Rayleigh-Schrödinger perturbation theory based on Gaussian wave functional approach

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(Received 16 January 2001; published 7 June 2001)

A Rayleigh-Schrödinger perturbation theory based on the Gaussian wave functional is constructed. The method can be used for calculating the energies of both the vacuum and the excited states. A model calculation is carried out for the vacuum state of the $\lambda\phi^4$ field theory.

DOI: 10.1103/PhysRevD.64.025006 PACS number(s): 11.10.Lm, 11.15.Tk

I. INTRODUCTION

The Gaussian wave-functional approach (GWFA) [1], or Gaussian approximation, has become a powerful and important tool to extract the nonperturbative results of quantum field theory [2–4], finite temperature field theory [5], and condensed matter systems [6,7] since Stevenson's advocation of the method about two decades ago [8]. The Gaussian effective potential (GEP) obtained from the GWFA provides a good starting point for further investigations of various systems [9–17]. Moreover, this approximation can also be used for realizing some novel ideas [18]. However, the GWFA is essentially a variational approximation, and hence improvement of the obtained result is not straightforward. So far, there exist mainly two methods to improve the GWFA. One way is to continue using the variational method with more elaborate, non-Gaussian trial wave functionals. For example, Kümmel and his collaborators developed the coupled cluster method and obtained results beyond the Gaussian approximation of the $\lambda\phi^4$ and ϕ^6 models [9]; Ritschel and his collaborators constructed a non-Gaussian trial wave functional through a nonlinear canonical transformation [10]; in order to investigate $(3+1)$-dimensional $\lambda\phi^4$ field theory, Yotsuyanagi proposed an improved scheme of the GWFA by adopting a BCS-type wave functional [11]. Another way of improving the GWFA is to use appropriate expansions which give the GEP in their lowest order. In this aspect, Okopińska developed an optimized expansion method to calculate the generating functional with the Euclidean formalism [12]. In the same Euclidean formalism, Stancu and Stevenson formulated a slightly different expansion scheme and calculated the post-GEP in the spirit of the background-field method [13]. Based on the GEP, Cea proposed a generalized GEP with a variational basis and carried out the calculation with the help of the standard perturbation technique in quantum field theory [14]. In the late 1990s, within the Minkowski formalism, one of the authors (Yee) and his collaborators developed the background field method to give an expansion of the effective action around the Gaussian approximate results [15]. Recently, in order to calculate the partition function of a fermionic system, two of the authors (Kim and Nahm) and their collaborator proposed a variational perturbation scheme based on the functional integral without resorting to the background field method [16]. Additionally, Solovtsov *et al.* proposed a kind of variational perturbation theory to calculate the effective potential [17]. All these investigations improve the GEP or Gaussian-approximate result with miscellaneous degrees of success. Nevertheless, further investigations are needed still to achieve better approximations.

We note that Rayleigh-Schrödinger perturbation theory is one of the basic techniques in nonrelativistic quantum mechanics [19]. It is also widely used in statistical mechanics [20] and, recently, generalized to quantum field theory [21]. Conventionally, Rayleigh-Schrödinger perturbation theory is based on an exactly solvable part of the Hamiltonian of a system. Recently, two of the authors (Kim and Nahm) and their collaborators calculated the energies of an anharmonic oscillator by combining Rayleigh-Schrödinger perturbation theory and variational method and obtained satisfactory results [22]. In this paper, we develop a variational Rayleigh-Schrödinger perturbation theory based on the Gaussian wave-functional approach (RSPTGA) in quantum field theory. Different from the schemes mentioned in the last paragraph, RSPTGA can be used to calculate the energies of both the vacuum and excited states. Applying RSPTGA to $\lambda\phi^4$ model, we calculate the vacuum state energy up to the third order. The result improves the GEP substantially and indicates a fast convergence. Comparing with the result in Ref. [14], our result up to second order is shown to introduce an additional term.

Next section, we construct a quasi-free-field eigenstate set based on the GWFA, and develop RSPTGA based on the eigenset. In Sec. III, the vacuum state energy of the $\lambda\phi^4$ field theory will be calculated using RSPTGA. Finally, physical implication of the present results will be discussed.

II. RAYLEIGH-SCHRÖDINGER PERTURBATION THEORY BASED ON THE GWFA

In this section, we briefly introduce the GWFA, and construct a quasi-free-field eigenstate set. Based on this eigenstate set, we construct RSPTGA.

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We consider a model with the Lagrangian density

$$\mathcal{L}=\frac{1}{2}\partial_\mu\phi_x\partial^\mu\phi_x-V(\phi_x), \tag{1}$$

where $x=(x^1,x^2,\ldots,x^D)$ represents a position in D-dimensional space, $\phi_x\equiv\phi(\vec{x})$ is the field at x, and the potential $V(\phi_x)$ has a Fourier representation in a sense of tempered distributions [23]. Many model potentials, such as polynomial models, sine-Gordon and sinh-Gordon models, possess this property. According to Ref. [4] (JPA), the GWFA produces the best trial vacuum wave functional:

$$|0\rangle^{(0)}=\mathcal{N}\exp\left\{-\frac{1}{2}\int_{x,y}(\phi_x-\varphi)f_{xy}(\phi_y-\varphi)\right\}, \tag{2}$$

where $\mathcal{N}$ is the normalization constant (i.e., ${}^{(0)}\langle 0|0\rangle^{(0)}=1$), $\int_{x,y}\equiv\int d^Dx d^Dy$, and $f_{xy}=\int d^Dp f(p)e^{ip(x-y)}$ with $p=(p^1,p^2,\ldots,p^D)$. The classical constant φ is equal to the Gaussian-vacuum expectation value of ϕ_x:

$$\varphi={}^{(0)}\langle 0|\phi_x|0\rangle^{(0)}. \tag{3}$$

Here,

$$f(p)=\sqrt{p^2+\mu^2(\varphi)}, \tag{4}$$

and

$$\mu^2(\varphi)=\int_{-\infty}^{\infty}\frac{d\alpha}{2\sqrt{\pi}}e^{-\alpha^2/4}V^{(2)}\left(\frac{\alpha}{2}\sqrt{I_1[\mu(\varphi)]}+\varphi\right), \tag{5}$$

where $V^{(n)}(z)=d^nV(z)/dz^n=\int_{-\infty}^{\infty}(dq/\sqrt{2\pi})(iq)^n\tilde{V}(q)e^{iqz}$ [$\tilde{V}(q)$ is the Fourier representation of $V(z)$] and $I_n(Q)=\int[d^Dp/(2\pi)^D][\sqrt{p^2+Q^2}/(p^2+Q^2)^n]$. The GEP of the system, Eq. (1), is given by

$$\mathcal{V}_G(\varphi)=\frac{1}{2}I_0(\mu)-\frac{\mu^2}{4}I_1(\mu)+\int_{-\infty}^{\infty}\frac{d\alpha}{2\sqrt{\pi}}e^{-\alpha^2/4}\times V\left(\frac{\alpha}{2}\sqrt{I_1(\mu)}+\varphi\right) \tag{6}$$

with μ chosen from the three possibilities: solution of Eq. (5), $\mu=0$ and $\mu\to\infty$ [4] (JPG). [Hereafter, we will write $\mu(\varphi)$ as μ for simplicity except for special cases.] We note that when $\mathcal{V}_G(\varphi)$ has the absolute minimum at φ_0, $\mu(\varphi_0)$ becomes just the physical mass and $\mathcal{V}_G(\varphi_0)$ represents the vacuum state energy. The symmetry of the vacuum can be discussed using Eqs. (3)–(6).

For the Gaussian vacuum, Eq. (2), one can construct the following annihilation and creation operators [4] (ZPC):

$$A_f(p)=\left(\frac{1}{2(2\pi)^Df(p)}\right)^{1/2}\int_x e^{-ipx}[f(p)(\phi_x-\varphi)+i\Pi_x] \tag{7}$$

and

$$A_f^\dagger(p)=\left(\frac{1}{2(2\pi)^Df(p)}\right)^{1/2}\int_x e^{ipx}[f(p)(\phi_x-\varphi)-i\Pi_x] \tag{8}$$

with the relations $[A_f(p),A_f^\dagger(p')]=\delta(p'-p)$ and $A_f(p)|0\rangle^{(0)}=0$. Here, $\Pi_x\equiv-i(\delta/\delta\phi_x)$ is the canonical conjugate operator to ϕ_x with the commutation relation, $[\phi_x,\Pi_y]=i\delta(x-y)$. Based on these operators, one can construct the quasi-free-field Hamiltonian,

$$H_0=\int dp f(p)A_f^\dagger(p)A_f(p)=\int_x\left[\frac{1}{2}\Pi_x^2+\frac{1}{2}(\partial_x\phi_x)^2+\frac{1}{2}\mu^2(\phi_x-\varphi)^2-\frac{1}{2}I_0(\mu)\right]. \tag{9}$$

The Gaussian vacuum, Eq. (2), is the ground state of H_0 with zero energy eigenvalue $E_0^{(0)}$. The excited states of H_0 are [21]

$$|n\rangle^{(0)}=\frac{1}{\sqrt{n!}}\prod_{i=1}^{n}A_f^\dagger(p_i)|0\rangle^{(0)},\quad n=1,2,\ldots,\infty, \tag{10}$$

with the corresponding energy eigenvalue

$$E_n^{(0)}=\sum_{i=1}^{n}f(p_i). \tag{11}$$

Note that Eq. (2) is not a naive vacuum, since it contains information on the interacting system, Eq. (1). Obviously, the wave functionals $|n\rangle^{(0)}$ and $|0\rangle^{(0)}$ are orthogonal and normalized to ${}^{(0)}\langle n|n\rangle^{(0)}=(1/n!)\Sigma_{P_i(n)}\Pi_{k=1}^{n}\delta(p_k'-p_{i_k})$ [here $P_i(n)$ represents a permutation of the set $\{i_k\}=\{1,2,\ldots,n\}$ and the summation is over all $P_i(n)$s]. $|n\rangle^{(0)}$ describes a n-particle state with the continuous momenta $p_1,p_2,\ldots,p_n$. $|0\rangle^{(0)}$ and $|n\rangle^{(0)}$ with $n=1,2,\ldots,\infty$ constitute the complete set for H_0, which we call quasi-free-field complete set.

Based on the above complete set, one can readily apply the conventional Rayleigh-Schrödinger perturbation technique to calculate the energy of the system, Eq. (1). In order to do so, we write the Hamiltonian of the system Eq. (1) as $H=H_0+H_I=H_0+(H-H_0)$ with

$$H_I=\int_x\left[-\frac{1}{2}\mu^2(\phi_x-\varphi)^2+\frac{1}{2}I_0(\mu)+V(\phi_x)\right]. \tag{12}$$

Following the Rayleigh-Schrödinger perturbation procedure [19,21], one can obtain the wave functionals and energies for the vacuum and excited states, respectively,

$$|n\rangle=\sum_{l=0}^{\infty}\left[Q_n\frac{1}{H_0-E_n^{(0)}}(E_n-E_n^{(0)}-H_I)\right]^l|n\rangle^{(0)}, \tag{13}$$

and

$$E_n(\varphi)=E_n^{(0)}+\sum_{l=0}^{\infty} \times {}^{(0)}\langle n|H_I\left[Q_n\frac{1}{H_0-E_n^{(0)}}(E_n-E_n^{(0)}-H_I)\right]^l|n\rangle^{(0)}, \tag{14}$$

with $Q_n=\Sigma_{j\neq n}^{\infty}\int d^Dp_1d^Dp_2\cdots d^Dp_j|j\rangle^{(0)\,(0)}\langle j|$.

For the case of $n=0$ and $l=0$, Eq. (14) gives the vacuum energy up to the first order $E_0^1=E_0^{(0)}+E_0^{(1)}=\int_x\mathcal{V}_G(\varphi)$ which is just the product of the GEP and the space volume. Thus, employing Eq. (14) with $n=0$, one can get the effective potential for the system, Eq. (1),

$$\mathcal{V}_{RS}(\Phi)=\frac{E_0(\varphi)}{\int d^Dx}, \tag{15}$$

with

$$\Phi=\frac{\langle 0|\phi_x|0\rangle}{\langle 0|0\rangle}, \tag{16}$$

which takes the GEP, Eq. (6), as the first-order approximation. Equation (16) implies that Φ should replace φ in the calculation of the effective potential. It is evident that φ in Eq. (3) is the zeroth order approximation, $\Phi^{(0)}$ of Φ. When Eq. (15) is truncated at nth order, one should also truncate Eq. (16) at the same order [12,13].

In the case of $n\neq 0$, Eq. (14) is the excited-state effective potential of the system [8]. Additionally, Eq. (14) becomes the vacuum and excited-state energies for the symmetric phase of the system if $\varphi=0$ is chosen in the scheme. In the next section, we apply RSPTGA to the vacuum state of the $\lambda\phi^4$ field theory.

III. APPLICATION TO $\lambda\phi^4$ FIELD THEORY

In this section, we consider the potential, $V(\phi_x)=\frac{1}{2}m^2\phi_x^2+(\lambda/4!)\phi_x^4$, which was widely studied in connection with the Gaussian approximation [14]. For this system, Eq. (5) gives rise to

$$\mu^2=m^2+\frac{1}{2}\lambda\varphi^2+\frac{1}{4}\lambda I_1(\mu), \tag{17}$$

and from Eq. (6), one readily obtains

$$\mathcal{V}_G(\varphi)=\frac{1}{2}m^2\varphi^2+\frac{1}{4!}\lambda\varphi^4+\frac{1}{2}I_0(\mu)-\frac{1}{32}\lambda I_1^2(\mu), \tag{18}$$

which is just Eq. (2.22) in Ref. [14] (PRD).[1] To obtain the effective potential of the $\lambda\phi^4$ field theory up to a given order according to Eq. (15), we first calculate the following matrix elements of H_I:

$${}^{(0)}\langle n|H_I|n\rangle^{(0)}={}^{(0)}\langle 0|H_I|0\rangle^{(0)\,(0)}\langle n|n\rangle^{(0)}+\frac{1}{n!}\frac{\lambda}{4(2\pi)^D}\sum_{\{P_{ij}(n-2)\}}\prod_{k=1}^{n-2}\Delta(p_{i_k}-p'_{j_k})\delta(p'_{j_{(n-1)}}+p'_{j_n}-p_{i_{(n-1)}}-p_{i_n}) \times[f(p'_{j_{(n-1)}})f(p'_{j_n})f(p_{i_{(n-1)}})f(p_{i_n})]^{-1/2}, \tag{19}$$

$${}^{(0)}\langle n|H_I|n-1\rangle^{(0)}=\frac{1}{\sqrt{n!(n-1)!}}\left\{\frac{\lambda\varphi}{2\sqrt{2(2\pi)^D}}\sum_{P_{ij}(n-2)}\prod_{k=1}^{n-2}\Delta(p_{i_k}-p'_{j_k})\delta(p'_{j_{(n-1)}}-p_{i_{(n-1)}}-p_{i_n}) \times[f(p'_{j_{(n-1)}})f(p_{i_{(n-1)}})f(p_{i_n})]^{-1/2}+\sqrt{\frac{(2\pi)^D}{2}}\left(\mu^2-\frac{\lambda}{3}\varphi^2\right)\varphi\sum_{P_{ij}(n-1)}\prod_{k=1}^{n-1}\Delta(p_{i_k}-p'_{j_k})\delta(p_{i_n}) \times[f(p_{i_n})]^{-1/2}\right\}, \tag{20}$$

$${}^{(0)}\langle n|H_I|n-2\rangle^{(0)}=\frac{1}{\sqrt{n!(n-2)!}}\frac{\lambda}{4(2\pi)^D}\sum_{P_{ij}(n-3)}\prod_{k=1}^{n-3}\Delta(p_{i_k}-p'_{j_k})\delta(p'_{j_{(n-2)}}-p_{i_{n-2}}-p_{i_{(n-1)}}-p_{i_n}) \times[f(p'_{j_{(n-2)}})f(p_{i_{n-2}})f(p_{i_{(n-1)}})f(p_{i_n})]^{-1/2}, \tag{21}$$

[1]Our notation $I_n(\mu)$ is different from Eq. (2.21) in Ref. [14] (PRD).

$$
{}^{(0)}\langle n|H_I|n-3\rangle^{(0)}=\frac{1}{\sqrt{n!(n-3)!}}\frac{\lambda\varphi}{2\sqrt{2(2\pi)^D}}\sum_{P_{ij}(n-3)}\prod_{k=1}^{n-3}\Delta(p_{i_k}-p'_{j_k})\,\delta(p_{2_{(n-2)}}+p_{i_{(n-1)}}+p_{i_n})
$$
$$
\times[f(p_{i_{(n-2)}})f(p_{i_{(n-1)}})f(p_{i_n})]^{-1/2} \tag{22}
$$

and

$$
{}^{(0)}\langle n|H_I|n-4\rangle^{(0)}=\frac{1}{\sqrt{n!(n-4)!}}\frac{\lambda}{4(2\pi)^D}\sum_{P_{ij}(n-4)}\prod_{k=1}^{n-4}\Delta(p_{i_k}-p'_{j_k})\,\delta(p_{i_{(n-3)}}+p_{i_{n-2}}+p_{i_{(n-1)}}+p_{i_n})
$$
$$
\times[f(p_{i_{(n-3)}})f(p_{i_{(n-2)}})f(p_{i_{(n-1)}})f(p_{i_n})]^{-1/2}, \tag{23}
$$

with

$$
\Delta(p_{i_k}-p'_{j_k})=\begin{cases}0 & \text{for } k<0,\\ 1 & \text{for } k=0,\\ \delta(p_{i_k}-p'_{j_k}) & \text{for } k>0.\end{cases}
$$

Here, the index $i_k\in\{1,2,\ldots,n\}$ with $k=1,2,\ldots,n$ corresponds to $|n\rangle^{(0)}$ and $j_k\in\{1,2,\ldots,n'\}$ with $k=1,2,\ldots,n'$ to ${}^{(0)}\langle n'|$. $P_{ij}(l)$ represents a given permutation of l momenta $p_{i_1},p_{i_2},\ldots,p_{i_l}$ paired respectively with $p'_{j_1},p'_{j_2},\ldots,p'_{j_l}$, and $\Sigma_{P_{ij}(l)}$ is over all different $P_{ij}(l)s$. For any $P_{ij}(l)$, $i_1,i_2,\ldots,i_l$ are different from one another, and so are $j_1,j_2,\ldots,j_l$. Employing the above matrix elements, a straightforward, yet lengthy calculation according to Eq. (15) gives the effective potential of the $\lambda\phi^4$ field theory up to the third order as

$$
\mathcal{V}^{iii}(\varphi)\equiv\frac{E_0^{iii}}{(2\pi)^D\delta(0)}=\mathcal{V}_G(\varphi)-\frac{1}{2}\frac{1}{\mu^2}\varphi^2\left(\mu^2-\frac{\lambda}{3}\varphi^2\right)^2-\frac{A}{48}\frac{\lambda^2}{\mu^2}\varphi^2-\frac{B}{384}\frac{\lambda^2}{\mu^2}+\frac{A+A_1+A_2}{48}\frac{\lambda^2}{\mu^4}\varphi^2\left(\mu^2-\frac{\lambda}{3}\varphi^2\right)
$$
$$
+\frac{2B_1+B_2}{128}\frac{\lambda^3}{\mu^4}\varphi^2+\frac{C}{512}\frac{\lambda^3}{\mu^4}, \tag{24}
$$

with

$$
A=\int\frac{dx}{(2\pi)^D}\frac{dy}{(2\pi)^D}[f_1(x)f_1(y)f_1(x+y)]^{-1}[f_1(x)+f_1(y)+f_1(x+y)]^{-1},
$$

$$
A_1=\int\frac{dx}{(2\pi)^D}\frac{dy}{(2\pi)^D}[f_1(x)f_1(y)f_1(x+y)]^{-1}[1+f_1(x)+f_1(y)+f_1(x+y)]^{-1},
$$

$$
A_2=\int\frac{dx}{(2\pi)^D}\frac{dy}{(2\pi)^D}[f_1(x)f_1(y)f_1(x+y)]^{-1}[f_1(x)+f_1(y)+f_1(x+y)]^{-1}
$$
$$
\times[1+f_1(x)+f_1(y)+f_1(x+y)]^{-1},
$$

$$
B=\int\frac{dx}{(2\pi)^D}\frac{dy}{(2\pi)^D}\frac{dz}{(2\pi)^D}[f_1(x)f_1(y)f_1(z)f_1(x+y+z)]^{-1}
$$
$$
\times[f_1(x)+f_1(y)+f_1(z)+f_1(x+y+z)]^{-1},
$$

$$B_1=\int \frac{dx}{(2\pi)^D}\frac{dy}{(2\pi)^D}\frac{dz}{(2\pi)^D}[f_1(x)f_1(y)f_1(z)f_1(x+y)f_1(x+y+z)]^{-1}$$
$$\times[f_1(x)+f_1(y)+f_1(x+y)]^{-1}[f_1(x)+f_1(y)+f_1(z)+f_1(x+y+z)]^{-1},$$
$$B_2=\int \frac{dx}{(2\pi)^D}\frac{dy}{(2\pi)^D}\frac{dz}{(2\pi)^D}[f_1(x)f_1(y)f_1(z)f_1(x+y)f_1(x+z)]^{-1}$$
$$\times[f_1(x)+f_1(y)+f_1(x+y)]^{-1}[f_1(x)+f_1(z)+f_1(x+z)]^{-1},$$
$$C=\int \frac{dx}{(2\pi)^D}\frac{dy}{(2\pi)^D}\frac{dz}{(2\pi)^D}\frac{d\omega}{(2\pi)^D}[f_1(x)f_1(y)f_1(z)f_1(\omega)f_1(x+y+z)f_1(x+y+\omega)]^{-1}$$
$$\times[f_1(x)+f_1(y)+f_1(z)+f_1(x+y+z)]^{-1}[f_1(x)+f_1(y)+f_1(\omega)+f_1(x+y+\omega)]^{-1}$$

and $f_1(w)=\sqrt{1+w^2}$. Here, we take φ as the zeroth order approximation of Φ. In Eq. (24), the second, third, and fourth terms are the second order correction and the last three terms the third order correction to the GEP.

From Eq. (24), one can see that after $\mathcal{V}_G(\varphi)$ is renormalized, its corrections will be finite and, accordingly, a further renormalization procedure is not needed for higher order corrections. In the case of $(0+1)$ dimensions, our second-order result with $\varphi=0$ is consistent with Eq. (14) in Ref. [22]. Furthermore, numerical calculation for the case of $(1+1)$ dimensions indicates that the effective potential with the second-order correction predicts existence of a second-order phase transition. We also note that the second-order correction improves GEP substantially, and the third order correction is vanishingly small. This can be seen by comparing the coefficients of the second, third and fourth terms of Eq. (24) with those of the last three terms of Eq. (24), respectively. This indicates that RSPTGA has a fast convergence in general.

IV. DISCUSSION AND CONCLUSION

In this paper, a Rayleigh-Schrödinger perturbation theory based on the GWFA within the framework of quantum field theory is proposed. Since the theory is based on the Gaussian approximation, it provides a systematical tool for controlling the Gaussian approximation. It can be used not only for calculating the effective potential but also for considering excited states. When one is interested in symmetric phase, the vacuum and excited-state energies can be calculated beyond the Gaussian approximation by RSPTGA using $\varphi=0$. Application of RSPTGA to the $\lambda\phi^4$ field theory shows that it can improve the GEP substantially with a fast convergence.

We note that RSPTGA predicts existence of a second-order phase transition in the $(1+1)$-dimensional $\lambda\phi^4$ field theory, although the critical coupling is very small. It may be attributed to the fact that for the second-order case we approximated $\Phi=\langle 0|\phi_x|0\rangle|_{l=2}$ as φ. We also note that the second-order result of Eq. (24) has an additional term $-\frac{1}{2}(1/\mu^2)\varphi^2[\mu^2-(\lambda/3)\varphi^2]^2$ which does not appear in Ref. [14]. In fact, Eq. (12) gives, for the $\lambda\phi^4$ field theory,

$$H_I=\int_x\left\{\mathcal{V}_G(\varphi)+\varphi\left(\mu^2-\frac{\lambda}{3}\varphi^2\right):(\phi_x-\varphi):+\frac{\lambda}{3!}\varphi:\right.$$
$$\left.\times(\phi_x-\varphi)^3:+\frac{\lambda}{4!}:(\phi_x-\varphi)^4:\right\}, \tag{25}$$

which is equal to Eq. (4.19) in Ref. [14] (PRD) except the $\int_x\mathcal{V}_G(\varphi)$ term, where the colons mean normal ordering with respect to the Gaussian vacuum, Eq. (2). The additional term in Eq. (24) arises from the linear term $\varphi[\mu^2-(\lambda/3)\varphi^2]:(\phi_x-\varphi):$, and disappears when the constraint Eq. (5.18) in Ref. [14] (PRD) is adopted in our scheme; that is, when Φ in Eq. (16) is taken as φ. However, we note that φ is simply the Gaussian-vacuum average value of the field operator [see Eq. (3)] and Φ is not equal to φ when higher order contributions are calculated [10,12,13]. Moreover, the variational procedure which led to the GEP, Eq. (18) produces the extremum condition, Eq. (17) [i.e., Eq. (2.19) in Ref. [14] (PRD)], and Eq. (5.18) in Ref. [14] (PRD) at any truncated order is not compatible with this extremum condition.

In closing the paper, we like to point out that it is straightforward to generalize RSPTGA to other cases, such as $O(N)$-symmetrical $\lambda\phi^4$ model [26], a Fermion field system, and so on. In fact, a Rayleigh-Schrödinger perturbation technique based on the variational results has been applied to a polaron problem [24]. Moreover, one of the authors (Lu) developed the GWFA in thermofield dynamics [25,5] (Lu). Based on it, it is possible to develop RSPTGA within the framework of thermofield dynamics which will be useful for treating finite temperature cases. Finally, instead of performing the variational procedure in the GWFA as described above, the extremization process with respect to μ can be carried out after truncating the series of Eq. (14) at some given order [12]. This procedure will lead to a slightly different variation of RSPTGA [8]. It may have its own advantages or peculiarities over the scheme developed here.

ACKNOWLEDGMENTS

This project was supported by the Korea Research Foundation (99–005–D00011). Lu's work was also supported in part by the National Natural Science Foundation of China under Grant No. 19875034.

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