}{4}B'(r)\Gamma_r\Gamma_m+\tilde{\omega}_m(\tilde{e}), \quad (64)$$

where we have defined $\omega_M\equiv\frac{1}{4}\omega_M^{AB}\gamma_{AB}$. And $\hat{\omega}_\mu(\hat{e})$ and $\tilde{\omega}_m(\tilde{e})$ are the spin connections constructed out of $\hat{e}_\mu$ and $\tilde{e}_m$, respectively. Using Eq. (64), the Dirac equation $[\Gamma^M D_M+m\varepsilon(r)]\Psi=0$ can be cast in the form

$$\left[\Gamma^r\left(\partial_r-\frac{p}{4}A'-\frac{n-1}{4}B'\right)+\Gamma^\mu(\partial_\mu+\hat{\omega}_\mu)+\Gamma^m(\partial_m+\tilde{\omega}_m)+m\varepsilon(r)\right]\Psi=0. \quad (65)$$

Let us find the massless zero-mode solution with the form of $\Psi(x^M)=\psi(x^\mu)u(r)\chi(y^m)$ such that $\Gamma^\mu\hat{D}_\mu\psi=\Gamma^m\tilde{D}_m\chi=0$ and the chirality condition $\Gamma^r\psi=\psi$ is imposed on the brane fermion. Then, Eq. (65) is reduced to a first-order differential equation for $u(r)$ and is easily solved to be

$$u(r)=u_0 e^{(p/4)A(r)+[(n-1)/4]B(r)-m\varepsilon(r)r}, \quad (66)$$

with an integration constant u_0.

In order to check the localization of this mode, let us plug this solution into the Dirac action (62). Then the action reduces to the form

$$\begin{aligned} S_{1/2}^{(0)}&=\int d^D x\sqrt{-g}\,\bar{\Psi}^{(0)} i[\Gamma^M D_M+m\varepsilon(z)]\Psi^{(0)}\\ &=u_0^2\int d^{n-1}y\sqrt{\tilde{g}}\chi^\dagger(y)\chi(y)\int_0^\infty dr e^{(1/2)A(r)-2m\varepsilon(r)r}\\ &\quad\times\int d^p x\sqrt{-\hat{g}}\,\bar{\psi} i\gamma^\mu\hat{D}_\mu\psi+\cdots. \end{aligned} \quad (67)$$

The condition of the trapping of the bulk spinor on an AdS_p brane requires that an integral over r has a finite value since an integral over y is finite. The integral is easily evaluated as follows:

$$\begin{aligned} I_{1/2}&=\int_0^\infty dr e^{(1/2)A(r)-2m\varepsilon(r)r}\\ &=\int_0^\infty dr\frac{1}{\cosh\omega r}e^{-2m\varepsilon(r)r}. \end{aligned} \quad (68)$$

This integral is obviously finite so the bulk spinor is confined to a brane by the gravitational interaction. In particular, in the case of massless fermion, the above integral can be integrated to be

$$I_{1/2}^{m=0}=\frac{\pi}{2\omega}. \quad (69)$$

Namely, even in the massless spinor, the bulk spinor is localized on a brane, whose fact should be contrasted with the case of a Minkowski brane, where only the massive bulk fermion with a ''kink'' profile is localized on the brane whereas the massless one is not. This fact can be traced in Eq. (69) since $I_{1/2}^{m=0}$ at $\omega=0$ is divergent. [Note that in both the Minkowski brane and the AdS brane, the form of the zero-mode solution of fermion, Eq. (66), is the same so this consideration is legitimate.] In the case at hand, irrespective of the presence of mass term, the bulk spinor can be localized on the brane through the gravitational interaction as long as the brane cosmological constant is nonvanishing.

However, there is a caveat. As seen in the concrete form of $u(r)$ in Eq. (66), the zero-mode $u(r)$ can be written to

$$u(r)=\frac{u_0}{R_0^{(n-1)/2}}(\cosh \omega r)^{-(D-1)/2}e^{-m\varepsilon(r)r}$$
$$\approx \frac{u_0}{R_0^{(n-1)/2}}2^{(D-1)/2}e^{-[(D-1)/2]\omega r-m\varepsilon(r)r}. \qquad (70)$$

The last expression was derived under the condition $\omega r \gg 1$. This expression tells us that provided that $\omega\sqrt{G_N}\ll 1$, the zero mode spreads more widely in a bulk in the massless limit. To avoid such a situation, we might also need the presence of a mass term with ''kink'' profile. Incidentally, note that the results obtained so far are independent of a concrete form of $B(r)$, so only the warp factor $e^{-A(r)}$ in front of the p-dimensional metric controls the results.

Next, let us consider the gravitino field of spin-3/2. The action for the spin-3/2 bulk gravitino is given by the Rarita-Schwinger action

$$S_{3/2}=\int d^Dx\sqrt{-g}\,\bar{\Psi}_M i\Gamma^{[M}\Gamma^N\Gamma^{R]}[D_N+\delta_N^r\Gamma_r m\varepsilon(r)]\Psi_R, \qquad (71)$$

where $D_M\Psi_N=\partial_M\Psi_N-\Gamma^R_{MN}\Psi_R+\frac{1}{4}\omega_M^{AB}\gamma_{AB}\Psi_N$ and the square bracket denotes the anti-symmetrization with weight 1. From the metric condition $D_M e_N^A=\partial_M e_N^A-\Gamma^R_{MN}e_R^A+\omega_M^{AB}e_{NB}=0$, we obtain the concrete expression for the affine connections $\Gamma^R_{MN}=e_A^R(\partial_M e_N^A+\omega_M^{AB}e_{NB})$. With the gauge condition $\Psi_r=0$ and assuming $\Psi_m=0$ for simplicity, the equations of motion $\Gamma^{[M}\Gamma^N\Gamma^{R]}[D_N+\delta_N^r\Gamma_r m\varepsilon(r)]\Psi_R=0$ can be cast to the form

$$g^{\mu\nu}\left[\Gamma^r\left(\partial_r-\frac{p-2}{4}A'(r)-\frac{n-1}{4}B'(r)\right)+m\varepsilon(r)\right]\Psi_\nu=0, \qquad (72)$$

where we have used equations $\gamma^\mu\Psi_\mu=\hat{D}^\mu\Psi_\mu=\gamma^{[\mu}\gamma^\nu\gamma^{\rho]}\hat{D}_\nu\Psi_\rho=\Gamma^m\tilde{D}_m\Psi_\mu=0$. Let us look for a solution with the form $\Psi_\mu(x^M)=\psi_\mu(x^\lambda)u(r)\chi(y^m)$. If the chirality condition $\Gamma^r\psi_\mu=\psi_\mu$ is utilized in Eq. (72), we can get a solution

$$u(r)=u_0 e^{[(p-2)/4]A(r)+[(n-1)/4]B(r)-m\varepsilon(r)r}, \qquad (73)$$

with an integration constant u_0.

Substituting this solution into the action (71), we arrive at the following expression:

$$S_{3/2}^{(0)}=\int d^Dx\sqrt{-g}\,\bar{\Psi}_M^{(0)}i\Gamma^{[M}\Gamma^N\Gamma^{R]}[D_N+\delta_N^r\Gamma_r m\varepsilon(r)]\Psi_R^{(0)}$$
$$=u_0^2\int d^{n-1}y\sqrt{\tilde{g}}\chi^2(y)\int_0^\infty dr\,e^{(1/2)A(r)-2m\varepsilon(r)r}$$
$$\times\int d^px\sqrt{-\hat{g}}\,\bar{\psi}_\mu i\gamma^{[\mu}\gamma^\nu\gamma^{\rho]}\hat{D}_\nu\psi_\rho+\cdots. \qquad (74)$$

Again the condition for the localization of the gravitino on a brane requires the integral over r to take a finite value. Namely, the following integral over r must be finite

$$I_{3/2}=\int_0^\infty dr\,e^{(1/2)A(r)-2m\varepsilon(r)r}$$
$$=\int_0^\infty dr\frac{1}{\cosh\omega r}e^{-2m\varepsilon(r)r}. \qquad (75)$$

Here let us notice that this condition has the same form as in spin-1/2 spinor field, Eq. (68), so the spin-3/2 gravitino is also localized on a brane. The form of $u(r)$ in (73), however, is similar to that of spin-1/2 spinor field (66), so as in the spinor field it might be necessary to include a mass term with a ''kink'' profile in order to have a sharp localization on a brane.

VII. DISCUSSIONS

In this article we have discussed locally localized gravity models in higher dimensions. As a solution to Einstein's equations with a set of scalar fields with *global* SO(n) symmetry, we have found two types of AdS$_p$ brane solution in a unified metric ansatz. Though these solutions have been already found in Ref. [22], our derivation is more concise than their derivation and we furthermore spelled out the physical properties of the solutions. An important issue that we have found in this paper is that in higher dimensions the solutions which are free from the naked curvature singularity and possess the property of gravity localization are very few. Apart from a type of trivial extension of the Randall-Sundrum solutions, in higher dimensions the physical solution corresponds to only an AdS$_p$ brane in a space-time with negative bulk cosmological constant. It is quite curious that there are no nontrivial solutions in higher dimensions which correspond to a dS$_p$ brane and a M$_p$ brane solution with needed physical properties. From this point of view, more study about an AdS brane seems to be warranted in the future. In higher dimensions, intersection brane solutions with two different warp factors might be needed in order to satisfy the physical properties [29].

Concerning the localization of various bulk fields on an AdS brane only by the gravitational interaction, we have explicitly considered spin-1/2 spinor, spin-1 vector, and spin-3/2 gravitino fields. We have also implicitly considered spin-2 graviton where we have stressed that a complete understanding of the gravity localization requires us to find a reasonable core model. The local fields which we have left aside are spin-0 scalar and higher-rank antisymmetric tensor fields. It is well known that a real scalar field satisfies the same equation of motion as that of the transverse, traceless graviton modes, so a real scalar field shares the common localization properties with the graviton. The treatment of higher rank tensor fields is completely parallel to that of the gauge fields, so we have skipped these cases.

It is worth stressing here that the localization mechanism that we have found in this paper, in particular, is new and novel for spin-1 vector and fermionic fields. For the former, it is known that the gauge field is not localized on a Minkowski brane in the original Randall-Sundrum model. On the other hand, in an anti–de Sitter brane, the gauge field *is* localized due to the presence of the brane cosmological constant. However, there is a caveat. Namely, although the gauge field is anyway localized near the brane, it is not sharply localized, by which we meet some phenomenological problems such as the violation of the charge conservation law in our world. Moreover, we have found a new phenomenon that the size of the brane cosmological constant is determined by that of the mass of ''*photon*'' on a brane. Also for the fermionic fields, the presence of the brane cosmological constant provides a novel localization mechanism where massless fermions are localized on an AdS brane whereas only massive fermion with a ''kink'' profile can be localized on an M brane as in the Randall-Sundrum model. However, as in the gauge field, the zero-mode of massless fermions also spreads rather widely in the bulk space-time, so we might need a mass term with a ''kink'' profile to have a sharply localized brane fermion.

Of course, if we wish to construct a fully successful brane world model in higher dimensions on the basis of global defects, it is essential to understand physics inside the core of the defects. Without knowledge of it, we cannot fully answer several questions such as stability of the defects. Another unsolved problem within the context of the present formulation is how to construct a model with two or more branes, which would be necessary to understand the mass hierarchy problem between the Planck scale and the electroweak scale. There are also future works of the construction of a supersymmetric model corresponding to the present model and of deriving the model at hand from superstring theory. We wish to clarify these important problems in a future publication.

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