Vacuum bubbles nucleation and dark matter production through gauge symmetry rearrangement

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Modern particle physics and cosmology support the idea that a background of invisible material pervades the whole universe, and identify in the cosmic vacuum the ultimate source of matter—energy, both seen and unseen. Within the framework of the theory of fundamental relativistic membranes, we suggest a self-consistent, *vacuum-energy-driven* mechanism for dark matter creation through gauge symmetry rearrangement.

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I. INTRODUCTION

The modern paradigms of physics are the standard big bang model of cosmology and the standard $SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$ model of the strong and electroweak interactions. During the past two decades both models have been refined with the addition of two key ingredients: *inflation* on the cosmological side [1] and *axions* as pseudo-Goldstone bosons associated with the spontaneous breakdown of the Peccei-Quinn symmetry in particle physics [2]. Inflation requires the existence of dark matter and axions have long been candidates for cold dark matter. A further refinement of the standard models stems from a recent analysis of the cosmic microwave background [3] added to the data from high-redshift supernova observations [4]. Together, they seem to support the idea that the universe is flat and is currently expanding at an accelerated rate.[1] As a result, dark matter *and* the cosmological constant [5], or some form of dark energy, have become the essential components of the new inflationary scenario.[2] In this paper we wish to suggest that those two components are connected, in a rather fundamental way, by a new mechanism of symmetry rearrangement that *requires* the creation of dark matter.

Cold dark matter, in axionic form, could be detected in an experiment capable of probing masses in the range 10^{-6}–10^{-3} eV. Where does that mass come from? The general consensus is that it comes from the quantum anomaly which violates the chiral $U(1)_{PQ}$ symmetry, thereby evading Goldstone's theorem. However, the chiral anomaly is just one of at least two possible loopholes by which the existence of a Goldstone boson can be avoided. The second loophole is the Higgs mechanism. In its conventional formulation, the Higgs mechanism essentially converts a gauge field, i.e., a massless spin-1 field, into a massive vector field while preserving the value of the spin as well as gauge invariance, thereby ensuring the renormalizability of the theory. Even though this is the mechanism that generates the mass of all known elementary particles within the standard model of particle physics, it is clearly unsuitable to describe the cosmological situation envisaged above, namely, the conversion of the *constant* vacuum energy into particles of matter. Thus we are led to ask: *How does one connect a ''constant energy background'' (nondynamical by definition) into material particles that are invisible but dynamical?* Our suggestion, in a nutshell, is as follows: first, turn the cosmological constant into a *nondynamical gauge field*, i.e., a gauge field with zero degrees of freedom; second, extract from that gauge field a *massive spin-0 field* according to the time honored procedure of symmetry breaking followed by restoration of gauge invariance. As we shall see, the new ingredients of that old procedure are relativistic extended objects (membranes) and their gauge partners (antisymmetric tensor gauge fields).

The topological nature of the new mechanism and its mathematical formulation were discussed in a recent article in connection with the broad issue of *electric-magnetic duality of p-branes* [7]. As stated above, we are presently interested in applying the notion of ''topological symmetry breaking'' to the new inflationary scenario. Accordingly, we have organized the paper as follows.

In Sec. II we introduce the concept of topological symmetry breaking. In Sec. III we discuss the case of ''electrodynamics in two dimensions,'' reinterpreted as ''bubble dynamics in two dimensions,'' as the simplest framework in which topological symmetry breaking can be implemented

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[1] At least the first acoustic peak in the power spectrum of temperature fluctuations of the cosmic microwave background, as determined by the Maxima and Boomerang observations, is best fit with $\Omega=1$ as required by inflation.

[2] Those revolutionary cosmological data were not available in 1991 when the authors first suggested the possibility that the cosmological constant and dark matter might be related [6].

together with the generation of mass. In Sec. IV, the details of the new mechanism are illustrated in four spacetime dimensions. There we first outline the three main steps leading to the creation of dark matter, and then discuss each step in a separate subsection. Instrumental to the overall mechanism is the false vacuum decay rate through bubble nucleation from the vacuum which we calculate in real spacetime and compare with the corresponding computation in Euclidean space. Section V concludes the paper with a summary of our discussion and a commentary on the applicability of the new mass generation mechanism to the inflationary cosmological scenario.

II. SYMMETRY BREAKING REVISITED

''Topological symmetry breaking'' and the concomitant mechanism of mass generation have never been discussed in the physics of point particles for the simple reason that the world history of a material particle is usually assumed to have no boundary; that is, it is usually assumed to be infinitely extended in time.[3] Typically, in a world of classical point particles described by a *local* field theory, reparametrization invariance of the world trajectory is tacitly assumed while gauge invariance is *explicitly* broken only by introducing a mass term in an otherwise invariant action. The asymmetry of the vacuum with respect to some global transformation (spontaneous symmetry breaking) provides a second possibility which, in turn, leads to the celebrated Nambu-Goldstone-Higgs mechanism of mass generation. *The existence of a boundary in the world history of an object provides an additional possibility of symmetry breaking.*

For the sake of illustration, consider the familiar case of a charged particle: the gauge invariance of the free Maxwell action may be broken either by introducing a mass term into the action,

$$S \equiv S_m + S_A , \tag{2.1}$$

$$S_m = -\mu_0 \int_0^\infty d\tau \sqrt{-\frac{dx^\mu}{d\tau}\frac{dx_\mu}{d\tau}}, \tag{2.2}$$

$$S_A = \int d^4x \left[-\frac{1}{4} F_{\mu\nu}F^{\mu\nu} + \frac{m^2}{2} A_\mu A^\mu - e\, J^\mu A_\mu \right], \tag{2.3}$$

or, more generally, by coupling the gauge potential $A_\mu(x)$ to a *non-conserved* current $J^\mu(x)$, i.e., $\partial_\mu J^\mu(x) \neq 0$. Thus, in either case: (i) $m \neq 0$, or (ii) $m = 0$, $\partial_\mu J^\mu \neq 0$, gauge invariance is violated. *The massless, non-conserved current case corresponds to a classical point-like particle whose world line* Γ_0 *has a free end point.* By definition, this represents a boundary condition that is not explicitly encoded into the action. Therefore, to the extent that there are no apparent symmetry violating terms in the action, we refer to this case as ''topological symmetry breaking.'' For instance,

$$\Gamma_0 : \quad x^\mu = x^\mu(\tau), \quad 0 \leqslant \tau \leqslant \infty \tag{2.4}$$

with

$$J^\mu(x) = \int_0^\infty d\tau \frac{dx^\mu}{d\tau} \delta^{4)}(x - x(\tau)) = \int_{\Gamma_0} dx^\mu\, \delta^{4)}(x - x(\tau)) \tag{2.5}$$

represents a semi-infinite spacetime trajectory Γ_0 that originates at x_0 and then extends forever. An extremal free end point physically represents a ''singular'' event in which a particle is either created or destroyed, so that the covariant conservation of the associated current is violated,[4]

$$J(x) \equiv \partial_\mu J^\mu(x) = \partial_\mu \int_{\Gamma_0} d[\delta^{4)}(x - y(\tau))] = \delta^{4)}(x - x_0) \neq 0. \tag{2.6}$$

Furthermore, under a gauge transformation of the action integral, the interaction term transforms as follows:

$$\delta_\Lambda S^{int} = e \int_{x_0}^\infty dx^\mu\, \partial_\mu \Lambda = -\Lambda(x_0) \tag{2.7}$$

assuming, as usual, that the gauge function vanishes at infinity.

In Sec. IV we shall extend the above considerations to the case of a relativistic bubble in $3+1$ dimensions. That is the natural setting for discussing the new inflationary scenario. There we shall argue that the corresponding classical action represents an *effective action* for the quantum bubble nucleation process that takes place within the background vacuum energy represented by the cosmological constant. The novelty here is that the cosmological constant is disguised as a ''Maxwell field strength.'' Because of the presence of a boundary of the bubble trajectory in spacetime, the process of nucleating an inflationary bubble [10,11,12,13] must be accompanied by the excitation of massive spinless particles. A possible quantum formulation of the same boundary mechanism using the path integral approach to the dynamics of a generic p-brane in an arbitrary number of spacetime dimensions is given in Ref. [7].

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[3] In the following, by ''boundary'' of an object (pointlike or spatially extended) we mean an extremal configuration (initial or final) of the world history of the object at a finite time, or, in other words, an extremal spacelike section of the object's trajectory in spacetime. The case of an object with a boundary that is *spatially* open can be treated along similar lines [8,9].

[4] A physical example of such a situation is the emission of an alpha particle by a radioactive nucleus. Because of quantum tunneling, the particle suddenly disappears from within the nucleus (its worldline comes to an end point) and reappears at a different point outside the parent nucleus. No *physical* trajectory connects the two branches of the particle world line. Thus, from the point of view of an external observer, the α particle world line is semi-infinite: it originates from a point outside the nucleus at a given instant of time and then evolves independently of the parent nucleus.

III. TWO DIMENSIONAL ELECTRODYNAMICS

''Electrodynamics in $1+1$ dimensions,'' also known in its early quantum formulation as ''the Schwinger model'' [14], means different things to different people. Formally, the action (or Lagrangian) of the model is the same as that of the familiar Maxwell electrodynamics in $3+1$ dimensions, hence the name. The physical content, however, is vastly different. This is because of the stringent kinematical constraints that exist in $1+1$ dimensions: since there is no ''transversality'' in one spatial dimension, the concept of spin is undefined, and the notion of ''vector field,'' massless or massive, is purely formal. Thus, there is no radiation field associated with the Maxwell tensor. There is, however, the same background vacuum energy and long range static interaction that we shall discuss in the next section for the membrane theory in $3+1$ dimensions. This is because in one spatial dimension a ''bubble'' degenerates into a particle-antiparticle pair, moving left and right, respectively, and the volume within the bubble is the linear distance between them [15]. Indeed, the main reason for the following discussion is to make it evident that those very kinematical constraints that exist in $1+1$ dimensions are intertwined with the production of mass and can be induced just as well in $3+1$ dimensions simply by increasing the spatial dimensions of the object: from a 0 brane in $1+1$ dimensions to a 2-brane, or bubble, in $3+1$ dimensions, indeed, to a generic p-brane embedded in a target space with $p+2$ dimensions. In other words, the familiar theory of electrodynamics in $3+1$ dimensions does not represent a unique generalization of the so called ''electrodynamics in $1+1$ dimensions.'' A more natural extension, especially from a cosmological standpoint, is the theory of a relativistic membrane coupled to a three index gauge potential. It is in the framework of bubble dynamics, regardless of the dimensionality of the target space, that the cosmological constant drives the creation of particles of matter, and the engine of that process, at least at the classical level, is the ''topological symmetry breaking'' due to the existence of a boundary in the world history of the membrane.

A. Massless phase

In the massless phase, the physical content of electrodynamics in $(1+1)$ dimensions is encoded into the gauge invariant action:

$$S=\int d^2x\left[\frac{1}{4}F^{\mu\nu}F_{\mu\nu}-\frac{1}{2}F^{\mu\nu}\partial_{[\mu}A_{\nu]}-e\,J^{\mu}A_{\mu}\right] \quad (3.1)$$

so that the current density J^{μ}, without further boundary conditions, is divergenceless: $\partial_{\mu}J^{\mu}=0$. The first order formulation of the action is not mandatory but makes it clear that, in two dimensions, the ''Maxwell tensor'' is assumed to be the covariant curl of the gauge potential, which is then treated as an independent variable.

Thus, variation of the action with respect to the potential A_{μ} leads to the Maxwell equation

$$\partial_{\mu}F^{\mu\nu}=e\,J^{\nu}. \quad (3.2)$$

The general solution of Eq. (3.2) is the sum of the free equation solution $(e=0)$, and a special solution of the inhomogeneous equation $(e\neq 0)$. The complete equation can be formally solved by the Green function method. The final result is

$$F^{\mu\nu}(x)=\sqrt{\Lambda}\,\epsilon^{\mu\nu}+e\,\partial^{[\mu}\frac{1}{\Box}J^{\nu]}$$
$$=\sqrt{\Lambda}\,\epsilon^{\mu\nu}+e\int d^2y\,\partial_x^{[\mu}G(x-y)J^{\nu]}(y). \quad (3.3)$$

Inserting the above solution into the action (3.1), and neglecting surface terms, we obtain

$$S=-\frac{1}{2}\int d^2x\left[\Lambda+e^2J^{\nu}\frac{1}{\Box}J_{\nu}\right]$$
$$=-\frac{1}{2}\int d^2x\,\Lambda+e^2\int d^2x\int d^2y\,J^{\nu}(x)G(x-y)J_{\nu}(y), \quad (3.4)$$

which we interpret as follows:

$$S=-\frac{1}{2}\int d^2x[\text{``cosmological constant''}$$
$$+\text{``Coulomb potential''}]. \quad (3.5)$$

The first term represents a constant energy background, or *cosmological term*, even though it can be ''renormalized away'' in the absence of gravity. The second term in Eq. (3.4) describes the long-range, ''Coulomb interaction'' in two spacetime dimensions. In reality, it represents the *linear confining potential* between point charges written in a manifestly covariant form. In such a covariant formulation, the existence of a boundary, even though not explicitly codified in the action (3.1), introduces a symmetry breaking condition since it implies that the world line of the ''charge'' has a free end point through which the symmetry leaks out, so that $\partial_{\mu}J^{\mu}=J\neq 0$. In that case, gauge invariance is topologically broken and the current density is no longer divergence free. Under such circumstances, the field equation (3.2) needs to be modified since the left-hand side is divergenceless, while the right-hand side is not.

B. Massive phase

The necessary remedy for the above inconsistency is the introduction of a mass term in the action (3.1). Paradoxically, the presence of mass is also necessary in order to restore gauge invariance, albeit in an extended form. As a matter of fact, the new action

$$S=\int d^2x\left[\frac{1}{4}F^{\mu\nu}F_{\mu\nu}-\frac{1}{2}F^{\mu\nu}\partial_{[\mu}A_{\nu]}+\frac{m^2}{2}A_{\mu}A^{\mu}-e\,J^{\mu}A_{\mu}\right] \quad (3.6)$$

reflects the fact that the original gauge invariance is not only topologically broken, i.e., *implicitly* broken by the boundary, but also *explicitly* broken by the presence of a mass term. However, we argue that there is a subtle interplay between those two mechanisms of symmetry breaking, so that manifest gauge invariance is actually restored. In order to further analyze the connection between the two mechanisms of symmetry breaking, it is convenient to separate the divergenceless, boundary free current from the nonconserved boundary current. In two dimensions a generic vector can be decomposed into the sum of a ''hatted,'' or divergence-free component, and a ''tilded,'' or curl-free component. Thus, we write

$$A_\mu = \hat{A}_\mu + \tilde{A}_\mu\,, \ : \ \partial_\mu \hat{A}^\mu = 0, \quad \partial_{[\mu}\tilde{A}_{\nu]} = 0 \tag{3.7}$$

and a similar decomposition holds for the current,

$$J^\mu = \left(J^\mu - \frac{\partial^\mu J}{\Box} \right) + \frac{\partial^\mu J}{\Box} \equiv \hat{J}^\mu + \tilde{J}^\mu, \ : \ \partial_\mu \hat{J}^\mu = 0, \quad \partial_\mu \tilde{J}^\mu = J. \tag{3.8}$$

In terms of this new set of fields and currents the action reads

$$S = \int d^2x \left[\frac{1}{4} F^{\mu\nu} F_{\mu\nu} - \frac{1}{2} F^{\mu\nu}\, \partial_{[\mu} \hat{A}_{\nu]} + \frac{m^2}{2} \hat{A}_\mu \hat{A}^\mu - e\, \hat{J}^\mu \hat{A}_\mu + \frac{m^2}{2} \tilde{A}_\mu \tilde{A}^\mu - e\, \tilde{J}^\mu \tilde{A}_\mu \right] \tag{3.9}$$

and we find two systems of decoupled field equations: the divergence-free vector field satisfies the Proca-Maxwell equation

$$\partial_\mu F^{\mu\nu} + m^2 \hat{A}^\nu = e\, \hat{J}^\nu \tag{3.10}$$

while the curl-free part must satisfy the constraint

$$m^2\, \partial_\nu \tilde{A}^\nu = e\, J \tag{3.11}$$

or, equivalently,

$$\tilde{A}_\mu = \frac{e}{m^2} \partial_\mu \frac{1}{\Box} J. \tag{3.12}$$

To the extent that the mass is linked to the divergence of the current, as shown by the above equations, it is also a measure of the ''symmetry leakage'' through the boundary. It is this connection between topological and explicit symmetry breaking that leads us to ask: Is there a way of restoring manifest gauge invariance in spite of the presence of a mass term in the action?

C. Massive, gauge invariant phase

The answer to the question raised in the previous subsection was suggested by Stueckelberg a long time ago [16]. The original Stueckelberg proposal was to recover gauge invariance by introducing a compensating scalar field θ so that the resulting action

$$S_A = \int d^2x \left[\frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \frac{1}{2} F^{\mu\nu}\, \partial_{[\mu} A_{\nu]} + \frac{m^2}{2} \left(A_\mu + \frac{1}{m} \partial_\mu \theta \right)^2 - e\, J^\mu \left(A_\mu + \frac{1}{m} \partial_\mu\, \theta \right) \right] \tag{3.13}$$

is invariant under the extended gauge transformation

$$A_\mu \to A'_\mu = A_\mu + \partial_\mu \lambda, \tag{3.14}$$

$$\theta \to \theta' = \theta - m\,\lambda. \tag{3.15}$$

In this case, the vector field equation

$$\partial_\mu F^{\mu\nu} + m^2 \left(A^\nu + \frac{1}{m} \partial^\nu\, \theta \right) = e\, J^\nu \tag{3.16}$$

is self-consistent because of the theta-field equation

$$m^2\, \partial_\nu \left(A^\nu + \frac{1}{m} \partial^\nu\, \theta \right) = e\, \partial_\nu J^\nu. \tag{3.17}$$

Evidently, the role of the constraint (3.17) is to combine the $\tilde{A}_\mu$ component of the vector potential with the compensator field in such a way that symmetry is restored with respect to the extended gauge transformation. In our geometric interpretation, this is equivalent to ''closing the world history'' by compensating for the leakage of symmetry through the boundary. In this sense, the generation of mass is the consequence of ''mixing'' two gauge fields, namely, the $\tilde{A}_\mu$ component of the vector potential with the θ field. As a matter of fact, Eq. (3.17) determines the mixed, gauge invariant field to be

$$\tilde{A}_\mu + \frac{1}{m} \partial_\mu\, \theta = \frac{e}{m^2} \partial_\mu \frac{1}{\Box} J. \tag{3.18}$$

Once the above equation (3.18) is inserted into the action (3.13) we obtain

$$S = \int d^2x \left[\frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \frac{1}{2} F^{\mu\nu}\, \partial_{[\mu} A_{\nu]} + \frac{m^2}{2} \hat{A}^2_\mu - e\, \hat{J}^\mu \hat{A}_\mu - \frac{e^2}{2m^2} J \frac{1}{\Box} J \right], \tag{3.19}$$

which represents an ''effective action'' for the only physical degree of freedom represented by $\hat{A}_\mu$.

IV. THREE STEPPING STONES OF ''DARK MATTER'' PRODUCTION: FORMULATION OF THE MECHANISM

In order to place our previous discussion in the right perspective and partly to justify the more technical approach in the following subsections, let us consider the inflationary idea that the early phase of the exponential expansion of the universe inflated a microscopic volume of space to a size much larger than the presently observable part of the universe; this idea can be formulated within the framework of general relativity as a special case of ''classical bubble dynamics'' (CBD), i.e., the study of the evolution of a vacuum bubble in the presence of gravity [10]. In our own formulation of CBD, inflation is driven by a gauge field $A_{\mu\nu\rho}(x)$ which is equivalent to a cosmological constant [11], and the boundary effects in CBD, completely similar to those discussed in the previous section, constitute the precise mechanism which extracts dark matter from the self-energy of $A_{\mu\nu\rho}$.

In short, how does that process take place? The following properties of $A_{\mu\nu\rho}$ constitute the crux of the boundary mechanism in the inflation-axion scenario:

(a) When massless, $A_{\mu\nu\rho}$ represents ''dark stuff'' by definition, since in 3+1 dimensions $A_{\mu\nu\rho}$ does not possess radiative degrees of freedom. In fact, the field strength $F_{\mu\nu\rho\sigma} \equiv \nabla_{[\mu} A_{\nu\rho\sigma]} = \partial_{[\mu} A_{\nu\rho\sigma]}$, as a solution of the classical field equation, is simply a constant disguised as a gauge field. This property, even though peculiar, is not new in field theory: it is shared by all d-potential forms in $(d+1)$-spacetime dimensions. For instance in two dimensions, $F_{\mu\nu} = \partial_{[\mu} A_{\nu]} = \epsilon_{\mu\nu} \Lambda$, while in four dimensions, $F_{\mu\nu\rho\sigma} = \epsilon_{\mu\nu\rho\sigma} f$, and f represents a constant background field in both cases by virtue of the field equations. What is then the meaning of ''f''? As a gauge field, $A_{\mu\nu\rho}$ is endowed with an energy momentum tensor and thus it couples to gravity [15]: the resulting equations are Einstein's equations with the cosmological term $\Lambda = 4\pi G f^2$. For this reason we call $A_{\mu\nu\rho}$ the ''*cosmological field*.'' This alternative interpretation of the cosmological constant can be traced back to Ref. [15] and its application to the inflationary scenario in Ref. [11]; it will be discussed in more detail in the following subsection.

(b) If the cosmological field acquires a mass, then it describes massive pseudoscalar particles, in contrast with the usual Higgs mechanism. Indeed, in the massive case the free field equation for $A_{\mu\nu\rho}$,

$$\partial_\lambda \partial^{[\lambda} A^{\mu\nu\rho]} + m^2 A^{\mu\nu\rho} = 0, \quad \Rightarrow \partial_\mu A^{\mu\nu\rho} = 0 \tag{4.1}$$

imposes the divergence-free constraint on the four components of $A_{\mu\nu\rho}$, leaving only one propagating degree of freedom. In other words, the introduction of a mass term ''excites'' a dynamical (pseudoscalar) particle of matter out of the cosmological energy background.

(c) Evidently, the transition from case (a) (massless, nondynamical field) to case (b) (massive propagating particles) requires a physical mechanism for its enactment. Here is where the idea of topological symmetry breaking and the concomitant rearrangement of gauge symmetry come into play. We hasten to say here, and expand our discussion in the following subsection, that the cosmological field $A_{\mu\nu\rho}$ does not interact directly with the ordinary matter fields that represent point-like particles. Rather, $A_{\mu\nu\rho}$ is the ''gauge partner'' of relativistic closed membranes, or bubbles, in the sense that it mediates the interaction between surface elements according to the same general principle of gauge invariance which dictates the coupling of point charges to vector gauge bosons, or the coupling of Kalb-Ramond potentials to elementary string-like objects. Clearly, this type of coupling to relativistic membranes as fundamental extended objects is a crucial assumption of the whole mechanism of mass generation advocated in this paper.

A. Massless field, closed membrane and vacuum energy density

In order to implement the three properties (a), (b), and (c) discussed above, we start from the action functional

$$S = \int d^4x \left(\frac{1}{2\times 4!} F_{\lambda\mu\nu\rho} F^{\lambda\mu\nu\rho} - \frac{1}{4!} F^{\lambda\mu\nu\rho} \partial_{[\lambda} A_{\mu\nu\rho]} - \frac{g}{3!} A_{\mu\nu\rho} J^{\mu\nu\rho} \right) - \mu^3 \int_M d^3\sigma \sqrt{-\gamma}. \tag{4.2}$$

This is a straightforward, but non-trivial, formal extension of the action for the electrodynamics of point charges, or the Kalb-Ramond action of ''string dynamics.'' More to the point, from our discussion in Sec. III, it represents a direct generalization to 3+1 dimensions of the same two-dimensional ''electrodynamics'' action [15] in a ''σ model'' inspired formulation where a fundamental extended object is coupled to its massless excitations. In order to keep our discussion as transparent as possible, we consider only an elementary, or structureless membrane with vanishing width, interacting with a single massless mode represented by $A_{\mu\nu\rho}$. Thus,

$$J^{\mu\nu\rho}(x) = \int \delta^{4)}[x - Y(\sigma)] dY^\mu \wedge dY^\nu \wedge dY^\rho \tag{4.3}$$

represents the current density associated with the world history of the membrane. More complex models, in which the membrane is some sort of collective excitation of an underlying field theory, while intriguing, are affected by highly non-trivial technical problems, such as renormalization [17], [8] and bosonization.[5] That approach, while conceivable in principle, is orthogonal to ours: here, we assume that the membranes under consideration are elementary geometric objects of a fundamental nature, on the same footing as points, strings and other p-branes that constitute the very fabric of quantum spacetime [18]. This principle of geometric democracy is reflected in the action functional (4.2) by our choice of the Nambu-Goto-Dirac action for a relativistic bubble in which γ stands for the determinant of the induced metric,

$$\gamma \equiv \det(\partial_m Y^\mu \partial_n Y_\mu) \tag{4.4}$$

and μ^3 represents the bubble surface tension. Notwithstanding the apparent simplicity of our model, we shall see in the

[5]Introducing fermionic degrees of freedom enables one to establish a correspondence between bosonic and fermionic variables, that is, $J^{\mu\nu\rho} \leftrightarrow \epsilon^{\mu\nu\rho\sigma} \overline{\psi} \gamma^5 \gamma_\sigma \psi$.

next subsection that it is possible to reproduce the correct false vacuum decay rate, without resorting to solitonic computational techniques.[6]

Gauge invariance of the action (4.2) is guaranteed whenever the bubble embedding equations $x^{\mu}=Y^{\mu}(\sigma)$ parametrize a world history without boundary, so that

$$\delta A_{\mu\nu\rho}=\partial_{[\mu}\Lambda_{\nu\rho]}, \tag{4.5}$$

$$\delta S_{\Lambda}=0 \leftrightarrow \partial_{\mu}J^{\mu\nu\rho}=0. \tag{4.6}$$

The divergence-free condition for the membrane current is the formal translation of the no-boundary condition. Essentially, it restricts the world history of the membrane to be [20]

(i) spatially closed;

(ii) either infinitely extended in the timelike direction (eternal membrane) or, compact without boundary (virtual membrane).

Consequently, the cosmological field $A_{\mu\nu\rho}$ couples in a gauge invariant way *only* to bubbles whose history extends from the remote past to the infinite future, or to objects that start as a point in the vacuum, expand to a maximum spatial volume, and then recollapse to a point in the vacuum. In such a case, variation of the action with respect to $A_{\mu\nu\rho}$ leads to Maxwell's equation

$$\partial_{\mu}F^{\mu\nu\rho\sigma}=g\,J^{\nu\rho\sigma}. \tag{4.7}$$

The general solution of Eq. (4.7) is the sum of the free equation solution $(g=0)$, and a special solution of the inhomogeneous equation $(g\neq 0)$. The complete formal solution is found by inverting the field equation according to the Green function method: taking into account that the Maxwell tensor is proportional to the epsilon *tensor*, we find

$$F^{\mu\nu\rho\sigma}=\epsilon^{\mu\nu\rho\sigma}\sqrt{\Lambda}+g\,\partial^{[\mu}\frac{1}{\Box}J^{\nu\rho\sigma]}. \tag{4.8}$$

Inserting the above solution back into the action (4.2), one finds apart from the Nambu-Goto-Dirac term

$$S=-\frac{1}{2}\int d^4x\left(\Lambda+\frac{g^2}{3!}J^{\nu\rho\sigma}\frac{1}{\Box}J_{\nu\rho\sigma}\right). \tag{4.9}$$

Exactly as in the $(1+1)$-dimensional case, we interpret the above expression as follows:

$$S=-\frac{1}{2}\int d^4x[\text{``cosmological constant''} + \text{``Coulomb potential''}]+\text{extra surface terms}. \tag{4.10}$$

[6]It seems to us that this result may well be a consequence of a *duality* between membranes as solitonic solutions of an underlying field theory and membranes as fundamental objects (for a comprehensive review, see Ref. [19]).

Indeed, as anticipated at the beginning of this section [property (a)], the first term in Eq. (4.9) is a solution of the *free* Maxwell equation and represents a constant energy density background. As a free field, that is, in the absence of gravity and any other interaction, that constant term can be "renormalized away" since it cannot be distinguished from the vacuum. However, it is equivalent to a cosmological term when gravity is switched on [15,13].

It may not be immediately evident that, even in the presence of a coordinate dependent metric $g_{\mu\nu}(x)$, the homogeneous solution of Eq. (3.2) still represents a constant background energy density. Phrased differently, it might appear that there is no longer a constant rank-4 tensor available to equate F to.

The loophole is in the covariant form of Eq. (3.2). Since we are considering the homogeneous solution, we may as well switch off the coupling to the current, so that

$$\nabla_{\mu}F^{\mu\nu\rho\sigma}=0. \tag{4.11}$$

Here, ∇_{μ} represents the covariant derivative compatible with the Riemannian metric $g_{\mu\nu}(x)$, i.e., the connection is chosen to be the Christoffel symbol. In four dimensions there is only one generally covariant and totally anti-symmetric tensor, namely, the covariant Levi-Civita tensor:

$$\varepsilon^{\mu\nu\rho\sigma}(x)\equiv\frac{1}{\sqrt{-g(x)}}\epsilon^{\mu\nu\rho\sigma}, \tag{4.12}$$

where $g(x)\equiv\det g_{\mu\nu}(x)$ and $\epsilon^{\mu\nu\rho\sigma}$ is the constant Levi-Civita tensor density. Thus, Eq. (4.11) may be solved by the ansatz

$$F^{\mu\nu\rho\sigma}\equiv\frac{1}{\sqrt{-g(x)}}\epsilon^{\mu\nu\rho\sigma}F(x), \tag{4.13}$$

where $F(x)$ is a scalar function to be determined by the field equations. The metric tensor $g_{\mu\nu}(x)$ and its determinant are both covariantly constant with respect to the Christoffel covariant derivative. Thus, the $\varepsilon(x)$ tensor has vanishing covariant derivative. By inserting the trial solution (4.13) in Eq. (4.11), one sees that the derivative operator bypasses the $\varepsilon(x)$ tensor and applies directly to the scalar function $F(x)$:

$$\varepsilon^{\mu\nu\rho\sigma}(x)\partial_{\mu}F(x)=0. \tag{4.14}$$

Thus, the solution of Eq. (4.14) is again

$$F(x)=\text{const}\equiv\sqrt{\Lambda}. \tag{4.15}$$

To conclude the proof that Λ represents a genuine cosmological constant, we need to compute the value of the classical action. This can be done by using the following property of the $\varepsilon(x)$ tensor:

$$\varepsilon_{\mu\nu\rho\sigma}(x)\equiv\sqrt{-g(x)}\,\epsilon_{\mu\nu\rho\sigma}, \quad \rightarrow\varepsilon_{\mu\nu\rho\sigma}(x)\varepsilon^{\mu\nu\rho\sigma}(x)=-4! \tag{4.16}$$

Thus,

$$-\frac{1}{2\times 4!}\int d^4x\sqrt{-g}F_{\lambda\mu\nu\rho}F^{\lambda\mu\nu\rho}\to\int d^4x\sqrt{-g}\,\frac{\Lambda}{2}. \tag{4.17}$$

Having clarified the physical meaning of the integration constant Λ, let us consider the second term in the action (4.9). Apparently, it describes a long-range, Coulomb interaction between the bubble surface elements. In reality, it represents the bubble volume energy density written in a manifestly covariant form. In fact, we can re-arrange that Coulomb term as follows. From the definition (4.3) and the condition (4.6), we deduce that

$$J^{\nu\rho\sigma}(x)=\partial_\mu K^{\mu\nu\rho\sigma}(x) \tag{4.18}$$

$$=\partial_\mu\int d^4\xi\,\delta^{4)}[x-Z(\xi)]dZ^\mu\wedge dZ^\nu\wedge dZ^\rho\wedge dZ^\sigma. \tag{4.19}$$

However, in four dimensions

$$K^{\mu\nu\rho\sigma}(x)=\epsilon^{\mu\nu\rho\sigma}\int d^4x\,\delta^{4)}[x-Z(\xi)]\equiv\epsilon^{\mu\nu\rho\sigma}\Theta(x), \tag{4.20}$$

where $\Theta(x)$ is referred to as the characteristic function of the spacetime open sub-manifold bounded by the membrane. Thus, the Coulomb term can be rewritten in terms of $\Theta(x)$

$$\begin{aligned}\frac{g^2}{3!}J^{\nu\rho\sigma}\frac{1}{\Box}J_{\nu\rho\sigma}&=\frac{g^2}{3!}\partial_\mu K^{\mu\nu\rho\sigma}\frac{1}{\Box}\partial^\tau K_{\tau\nu\rho\sigma}\\&=\frac{g^2}{3!}\partial_\mu\,\epsilon^{\mu\nu\rho\sigma}\Theta(x)\frac{1}{\Box}\partial^\tau\epsilon_{\tau\nu\rho\sigma}\Theta(x)\\&=-g^2\partial_\mu\,\Theta\frac{1}{\Box}\partial^\mu\,\Theta\\&=g^2\Theta(x),\end{aligned} \tag{4.21}$$

where we have made use of the formal identity $\Theta^2(x)\equiv\Theta(x)$ and discarded a total divergence. Thus, the classical solution (4.8) and the action (4.2) show that the cosmological field $A_{\mu\nu\rho}$ does not describe the propagation of material particles; rather, it represents a constant energy density background with two different values inside and outside the membrane. Indeed, using the previous result, one may calculate the value of the classical action corresponding to the solution (4.8),

$$S=-\frac{1}{2}\int d^4x[\Lambda+g^2\,\Theta(x)]-\mu^3\int_M d^3\sigma\sqrt{-\gamma}. \tag{4.22}$$

Once again, we note that in the absence of gravity one is at liberty to choose the ''zero'' of the energy density scale, and thus measure the energy density with respect to the constant background represented by Λ. With that observation in mind, the classical action turns out to be a pure volume term, as announced:

$$\begin{aligned}S_\Lambda+\mu^3\int_M d^3\sigma\sqrt{-\gamma}&\equiv S(\Lambda;g)-S(\Lambda;g=0)\\&=\frac{g^2}{2}\int d^4x\,\Theta(x)\\&=\frac{g^2}{2}\int d^4x\int_B d^4\sigma\,\delta^{4)}[x-Y(\sigma)].\end{aligned} \tag{4.23}$$

B. Nucleation rate, symmetry breaking and mass

With the results of the previous subsection in hand, we can finally relate the nucleation rate of vacuum bubbles with the idea of topological symmetry breaking and mass generation. A common procedure for computing the nucleation rate of vacuum bubbles amounts, in our present formulation, to analytically continuing the action (4.23) to imaginary time[7]

$$S_E(F;g^2)=\frac{1}{2}\int d^4x[\Lambda-g^2\,\Theta_B(x)]-\mu^3\int d^p\sigma\sqrt{-\gamma}. \tag{4.25}$$

A semi-classical estimate for the nucleation rate of a spherical bag of radius R can be obtained through a saddle point estimate of S_E:

$$\begin{aligned}e^{-\Gamma}&\simeq e^{-[S(\Lambda;g)-S(\Lambda;0)]}\equiv e^{-S(R)}\\&=\exp\left[-\left(\frac{\pi^2}{4}g^2R^4-2\pi^2\mu^3R^3\right)\Bigg|_{R=R_0}\right],\end{aligned} \tag{4.26}$$

where the nucleation radius R_0 is a stationary point

$$\left(\frac{\partial S(R)}{\partial R}\right)_{R=R_0}\to R_0=\frac{6\mu^3}{g^2}. \tag{4.27}$$

Then, one finds

$$e^{-B}=\exp\left(-\frac{\pi^2}{2}\mu^3R_0^3\right) \tag{4.28}$$

which is the original Coleman–De Luccia result for the false vacuum decay rate [22]. Apart from confirming the validity of our approach against a well-tested calculation, the Euclidean description of vacuum decay shows how a vacuum bubble may materialize in Minkowski spacetime as a spacelike domain at a finite time. This is precisely the ''extremal,'' i.e., initial boundary of the membrane world manifold, and leads us to conclude, by the argument of the previous section, that the current associated with the bubble nucleation process cannot be divergence free. Thus, a

[7] $\epsilon^{\mu\nu\rho\sigma}\to i\epsilon^{\mu\nu\rho\sigma}$ under Wick rotation. Thus,

$$\partial_\mu K^{\mu\nu\rho\sigma}\frac{1}{\Box}\partial^\tau K_{\tau\nu\rho\sigma}\to+g^2\partial_\mu\,\Theta(x)\frac{1}{\Box}\partial_\mu\,\Theta(x). \tag{4.24}$$

Minkowskian description of the bubble nucleation process seems to be in conflict with the requirement of gauge invariance.

Against this background, we wish to show that a consistent description, in real spacetime, of the (quantum) nucleation process, can be achieved by restoring the gauge invariance of the original action. However, restoring gauge invariance is tantamount to ‘‘closing’’ the world history of the membrane, so that no boundary exists. There are essentially two ways to achieve this: in the Euclidean formulation one bypasses the problem by ‘‘closing the free boundary in imaginary time,’’ so that the resulting Euclidean world manifold is again without boundary. Somewhat paradoxically, the alternative procedure in real spacetime is to include in the action an additional source of symmetry violation in the form of a mass term for the cosmological field. We hasten to emphasize, before proceeding further, that the inclusion of a mass term is not a matter of choice. As we have seen in $1+1$ dimensions, it is actually dictated by the self-consistency of the field equations. There, we have shown how the explicit symmetry breaking due to the presence of mass and the topological symmetry breaking due to the presence of a boundary actually conspire to produce an action which is gauge invariant, albeit in an extended form. Thus, in the last analysis, *it is the self-consistency of the theory that forces upon us the introduction of a massive particle.*

Consider the coupling of the cosmological field $A_{\mu\nu\rho}(x)$ to a quantum mechanically nucleated relativistic membrane. According to the discussion in the previous section, the history of such an object is spatially closed, but only semi-infinite along the timelike direction because the membrane comes into existence at a finite instant of time. The nucleation event provides a spacelike boundary that consists of a two-surface where symmetry ‘‘leaks out’’ and gauge invariance is broken. Therefore, the apparently gauge invariant action

$$S_0=\int d^4x\left(\frac{1}{2\times 4!}F_{\lambda\mu\nu\rho}F^{\lambda\mu\nu\rho}-\frac{1}{4!}F^{\lambda\mu\nu\rho}\,\partial_{[\lambda}A_{\mu\nu\rho\,]}\right.$$
$$\left.-\frac{g}{3!}J^{\mu\nu\rho}A_{\mu\nu\rho}\right) \tag{4.29}$$

leads to field equations

$$\partial_\lambda F^{\lambda\mu\nu\rho}=g\,J^{\mu\nu\rho}(x) \tag{4.30}$$

that are inconsistent. This is because the left-hand side of Eq. (4.30) is divergence free everywhere due to the antisymmetry of the Maxwell tensor, whereas the membrane current is divergenceless everywhere except at the nucleation event where

$$\partial_\mu J^{\mu\nu\rho}=j^{\nu\rho}\neq 0. \tag{4.31}$$

Here $j^{\nu\rho}$ represents the *boundary current* localized on the initial two-surface. For a Minkowskian observer the membrane is created *ex nihilo*, and its current suddenly jumps from zero to a non-vanishing value. Therefore, it cannot be ‘‘conserved’’ and the amount of (topological) symmetry breaking is taken into account by $j^{\nu\rho}$. Thus, what we learn from Eq. (4.30) is that *the massless cosmological field cannot couple to the current of a relativistic membrane which is nucleated from the vacuum.*

In order to write down a self-consistent model for interacting semi-infinite world histories, the coupling must involve a massive tensor field:

$$S=\int d^4x\left(\frac{1}{2\times 4!}F_{\lambda\mu\nu\rho}F^{\lambda\mu\nu\rho}-\frac{1}{4!}F^{\lambda\mu\nu\rho}\,\partial_{[\lambda}A_{\mu\nu\rho]}\right.$$
$$\left.+\frac{m^2}{2\times 3!}A_{\mu\nu\rho}A^{\mu\nu\rho}-\frac{g}{3!}J^{\mu\nu\rho}A_{\mu\nu\rho}\right). \tag{4.32}$$

Inspection of the above action tells us that the physical spectrum consists of massive spin-0 particles, in agreement with property (b) listed in the previous section. However, in order to extract the full physical content of the system (4.32) we can proceed as follows. From the above action we derive the field equations

$$\partial_\lambda F^{\lambda\mu\nu\rho}+m^2A^{\mu\nu\rho}=g\,J^{\mu\nu\rho}, \tag{4.33}$$

$$\partial_\mu A^{\mu\nu\rho}=\frac{g}{m^2}j^{\nu\rho}. \tag{4.34}$$

Next, we use the identity

$$J^{\mu\nu\rho}=\left(J^{\mu\nu\rho}-\partial^{[\mu}\frac{1}{\Box}j^{\nu\rho]}\right)+\partial^{[\mu}\frac{1}{\Box}j^{\nu\rho]}\equiv\hat{J}^{\mu\nu\rho}+\tilde{J}^{\mu\nu\rho} \tag{4.35}$$

in order to split the current into two parts

$$\partial_\mu\hat{J}^{\mu\nu\rho}=0,\quad \partial_\mu\tilde{J}^{\mu\nu\rho}=j^{\nu\rho}. \tag{4.36}$$

Evidently, $\hat{J}^{\mu\nu\rho}(x)$ is the divergenceless, boundary free current, while $\tilde{J}^{\mu\nu\rho}(x)$ represents the pure boundary current.

In a similar fashion we decompose the tensor gauge field into the sum of a divergence-free and a curl-free part:

$$A^{\mu\nu\rho}\equiv\hat{A}^{\mu\nu\rho}+\tilde{A}^{\mu\nu\rho} \tag{4.37}$$

$$\partial_\mu\hat{A}^{\mu\nu\rho}=0,\quad \partial_{[\lambda}\tilde{A}_{\mu\nu\rho]}=0. \tag{4.38}$$

Consequently, the ‘‘Proca-Maxwell’’ equations (4.33), (4.34) split into the following set:

$$\partial_\lambda F^{\lambda\mu\nu\rho}+m^2(\hat{A}^{\mu\nu\rho}+\tilde{A}^{\mu\nu\rho})=g(\hat{J}^{\mu\nu\rho}+\tilde{J}^{\mu\nu\rho}), \tag{4.39}$$

$$\partial_\mu\tilde{A}^{\mu\nu\rho}=\frac{g}{m^2}j^{\nu\rho}. \tag{4.40}$$

From Eq. (4.40) we obtain

$$\tilde{A}^{\mu\nu\rho}=\frac{g}{m^2}\partial^{[\mu}\frac{1}{\Box}j^{\nu\rho]}. \tag{4.41}$$

Then, Eq. (4.39) becomes

$$\partial_\lambda F^{\lambda\mu\nu\rho}+m^2\hat{A}^{\mu\nu\rho}=g\,\hat{J}^{\mu\nu\rho}, \tag{4.42}$$

which gives

$$F^{\mu\nu\rho\sigma}=\sqrt{\Lambda}\epsilon^{\mu\nu\rho\sigma}+g^2\,\partial^{[\mu}\frac{1}{\Box}\hat{J}^{\nu\rho\sigma]}-m^2\,\partial^{[\mu}\frac{1}{\Box}\hat{A}^{\nu\rho\sigma]}$$

$$\equiv F_0^{\mu\nu\rho\sigma}-m^2\,\partial^{[\mu}\frac{1}{\Box}\hat{A}^{\nu\rho\sigma]}, \tag{4.43}$$

where $F_0^{\mu\nu\rho\sigma}$ represents the solution of Maxwell's equation

$$\partial_\mu F_0^{\mu\nu\rho\sigma}=g\,\hat{J}^{\nu\rho\sigma} \tag{4.44}$$

obtained in the previous massless case.

Substituting the solution (4.43) into the action (4.32), and following step by step the same procedure outlined in the previous subsection for the massless case, we obtain the corresponding result for the massive cosmological field:

$$S=\int d^4x\Bigg[\frac{1}{2\times 4!}F_{0\,\mu\nu\rho\sigma}F_0^{\mu\nu\rho\sigma}-\frac{m^2}{2\times 3!}\hat{A}^{\mu\nu\rho}\left(\frac{\Box+m^2}{\Box}\right)\hat{A}_{\mu\nu\rho}+\frac{g\,m^2}{3!}\hat{J}^{\mu\nu\rho}\frac{1}{\Box}\hat{A}_{\mu\nu\rho}+\frac{g^2}{4m^2}j^{\mu\nu}\frac{1}{\Box}j_{\mu\nu}\Bigg]$$
$$=\int d^4x\Bigg[\frac{1}{2}[\Lambda-g^2\,\Theta(x)]-\frac{m^2}{2\times 3!}\hat{A}^{\mu\nu\rho}\left(\frac{\Box+m^2}{\Box}\right)\hat{A}_{\mu\nu\rho}+\frac{g\,m^2}{3!}\hat{J}^{\mu\nu\rho}\frac{1}{\Box}\hat{A}_{\mu\nu\rho}+\frac{g^2}{4m^2}j^{\mu\nu}\frac{1}{\Box}j_{\mu\nu}\Bigg]. \tag{4.45}$$

The first term is of the same form as in Eq. (4.22). It represents the bubble ''volume action'' with respect to the constant energy density background that determines the false vacuum decay rate.

The second and third terms govern the dynamics of $\hat{A}_{\mu\nu\rho}$ according to the equation

$$(\Box+m^2)\hat{A}_{\mu\nu\rho}=g\,\hat{J}^{\mu\nu\rho}. \tag{4.46}$$

We emphasize that this massive mode represents the *only* propagating degree of freedom, exactly as in the $(1+1)$-dimensional case. As a matter of fact, the last term in the action (4.45) represents a boundary induced Coulomb interaction [21]. Indeed, a direct calculation taking into account $\partial_\mu j^{\mu\nu}=0$ gives

$$\frac{g^2}{4m^2}\int d^4x\,j^{\mu\nu}\frac{1}{\Box}j_{\mu\nu}$$
$$=\frac{g^2}{2m^2}\int dx^0\,d^3x\,j^{0k}(x^0,\vec{x})\,\frac{1}{\nabla^2}j^{0k}(x^0,\vec{x})$$
$$=\frac{g^2}{2m^2}\int dx^0\int d^3x\int d^3y\,j^{0k}(x^0,\vec{x})\,\frac{1}{\nabla^2}\delta^{3)}(\vec{x}-\vec{y})j^{0k}(x^0,\vec{y})$$
$$=-\frac{g^2}{4\pi m^2}\int dx^0\int d^3x\int d^3y\,j^{0k}(x^0,\vec{x})\times\frac{1}{|\vec{x}-\vec{y}|}j^{0k}(x^0,\vec{y}) \tag{4.47}$$

showing that there is no physical particle mediating such an interaction.

C. Restoring gauge invariance

In the previous subsections we have developed a self-consistent model for membranes with a spacelike boundary, coupled to a massive tensor field. The price for that result is the apparent loss of manifest gauge invariance. As in the case of ''bubble dynamics'' in $1+1$ dimensions, this leads us to the question: is there a way of introducing a mass term into the action without spoiling manifest gauge invariance? Once again, we follow the original Stueckelberg proposal [16] of restoring gauge invariance by introducing a mass term together with a compensating scalar field. Presently, however, we need a modification of Stueckelberg's approach that is suitable for our massive tensor theory of bubble dynamics. The procedure is straightforward. The only novel aspect is that the role of compensating field is now played by a two-index Kalb-Ramond potential $B_{\nu\rho}(x)$ [23]. Accordingly, we modify the action (4.32) as follows:

$$S=\int d^4x\Bigg[\frac{1}{2\times 4!}F_{\lambda\mu\nu\rho}F^{\lambda\mu\nu\rho}-\frac{1}{4!}F^{\lambda\mu\nu\rho}\,\partial_{[\lambda}A_{\mu\nu\rho]}-\frac{g}{3!}J^{\mu\nu\rho}\left(A_{\mu\nu\rho}+\frac{1}{m}\partial_{[\mu}B_{\nu\rho]}\right)-\frac{m^2}{2\times 3!}\left(A_{\mu\nu\rho}+\frac{1}{m}\partial_{[\mu}B_{\nu\rho]}\right)\left(A^{\mu\nu\rho}+\frac{1}{m}\partial^{[\mu}B^{\nu\rho]}\right)\Bigg]. \tag{4.48}$$

This action is invariant under the extended tensor gauge transformation

$$\delta A_{\mu\nu\rho}=\partial_{[\mu}\Lambda_{\nu\rho]} \tag{4.49}$$

$$\delta B_{\nu\rho}=-m\,\Lambda_{\nu\rho}. \tag{4.50}$$

Note that the (gauge invariant) kinetic term for B makes A massive: from a dynamical point of view, the presence of a boundary, or a nonconserved current, introduces a mass term for the gauge field that the current is coupled to.

The field equations become

$$\partial_\lambda F^{\lambda\mu\nu\rho}+m^2\left(A^{\mu\nu\rho}+\frac{1}{m}\partial^{[\mu}B^{\nu\rho]}\right)=g\,J^{\mu\nu\rho}, \tag{4.51}$$

$$\partial_\mu\left(A^{\mu\nu\rho}+\frac{1}{m}\partial^{[\mu}B^{\nu\rho]}\right)=\frac{g}{m^2}J^{\nu\rho}. \tag{4.52}$$

Equation (4.52) assures the self-consistency of Eq. (4.51). Moreover, using the same field decomposition as in the previous section, we see that Eq. (4.52) fixes the divergenceless component $\tilde{A}_{\mu\nu\rho}$:

$$\tilde{A}^{\mu\nu\rho}+\frac{1}{m}\partial^{[\mu}B^{\nu\rho\,]}=\frac{e}{m^2}\partial^{[\mu}\frac{1}{\Box}J^{\nu\rho]} \tag{4.53}$$

while Eq. (4.51) leads to the following expression for $F^{\lambda\mu\nu\rho}$:

$$F^{\mu\nu\rho\sigma}=\sqrt{\Lambda}\epsilon^{\mu\nu\rho\sigma}+g^2\,\partial^{[\mu}\frac{1}{\Box}\hat{J}^{\nu\rho\sigma]}-m^2\,\partial^{[\mu}\frac{1}{\Box}\hat{A}^{\nu\rho\sigma]}$$

$$\equiv F_0^{\mu\nu\rho\sigma}-m^2\,\partial^{[\mu}\frac{1}{\Box}\hat{A}^{\nu\rho\sigma]}. \tag{4.54}$$

Substituting the above expression into the action, we obtain after some rearrangement

$$S=\int d^4x\left[\frac{1}{2\times 4!}F_{0\ \mu\nu\rho\sigma}F_0^{\mu\nu\rho\sigma}\right.$$
$$-\frac{m^2}{2\times 3!}\hat{A}^{\mu\nu\rho}\left(\frac{\Box+m^2}{\Box}\right)\hat{A}_{\mu\nu\rho}$$
$$\left.+\frac{g\,m^2}{3!}\hat{J}^{\mu\nu\rho}\frac{1}{\Box}\hat{A}_{\mu\nu\rho}+\frac{g^2}{4m^2}j^{\mu\nu}\frac{1}{\Box}j_{\mu\nu}\right] \tag{4.55}$$

$$=-\frac{1}{2}\int d^4x\left[\Lambda+\frac{g^2}{2\times 3!}\hat{J}^{\mu\nu\rho}\frac{1}{\Box+m^2}\hat{J}_{\mu\nu\rho}\right.$$
$$\left.-\frac{g^2}{4m^2}j^{\mu\nu}\frac{1}{\Box}j^{\mu\nu}\right]. \tag{4.56}$$

The final form of the action (4.56) shows how the introduction of a compensating Kalb-Ramond field, which is necessary for restoring gauge invariance, leads to an ''effective closure'' of the membrane in the physical Minkowskian spacetime and is, in fact, an alternative to the Euclidean procedure of closing the membrane in imaginary time.

However, it seems to us that a careful consideration of the boundary effect in the nucleation process of a vacuum bubble in real spacetime has a clear advantage over the Euclidean formulation in that it brings out the existence of a massive pseudo-scalar degree of freedom which is otherwise hidden in the energy background provided by the cosmological field.

V. CONCLUSIONS AND OUTLOOK

To the extent that the presence or absence of a boundary constitutes a topological property of a manifold, we may refer to the idea underlying the whole discussion in this paper as *topological symmetry breaking.* The effect of this new mechanism on the inflationary-axion scenario is apparent in our formulation of bubble dynamics, and was illustrated in four as well as in two spacetime dimensions. Indeed, the action functional of bubble dynamics can be defined in any number of dimensions as a generalization of the Einstein-Maxwell action for the dynamics of point charges on a Riemannian manifold. As a matter of fact, gravity plays no special role in it: even though we have formulated the model for a bubble in $3+1$ Minkowski spacetime, it can be extended to a generic p-brane embedded in a target space with $D=p+2$ dimensions. In four dimensions and under the assumption of spherical symmetry, the field equations of bubble dynamics are integrable [15]: the net physical result of the $A_{\mu\nu\rho}$ coupling to the membrane degree of freedom is the nucleation of a bubble whose boundary separates two vacuum phases characterized by two effective and distinct cosmological constants, one inside and one outside the bubble.[8]

The three-index representation of the cosmological field is the key to the whole formulation of topological symmetry breaking and self-consistent generation of mass. That field, we have argued, represents the ultimate source of energy in the bubble universe. But how does matter manage to ''bootstrap'' itself into existence out of that source of latent energy? Here is where the difference between the conventional ''cosmological constant'' and the cosmological field comes into play: the original cosmological constant introduced by Einstein plays a somewhat passive role, in that it is ''frozen'' within the Hilbert action of general relativity; the cosmological field, on the contrary, even though it represents a nondynamical gauge field, will *interact* with gravity and combine with a bubble single degree of freedom thereby acquiring mass as a consequence of topological symmetry breaking. In fact, there are some similarities with the Higgs mechanism which may help to clarify the boundary mechanism of mass production. For instance, the extended gauge symmetry (4.49),(4.50) represents the end result of a process that begins with the violation of gauge invariance due to the presence of a boundary. Then, the Kalb-Ramond field $B_{\mu\nu}$, prescribed by the Stueckelberg procedure, represents massless, spinless particles that play the role of Goldstone bosons. However, while in the usual Higgs mechanism the spin content of the gauge field is the same before and after the appearance of mass, a new effect occurs when the gauge field is $A_{\mu\nu\rho}$: when massless, $A_{\mu\nu\rho}$ carries no degrees of freedom, while Eq. (4.1) describes massive spin-0 particles in a representation which is *dual* to the familiar Proca representation of massive, spin-1 particles [24]. In light of this formal analogy with the Higgs mechanism, axions are interpreted as massless spin-0 Goldstone bosons represented by a Kalb-Ramond field while the physical spectrum consists of mas-

[8]The evolution of the bubble, which is controlled by the two cosmological constants and by the surface tension, can be simulated by the one-dimensional motion of a fictitious particle in a potential [11]; furthermore, a well defined algorithm exists that is capable of determining all possible types of solutions, including inflationary ones, together with the region in parameter space where families of solutions can exist [13].

sive spin-0 particles represented by the $\hat{A}_{\mu\nu\rho}$ field. In this sense, topological symmetry breaking by the boundary has the same effect as the breaking of the Peccei-Quinn symmetry in the local standard model of particle physics.

We do not have at present a fully fledged quantum theory of bubble dynamics even though we have taken several steps on the way to that formulation [7,25]. However, if bubble dynamics in two dimensions is any guide, one can anticipate the main features of the quantum theory: as the volume of the bubble universe increases exponentially during the inflationary phase, so does the total (volume) energy of the interior ''de Sitter vacuum,'' at least classically. Quantum mechanically there is a competitive effect which is best understood in terms of an analogous effect in $1+1$ dimensions. As we have argued in the previous section, the''volume'' within a one-dimensional bubble is the linear distance between the two end-point charges. As the distance increases, so does the potential energy between them. Quantum mechanically, however, it is energetically more favorable to polarize the vacuum through the process of pair creation [26], which we interpret as the nucleation of secondary bubbles out of the vacuum enclosed by the original bubble. The net physical result of this mechanism is the production of massive spin-0 particles [14]. The same mechanism can be lifted to $3+1$ dimensions and reinterpreted in the cosmological context: the $A_{\mu\nu\rho}$ field shares the same properties of the gauge potential A_{μ} in two dimensions and polarizes the vacuum via the formation of secondary bubbles. Consider now a spherical bubble and focus on the radial evolution alone. The intersection of any diameter with the bubble surface evolves precisely as a particle–antiparticle pair in $1+1$ dimensions. However, since there is no preferred direction, the mechanism operates on concentric shells inside the original bubble. Remarkably, the final result is again the production of massive pseudoscalar particles in the bubble universe. However, while in two dimensions Goldstone bosons do not exist, in $3+1$ dimensions they do exist and have a direct bearing on the axion mass problem. Born out of the darkness of the cosmic vacuum, axions were invisible to begin with and remain invisible to the extent that they are ''absorbed'' by the cosmological field. According to this interpretation, one of the possible forms of dark matter in the universe, in addition to massive compact halo objects (MACHO's), emerges as the *necessary* end product of a process, driven by the cosmic vacuum energy, according to which gauge invariance and vacuum decay conspire to extract massive particles out of the cosmological field of dark energy.

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