

Brane-world cosmology of modulus stabilization with a bulk scalar field

James M. Cline and Hassan Firouzjahi

Physics Department, McGill University, 3600 University Street, Montréal, Québec, Canada H3A 2T8

(Received 24 May 2000; published 11 June 2001)

We point out that the potential of Goldberger and Wise for stabilizing the distance between two 3-branes, separated from each other along an extra dimension with a warp factor, has a metastable minimum when the branes are infinitely separated. The classical evolution of the radion (brane separation) will place it in this false minimum for generic initial conditions. In particular, inflation could do this if the expansion rate is sufficiently large. We present a simplified version of the Goldberger-Wise mechanism in which the radion potential can be computed exactly, and we calculate the rate of thermal transitions to the true minimum, showing that model parameters can be chosen to ensure that the universe reaches the desired final state. Finiteness of bulk scalar field brane potentials can have an important impact on the nucleation rate, and it can also significantly increase the predicted mass of the radion.

DOI: 10.1103/PhysRevD.64.023505

PACS number(s): 98.80.Cq

I. INTRODUCTION

One of the most striking proposals in current elementary particle theory is the existence of extra dimensions which are hidden from us, not necessarily by their smallness, but by our confinement to a four-dimensional slice (a 3-brane) of the full spacetime [1]. Randall and Sundrum [2] have produced a version of this scenario which is particularly attractive because of its apparent links to deep theoretical developments: the conformal field theory/5D anti-de Sitter correspondence and holography [3]. On a more practical level, their idea provides an explanation for why the Planck scale is so much greater than the weak scale, independently of supersymmetry. The cosmological implications of this model have also been the subject of vigorous study [4].

The Randall-Sundrum proposal involves a “Planck brane” located at a position $y=0$ in a single additional dimension, and a second “TeV brane” located at $y=1$, in our conventions. The extra dimension is permeated by a negative bulk vacuum energy density, so that the space between the branes is a slice of anti-de Sitter space. Solving the 5D Einstein equations results in the line element

$$ds^2 = e^{-2\sigma(y)}(dt^2 - d\vec{x}^2) - b^2 dy^2, \quad (1)$$

$$\sigma(y) = kby, \quad y \in [0,1]. \quad (2)$$

The constant k is related to the 4D and 5D Planck masses, M and M_p , respectively, by

$$k = \frac{M^3}{M_p^2} \quad (3)$$

where $M_p^{-2} = 8\pi G_N$ and the 5D gravitational action is $S = \frac{1}{2}M^3 \int d^5x \sqrt{-g}R$. The warp factor $e^{-\sigma(y)}$ determines the physical masses of particles on the TeV brane: even if a bare mass parameter m_0 in the TeV brane Lagrangian is of order M_p , the physical mass gets rescaled by

$$m_{\text{phys}} = e^{-\bar{\sigma}} m_0, \quad \bar{\sigma} \equiv \sigma(1) = kb, \quad (4)$$

as can be easily derived starting from a scalar field action written covariantly in terms of the metric (1), $S = \frac{1}{2} \int d^4x e^{-4\bar{\sigma}} [e^{2\bar{\sigma}} (\partial\phi)^2 - m_0^2 \phi^2]$, and rescaling ϕ so that the kinetic term becomes canonically normalized.

In order for m_0 to be of order 100 GeV, the combination kb must be approximately 36, so that $e^{-\bar{\sigma}} \sim 10^{-16}$. Yet in the original proposal, the value of b , which is the size of the extra dimension, was completely undetermined. It is a modulus with no potential, which is phenomenologically unacceptable. For one thing, the particle associated with 4D fluctuations of b , the radion, would couple to matter on the TeV brane similarly to gravitons, but more strongly by a factor of $e^{\bar{\sigma}}$ [5]. This would lead to a fifth force which could easily have been detected. Furthermore, a massless radion leads to problems with cosmology: our brane universe would have to have a negative energy density to expand at the expected rate, assuming that energy densities on the branes are tuned to give a static extra dimension [6]. It was shown in Refs. [7,8] (see also [9]) that this problem disappears when the size of the extra dimension is stabilized.

Radion stabilization is therefore a crucial ingredient of the Randall-Sundrum idea. Goldberger and Wise have presented an elegant mechanism for accomplishing this [10], using a bulk scalar field. Self-interactions of the field on the branes forces it to take nonvanishing vacuum expectation values (VEV's), v_0 and v_1 respectively, which are generally different from each other. The field thus has a gradient in the extra dimension, and the competition between the gradient and potential energies gives a preferred value for the size of the extra dimension. In other words, a potential for the radion is generated, which has a nontrivial minimum. It is easy to get the correct brane separation using natural values of the parameters in the model.

Roughly speaking, the radion potential has the form

$$V(\phi) \cong \lambda \phi^4 \left[\left(\frac{\phi}{f} \right)^\epsilon - \frac{v_1}{v_0} \right]^2 \quad (5)$$

with a nontrivial minimum at $\phi = f(v_1/v_0)^{1/\epsilon}$. However, there is another minimum at $\phi = 0$, which represents an in-

finitely large extra dimension. This could not describe our universe, because then $e^{-\bar{\sigma}}$ would be zero, corresponding to vanishing particle masses on the TeV brane. In the more exact expression for the potential, we will show that $\phi=0$ is actually a false vacuum—it has higher energy than the non-trivial minimum. Nevertheless, it is quite likely that the metastable state could be reached through classical evolution in the early universe. The question then naturally arises whether tunneling or thermal transitions to the desired state occurs. This is the subject of our study.

Such detailed questions about the viability of the Goldberger-Wise mechanism are important because there are few attractive alternatives at the moment. Reference [11] recently studied Casimir energies of fields between the branes as a possible origin of a stabilizing potential. They found that stabilization in this way is indeed possible, but that the resulting radion mass is too small to be phenomenologically consistent if the size of the extra dimension is that taken to be that dictated by the hierarchy problem.

In Sec. II we derive the Goldberger-Wise potential for the radion in a slightly simplified model which allows for the exact solution of the potential. The classical evolution of the radion field is considered in Sec. III, where we show that for generic initial conditions, the universe reaches a state in which the radion is not stabilized, but instead the extra dimension has become infinitely large. This is a metastable state however, and in Sec. IV the rate of transitions to the minimum energy state, with finite brane separation, is computed. Conclusions are given in Sec. V.

II. RADION POTENTIAL

Let $\psi(y)$ be the bulk scalar field which will be responsible for stabilizing the radion. Its action consists of a bulk term plus interactions confined to the two branes, located at coordinate positions $y=0$ and $y=1$, respectively. Using the variable σ of Eq. (2) in place of y , the 4D effective Lagrangian for a static ψ configuration can be written as

$$\mathcal{L} = -\frac{k}{2} \int_{-\bar{\sigma}}^{\bar{\sigma}} e^{-4\sigma} [(\partial_\sigma \psi)^2 + \hat{m}^2 \psi^2] d\sigma - m_0(\psi_0 - v_0)^2 - e^{-4\bar{\sigma}} m_1(\psi_1 - v_1)^2, \quad (6)$$

where \hat{m} is the dimensionless mass m/k , ψ_i are the values of ψ at the respective branes, and the orbifold symmetry $\psi(\sigma) = \psi(-\sigma)$ is to be understood. In 5D, the field ψ , as well as the VEV's v_i on the two branes, have dimensions of (mass)^{3/2}, while the parameters m_i have dimensions of mass. Denoting $\partial_\sigma \psi = \psi'$, the Euler-Lagrange equation for ψ is

$$k(\psi'' - 4\psi' - \hat{m}^2 \psi) = 2m_0(\psi_0 - v_0)\delta(\sigma) + 2m_1(\psi_1 - v_1)\delta(\sigma - \bar{\sigma}). \quad (7)$$

The general solution has the form

$$\psi = e^{2\sigma}(Ae^{\nu\sigma} + Be^{-\nu\sigma}), \quad \nu = \sqrt{4 + \hat{m}^2} \equiv 2 + \epsilon. \quad (8)$$

To get the correct hierarchy between the Planck and weak scales, it is necessary to take \hat{m}^2 to be small, hence the notation ϵ .

The brane terms induce boundary conditions specifying the discontinuity in ψ' at the two branes. Imposing the orbifold symmetry $\psi(-\sigma) = \psi(\sigma)$, this implies that

$$\begin{aligned} \psi'(0) &= \hat{m}_0(\psi_0 - v_0), \\ \psi'(\bar{\sigma}) &= -\hat{m}_1(\psi_1 - v_1), \end{aligned} \quad (9)$$

where we defined $\hat{m}_i = m_i/k$. Substituting the solution (8) into Eq. (9) allows us to solve for the unknown coefficients A and B exactly. In this respect, the present model is simpler than that originally given in Ref. [10]. There the brane potentials were taken to be quartic functions, so that A and B could only be found in the approximation where the field values ψ_i were very strongly pinned to their minimum energy values, v_i . In our model this would occur in the limit $m_i \rightarrow \infty$. However, we can easily explore the effect of leaving these parameters finite since A and B can be determined exactly. Let us denote

$$\hat{\phi} = e^{-kb} = e^{-\bar{\sigma}}, \quad (10)$$

which will be convenient in the following, because $\hat{\phi}$ is proportional to the canonically normalized radion field. Then A and B are given by

$$\begin{aligned} A &= (-C_1 \hat{\phi}^\nu + C_2 \hat{\phi}^2) \hat{\phi}^\nu / D(\hat{\phi}) \\ B &= (C_3 \hat{\phi}^{-\nu} - C_4 \hat{\phi}^2) \hat{\phi}^\nu / D(\hat{\phi}) \end{aligned} \quad (11)$$

where

$$C_1 = \hat{m}_0 v_0 (\hat{m}_1 - \epsilon)$$

$$C_2 = \hat{m}_1 v_1 (\hat{m}_0 + \epsilon)$$

$$C_3 = \hat{m}_0 v_0 (\hat{m}_1 + 4 + \epsilon)$$

$$C_4 = \hat{m}_1 v_1 (\hat{m}_0 - 4 - \epsilon)$$

$$D(\hat{\phi}) = \frac{(C_2 C_3 - C_1 C_4 \hat{\phi}^{2\nu})}{(\hat{m}_0 \hat{m}_1 v_0 v_1)}. \quad (12)$$

It can be checked that in the limit $\hat{m}_i \rightarrow \infty$, the field values on the branes approach $\psi_i \rightarrow v_i$. For finite \hat{m}_i , the competing effect of minimizing the bulk energy causes departures from these values, however.

The solution for ψ can be substituted back into the Lagrangian (6) to obtain the effective 4D potential for the radion, $V(\hat{\phi})$. However, rather than substituting directly, one can take advantage of the fact that ψ is a solution to the Euler-Lagrange equation. Doing a partial integration and using the boundary terms in Eq. (7) gives a simpler expression

for $V(\hat{\phi})$. In the general case where the brane potentials are denoted by $V_i(\psi)$, one can easily show that

$$\mathcal{L} = - \sum_i e^{-4\sigma_i} [V_i(\psi_i) - \frac{1}{2} \psi_i V'_i(\psi_i)]. \quad (13)$$

In the present case, we obtain

$$\begin{aligned} V(\hat{\phi}) &= -\mathcal{L} = m_0 v_0 (v_0 - \psi_0) + \hat{\phi}^4 m_1 v_1 (v_1 - \psi_1) \\ &= m_0 v_0 [v_0 - (A+B)] + \hat{\phi}^4 m_1 v_1 \\ &\quad \times [v_1 - \hat{\phi}^{-2} (A \hat{\phi}^{-\nu} + \hat{\phi}^{\nu} B)]. \end{aligned} \quad (14)$$

In the following, we will be interested in the situation where $V(\hat{\phi})$ has a nontrivial minimum for very small values of $\hat{\phi} \sim 10^{-16}$, as needed to address the weak scale hierarchy problem. It is therefore a good approximation to expand $V(\hat{\phi})$ near $\hat{\phi}=0$, keeping only the terms $\hat{\phi}^n$ with $n \leq 2\nu$. This is accomplished by expanding the denominator $D(\hat{\phi})$ in Eqs. (11), after which the potential can be written in the simple form

$$V(\hat{\phi}) = \Lambda \hat{\phi}^4 [(1 + \epsilon_4 - \epsilon_1) \hat{\phi}^{2\epsilon} - 2\eta(1 + \epsilon_4) \hat{\phi}^{\epsilon} + \eta^2] \quad (15)$$

where we introduce the notation

$$\epsilon_4 = \frac{\epsilon}{4}, \quad \epsilon_0 = \frac{\epsilon}{\hat{m}_0}, \quad \epsilon_1 = \frac{\epsilon}{\hat{m}_1}, \quad (16)$$

$$\eta = (1 + \epsilon_0) \frac{v_1}{v_0} \quad (17)$$

and

$$\Lambda = 4k v_0^2 \frac{(1 + \epsilon_4)(1 + \epsilon_0)^{-2}}{\left(1 + \frac{4}{\hat{m}_1} + \epsilon_1\right)}. \quad (18)$$

In the following it will be convenient for us to rewrite $V(\hat{\phi})$ in the form

$$V(\hat{\phi}) = \Lambda' \hat{\phi}^4 (\hat{\phi}^{\epsilon} - c_+) (\hat{\phi}^{\epsilon} - c_-) \quad (19)$$

where c_{\pm} are given by

$$c_{\pm} = \eta \left(\frac{(1 + \epsilon_4) \pm \sqrt{(1 + \epsilon_4)^2 - (1 + \epsilon_4 - \epsilon_1)}}{(1 + \epsilon_4 - \epsilon_1)} \right) \quad (20)$$

and $\Lambda' = \Lambda(1 + \epsilon_4 - \epsilon_1)$.

III. PHENOMENOLOGY AND EARLY COSMOLOGY OF THE MODEL

The warp factor which determines the hierarchy between the weak and Planck scales can be found by minimizing the potential (19). Expanding in ϵ , it has a global minimum and a local maximum at the respective values $\hat{\phi}_+$ and $\hat{\phi}_-$:

$$\begin{aligned} \hat{\phi}_{\pm} &= \left(\frac{v_1}{v_0} \right)^{1/\epsilon} \left(\frac{1 + \epsilon_0}{1 + \epsilon_4 - \epsilon_1} \right)^{1/\epsilon} (1 \pm \sqrt{\epsilon_1 + \epsilon_4 - \frac{1}{2}\epsilon\epsilon_1})^{1/\epsilon} \\ &\equiv \left(\frac{v_1}{v_0} \right)^{1/\epsilon} \exp \left[\pm \sqrt{\frac{1}{\epsilon} \left(\frac{1}{\hat{m}_1} + \frac{1}{4} \right)} + \frac{1}{\hat{m}_0} + \frac{1}{\hat{m}_1} - \frac{1}{4} \right]. \end{aligned} \quad (21)$$

The last approximation holds in the limit of small ϵ , ϵ_0 and ϵ_1 ; it is not always an accurate approximation for the parameter values of interest, so we will use the exact expression in any computations which might be sensitive to the actual value. The positions of the zeros of V , $\hat{\phi} = c_{\pm}^{1/\epsilon}$, are slightly greater than $\hat{\phi}_{\pm}$, by the factor $(1 + \epsilon_4)^{1/\epsilon} \cong e^{1/4}$, as can be seen by comparing Eq. (21) with Eq. (20). The large hierarchy of $\hat{\phi}_+ \sim 10^{-16}$ is achieved by taking a moderately small ratio $v_1/v_0 < 1$ and raising it to a large power,¹ $1/\epsilon$. This leads to the mass scale which functions like the cutoff on the TeV brane,

$$\hat{\phi}_+ M_p \equiv M_{\text{TeV}} \quad (22)$$

where $M_{\text{TeV}}/(1 \text{ TeV})$ is a number of order unity, which we will take to be one of the phenomenological free parameters of the model. The choice of M_{TeV} specifies precisely where we want our cutoff scale to be. In Ref. [10] the exponential corrections to $(v_1/v_0)^{1/\epsilon}$ in Eq. (21) were not considered; these change somewhat the values one might choose for v_1/v_0 and ϵ to get the correct hierarchy. The factor $e^{\pm \sqrt{(1/\hat{m}_1 + 1/4)/\epsilon}}$ in particular can be significant.

References [7] and [5] showed that the canonically normalized radion field is $\phi = f\hat{\phi}$, where $f = \sqrt{6M^3/k}$ is another scale of order M_p .² The 4D effective action for the radion and gravity is

$$\begin{aligned} S &= \frac{M^3}{2k} \int d^4x \sqrt{-g} (1 - \hat{\phi}^3) R \\ &\quad + \int d^4x \sqrt{-g} \left[\frac{1}{2} f^2 \partial_{\mu} \hat{\phi} \partial^{\mu} \hat{\phi} - V(\hat{\phi}) \right], \end{aligned} \quad (23)$$

where $V(\hat{\phi})$ is the potential (19) and R is the Ricci scalar. To get the correct strength of gravity, we must therefore have

$$\frac{M^3}{k} = M_p^2 = \frac{1}{8\pi G_N}, \quad f = \sqrt{6} M_p. \quad (24)$$

¹An alternative possibility, taking $v_1/v_0 > 1$ and $\epsilon < 0$, corresponding to a negative squared mass in the bulk Lagrangian (6), does not work. Although the negative m^2 does not necessarily lead to any instability, since the field is prevented from going to infinity by the potentials on the branes, the radion potential has no non-trivial minimum in this case.

²The choice $f = \sqrt{24M^3/k}$ in Ref. [5] seems to correspond to an unconventional normalization for M_p .

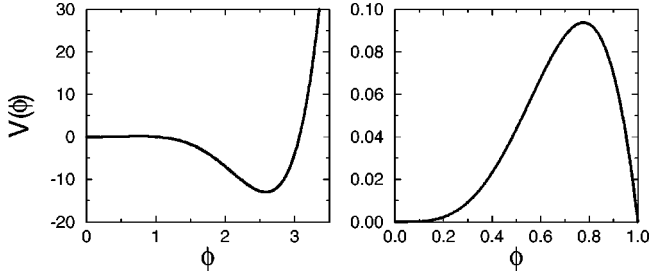


FIG. 1. $V(\phi)$ versus ϕ for $\epsilon=0.2$, showing the smallness of the barrier (right) relative to the minimum (left). Notice the difference in vertical scales.

The radion mass is f^{-2} times the second derivative of V evaluated at its minimum. We find the value

$$m_\phi^2 = 4 \frac{\Lambda}{f^2} \left(1 + \frac{\epsilon}{4} - \frac{\epsilon}{\hat{m}_1} \right)^{-1} \left(\sqrt{1 + \frac{4}{\hat{m}_1}} + 2 \frac{\sqrt{\epsilon}}{\hat{m}_1} \right) \eta^2 \hat{\phi}_+^2 \epsilon^{3/2}, \quad (25)$$

which implies that m_ϕ is of typically of order $\epsilon^{3/4}$ times the TeV scale. The factor of $\epsilon^{3/4}$ leads to the prediction that the radion will be lighter than the Kaluza-Klein excitations of the graviton, which would also be a signal of the new brane physics [5]. However, we see that the corrections due to finite \hat{m}_1 , which were not explicitly considered in [5], can possibly compensate this and make the radion heavier, if $\hat{m}_1 \sim \epsilon$.

Now let us turn to the evolution of $\hat{\phi}$ in the early universe. For this purpose it is important to understand that the depth of the potential at its global minimum, as well as the height of the bump separating the minimum from $\hat{\phi}=0$, is set by the TeV scale. The values of the potential at these extrema are approximately (to leading order in ϵ_4 , but exact in ϵ_1)

$$V(\hat{\phi}_\pm) \cong \mp 2 \eta^2 \Lambda \hat{\phi}_\pm^4 \frac{\epsilon_4 \sqrt{\epsilon_1 + \epsilon_4}}{1 + \epsilon_4 - \epsilon_1} (1 \pm \sqrt{\epsilon_1}). \quad (26)$$

Since $\Lambda \sim M_p^4$, the depth at the minimum is $V(\hat{\phi}_+) \sim -\epsilon^{3/2} O(\text{TeV})^4$ —suppressed slightly by the factor of $\epsilon^{3/2}$. The height of the bump at $\hat{\phi}_-$ can be considerably smaller because of the exponential factors in Eq. (21). Ignoring factors of order unity, for small ϵ ,

$$\left| \frac{V(\hat{\phi}_-)}{V(\hat{\phi}_+)} \right| \sim \left(\frac{\hat{\phi}_-}{\hat{\phi}_+} \right)^4 \equiv \Omega^4 \quad (27)$$

where

$$\Omega \equiv \left(\frac{1 - \sqrt{\epsilon_1 + \epsilon_4}}{1 + \sqrt{\epsilon_1 + \epsilon_4}} \right)^{1/\epsilon} \sim \exp \left[- \sqrt{\frac{1}{\epsilon}} \left(1 + \frac{4}{\hat{m}_1} \right) \right]. \quad (28)$$

For example, if $\epsilon=0.01$ as suggested by [10], Ω^4 is less than 10^{-17} . If the brane potential parameter m_1 is not large, so that $\hat{m}_1 \lesssim 1$, the suppression will be much greater. Figure 1

illustrates the flatness of the potential for the case $\epsilon=0.2$, where the barrier is not so suppressed. The new mass scale $\Omega \text{ TeV} \ll 1 \text{ TeV}$ is due to the small curvature of the radion potential at the top of the barrier, and its smallness will play an important role in the following.

Thus the barrier separating the true minimum at $\hat{\phi}_+$ from the false one at $\hat{\phi}=0$ is extremely shallow. Moreover a generic initial condition for the radion is a value like $\hat{\phi} \sim 1$, quite different from the one we want to end up with, $\hat{\phi} \sim 10^{-16}$. Clearly, the shape of the potential is such that, if we started with a generic initial value for $\hat{\phi}$, it would easily roll past the global minimum and the barrier, hardly noticing their presence. The point $\hat{\phi}=0$ toward which it rolls is the limit of infinite brane separation, phenomenologically disastrous since gravity no longer couples at all to the visible brane in this limit.

One might wonder whether inflation could prevent this unwanted outcome, since then there would be a damping term in the ϕ equation of motion, possibly causing it to roll slowly:

$$\ddot{\phi} + 3H\dot{\phi} + V'_{\text{eff}}(\phi) = 0. \quad (29)$$

Indeed, with sufficiently large Hubble rate H , the motion could be damped so that ϕ would roll to its global minimum. The condition for slow-roll is that

$$V''_{\text{eff}} \ll 9H^2. \quad (30)$$

However, inflation is a two-edged sword in this instance, because the effective potential V_{eff} for the radion gets additional contributions from the curvature of the universe during inflation. From Eq. (23) one can see that

$$\begin{aligned} V_{\text{eff}}(\hat{\phi}) &= V(\hat{\phi}) + \frac{M^3}{2k} R \hat{\phi}^2 = V(\hat{\phi}) + 6 \frac{M^3}{k} H^2 \hat{\phi}^2 \\ &= V(\hat{\phi}) + H^2 \hat{\phi}^2 \end{aligned} \quad (31)$$

using the relation $R = 12H^2$ which applies for de Sitter space and Eq. (24). The new term tends to destroy the nontrivial minimum of the radion potential. One can estimate the relative shift in the position of the minimum as

$$\frac{\delta \hat{\phi}_+}{\hat{\phi}_+} = - \frac{\delta V'(\hat{\phi}_+)}{\hat{\phi}_+ V''(\hat{\phi}_+)} = - \frac{H^2}{m_\phi^2}. \quad (32)$$

This should be less than unity to avoid the disappearance of the minimum altogether.

Combining the requirement that the global minimum survives with the slow-roll condition (30), evaluated near the minimum of the potential, we find the following constraint on the Hubble rate:

$$\frac{1}{9} m_\phi^2 \ll H^2 < m_\phi^2. \quad (33)$$

This is a narrow range, if it exists at all. In fact, one never expects such a large Hubble rate in the Randall-Sundrum scenario since the TeV scale is the cutoff: H should never

exceed $T^2/M_p \sim 10^{-16}$ TeV if the classical equations are to be valid. The problem of radion stabilization might also be exacerbated because contributions to the energy density of the universe which cause inflation can give additional terms of the type $\hat{\phi}^2$ to V_{eff} which are not considered in the above argument. For example, a field in the bulk which does not have its end points fixed on the branes has a 5D energy density which is peaked on the visible brane [12], $\rho_5(y) \sim \rho_0 e^{2kby}$, and gives a contribution to V_{eff} of

$$\delta V \sim \rho_0 \int_0^1 dy b e^{-4kby} \rho_0 e^{2kby} = b \rho_0 \hat{\phi}^2, \quad (34)$$

remembering that $e^{-kb} = \hat{\phi}$. Such a contribution could destroy the nontrivial minimum even if Eq. (33) is satisfied. In any case, it does not appear to be natural to tune the Hubble rate during inflation to try to solve the stabilization problem.

IV. PHASE TRANSITION TO THE TRUE VACUUM

Since the barrier of the radion potential is too small to prevent the radion from rolling into the false minimum, perhaps we can take advantage of this smallness to get tunneling or thermal transitions back into the true vacuum. The situation is quite similar to that of “old inflation” [13], except that in the latter, the transition was never able to complete because the universe expanded too rapidly compared to the rate of nucleation of bubbles of the true vacuum. In the present case this problem can be avoided because we are not trying to use the radion for inflation. Indeed, a small amount of inflation may take place before the tunneling occurs, since the radion potential is greater than zero at $\phi=0$,³ but we will not insist that this be sufficient to solve the cosmological problems inflation is intended to solve—otherwise we would be stuck with the problems of old inflation. Instead we will assume that inflation is driven by some other field, and consider the transitions of the radion starting from the time of reheating. The criterion that the phase transition completes is that the rate of bubble nucleation per unit volume, Γ/V , exceeds the rate of expansion of the universe per Hubble volume:

$$\frac{\Gamma}{V} \gtrsim H^4. \quad (35)$$

The reason is that the bubbles expand at nearly the speed of light, so the relevant volume is determined by the distance which light will have traveled by a given time, which is of order $1/H$.

³In Eqs. (19) and (31) we have neglected an additive constant in the potential which is needed to make $V=0$ at the true minimum, so that the cosmological constant is zero. This neglect is necessary for finding the bounce solution, since otherwise the energy of the bounce would diverge, but one must remember that the false minimum is not at zero energy when computing the Hubble rate.

A. Euclidean bounce

To compute the nucleation rate Γ/V , one must construct the bounce solution which is a saddle point of the Euclideanized action [14], in other words, with the sign of the potential reversed. This is a critical bubble solution with the boundary conditions

$$\begin{aligned} \phi(r)|_{r=0} &= \phi_0, & \phi'(r)|_{r=0} &= 0, \\ \phi(r)|_{r \rightarrow \infty} &= 0, & \phi'(r)|_{r \rightarrow \infty} &= 0. \end{aligned} \quad (36)$$

The value of ϕ_0 which ensures the desired behavior as $r \rightarrow \infty$ cannot be computed analytically because the motion of the field is damped by the term ϕ'/r in the equation of motion. We will consider bubble nucleation at finite temperature in the high T limit, where the bounce solutions are three dimensional. The equation of motion is

$$\frac{1}{r^2} (r^2 \phi')' = +V'_{\text{eff}}(\phi), \quad (37)$$

where now V_{eff} includes thermal corrections, which are much larger than the $H^2 \phi^2$ term considered in Eq. (31):

$$\begin{aligned} V_{\text{eff}}(\phi) &\cong V(\phi) + \frac{T^2}{24} m_{\text{th}}^2(\phi) - \frac{T}{12\pi} m_{\text{th}}^3(\phi) \\ &\quad + \frac{c_b}{64\pi^2} m_{\text{th}}^4(\phi) - V_0, \end{aligned} \quad (38)$$

$$m_{\text{th}}^2(\phi) = V''(\phi) + \frac{1}{24} T^2 V^{(4)}(0) \quad (39)$$

where $c_b \cong 3.9$ if the renormalization scale is taken to be equal to T . The leading thermal correction is of order $T^2 \phi^2$, whereas $H^2 \phi^2$ is suppressed by T^2/M_p^2 relative to this. The cubic term m_{th}^3 becomes imaginary if $V''(\phi) + (T^2/24)V^{(4)}(0)$ becomes negative; we take the real part only. The term $\frac{1}{24} T^2 V^{(4)}(0)$ represents the thermal mass, which appears in the cubic and quartic terms when ring diagrams are resummed [15]. We subtract a constant term V_0 from V_{eff} so that $V_{\text{eff}}(0)=0$, as is needed to properly compute the action associated with the bounce solution.

The thermal corrections to the effective radion potential cause the bounce solution to fall exponentially at large r :

$$\phi \sim \frac{c}{r} e^{-r/r_0}, \quad (40)$$

where $1/r_0 \sim \eta \sqrt{\Lambda/f^4} T$ if $\Lambda/f^4 \ll 1$ [the exact expression is $1/r_0 = \sqrt{\lambda U_*} T$, in terms of quantities to be defined below, in Eqs. (43) and (55)]. Once the bounce solution is known, it must be substituted back into the action, which can be written as

$$S = \frac{4\pi}{T} \int_0^\infty dr r^2 \left[\frac{1}{2} \phi'^2 + V_{\text{eff}}(\phi) \right]. \quad (41)$$

The nucleation rate is given by

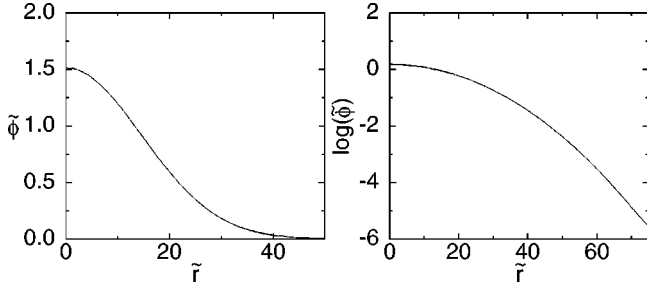


FIG. 2. The bounce solution, for the parameters $\epsilon=0.01$, $\hat{m}_1=0.1$, $m_\phi=T=100$ GeV and $M_{\text{TeV}}=1$ TeV. The rescaling $\phi \rightarrow \tilde{\phi}$ and $r \rightarrow \tilde{r}$ is given in Eqs. (43),(44).

$$\frac{\Gamma}{V} = \frac{|\omega_-|}{2\pi} \left(\frac{S}{2\pi} \right)^{3/2} |\mathcal{D}|^{-1/2} e^{-S} \quad (42)$$

where ω_- is the imaginary frequency of the unstable mode of fluctuations around the bounce solution, and \mathcal{D} is the fluctuation determinant factor, to be described below. A typical profile for the bounce solution is shown in Fig. 2.

For the numerical determination of the bounce solution and action, as well as understanding their parametric dependences, it is convenient to rescale the radius and the field by

$$r = \frac{\tilde{r}}{\sqrt{\lambda}T}, \quad \lambda = \frac{\Lambda'}{f^4} c_-^2, \quad (43)$$

$$\phi = ZT\tilde{\phi}, \quad Z = \frac{fc_-^{1/\epsilon}}{T}. \quad (44)$$

Then the action takes the form

$$S = \frac{Z^2}{\sqrt{\lambda}} \tilde{S}(\epsilon, \hat{m}_1, \lambda, Z), \quad (45)$$

$$\begin{aligned} \tilde{S} = 4\pi \int_0^\infty d\tilde{r} \tilde{r}^2 \left\{ \frac{1}{2} (\tilde{\phi}'^2 + \tilde{\phi}^2 f_2) + Z^2 \tilde{\phi}^4 f_0 \right. \\ \left. - \frac{1}{12\pi} \frac{\sqrt{\lambda}}{Z^2} \left[\left(\frac{c_+}{c_-} + 12Z^2 \tilde{\phi}^2 f_2 \right)^{3/2} - \left(\frac{c_+}{c_-} \right)^{3/2} \right] \right. \\ \left. + \frac{3c_b}{8\pi^2} \lambda f_2 \tilde{\phi}^2 \left(\frac{c_+}{c_-} + 6Z^2 \tilde{\phi}^2 f_2 \right) \right\} \end{aligned} \quad (46)$$

where

$$\begin{aligned} f_0(\tilde{\phi}) &= (\tilde{\phi}^\epsilon - 1) \left(\tilde{\phi}^\epsilon - \frac{c_+}{c_-} \right) \\ f_{n+1}(\tilde{\phi}) &= \left(1 + \frac{1}{4-n} \phi \partial_\phi \right) f_n(\tilde{\phi}). \end{aligned} \quad (47)$$

In the following, it will be helpful to keep in mind that Z can be extremely small, of order Ω in Eq. (28) when ϵ is small, whereas $\sqrt{\lambda}$ tends to be closer to unity, depending on the mass of the radion and the definition of the TeV scale (22):

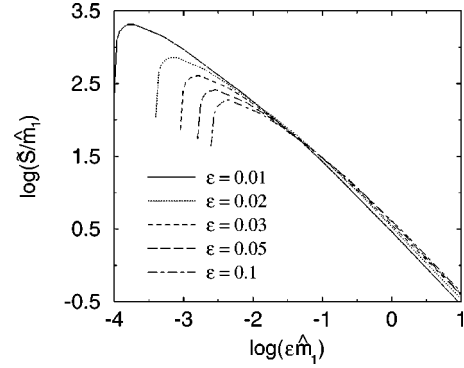


FIG. 3. $\log_{10}(\tilde{S}/\hat{m}_1)$ versus $\log_{10}(\epsilon\hat{m}_1)$, where \tilde{S} is the rescaled bounce action, Eq. (46). The other parameters are $m_\phi=T=100$ GeV and $M_{\text{TeV}}=1$ TeV.

$$\sqrt{\lambda} = \frac{\epsilon^{-3/4}}{2\sqrt{6}} \frac{m_\phi}{M_{\text{TeV}}} \frac{(1 - \sqrt{\epsilon_1 + \epsilon_4})}{\left(\sqrt{1 + \frac{4}{\hat{m}_1} + 2\frac{\sqrt{\epsilon}}{\hat{m}_1}} \right)^{1/2}}. \quad (48)$$

In the rescaled variables, the value $\tilde{\phi}=1$ corresponds to the first zero of the potential, which would be the starting point of the bounce if energy was conserved in the mechanical analog problem, i.e., if there was no viscous damping term ϕ'/r in the equation of motion. The actual starting point turns out to have a value in the range $\tilde{\phi}_0 \sim 1.5-3$ because of this. The rescaled action \tilde{S} depends mainly on the model parameters ϵ and \hat{m}_1 , for the parameter values which are of interest to us. All the sensitive exponential dependence on ϵ , namely the factor $c_-^{1/\epsilon}$, is removed from \tilde{S} . Numerically we find that

$$\tilde{S} \cong \frac{2}{3} \hat{m}_1 (\epsilon \hat{m}_1)^{-3/4}, \quad (49)$$

except when \hat{m}_1 becomes close to ϵ . For \hat{m}_1 slightly smaller than ϵ , ϵ_1 starts to exceed 1, and c_- becomes negative, signaling the onset of an instability in the radion potential toward coincidence of the two brane positions. Figure 3 shows the dependence of $\log(\tilde{S}/\hat{m}_1)$ versus $\log(\epsilon\hat{m}_1)$.

We have computed the bounce solution and the corresponding action for a range of parameters ϵ and \hat{m}_1 which can be consistent with the solution to the hierarchy problem (i.e., that $\tilde{\phi} \sim 10^{-16}$ at the global minimum). The size of the bounce in position space, measured as the width at half maximum, is small near $\epsilon = \hat{m}_1$, and reaches a larger constant value as $\hat{m}_1 \rightarrow \infty$. Using the rescaled radial variable $\tilde{r} = r\sqrt{\lambda}T$, the dependence of the width on ϵ and \hat{m}_1 is shown in Fig. 4.

The action of the bounce can be much greater than or much less than 1, depending on the parameters: for $\epsilon \sim \hat{m}_1 \ll 1$, $S \ll 1$, while for larger values of ϵ and \hat{m}_1 , $S \gg 1$. Where the crossover occurs ($S \sim 1$) depends on m_ϕ , T and

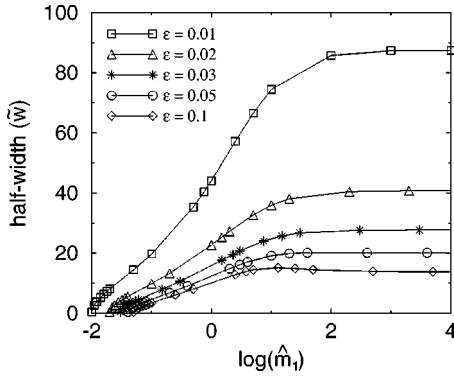


FIG. 4. Half width \tilde{w} of the bounce solution, in terms of the rescaled radial distance $\tilde{r} = r\sqrt{\lambda}T$, versus $\log_{10}(\hat{m}_1)$, for the same parameters as in Fig. 3.

M_{TeV} . This behavior can be inferred from Fig. 3 (showing \tilde{S}) and the dependences of the coefficient in the relation $S = (Z^2/\sqrt{\lambda})\tilde{S}$. Rather than presenting further results for S directly, however, we will turn to the more relevant quantity, the rate of bubble nucleation. For this we need to determine the prefactor of e^{-S} in the rate.

B. Prefactor of the bubble nucleation rate

The bounce action is the most important quantity determining the rate of tunneling, since it appears in the exponent of the rate (42). Since we do not have a model for the inflation and reheating of the universe which must occur prior to the bubble nucleation, hence an exact prediction for the reheating temperature which enters the rate, it would not be worthwhile to compute the prefactors in Eq. (42) very accurately; however, we can estimate their size.

First, consider the frequency ω_- of the unstable mode. ω_-^2 is the negative eigenvalue of the Schrödinger-like equation for small fluctuations $\delta\phi$ around the bounce solution, which we will denote by $\phi_b(r)$:

$$-\left(\delta\phi'' + \frac{2}{r}\delta\phi'\right) + \frac{\partial^2 V_{\text{eff}}}{\partial\phi^2}\bigg|_{\phi_b(r)} \delta\phi = \omega_-^2 \delta\phi. \quad (50)$$

Rescaling the radius and background field exactly as in Eqs. (43),(44), Eq. (50) becomes

$$-\left(\delta\phi'' + \frac{2}{r}\delta\phi'\right) + U(\tilde{r})\delta\phi = \frac{\omega_-^2}{\lambda T^2} \delta\phi, \quad (51)$$

$$\begin{aligned} U(\tilde{r}) &= \frac{1}{\lambda T^2} \frac{\partial^2 V_{\text{eff}}}{\partial\phi^2}(\tilde{\phi}_b(r)) \\ &= f_4 + X - \frac{3\sqrt{\lambda}}{\pi} \left(f_4 \sqrt{\frac{c_+}{c_-}} + X + \frac{12(Z\tilde{\phi}f_3)^2}{\sqrt{\frac{c_+}{c_-}} + X} \right) \\ &\quad + \frac{6c_b}{8\pi^2} \lambda \left(\frac{c_+}{c_-} f_4 + X(f_4 + 2f_3^2/f_2) \right) \end{aligned} \quad (52)$$

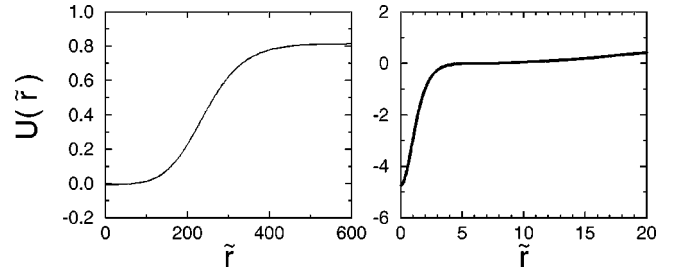


FIG. 5. The potential for small fluctuations around the bounce solution, $U(\tilde{r}) = X^2 V''(\phi_b(\tilde{r}))$, as a function of \tilde{r} , for the parameters $\epsilon=0.02$, $\hat{m}_1=5$ (left) and $\epsilon=0.1$, $\hat{m}_1=25$ (right).

where primes now denote $d/d\tilde{r}$, $X = 12Z^2\tilde{\phi}^2 f_2$, and the f_i are defined in Eq. (47). Except when the radion mass is significantly larger than 100 GeV, λ is much smaller than 1, and U is dominated by the first two terms in Eq. (51). Of these, the first (f_4) always dominates if $Z \ll 1$, while the second (X) can be important near $r=0$ if $Z \gtrsim 1$. The two different cases are illustrated in Fig. 5. Since $1 < \tilde{\phi}_0^\epsilon < c_+/c_-$, both f_4 and X are negative at $r=0$, so that

$$\begin{aligned} U_0 \equiv U(0) &\cong -\epsilon^{3/2} \sqrt{1 + \frac{4}{\hat{m}_1}} [2 + \ln \tilde{\phi}_0 \\ &\quad + 12Z^2\tilde{\phi}_0^2(\frac{1}{2} + \ln \tilde{\phi}_0)] \end{aligned} \quad (53)$$

and thus the smallest eigenvalue of Eq. (50) is negative. This is the unstable mode of the saddle point solution, with imaginary frequency of order

$$\omega_-^2 \sim U_0 \lambda T^2. \quad (54)$$

Recall that $|\omega_-|$ appears in the prefactor of the nucleation rate Γ/V .

As $\tilde{r} \rightarrow \infty$, $U(\tilde{r})$ approaches a maximum value

$$U_* \equiv U(\infty) = \frac{c_+}{c_-} \left(1 - \frac{3}{\pi} \sqrt{\lambda \frac{c_+}{c_-}} + \frac{3c_b}{4\pi^2} \lambda \frac{c_+}{c_-} \right), \quad (55)$$

which determines the asymptotic behavior of the fluctuations at large \tilde{r} : $\delta\phi \sim e^{-\sqrt{U_*}\tilde{r}}$. The fluctuations around the false vacuum state ($\phi=0$) thus have a mass given by

$$m^2 = V''_{\phi=0} = U_* \lambda T^2, \quad (56)$$

which will be relevant for the following.

Next we must estimate the functional determinant factor

$$\mathcal{D} = \frac{\det'[-\partial_\tau^2 - \nabla^2 + V''(\phi_b)]}{\det[-\partial_\tau^2 - \nabla^2 + V''(0)]}, \quad (57)$$

where τ is imaginary time ($\tau \in [0, 1/T]$), ∇^2 is the three-dimensional Laplacian, ϕ_b is the bounce solution, and the prime on \det' means that the three translational zero-mode

eigenvalues must be omitted from the determinant for fluctuations around the bounce. These zero modes correspond to spatial translations of the bubble solution. Because of their removal, \mathcal{D} has dimensions of $(\text{energy})^{-6}$, as is required to get a rate per unit volume in Eq. (42).

Reference [16] has given a thorough account of how to compute \mathcal{D} by a method which was discussed for one-dimensional systems in [17]. In 3D one should classify the eigenvalues of the fluctuation operators by the quantum numbers n and l , denoting Matsubara and angular momentum excitations, respectively. Then \mathcal{D} can be written as a product, $\mathcal{D} = \prod_{n,l} \mathcal{D}_{n,l}$.

Reference [16] shows that the contribution to \mathcal{D} from the l th partial wave can be expressed, to leading order in a perturbative expansion in the potential $U(r)$, as

$$\mathcal{D}_{n,l} \cong (1 + h_l^{(1)})^{2l+1}. \quad (58)$$

The quantity $h_l^{(1)}$ has the Green's function solution

$$\begin{aligned} h_l^{(1)} &= 2 \int_0^\infty dr r I_{l+1/2}(\kappa r) K_l(\kappa r) [V''(\phi_b(r)) - m^2] \\ &= 2 \int_0^\infty d\tilde{r} r I_{l+1/2}\left(\frac{\kappa \tilde{r}}{\sqrt{\lambda T}}\right) K_l\left(\frac{\kappa \tilde{r}}{\sqrt{\lambda T}}\right) [U(\tilde{r}) - U_*] \end{aligned} \quad (59)$$

using the modified Bessel functions I and K , and the mass m of the field in the false vacuum, Eq. (56). For general Matsubara frequencies, $\nu = 2\pi nT$, one defines $\kappa = \sqrt{m^2 + \nu^2}$.

The subdeterminant for the $n=0$ (zero-temperature) sector of the theory has the usual ultraviolet divergences of quantum field theory, namely the vacuum diagram \bigcirc [the dot represents one insertion of $V(\phi)$], which should be absorbed by renormalization of the zero of energy for the radion potential. Since we are not attempting to solve the cosmological constant problem here, we are going to ignore all of this and compute only the factor $\mathcal{D}_{0,1}$, which contains the translational zero modes—or more precisely, which has the zero modes removed. This removal is accomplished by replacing

$$1 + h_1^{(1)} \rightarrow \frac{dh_1^{(1)}}{d\kappa^2}. \quad (60)$$

Notice that this quantity has dimensions of $(\text{mass})^{-2}$, and there are $2l+1=3$ such factors, so that $|\mathcal{D}|^{-1/2}$ has dimensions of $(\text{mass})^3$, as required. From Eq. (59) one can show that

$$\begin{aligned} \frac{dh_1^{(1)}}{d\kappa^2} &= \frac{1}{\lambda T^2 U_*^2} I_U, \\ I_U &\equiv \int_0^\infty dy y^2 [I_{3/2}(y) K_1(y)] \left[U\left(\frac{y}{\sqrt{U_*}}\right) - U_* \right]. \end{aligned} \quad (61)$$

We have numerically evaluated the integral I_U for each set of parameters. Our estimate for the fluctuation determinant factor in the nucleation rate can then be written as

$$|\mathcal{D}|^{-1/2} \sim \left(\frac{\lambda T^2 U_*^2}{I_U} \right)^{3/2}. \quad (62)$$

C. Results for the nucleation rate

Putting the above ingredients together to find the rate of bubble nucleation per unit volume, Γ/V , we see that the latter depends on five undetermined parameters: ϵ , \hat{m}_1 , m_ϕ , M_{TeV} and the temperature T . Reference [7] showed that, as long as the energy density on the TeV brane is much less than M_{TeV}^4 , the usual 4D effective theory governs the Hubble rate:

$$H^2 = \frac{\rho}{3M_p^2}, \quad (63)$$

where ρ is the total energy density,

$$\begin{aligned} \rho &= g_* \frac{\pi^2}{30} T^4 + \rho_\phi, \\ \rho_\phi &= \frac{1}{2} \dot{\phi}^2 + V(\phi) - V(\phi_+) \\ &\cong \frac{3}{8} M_{\text{TeV}}^2 m_\phi^2 \frac{\sqrt{1 + \frac{4}{\hat{m}_1}(1 + \sqrt{\epsilon_1})}}{\sqrt{1 + \frac{4}{\hat{m}_1} + 2\frac{\sqrt{\epsilon}}{\hat{m}_1}}}. \end{aligned} \quad (64)$$

We take the number of relativistic degrees of freedom, g_* , to be 100. The kinetic energy of the radion is zero since $\dot{\phi} = 0$ in the false vacuum, so ρ_ϕ is essentially the potential energy of the radion in the false vacuum, assuming that the 4D cosmological constant is zero: $V(\phi) - V(\phi_+) = |V(\dot{\phi}_+)|$, which is given by Eq. (26). Depending on the parameters, this can be comparable in size or dominate over the energy density of radiation. Using our estimates for the prefactor of the tunneling rate, the logarithm of the ratio of Γ/V to H^4 can be written as

$$\ln \frac{\Gamma}{VH^4} \cong \ln \left[\frac{\lambda^2 \sqrt{U_0} U_*^3}{(2\pi)^{5/2}} T^4 \left(\frac{3M_p^2}{\rho} \right)^2 \left(\frac{S}{I_U} \right)^{3/2} \right] - S \quad (65)$$

where S is the action of the bounce solution. The criterion for completion of the phase transition to the true vacuum state is that $\ln(\Gamma/VH^4) > 0$. The saddle point approximation leading to Eq. (42) is only valid if the action S is not much less than 1. Otherwise, the barrier is not effective for preventing the field from rolling to the true minimum, as in a second order phase transition. This situation occurs in the vicinity of $\ln(\Gamma/VH^4) \sim 150$ in the following results; thus the transition region where $\Gamma/VH^4 = 1$ is well within the realm of validity of the approximation.

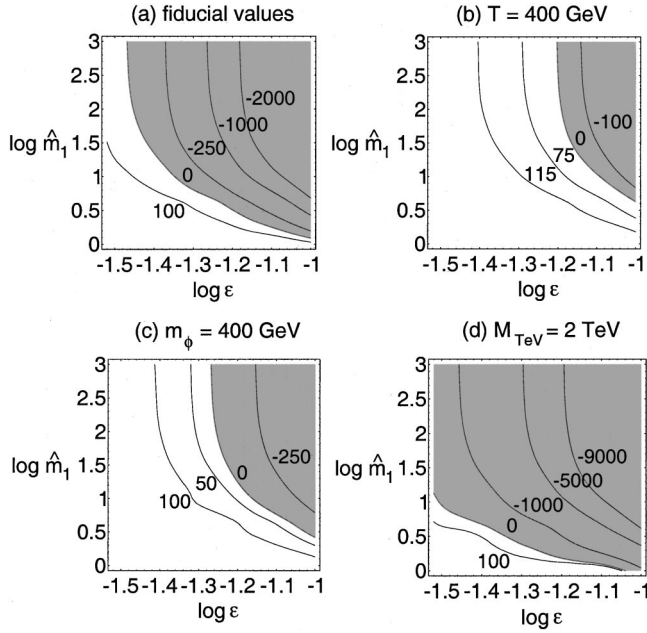


FIG. 6. Contours of $\ln(\Gamma/VH^4)$ in the plane of $\log_{10}(\hat{m}_1)$ versus $\log_{10}(\epsilon)$. The shaded regions are where the tunneling rate is too small for the phase transition to complete. (a) has $T = m_\phi = 100$ GeV, $M_{\text{TeV}} = 1$ TeV. The other figures are the same except for the following changes: (b) $T = 400$ GeV, (c) $m_\phi = 400$ GeV, (d) $M_{\text{TeV}} = 2$ TeV.

In Fig. 6 we show the contours of constant $\ln(\Gamma/VH^4)$ in the plane of $\log_{10}(\hat{m}_1)$ and $\log_{10}(\epsilon)$, starting with the fiducial values $T = m_\phi = 100$ GeV, $M_{\text{TeV}} = 1$ TeV for the other parameters, and showing how the results change when any one of these is increased. There is no explicit dependence on \hat{m}_0 because it appears only in the formula for the radion mass. The dependences can be understood from the prefactor $Z^2/\sqrt{\lambda}$ in the action, Eq. (45):

$$\frac{Z^2}{\sqrt{\lambda}} \sim \epsilon^{3/4} \frac{M_{\text{TeV}}^3 \Omega^2}{T^2 m_\phi}, \quad (66)$$

where we recall that Z and λ are given by Eqs. (43),(44) and Ω by Eq. (28). The factor Ω is responsible for suppressing the bounce action when $\epsilon \ll 1$ or $\epsilon \hat{m}_1 \ll 1$, explaining the shape of the allowed regions in each graph. Nucleation of bubbles containing the true minimum becomes faster when the temperature or the radion mass is increased, but slower if the definition of the TeV scale is increased. These dependences are dictated not only by the size of the barrier between the two minima in the effective potential, but also by the size of the bubbles.

Interestingly, the borderline between allowed and forbidden regions of parameter space falls within the range which is relevant from the point of view of building a model of radion stabilization. That is, some choices which would otherwise have been natural and acceptable are ruled out by our considerations. We see furthermore that the choice of \hat{m}_1

$\rightarrow \infty$, as was effectively focused on in Ref. [10], is not the optimal one for achieving a large nucleation rate.

It might be thought that our analysis is rendered less important by the fact that one can always obtain fast nucleation simply by going to high enough temperatures. However, it must be remembered that the TeV scale functions as the high-energy cutoff in the Randall-Sundrum scenario: the whole semiclassical description breaks down at super-TeV scales, where quantum gravity effects start to become important. From this point of view, the temperatures of 100–300 GeV which we are discussing are already rather high, and a fairly efficient mechanism of reheating at the end of inflation will be needed to generate them.

V. DISCUSSION

In this paper we have presented a somewhat simpler model of radion stabilization by a bulk field (ψ) than that of Goldberger and Wise [10]; although the physics is qualitatively identical, we are able to write the radion potential exactly, and thus explore the effect of letting the stabilizing field's VEV's on the branes be pinned more or less strongly to their minimum energy values. One such effect is that the mass of the radion can be significantly increased for small values of the parameter m_1 , which is the coefficient of the potential for ψ on the TeV brane. Moreover if $m_1/k \equiv \hat{m}_1$ is accidentally close to ϵ , approximately the minimum value consistent with a stable potential, the radion mass can start to diverge, by the factor $(1 - \epsilon/4 - \epsilon/\hat{m}_1)^{-1/2}$. This modifies somewhat the expectation expressed in Ref. [5] that the radion mass will be small relative to the TeV scale, due to a factor of $\epsilon^{3/4}$.

Our main focus was on the problem that the radion potential has a local minimum at infinite brane separation, and that the barrier between the true and false minima is so small that for generic initial conditions, one would expect the true minimum to be bypassed as the radion field rolls through it. We showed that for a large range of parameters, the high-temperature phase transition to the true minimum is able to complete, thus overcoming the problem. There are however significant constraints on the model parameters, and the initial temperature after inflation, to insure this successful outcome.

There remain some outstanding issues. The form of the radion effective potential is such that the field is able to reach $\phi = 0$ in a finite amount of time; yet $\phi = 0$ represents infinite brane separation in the extra dimension. This paradoxical situation may be due to the assumption that the stabilizing field, ψ , is always in its minimum energy configuration at any given moment. In reality ψ must require a finite amount of time to respond to changes in the radion. Thus one should solve the coupled problem for time-varying ϕ and ψ to do better. This is probably a difficult problem, which we leave to future study.

A related question is whether it is correct to treat thermal fluctuations of the radion field ϕ analogously to a normal scalar field with values in the range $(-\infty, \infty)$. Since ϕ is related to the size of the extra dimension by $\phi = f e^{-kb}$, its

range is $[0, f]$. We have not studied what effect this might have on the thermal part of the effective potential; instead we assumed that the usual treatment suffices.

Another approximation we made was to ignore the backreaction of the stabilizing field on the geometry. Reference [18] has given a method of finding exact solutions to the coupled equations for the warp factor $a(y)$ and the stabilizing field $\psi(y)$. This method cannot be applied in the present case because it works only for bulk scalar potentials with a special form that, among other things, requires them to be unbounded from below. Moreover, since the method of [18] generates only static solutions to the equation of motion, it cannot be used to deduce the radion potential, which is a probe of the response of the geometry when it is perturbed away from a static solution. On the other hand, [18] does show that the neglect of the backreaction is justified for the parameter values which most closely resemble the Goldberger-Wise model.

One might at first feel uneasy about using a 4D effective description of the problem when in reality our initial condition is a universe with an infinitely large extra dimension. In the Randall-Sundrum scenario, however, this is justified because the graviton is trapped on the transverse length scale of $1/k \sim 1/M_p$, rather than the size of the 5th dimension (see also [19]). Moreover the 4D Friedmann equation (63) was shown by Ref. [7] to be valid without actually assuming the

radion to be stabilized.⁴ Difficulties with the “wrong” rate of expansion ($H \propto \rho$ instead of $H \propto \sqrt{\rho}$ [20]) arise only when one fine-tunes the brane energy densities to prevent radion motion even in the absence of stabilization, which we are not doing here. In any case, changing the form of the expansion rate would have a small effect on our results since this alters only the logarithmic term in Eq. (65), not the overwhelmingly dominant term S .

The problem of shallow barriers in moduli potentials is not unique to the Randall-Sundrum scenario, and a new idea for addressing it was recently presented in Ref. [21]. The coupling of the kinetic terms of matter fields to the modulus can give the damping of the modulus motion needed to make it settle in the true minimum in some cases. This effect might provide an alternative to the thermal mechanism we have discussed here.

ACKNOWLEDGMENTS

We thank Mark Wise and Rob Myers for enlightening discussions, Guy Moore for helpful correspondence and Geneviève Boisvert for perceptive criticisms of the manuscript.

⁴The extra dimension is free to expand in this case, and the kinetic energy of the radion simply appears as an additional contribution to the energy density of the universe, as in Eqs. (63),(64).

-
- [1] I. Antoniadis, Phys. Lett. B **246**, 377 (1990); N. Arkani-Hamed, S. Dimopoulos, and G. Dvali, *ibid.* **429**, 263 (1998); Phys. Rev. D **59**, 086004 (1999); I. Antoniadis, N. Arkani-Hamed, S. Dimopoulos, and G. Dvali, Phys. Lett. B **436**, 257 (1998); K. R. Dienes, E. Dudas, and T. Gerghetta, *ibid.* **436**, 55 (1998); N. Arkani-Hamed, S. Dimopoulos, G. Dvali, and N. Kaloper, Phys. Rev. Lett. **84**, 586 (2000).
 - [2] L. Randall and R. Sundrum, Phys. Rev. Lett. **83**, 3370 (1999); **83**, 4690 (1999).
 - [3] H. Verlinde, Nucl. Phys. **B580**, 264 (2000); M. J. Duff and J. T. Liu, Phys. Rev. Lett. **85**, 2052 (2000).
 - [4] P. Kanti, K. A. Olive, and M. Pospelov, Phys. Rev. D **62**, 126004 (2000); Phys. Lett. B **481**, 386 (2000); C. van de Bruck, M. Dorca, R. H. Brandenberger, and A. Lukas, Phys. Rev. D **62**, 123515 (2000); C. Park and S. Sin, Phys. Lett. B **485**, 239 (2000); B. Grinstein, D. R. Nolte, and W. Skiba, Phys. Rev. D **62**, 086006 (2000); R. Maartens, *ibid.* **62**, 084023 (2000); H. Stoica, S. H. Tye, and I. Wasserman, Phys. Lett. B **482**, 205 (2000); J. E. Kim, B. Kyae, and H. M. Lee, Nucl. Phys. **B582**, 296 (2000); **B591**, 587(E) (2000); Phys. Rev. D **62**, 045013 (2000); A. Mazumdar and J. Wang, Phys. Lett. B **490**, 251 (2000); C. Barcelo and M. Visser, Nucl. Phys. **B584**, 415 (2000); I. I. Kogan and G. G. Ross, Phys. Lett. B **485**, 255 (2000); U. Ellwanger, hep-th/0001126; Phys. Lett. B **473**, 233 (2000); Z. Chacko and A. E. Nelson, Phys. Rev. D **62**, 085006 (2000); D. Ida, J. High Energy Phys. **09**, 014 (2000); P. Binétruy, C. Deffayet, U. Ellwanger, and D. Langlois, Phys. Lett. B **477**, 285 (2000); A. G. Cohen and D. B. Kaplan, *ibid.* **470**, 52 (1999); J. Cline, C. Grojean, and G. Servant, *ibid.* **472**, 302 (2000); Nucl. Phys. **B578**, 259 (2000); H. B. Kim and H. D. Kim, Phys. Rev. D **61**, 064003 (2000); D. J. Chung and K. Freese, *ibid.* **61**, 023511 (2000).
 - [5] W. D. Goldberger and M. B. Wise, Phys. Lett. B **475**, 275 (2000).
 - [6] C. Csaki, M. Graesser, C. Kolda, and J. Terning, Phys. Lett. B **462**, 34 (1999); J. M. Cline, C. Grojean, and G. Servant, Phys. Rev. Lett. **83**, 4245 (1999).
 - [7] C. Csaki, M. Graesser, L. Randall, and J. Terning, Phys. Rev. D **62**, 045015 (2000).
 - [8] P. Kanti, I. I. Kogan, K. A. Olive, and M. Pospelov, Phys. Rev. D **61**, 106004 (2000); Phys. Lett. B **468**, 31 (1999).
 - [9] H. B. Kim, Phys. Lett. B **478**, 285 (2000). R. N. Mohapatra, A. Perez-Lorenzana, and C. A. de Sousa Pires, hep-ph/0003328; J. Lesgourgues, S. Pastor, M. Peloso, and L. Sorbo, Phys. Lett. B **489**, 411 (2000).
 - [10] W. D. Goldberger and M. B. Wise, Phys. Rev. Lett. **83**, 4922 (1999).
 - [11] J. Garriga, O. Pujolas, and T. Tanaka, hep-th/0004109.
 - [12] W. D. Goldberger and M. B. Wise, Phys. Rev. D **60**, 107505 (1999).
 - [13] A. H. Guth, Phys. Rev. D **23**, 347 (1981).
 - [14] S. Coleman, Phys. Rev. D **15**, 2929 (1977); C. Callan and S. Coleman, *ibid.* **16**, 1762 (1977).
 - [15] P. Arnold and O. Espinosa, Phys. Rev. D **47**, 3546 (1993).
 - [16] J. Baacke and V. G. Kiselev, Phys. Rev. D **48**, 5648 (1993).
 - [17] S. Coleman, lecture delivered at 1977 International School of

- Subnuclear Physics, Erice, Italy, 1977 Report No. HUTP-78/A004.
- [18] O. DeWolfe, D. Z. Freedman, S. S. Gubser, and A. Karch, Phys. Rev. D **62**, 046008 (2000).
- [19] J. Lykken and L. Randall, J. High Energy Phys. **06**, 014 (2000).
- [20] A. Lukas, B. A. Ovrut, K. S. Stelle, and D. Waldram, Phys. Rev. D **59**, 086001 (1999); A. Lukas, B. A. Ovrut, and D. Waldram, *ibid.* **60**, 086001 (1999); N. Kaloper and A. Linde, *ibid.* **59**, 101303 (1999). N. Kaloper, *ibid.* **60**, 123506 (1999); P. Binétruy, C. Deffayet, and D. Langlois, Nucl. Phys. **B565**, 269 (2000); E. E. Flanagan, S. H. Tye, and I. Wasserman, Phys. Rev. D **62**, 044039 (2000).
- [21] G. Huey, P. J. Steinhardt, B. A. Ovrut, and D. Waldram, Phys. Lett. B **476**, 379 (2000).