Constraints on the mass and mixing of the fourth generation quark from direct CP violation ϵ'/ϵ and rare K decays

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(Received 22 May 2000; published 8 June 2001)

We investigate the ϵ'/ϵ for $K\rightarrow\pi\pi$ in a sequential fourth generation model. By giving the basic formulas for ϵ'/ϵ in this model, we analyze the numerical results which are dependent on $m_{t'}$ and the imaginary part of the fourth CKM factor, $\mathrm{Im}V_{t's}^{*}V_{t'd}$ (or $V_{t's}^{*}V_{t'd}$ and the fourth generation CKM matrix phase θ). We find that, unlike the SM, when taking the central values of all parameters for ϵ'/ϵ, the values of ϵ'/ϵ can easily fit to the current experimental data for all values of hadronic matrix elements estimated from various approaches. Also, we show that the experimental values of ϵ'/ϵ and rare K decays can provide a strong constraint on both mass and mixing of the fourth generation quark. When taking the values of hadronic matrix elements from the lattice or $1/N$ expansion calculations, a large region of the up-type quark mass $m_{t'}$ is excluded.

DOI: 10.1103/PhysRevD.64.016009 PACS number(s): 11.30.Er, 12.60.−i, 13.25.Es, 14.80.−j

I. INTRODUCTION

Although the standard model (SM) is very successful for explaining the particle physics experiments, it has to face the difficulties of many interesting open questions, such as CP violation. The new experimental results for ϵ'/ϵ, which measures direct CP violation in $K\rightarrow\pi\pi$ decays, have been reported by the KTeV Collaboration at Fermilab [1] and NA48 Collaboration at CERN [2],

$$\mathrm{Re}(\epsilon'/\epsilon)=(28.0\pm4.1)\times10^{-4}\quad \mathrm{KTeV}, \tag{1}$$

$$\mathrm{Re}(\epsilon'/\epsilon)=(18.5\pm7.3)\times10^{-3}\quad \mathrm{NA48}, \tag{2}$$

while the new world average reads [2,3]

$$\mathrm{Re}(\epsilon'/\epsilon)=(21.1\pm4.6)\times10^{-4}. \tag{3}$$

This establishment of direct CP violation rules out old superweak models [4]. Yet while the SM predicts a nonvanishing ϵ'/ϵ, the values in Eqs. (1), (2), and (3) exceed most theoretical predictions of SM [5,6]. Someone has to face and resolve this discrepancy. Some possibilities to accommodate the data in SM have been pointed out [7,8].

The SM makes precise assumptions on the mechanism that generates the CP violation. The only source of CP violating phase originates from the elements $V_{u_i d_j}$ of the Cabibbo-Kobayashi-Maskawa (CKM) matrix with three quark generations. In the SM, there are both indirect (ϵ) and direct (ϵ') CP violation. The analysis of ϵ'/ϵ can be divided into the short-distance (perturbative) part and long-distance (nonperturbative) parts. Using the effective Hamiltonian [$\mathcal{H}_W=\Sigma_i C_i(\mu)Q_i(\mu)$ [9,10]], one can obtain an expression of ϵ'/ϵ that involves CKM parameters ($V_{u_i d_j}$), Wilson coefficients (y_i) and local operator matrix elements ($\langle Q_i\rangle_I$). The source of most theoretical uncertainties for ϵ'/ϵ is mainly from the difficulty in calculating nonperturbative part (local operator matrix elements), comparing with the phenomenological determination of CKM parameters [11] and the calculation of the Wilson coefficients at a next leading order (NLO) level [10]. For ϵ'/ϵ, one of the goals of the SM is to determine the hadronic matrix elements [12–17].

The interest in this paper is not in this nonperturbative part but the new effects with the fourth sequential generation particles in the short-distance part. Except for the SM explanation, there are many directions in the search for new physics beyond the SM [18–24] to resolve CP violation. Unlike the SM, almost any extension of SM has, in general, new CP violating phases. That is to say, they give new CP violation sources. The new physics on CP violation beyond the SM includes CP violation in supersymmetry models [19] and extensions of fermion sector [20,23,24], scalar sector [21] and gauge sector [22] of the SM. In extensions of fermion sector, there are many models, such as vectorlike quark models [23], sterile neutrino models [24], proposed for probing new effects on CP violation.

In this paper, as in Ref. [25], we consider a sequential fourth generation model [25,26], in which an up-type quark t', down-type quark b', lepton τ', and heavy neutrino ν' are added into the SM. The properties of these new fermions are all the same as their corresponding counterparts of other three generations except their masses and CKM mixing, see Table I.

As the SM does not fix the number of generations, so far we do not know why there is more than one generation and what law of nature determines their number. On the one hand, the purely sequential fourth generation is constrained,

TABLE I. The elementary particle spectrum of SM4.

	uplike quark	downlike quark	charged lepton	neutral lepton
	u	d	e	ν_e
SM fermions	c	s	μ	ν_μ
	t	b	τ	ν_τ
new fermions	t'	b'	τ'	$\nu_{\tau'}$

even excluded in much of the literature [27]. For example, in Refs. [28,29] the method of Padé approximates is used to show that for a large fermion mass, it is possible to dynamically generate *p*-wave resonance and then the *S* parameter bound can serve to exclude a heavy fourth generation of fermions [29]. Reference [30] found that there is no violation of the *S* parameter upper bound for any value of the heavy fermion mass and that elastic unitarity, imposed as a constraint on strong $W_L W_L$ scattering, yields no information concerning and sheds no light on the existence of a heavy fourth generation. The Ref. [31] compared various precision determinations of the Femi constant G_F to get the rather stringent bound of third and fourth generation lepton mixing angle θ_{34}. It found that the fourth charged lepton is too heavy and seems nonexistent. The precision electroweak measurements can also give the strong constraints to the sequential fourth generation, in particular the *S* parameter excludes it to 99.8% C.L. if is degenerate, and if not a small *T* parameter is allowed and then it is excluded to 98.2% C.L. [32].

However, on the other hand, experimentally, the CERN e^+e^- collider LEP determinations of the invisible partial decay width of the Z^0 gauge boson only show that there are certainly three *light* neutrinos of the usual type with mass less than $M_Z/2$ [33]. But the existence of the fourth generation with a heavy neutrino, i.e., $m_{\nu_4} \geqslant M_Z/2$ [34] is not yet excluded. Perhaps there exists some more deep or mechanism to give the room of the sequential fourth generation, because we really do not know why there are only three generations. So, it is not invaluable to research these new generation as one of the new physics. Before having a more fundamental reason for three generations, one may investigate phenomenologically whether the existing experimental data allow the existence of the fourth generation. This is also the main purpose of this paper. There are a number of papers [26,27] for discussing the fourth generation phenomena.

In our previous paper [35], we have investigated the constraints on the fourth generation from the inclusive decays of $B \to X_s l^+ l^-$ and $B \to X_s \gamma$. In this paper, we further study its effects on direct *CP*-violating parameter ϵ'/ϵ in $K \to \pi\pi$ decays as well as possible new constraints from ϵ'/ϵ and rare *K* decays. We limit ourselves to the non-SUSY case in order to concentrate on the phenomenological implication of the fourth generation and will call this model as SM4 hereafter for the sake of simplicity.

CP-violating parameter ϵ'/ϵ is a short distance dominated process and is sensitive to new physics. In the SM4 model, there are not new operators produced. The new particle involved is only the fourth generation up-type quark t'. The heavy mass of t' propagating in the loop diagrams of penguin and box enters the Wilson coefficients y_i, as well as top quark *t* and *W* boson. The effects of the fourth generation particles can only modify y_i. Each new Wilson coefficient $y_i^{\rm new}(\mu)$ is the sum of $y_i^{\rm sm}(\mu)$ and $y_i^{(4)}(\mu)$ contributed by *t* and t' correspondingly. We can get $y_{t'}^{(4)}$ by taking the mass of t' as one of the input parameter. Moreover, for obtaining ϵ'/ϵ in SM4, we must know something about elements $V_{t'd_j}$ of the fourth generation 4×4 CKM matrix which now contains nine parameters, i.e., six angles and three phases. But there are not any direct experimental measurements of them. So we have to get their information indirectly from some meson decays. We investigate three rare *K* decays, $K^+ \to \pi^+ \nu\bar{\nu}$, $K_L \to \pi^0 \nu\bar{\nu}$, and $K_L \to \mu^+\mu^-$ [36], in SM4. These decays can give the constraint of the fourth CKM factor $\mathrm{Im}\, V_{t's}^* V_{t'd}$ (or $V_{t's}^* V_{t'd}$ and a fourth generation phase θ), which is needed for calculating $(\epsilon'/\epsilon)^{\rm new}$. We shall take it as an additional input parameter. As a consequence, the total (ϵ'/ϵ) is the sum of $(\epsilon'/\epsilon)^{\rm sm}$ and $(\epsilon'/\epsilon)^4$ contributed by the SM and the new particle t' correspondingly. Unlike the SM, when taking the central values of all parameters for ϵ'/ϵ, the new value of (ϵ'/ϵ) can reach the range of the current experimental results whatever values of the nonperturbation part, hadronic matrix elements, are taken in all known cases. Also, the experimental values of $(\epsilon'/\epsilon)^{\rm exp}$ impose strong constraints on the parameter space of $\mathrm{Im}\, V_{t's}^* V_{t'd}$ and $m_{t'}$.

In Sec. II, we give the basic formulas for ϵ'/ϵ with the fourth uplike quark t' in SM4. In Sec. III, we analyze the constraints on the fourth generation CKM matrix factor $\mathrm{Im}\, V_{t's}^* V_{t'd}$ which is necessary for calculating ϵ'/ϵ in SM4. Section IV is devoted to the numerical analysis. Finally, in Sec. V, we give our conclusion.

II. BASIC FORMULAS FOR ϵ'/ϵ AND WILSON COEFFICIENTS $y_I^{(4)}(\mu)$ IN SM4

The essential theoretical tool for the calculation of ϵ'/ϵ is the $\Delta S = 1$ effective Hamiltonian [9,10],

$$\mathcal{H}_W = \sum_i \frac{G_F}{\sqrt{2}} V_{ud} V_{us}^* [z_i(\mu) + \tau y_i(\mu)] Q_i(\mu) \tag{4}$$

with $\tau = V_{ts}^* V_{td}/(V_{us}^* V_{ud})$. The direct *CP* violation in $K \to \pi\pi$ is described by ϵ'. The parameter ϵ' is given in terms of the amplitudes $A_0 \equiv A[k \to (\pi\pi)_{I=0}]$ and $A_2 \equiv A[k \to (\pi\pi)_{I=2}]$ as follows:

$$\epsilon' = -\frac{1}{\sqrt{2}} \xi (1-\Omega) \exp(i\Phi), \tag{5}$$

where

$$\xi = \frac{\mathrm{Im}\, A_0}{\mathrm{Re}\, A_0}, \quad \omega = \frac{\mathrm{Re}\, A_2}{\mathrm{Re}\, A_0}, \quad \Omega = \frac{1}{\omega} \frac{\mathrm{Im}\, A_2}{\mathrm{Im}\, A_0} \tag{6}$$

and $\Phi = \pi/2 + \delta_2 - \delta_0 \approx \pi/4$. With the effective Hamiltonian (4), we can cast Eq. (5) into the form

$$\frac{\epsilon'}{\epsilon} = \mathrm{Im}\, \lambda_t [P^{(1/2)} - P^{(3/2)}], \tag{7}$$

where

$$P^{(1/2)} = \sum P_i^{(1/2)} = r \sum y_i \langle Q_i \rangle_0 (1 - \Omega_{\eta+\eta'}), \tag{8}$$

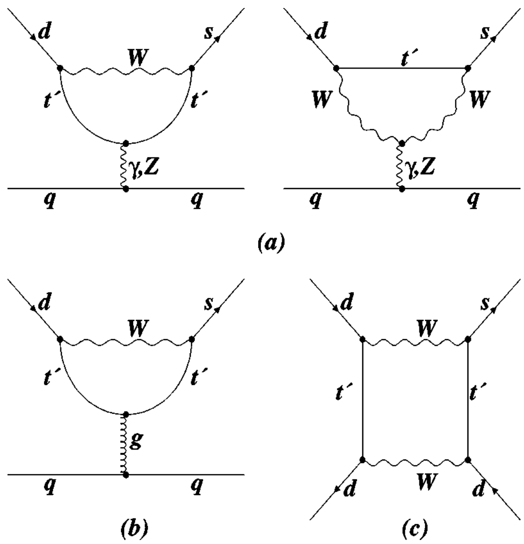

FIG. 1. The additional (a) EW, (b) QCD penguins, and (b) box diagrams with t'.

$$P^{(3/2)}=\sum P_i^{(3/2)}=\frac{r}{\omega}\sum y_i\langle Q_i\rangle_2, \tag{9}$$

with $r=G_F\omega/(2|\mathrm{Re}\,A_0|)$. y_i are the Wilson coefficients and the hadronic matrix elements are

$$\langle Q_i\rangle_I \equiv \langle(\pi\pi)_I|Q_i|K\rangle. \tag{10}$$

The operators Q_i and $\langle Q_i\rangle_I$ are given explicitly in many reviews [9,10].

When including the contributions from the fourth generation up-type quark t', the above equations will be modified. The corresponding effective Hamiltonian can be expressed as

$$\mathcal{H}_W=\sum_i \frac{G_F}{\sqrt{2}}V_{ud}V_{us}^*[z_i(\mu)+\tau y_i^{\mathrm{SM}}(\mu)+\tau' y_i^{(4)}(\mu)]Q_i(\mu) \tag{11}$$

with $\tau=V_{ts}^*V_{td}$ and $\tau'=V_{t's}^*V_{t'd}$. In comparison with the SM, one may introduce the new effective coefficient functions $y_i^{\mathrm{new}}(\mu)$

$$y_i^{\mathrm{new}}(\mu)=y_i^{\mathrm{SM}}+\frac{\tau'}{\tau}y_i^{(4)}(\mu), \tag{12}$$

where $y_i(\mu)$ are the Wilson coefficient functions in the SM and $y_i^{(4)}$ are the ones due to fourth generation quark contributions. The evolution for $y_i^{(4)}(\mu)$ is an analogy to the one $y_i^{\mathrm{SM}}(\mu)$ in SM [9,10] except replacing the t quark by t' quark. The corresponding diagrams of penguin and box are shown in Fig. 1.

Using Eqs. (11) and (12), Eq. (7) can be written as

$$\left(\frac{\epsilon'}{\epsilon}\right)=\left(\frac{\epsilon'}{\epsilon}\right)^{\mathrm{SM}}+\left(\frac{\epsilon'}{\epsilon}\right)^{(4)},$$

$$\left(\frac{\epsilon'}{\epsilon}\right)^{(4)}=\mathrm{Im}\,\lambda_t'[P'^{(1/2)}-P'^{(3/2)}], \tag{13}$$

where the definitions of $P'^{(1/2)}$ and $P'^{(3/2)}$ are the same as Eqs. (8) and (9) only by changing $y_i(\mu)$ into $y_i^{(4)}(\mu)$, and

$$\mathrm{Im}\,\lambda_{t'}=\mathrm{Im}\,V_{t's}^*V_{t'd}. \tag{14}$$

Thus the main test of evaluating ϵ'/ϵ in the SM4 is to calculate the Wilson coefficients $y_i^{(4)}(\mu)$ and to provide the possible constraints on $\mathrm{Im}\,\lambda_{t'}$. The constraints of $\mathrm{Im}\,\lambda_{t'}$ will be discussed in the next section. The calculation of $y_i^{(4)}(\mu)$ is the same as their counterpart $y_i^{\mathrm{SM}}(\mu)$ in the SM and can be simply done by changing m_t to $m_{t'}$, which is easy to be found in any corresponding reviews [9,10]. Here we repeat the same calculations and only provide the numerical results for $y_i^{(4)}(\mu)$ as the functions of the mass m_t'. In the numerical calculations we take a large range for t'-quark mass $m_{t'}=50$, 100, 150, 200, 250, 300, 400 GeV [26], see Table II.

TABLE II. $\Delta S=1$ Wilson coefficients at $\mu=1.0$ GeV for $\Lambda^{(4)}=340$ MeV and $f=3$ effective flavors at leading order. $y_1^4=y_2^4\equiv 0$.

m_t' (GeV)	50	100	150	200	250	300	350	400
z_1^4	−0.594	−0.594	−0.594	−0.594	−0.594	−0.594	−0.594	−0.594
z_2^4	0.323	0.323	0.323	0.323	0.323	0.323	0.323	0.323
y_3^4	0.028	0.032	0.036	0.042	0.048	0.055	0.064	0.074
y_4^4	−0.049	−0.052	−0.056	−0.059	−0.064	−0.069	−0.075	−0.081
y_5^4	0.011	0.011	0.012	0.012	0.013	0.013	0.014	0.014
y_6^4	−0.089	−0.097	−0.112	−0.104	−0.107	−0.111	−0.114	−0.118
y_7^4/α	−0.114	−0.076	−0.004	0.092	0.210	0.348	0.506	0.686
y_8^4/α	−0.034	0.011	0.097	0.210	0.350	0.514	0.704	0.917
y_9^4/α	−0.367	−0.825	−1.335	−1.913	−2.571	−3.318	−4.159	−5.098
y_{10}^4/α	0.172	0.397	0.6475	0.932	1.255	1.622	2.037	2.498

TABLE III. Comparison of $\mathrm{Br}(K^+\to\pi^+\nu\overline{\nu})$, $\mathrm{Br}(K_L\to\pi^0\nu\overline{\nu})$, and $\mathrm{Br}(K_L\to\pi^0\nu\overline{\nu})$ among the experimental values and SM predictions with maximum mixing.

	$\mathrm{Br}(K^+\to\pi^+\nu\overline{\nu})$	$\mathrm{Br}(K_L\to\pi^0\nu\overline{\nu})$	$\mathrm{Br}(K_L\to\mu^+\mu^-)$
Experiment	$<2.4\times10^{-9}$ [37]	$<1.6\times10^{-6}$ [38]	$(6.9\pm0.4)\times10^{-9}$ [39]
	$(4.2+9.7-3.5)\times10^{-10}$ [44]	$<6.1\times10^{-9}$ [40]	$(7.9\pm0.7)\times10^{-9}$ [41]
SM	$(8.2\pm3.2)\times10^{-11}$ [43]	$(3.1\pm1.3)\times10^{-11}$ [43]	$(1.3\pm0.6)\times10^{-9}$ [42]

III. CONSTRAINTS ON CKM FACTOR $V^*_{t's}V_{t'd}$ IN SM4

Though we have no direct information for the additional fourth generation CKM matrix elements, while constraints may be obtained from some rare meson decays. In Ref. [35], we obtained the values of the fourth CKM factor $V^*_{t's}V_{t'b}$ from the decay of $B\to s\gamma$. In this paper, we shall investigate three rare K meson decays: two semileptonic decays, $K^+\to\pi^+\nu\overline{\nu}$ and $K_L\to\pi^0\nu\overline{\nu}$, and one leptonic decay $K_L\to\mu^+\mu^-$ [36] within SM4. These decays can provide certain constraints on the fourth generation CKM factors $V^*_{t's}V_{t'd}$, $\mathrm{Im}\,V^*_{t's}V_{t'd}$ and $\mathrm{Re}\,V^*_{t's}V_{t'd}$, respectively.

Within the SM, the decays $K\to\pi\nu\overline{\nu}$ are loop-induced semileptonic FCNC processes determined only by Z^0-penguin and box diagrams. These decays are the theoretically cleanest decays in rare K decays. The great virtue of $K_L\to\pi^0\nu\overline{\nu}$ is that it proceeds almost exclusively through direct CP violation [45] which is very important for the investigation of ϵ'/ϵ in SM4. The precise calculation of these two decays at the NLO in the SM can be found in Ref. [46]. While experimentally, its branching ratio has not yet been well measured, only an upper bound has been given and is larger by one order of magnitude than the one in the SM (see Table III).[1] This remains allowing the new physics to dominate their decay amplitude [18]. Moreover, unlike the previous two semileptonic decays, the branching ratio $\mathrm{Br}(K_L\to\mu^+\mu^-)$ has already been measured with a very good precision, while its experimental result is several times larger than theoretical prediction in SM (see Table III). This also provides a window for new physics.

In the SM4, the branching ratios of the three decay modes mentioned above receive additional contributions from the up-type quark t' [47]:

$$\mathrm{Br}(K^+\to\pi^+\nu\overline{\nu})=\kappa_+\left|\frac{V_{cd}V^*_{cs}}{\lambda}P_0+\frac{V_{td}V^*_{ts}}{\lambda^5}\eta_t X_0(x_t)+\frac{V_{t'd}V^*_{t's}}{\lambda^5}\eta_{t'}X_0(x_{t'})\right|^2, \tag{15}$$

$$\mathrm{Br}(K_L\to\pi^0\nu\overline{\nu})=\kappa_L\left|\frac{\mathrm{Im}\,V_{td}V^*_{ts}}{\lambda^5}\eta_t X_0(x_t)+\frac{\mathrm{Im}\,V_{t'd}V^*_{t's}}{\lambda^5}\eta_{t'}X_0(x_{t'})\right|^2, \tag{16}$$

$$\mathrm{Br}(K_L\to\mu\overline{\mu})_{\mathrm{SD}}=\kappa_\mu\left[\frac{\mathrm{Re}(V_{cd}V^*_{cs})}{\lambda}P'_0+\frac{\mathrm{Re}(V_{td}V^*_{ts})}{\lambda^5}Y_0(x_t)+\frac{\mathrm{Re}(V_{t'd}V^*_{t's})}{\lambda^5}Y_0(x_{t'})\right]^2, \tag{17}$$

where $\kappa_+,\kappa_L,\kappa_\mu,X_0(x_t),X_0(x_{t'}),Y_0(x_t),Y_0(x_{t'}),P_0,P'_0$ may be found in Refs. [9,10]. The QCD correction factors are taken to be $\eta_t=0.985$ and $\eta_{t'}=1.0$ [47].

To solve the constrains of the fourth generation CKM matrix factors $V^*_{t's}V_{t'd}$, $\mathrm{Im}\,V^*_{t's}V_{t'd}$, and $\mathrm{Re}\,V^*_{t's}V_{t'd}$, we must calculate the Wilson coefficients $X_0(x_{t'})$ and $Y_0(x_{t'})$. They are the functions of the mass of the fourth generation top quark $m_{t'}$. Here we give their numerical results accord-

TABLE IV. Wilson coefficients $X_0(x_{t'}),Y_0(x_{t'})$ to $m_{t'}$.

$m_{t'}$ (GeV)	50	100	150	200	250	300	400	500	600
$X_0(x_{t'})$	0.404	0.873	1.357	1.884	2.474	3.137	4.703	6.615	8.887
$Y_0(x_{t'})$	0.144	0.443	0.833	1.303	1.856	2.499	4.027	5.919	8.179

[1]From Ref. [40], one can easily derive by means of isospin symmetry the following model independent bound:

$$\mathrm{Br}(K^0\to\pi^+\nu\overline{\nu})<4.4\times\mathrm{Br}(K_L\to\pi^+\nu\overline{\nu}),$$

which gives

$$\mathrm{Br}(K^0\to\pi^+\nu\overline{\nu})<6.1\times10^{-9}.$$

This bound is much stronger than the direct experimental bound.

TABLE V. Phenomenological values of B_i. An asterisk marks an educated guess.

$B_1^{(1/2)}$	$B_2^{(1/2)}$	$B_3^{(1/2)}$	$B_4^{(1/2)}$	$B_5^{(1/2)}$	$B_6^{(1/2)}$	$B_7^{(1/2)}$	$B_8^{(1/2)}$	$B_9^{(1/2)}$	$B_{10}^{(1/2)}$
13.0	6.1±1.0	1.0*	5.2*	$B_6^{(1/2)}$	INPUT	1.0*	1.0*	7.0*	7.5*
$B_1^{(3/2)}$	$B_2^{(3/2)}$	$B_3^{(3/2)}$	$B_4^{(3/2)}$	$B_5^{(3/2)}$	$B_6^{(3/2)}$	$B_7^{(3/2)}$	$B_8^{(3/2)}$	$B_9^{(3/2)}$	$B_{10}^{(3/2)}$
0.48	0.48	1.0*	5.2*	1.0±0.3*	1.0±0.3*	1.0*	INPUT	0.48	0.48

ing to several values of $m_{t'}$ (see Table IV). We found that the Wilson coefficients $X_0(x_{t'})$ and $Y_0(x_{t'})$ increase with the $m_{t'}$. To get the largest constraint of the factors in Eqs. (15), (16), and (17), we must use the little value of $m_{t'}$. Considering that the fourth generation particles must have the mass larger than $M_Z/2$ [33], we take $m_{t'}$ with 50 GeV to get our constraints of those three factors.

Then, from Eqs. (15), (16), and (17), we arrive at the following constraints:

$$|V^*_{t's}V_{t'd}|\leqslant 2\times 10^{-4}, \tag{18}$$

$$|\mathrm{Im}\, V^*_{t's}V_{t'd}|\leqslant 1.2\times 10^{-4}, \tag{19}$$

$$|\mathrm{Re}\, V^*_{t's}V_{t'd}|\leqslant 1.0\times 10^{-4}. \tag{20}$$

For the numerical calculations, we will take $|\mathrm{Im}\, V^*_{t's}V_{t'd}| \leqslant 1.2\times 10^{-4}$.

It is easy to check that Eq. (18) obeys the CKM matrix unitarity constraint, which states that any pair of rows, or any pair of columns, of the CKM matrix are orthogonal [11]. The relevant one to those decay channels is

$$V^*_{us}V_{ud}+V^*_{cs}V_{cd}+V^*_{ts}V_{td}+V^*_{t's}V_{t'd}=0. \tag{21}$$

Here we have taken the average values of the SM CKM matrix elements from Ref. [11]. Considering the fact that the data of the CKM matrix is not yet very accurate, there still exists a sizable error for the sum of the first three terms. Using the value of $V^*_{t's}V_{t'd}$ obtained from Eq. (18), the sum of the four terms in the left-hand side of Eq. (21) can still be close to 0, because the values of $V^*_{t's}V_{t'd}$ are about 10^{-4} order, ten times smaller than the sum of the first three ones in the left of Eq. (21). Thus, the values of $V^*_{t's}V_{t'd}$ remain satisfying the CKM matrix unitarity constraints in SM4 within the present uncertainties.

IV. THE NUMERICAL ANALYSIS

In the calculation of ϵ'/ϵ, the main source of uncertainty are the hadronic matrix elements $\langle Q_i\rangle_I$. They depend generally on the renormalization scale μ and on the scheme used to renormlize the operators Q_i. But the calculation of $\langle Q_i\rangle_I$ is much beyond the perturbative method. They only can be treated by nonperturbative methods, such as lattice methods, $1/N$ expansion, chiral quark models, and chiral effective Lagrangians, which is not sufficient to obtain the high accuracy. We shall present the analysis on t'-quark effects when considering the uncertainties of $\langle Q_i\rangle_I$ due to model-dependent calculations.

It is customary to express the matrix elements $\langle Q_i\rangle_I$ in terms of nonperturbative parameters $B_i^{(1/2)}$ and $B_i^{(3/2)}$ as follows:

$$\langle Q_i\rangle_0\equiv B_i^{(1/2)}\langle Q_i\rangle_0^{(\mathrm{vac})},\quad \langle Q_i\rangle_2\equiv B_i^{(3/2)}\langle Q_i\rangle_2^{(\mathrm{vac})}. \tag{22}$$

The full list of $\langle Q_i\rangle_I$ is given in Ref. [12]. We take the phenomenological values of B_i [17] (see Table V) except for $B_6^{(1/2)}$ and $B_8^{(3/2)}$ which are taken as input parameters with values calculated by three different nonperturbative methods. Other numerical input parameters are given in Table VI.

We take the values of $B_6^{(1/2)}$and $B_8^{(3/2)}$ in three nonperturbative approaches: lattice methods, $1/N$ expansion, and chiral quark models (see Table VII and Figs. 2–4) in each case.

The numerical results are shown in Figs. 2–4 which correspond to the three cases of calculating hadronic matrix elements: lattice method, 1/N expansion, and chiral quark model. We now present a study for ϵ'/ϵ as functions of $\mathrm{Im}\,\lambda_{t'}$ and $m_{t'}$: ϵ'/ϵ versus $\mathrm{Im}\,\lambda_{t'}$ with fixing $m_{t'}$ is plotted in Figs. 2(a)–4(a); ϵ'/ϵ versus $m_{t'}$ with fixing $\mathrm{Im}\,\lambda_{t'}$ is plotted in Figs. 2(b)–4(b); and the allowed parameter space of $\mathrm{Im}\,\lambda_{t'}$ and $m_{t'}$ is plotted in Figs. 2(c)–4(c). We shall analyze each case in detail as follows.

In Figs 2(a)–4(a) we plot eight lines corresponding to $m_{t'}=50$, 100, 150, 200 250, 300, 350, 400 GeV, respectively. First, we notice that the slope of the line decreases as

TABLE VI. Numerical values of the input parameters.

$\mathrm{Re}\,A_0$	3.33×10^{-7} GeV	$\Omega_{\eta\eta'}$	0.25	G_F	1.166×10^{-5} GeV^{-2}
$\mathrm{Re}\,A_2$	1.50×10^{-8} GeV	ω	0.045	$\mathrm{Im}\,\lambda_t$	1.34×10^{-4}
$m_d(m_c)$	8 MeV	m_π	138 MeV	$\Lambda^4_{\overline{\mathrm{MS}}}$	340 MeV
$m_s(m_c)$	130 GeV	m_K	498 MeV	M_W	80.2 GeV
$m_c(m_c)$	1.3 GeV	F_π	131 MeV	$\alpha_s(M_Z)$	0.117
$m_b(m_b)$	4.8 GeV	F_K	160 MeV	α	1/129
$m_t(m_t)$	175 GeV			$\sin\theta_W$	0.23

TABLE VII. The input values of $B_6^{(1/2)}$ and $B_8^{(3/2)}$ in three cases.

	lattice method	$1/N$ expansion	chiral quark models
$B_6^{(1/2)}$	$1.0\pm0.02\pm0.05$ [48]	0.81 [6]	1.6 ± 0.3 [7]
$B_8^{(3/2)}$	0.8 ± 0.15 [14]	0.49 [6]	0.92 ± 0.002 [7]

$m_{t'}$ increases. At a value of $m_{t'}$, about 230 GeV, the slope is zero because the second part in the right-hand side of Eq. (13) vanishes. The reason is similar to that in SM, i.e., with increasing $m_{t'}$ the EW penguin diagrams become increasingly important and their contributions to ϵ'/ϵ are with the opposite sign to those of QCD penguin diagrams so that at some values of $m_{t'}$ there is a cancellation. The behavior comes essentially once $m_{t'}$ becomes larger than 230 GeV, the slope is negative. Its absolute value increases with $m_{t'}$. Such a behavior comes essentially from the change of the Wilson coefficients $y_i^{(4)}$ as $m_{t'}$. Second, from Figs. 2(a)–4(a), we found, within the constraints on $\mathrm{Im}\,\lambda_{t'}$ from the three rare K meson decays, that ϵ'/ϵ can generally be consistent with the experimental average except for some ranges of $m_{t'}$ once the nonperturbative parameters $B_6^{(1/2)}$ and $B_8^{(3/2)}$ are taken from values calculated based on the lattice gauge theory and $1/N$ expansion. Such a range roughly ranges from 170 to 300 GeV, which can be seen from Figs. 2(a) and 3(a). There is no excluded range for the case of the chiral quark model. This is because in the first two cases, the SM values $(\epsilon'/\epsilon)^{\rm SM}$ are about 8.8×10^{-4}, which is much lower than the experimental average. For a large range of $m_{t'}$, $(\epsilon'/\epsilon)^{(4)}$ is not large enough to make total ϵ'/ϵ reach the experimental average. But in the chiral quark model, the SM value is about 18.8×10^{-4} which is in the 1σ error range of the present experimental average so that ϵ'/ϵ can reach the experimental average for all values of $m_{t'}$ in the reasonable region. Thus once the nonperturbative method calculations become more reliable and the experimental measurements get more accuracy, it may provide more strong constraints on the forth generation quark from the study on ϵ'/ϵ. Unfortunately, we can't get any information on the upper bound of $m_{t'}$.

We also plot in Figs. 2(b)–4(b) eight curves corresponding to $\mathrm{Im}\,\lambda_{t'}=1.0$, 0.75, 0.5, 0.25, -0.25, -0.5, -0.75, and -1.0×10^{-4}, respectively. Thus similar results as those in Figs 2(a)–4(a) are arrived. These curves are divided into two types determined by the sign of the fourth generation CKM factor $\mathrm{Im}\,\lambda_{t'}$. The reason is also similar to the analysis for Figs. 2(a)–4(a). Figures 2(b)–4(b) also show the constraints on $\mathrm{Im}\,\lambda_{t'}$. It is interesting to see that there is an excluded region from 0 to 0.6×10^{-4} based on lattice gauge theory results and from 0 to 0.76×10^{-4} based on the $1/N$ expansion results, while there is no such excluded region based on the chiral quark model results. The reason is the same as that in the analysis of Figs. 2(a)–4(a). Moreover, it seems that $\mathrm{Im}\,\lambda_{t'}$ favors the negative values which may be interesting since the negative value of $\mathrm{Im}\,\lambda_{t'}$ is better to satisfy the unitarity constraints of the CKM matrix [see Eq. (21)]. Therefore if a fourth generation exists, from both the theo-

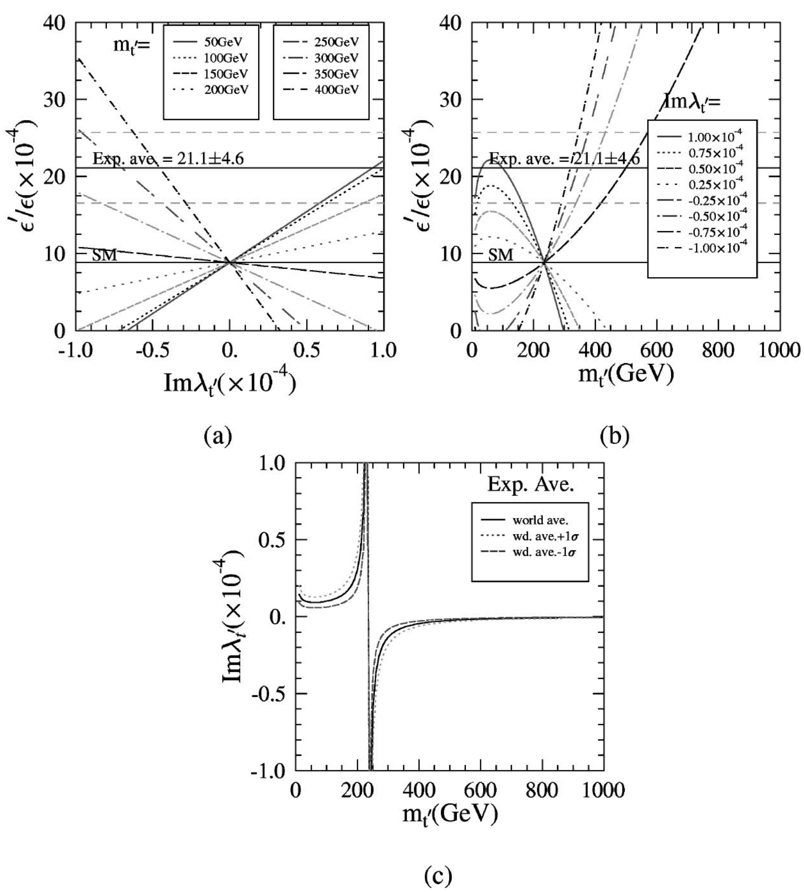

FIG. 2. The diagrams of (a) ϵ'/ϵ to $\mathrm{Im}\,\lambda_{t'}$, (b) ϵ'/ϵ to $m_{t'}$, and (c) parameter space: $\mathrm{Im}\,\lambda_{t'}$ to $m_{t'}$ with $B_6^{(1/2)}$ and $B_8^{(3/2)}$ in lattice gauge theory.

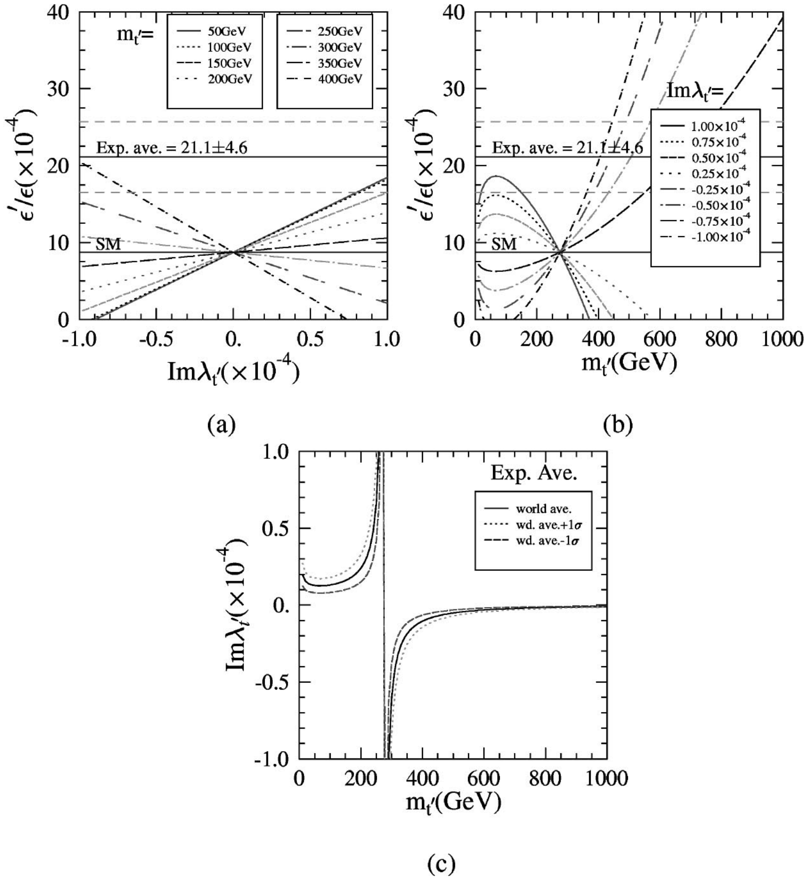

FIG. 3. Same as Fig. 2 in $1/N$ expansion.

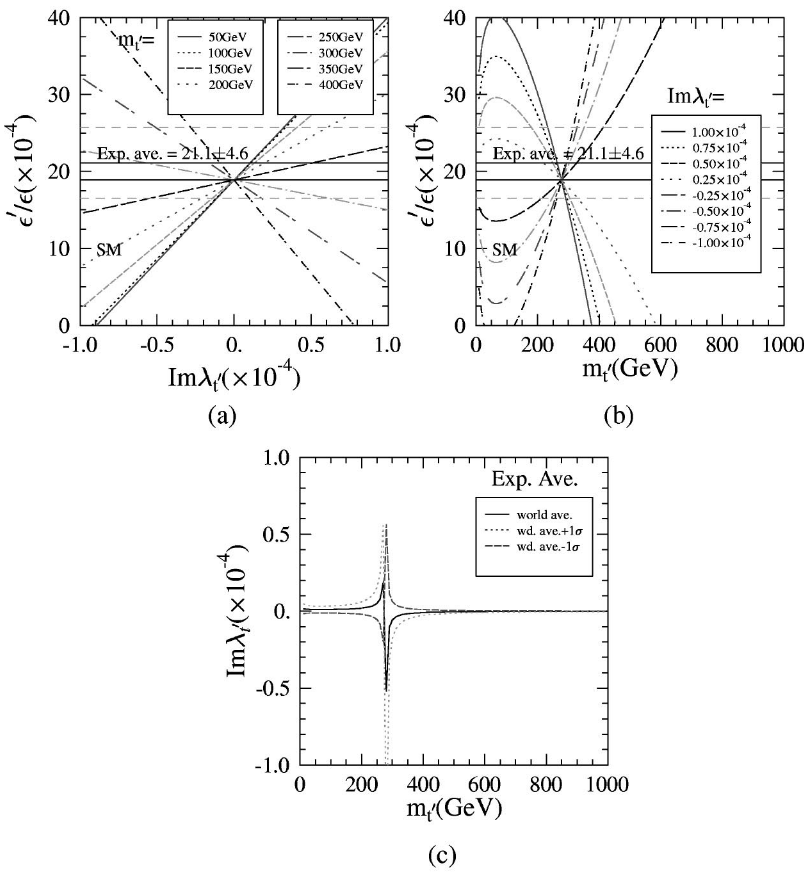

FIG. 4. Same as Fig. 2 in chiral quark model.

retical and experimental parts, one might be able get useful information on the fourth generation CKM matrix elements, such as $V^*_{t's}V_{t'b}$ which has been studied in our previous paper [35].

In Figs. 2(c)–4(c), we show the correlation between $\mathrm{Im}\,\lambda_{t'}$ and $m_{t'}$. The three curves in the figure correspond to the experimental values of the new world average and its 1σ error, respectively. It is seen that the allowed parameter space is strongly limited for all three cases when the ratio ϵ'/ϵ is around the present experimental average within 1 σ error. The allowed parameter space is divided into two pieces except in the chiral quark model. This is in agreement with the analyses in Figs. 2(a),2(b),3(a),3(b),4(a),4(b). Such a small parameter space indicates that ϵ'/ϵ may impose a very strong constraint on the mass and mixing of the fourth generation up-type quark.

V. CONCLUSION

In summary, we have investigated the direct CP-violating parameter ϵ'/ϵ in K^0-$\overline{K}^0$ system with considering the up-type quark t' in SM4. The basic formulas for ϵ'/ϵ in SM4 have been presented and the Wilson coefficient functions in the SM4 have also been evaluated. The numerical results of the additional Wilson coefficient functions have been given as functions of the mass $m_{t'}$. We have also studied the relevant rare K meson decays: two semileptonic decays $K^+\to\pi^+\nu\overline{\nu}$ and $K_L\to\pi^0\nu\overline{\nu}$, and one leptonic decay $K_L\to\mu^+\mu^-$, which allow us to obtain the bounds on the fourth generation CKM matrix factor $V^*_{t's}V_{t'd}$. In particular, we have analyzed the numerical result of ϵ'/ϵ as the function of $m_{t'}$ and imaginary part of the fourth CKM factor $\mathrm{Im}\,V^*_{t's}V_{t'd}$ (or $V^*_{t's}V_{t'd}$ and a fourth generation CKM matrix phase θ). The correlation between ϵ'/ϵ and $\mathrm{Im}\,V^*_{t's}V_{t'd}$ has been studied in detail with different hadronic matrix elements calculated from various approaches, such as lattice gauge method, $1/N$ expansion, and chiral quark model. It has been seen that, unlike the SM, when taking the central values of all parameters, the values of ϵ'/ϵ can be easily made to be consistent with the current experimental data for all estimated values of the relevant hadronic matrix elements from various approaches. Especially, we have also investigated the allowed parameter space of $m_{t'}$ and $\mathrm{Im}\,V^*_{t's}V_{t'd}$, as a consequence, when considering 1σ error of the current experimental data for ϵ'/ϵ, the allowed parameter space for $m_{t'}$ and $\mathrm{Im}\,V^*_{t's}V_{t'd}$ is very small and strongly restricted. This implies that the experimental data in the K system can provide strong constraints on the mass of the t' quark and also on the fourth generation quark mixing matrix.

ACKNOWLEDGMENTS

This research was supported in part by the National Science Foundation of China.

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