New prediction for the direct CP-violating parameter ε'/ε and the $\Delta I=1/2$ rule

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The low-energy dynamics of QCD is investigated with special attention paid to the matching between QCD and chiral perturbation theory (ChPT), and also to some useful algebraic chiral operator relations which survive even when we include chiral loop corrections. It then allows us to evaluate the hadronic matrix elements below the energy scale $\Lambda_\chi \simeq 1$ GeV. Based on the new analyses, we present a consistent prediction for both the direct CP-violating parameter ε'/ε and the $\Delta I=1/2$ rule in kaon decays. In the leading $1/N_c$ approximation, the isospin amplitudes A_0 and A_2 are found to agree well with the data, and the direct CP-violating parameter ε'/ε is predicted to be large, which also confirms our earlier conclusion. Its numerical value is $\varepsilon'/\varepsilon = 23.6^{+12.4}_{-7.8}\times 10^{-4}(\mathrm{Im}\,\lambda_t/1.2\times 10^{-4})$ which is no longer sensitive to the strange quark mass due to the matching conditions. Taking into account a simultaneous consistent analysis on the isospin amplitudes A_0 and A_2, the ratio ε'/ε is in favor of the values $\varepsilon'/\varepsilon=(20\pm 9)\times 10^{-4}$.

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I. INTRODUCTION

To make a consistent prediction for the direct CP-violating parameter ε'/ε caused by the Kobayashi-Maskawa CP-violating phase [1] which can arise from explicit CP violation in the standard model (SM) or originate from spontaneous CP violation [2] in the simple extension of the SM with two Higgs doublets (S2HDM) [3], it is thought to be necessary to understand simultaneously the longstanding puzzle of the $\Delta I=1/2$ rule in kaon decays as they involve the long-distance evolution of common hadronic matrix elements. It is believed that the low-energy dynamics of QCD should play a crucial role for a consistent analysis. During the past few years, both theoretical and experimental efforts on direct CP violation in the kaon decays have made important progress. As a consequence, agreement between the experimental results [4–8] and the theoretical predictions has been reached within the framework of chiral perturbation theory (ChPT) [9,10] and the chiral quark model [11]. On the experimental side, two improved new experiments [6,7] with higher precision have reported results which are consistent with each other and also agree with the earlier result [4]. On the theoretical side, there have been some developments which are mainly based on QCD of quarks and cutoff ChPT at low energies for mesons. The renormalization coefficients of all the relevant four quark operators, which characterize the short-distance effects of the effective Hamiltonian generated from renormalization of the weak interactions, have been computed and extended from the leading order [13,14] to the next-to-leading order [15,16] QCD corrections. The results agree with each other. The long-distance effects have been evaluated from the ChPT inspired from $1/N_c$ expansion [17,18] up to the chiral one-loop level [19,9,10] as well as from chiral quark model [11]. Recently they have been recalculated within the same framework of ChPT but with a different calculating scheme [20–23]. The important issue concerned in all the calculations is the matching problem to QCD. In ChPT, it requires the matching between the short-distance operator evolution from QCD with infrared cutoff and long-distance operator evolution from ChPT with ultraviolet cutoff. Practically, the renormalization scale μ dependence of the Wilson coefficient functions $c_i(\mu)$ from perturbative QCD should cancel the one of the corresponding operators $Q_i(\mu)$ from nonperturbative contributions. In the chiral quark model the operator evolution leads to the results which are expected to be valid only at a special value of the energy scale μ. Alternatively, in the ChPT approach, its attractive advantage is that chiral loops with an ultraviolet cutoff, denoted by M, introduce a scale dependence for long-distance operator evolution. As a simple consideration, the ultraviolet cutoff M might naively be identified to the infrared cutoff μ to improve the matching. Consequently, both the $\Delta I=1/2$ rule and direct CP-violating parameter ε'/ε can be enhanced to be more consistent with the present experimental data. Nevertheless, in the existing treatments of the approach, there remain some open questions which need to be further clarified. First, the momentum cutoff M in the long-distance operator evolution from meson loops can in general only be extended to the energy scale which must be smaller than the dynamical chiral symmetry breaking scale Λ_f, i.e., $M<\Lambda_f \sim 1$ GeV, whereas the short-distance operator evolution from perturbative QCD (by using renormalization group equation) requires that the energy scale should be above the confining scale, i.e., $\mu>1$ GeV. Thus naively identifying the ultraviolet cutoff M in ChPT to the infrared cutoff μ in perturbative QCD may become inappropriate. Secondly, there appear some discrepancies between Refs. [19] and [20,22] for the matrix elements Q_1 and Q_2 even if the same chiral Lagrangian has been used and the same loop diagrams have been considered. It is noticed that the discrepancies only occur in the coefficients of the quadratic terms of the cutoff energy scale M and in the constant terms. Such discrepancies mainly arise from different calculating schemes. In Refs. [19,9,10], all the chiral one-loop contributions were considered to be summed up with the cutoff regularization, and the coupling constants are replaced by the renormalized ones. Such a treatment is the standard one as adopted in the quantum field theory. In the recent calculations [21–23], the chiral one-loop diagrams have been separated into two classes, i.e., so-called factorized and nonfactorized diagrams. For the nonfactorized diagrams, a virtual momentum flow has been artificially added to the propagators. As a consequence, such an alternative treatment is equivalent to the change of the cutoff energy scale. When taking the cutoff energy scale to be infinity as the case for a renormalized field theory such as QCD and QED, the treatment has no effects. However, for a finite cutoff integral, the change of the variables of the integrand will result in different results. This is the main reason why two calculations led to different results in the quadratic terms of cutoff scale and in the constant terms. In fact, one can simply rescale the cutoff scale Λ_c in Ref. [22] via $\Lambda_c^2=2M^2/3$ to obtain the results in Ref. [19] for Q_1 and Q_2. For a similar reason, for Q_6 there also exist discrepancies between Refs. [9,10] and [23] for the quadratic and constant terms, however, two results cannot be simply related by the same rescaling factor as the one for Q_1 and Q_2. The reason is that the results in Ref. [23] were obtained only by evaluating part of the so-called nonfactorized diagrams. Notice that it may not be so clear to separate the factorized and nonfactorized diagrams for the loop corrections of the left-right structure four quark operators generated from the penguin diagrams in which the intermediate quarks form a closed loop with the gauge bosons. In fact, for the density$\times$density operators, the so-called factorized diagrams do provide contributions to the anomalous dimension of the operators in QCD evolution. The μ dependence of such factorized contributions is exactly cancelled by the one of explicit quark mass factor appearing in the corresponding chiral operators, but not by the one of the corresponding factorized chiral loop, since the quark mass factor does not arise from the chiral loop contributions. Therefore, for the density$\times$density operators, or more general for the operators with left-right structure, it is not necessary to have one to one correspondings between QCD loop and chiral loop due to the μ dependence of the low-energy coupling constants in the chiral Lagrangian. Nevertheless, physics observables should be independent of the calculating schemes.

Thus, clarifying the above two open questions comes to one of the main purposes of this paper. Our paper is organized as follows. In Sec. II, we describe the basic motivations for evaluating the long-distance contributions of the hadronic matrix elements within the framework of ChPT. Especially, a functional cutoff momentum is introduced for the purpose of matching, namely, the cutoff momentum M is in general considered as the function of the QCD running scale μ, i.e., $M\equiv M(\mu)$. In Sec. III we explicitly write down the chiral representation of four quark operators and emphasize some useful algebraic chiral operator relations. In Sec. IV we investigate the matching between QCD and ChPT, where the chiral operators are explicitly evaluated in the functional cutoff momentum scheme, two useful matching conditions will be obtained. Of interest, the strange quark mass is found to be fixed from the matching condition and algebraic chiral operator relation. The long-distance chiral operator evolution is carried out in Sec. V. In Sec. VI we present our numerical predictions for the direct CP-violating parameter ε'/ε and $\Delta I=1/2$ rule, they are found to be remarkably consistent with the data. Our conclusions and remarks are made in the last section.

II. BASIC MOTIVATION

Our considerations are mainly based on the following basic points.

In the large N_c limit but with the combination $\alpha_s N_c\equiv\alpha_0$ being held fixed. The QCD loop corrections which are proportional to α_s are then corresponding to a large N_c expansion, $\alpha_s\sim 1/N_c$ [17].

Chiral symmetry is supposed to be broken dynamically due to attractive gauge interactions, namely, the chiral condensates $\langle\bar{q}q\rangle$ exist and lead to the Goldstone-like pseudoscalar mesons π,K,η. The chiral symmetry breaking scale Λ_f is characterized by the condensate, $\Lambda_f\simeq 4\pi\sqrt{-2\langle\bar{q}q\rangle/r}\sim 1$ GeV with $r=m_{\pi_0}^2/\hat{m}$ [$\hat{m}=(m_u+m_d)/2$].

The chiral Lagrangian is considered to describe the low-energy dynamics of QCD in large N_c limit and is going to be treated as a cutoff effective field theory. The cutoff momentum M is expected to be below the chiral symmetry breaking scale Λ_f.

The chiral meson loop contributions are characterized by the powers of p^2/Λ_f^2 with $\Lambda_f=4\pi f$. Here $f^2\simeq -2\langle\bar{q}q\rangle/r\sim N_c$ is at the leading N_c order and fixed by the π decay coupling constant $f\sim F_\pi$. Thus the chiral meson loop contributions are also corresponding to a large N_c expansion of QCD, $p^2/\Lambda_f^2\sim 1/N_c\sim\alpha_s$. Therefore both chiral loop and QCD loop contributions must be matched to each other, at least in the sense of large N_c limit. Thus the final physical results should be independent of the cutoff schemes.

The cutoff momentum M of loop integrals should not be naively identified to the renormalization scale μ appearing in the perturbative QCD in large N_c limit. It is in general taken to be a function of μ, i.e., $M\equiv M(\mu)$, which may be regarded as a functional cutoff momentum, its form is determined by the matching between the Wilson coefficients of QCD and hadronic matrix elements evaluated via ChPT. It is seen that the matching relates the chiral cutoff momentum to the strong coupling constant so that the results become scheme independent.

From these points of view, the ChPT with functional cutoff momentum is going to be treated, in certain sense, as a low-energy effective field theory of QCD in the large N_c limit. With such a treatment, it is in general not necessary to distinguish the so-called factorized and nonfactorized contributions since the renormalization of field theory should well cover both of their contributions automatically. In this paper, we will give up the calculating scheme of separating the factorized and nonfactorized contributions, and adopt the calculating scheme first proposed by Bardeen, Buras, and Gérard [19] in the ChPT inspired by the $1/N_c$ expansion, but with a functional cutoff momentum $M(\mu)$ instead of naively identifying the cutoff momentum to the QCD running scale μ.

III. CHIRAL REPRESENTATION AND ALGEBRAIC RELATIONS

In the standard model, the $\Delta S=1$ low-energy ($\mu<m_c$) effective Hamiltonian for calculating $K\to\pi\pi$ decay amplitudes can be written as

$$\mathcal{H}_{\rm eff}^{\Delta S=1}=\frac{G_F}{\sqrt{2}}\lambda_u\sum_{i=1}^{8}c_i(\mu)Q_i(\mu)\quad(\mu<m_c),\tag{1}$$

with Q_i the four quark operators

$$Q_1=4\overline{s_L}\gamma^\mu d_L\overline{u_L}\gamma_\mu u_L\,,\quad Q_2=4\overline{s_L}\gamma^\mu u_L\overline{u_L}\gamma_\mu d_L\,,$$

$$Q_3=4\sum_q\overline{s_L}\gamma^\mu d_L\overline{q_L}\gamma_\mu q_L\,,\quad Q_4=4\sum_q\overline{s_L}\gamma^\mu q_L\overline{q_L}\gamma_\mu d_L\,,$$

$$Q_5=4\sum_q\overline{s_L}\gamma^\mu d_L\overline{q_R}\gamma_\mu q_R\,,\quad Q_6=-8\sum_q\overline{s_L}q_R\overline{q_R}d_L\,,\tag{2}$$

$$Q_7=4\sum_q\frac{3}{2}e_q\overline{s_L}\gamma^\mu d_L\overline{q_R}\gamma_\mu q_R,$$

$$Q_8=-8\sum_q\frac{3}{2}e_q\overline{s_L}q_R\overline{q_R}d_L,$$

with $q_{R,L}=\frac{1}{2}(1\pm\gamma_5)q$. Where the sum goes over the light flavors ($q=u,d,s$) and e_q is the charge of the corresponding light quarks. $Q_3,\ldots,Q_6$ arise from strong penguin diagrams. They transform as $(8_L,1_R)$ under $SU(3)_L\times SU(3)_R$ and solely contribute to $\Delta I=1/2$ transitions. Note that only seven operators are independent as the linear relation $Q_4=Q_2-Q_1+Q_3$. Q_7 and Q_8 originate from electroweak penguin diagrams. $c_i(\mu)$ are Wilson coefficient functions

$$c_i(\mu)=z_i(\mu)+\tau y_i(\mu),\tag{3}$$

where $\tau=-\lambda_t/\lambda_u$ with $\lambda_q=V_{qs}^*V_{qd}$. The Wilson coefficient functions $z_i(\mu)$ and $y_i(\mu)$ have been evaluated up to the next-to-leading order QCD corrections. The $K\to\pi\pi$ decay amplitudes A_I with isospin I are given by

$$A_Ie^{i\delta_I}=\langle\pi\pi|\mathcal{H}_{\rm eff}^{\Delta S=1}|K\rangle\equiv\frac{G_F}{\sqrt{2}}\lambda_u\sum_{i=1}^{8}c_i(\mu)\langle Q_i(\mu)\rangle_I\,,\tag{4}$$

where δ_I are the final state strong interaction phases. It is a hard task for calculating the hadronic matrix elements $\langle Q_i(\mu)\rangle_I$ for $\mu<\Lambda_\chi=1$ GeV which is at the order of chiral symmetry breaking scale. This is because perturbative QCD becomes unreliable in such a low-energy scale. In this paper we adopt the ChPT with functional cutoff momentum to evaluate $\langle Q_i(\mu)\rangle_I$ when $\mu<\Lambda_\chi$. To do that, the procedure is as follows: one first represents the current$\times$current or density$\times$density four quark operators Q_i by bosonized chiral fields from the chiral Lagarangian, then calculate loop contributions by using the functional cutoff momentum scheme. Finally, one matches the two results obtained from QCD and ChPT with functional cutoff momentum by requiring scale independence of the physical results.

The general form of the chiral Lagrangian can be expressed in terms of the expansions of the momentum p and quark mass to the energy scale Λ_χ. Here we only use the chiral Lagrangian which is relevant to the $K\to\pi\pi$ decays (for the most general one, see Ref. [12]),

$$\begin{aligned}\mathcal{L}_{\rm eff}=\frac{f^2}{4}\bigg\{&{\rm tr}(D_\mu U^\dagger D^\mu U)+\frac{m_\alpha^2}{4N_c}{\rm tr}(\ln U^\dagger-\ln U)^2\\&+r\,{\rm tr}(\mathcal{M}U^\dagger+U\mathcal{M}^\dagger)+r\frac{\chi_5}{\Lambda_\chi^2}{\rm tr}[D_\mu U^\dagger D^\mu U(\mathcal{M}^\dagger U\\&+U^\dagger\mathcal{M})]+r^2\frac{\chi_8}{\Lambda_\chi^2}{\rm tr}(\mathcal{M}^\dagger U\mathcal{M}^\dagger U+\mathcal{M}U^\dagger\mathcal{M}U^\dagger)\\&+r^2\frac{\kappa_2}{\Lambda_\chi^2}{\rm tr}(\mathcal{M}^\dagger\mathcal{M})\bigg\}\end{aligned}\tag{5}$$

with

$$D_\mu U=\partial_\mu U-ir_\mu U+iUl_\mu\,,$$

and

$$\mathcal{M}={\rm diag}(m_u,m_d,m_s).$$

l_μ and r_μ are left- and right-handed gauge fields, respectively. The unitary matrix U is a nonlinear representation of the pseudoscalar meson nonet given as $U=e^{i\Pi/f}$ with $\Pi=\pi^a\lambda_a$ and ${\rm tr}(\lambda_a\lambda_b)=2\delta_{ab}$. Here we keep the leading terms at the large N_c limit except the anomaly term which arises from the order of $1/N_c$. Note that in order to make clear for two independent expansions, namely, $1/N_c$ expansion characterized by p^2/Λ_f^2 in the large N_c limit, and the momentum expansion described by p^2/Λ_χ^2, we have introduced a scaling factor $\Lambda_\chi\simeq1$ GeV and redefined the low-energy coupling constants L_i introduced in Ref. [12] via $L_i=\chi_if^2/4\Lambda_\chi^2$ and $H_j=\kappa_jf^2/4\Lambda_\chi^2$, so that the coupling constants χ_i ($i=3,5,8$) and Λ_χ are constants in the large N_c limit and the whole Lagrangian is multiplied by f^2 and is of order N_c, except the U(1) anomalous term. This is because when applying the large N_c counting rules to all terms in Lagrangian, it shows that $L_i=O(N_c)$ ($i\neq7$) and $H_j=O(N_c)$. Here L_7 is at order of N_c^2. Numerically, one sees that $\chi_i=O(1)$ for $\Lambda_\chi=1$ GeV.

As the first step, we represent the quark currents and densities by the chiral fields

$$\begin{aligned}\overline{q}_{jL}\gamma^\mu q_{iL}\equiv\frac{\delta\mathcal{L}}{\delta(l_\mu(x))_{ji}}=-i\frac{f^2}{2}\bigg\{&U^\dagger\partial^\mu U-r\frac{\chi_5}{2\Lambda_\chi^2}(\partial^\mu U^\dagger\mathcal{M}\\&-\mathcal{M}^\dagger\partial^\mu U+\partial^\mu U^\dagger U\mathcal{M}^\dagger U-U^\dagger\mathcal{M}U^\dagger\partial^\mu U)\bigg\}_{ij},\end{aligned}\tag{6}$$

$$\overline{q}_{jR}q_{iL} \equiv -\frac{\delta\mathcal{L}}{\delta\mathcal{M}_{ji}} = -r\frac{f^2}{4}\left(U^\dagger + \frac{\chi_5}{\Lambda_\chi^2}\partial_\mu U^\dagger\partial^\mu U U^\dagger + 2r\frac{\chi_8}{\Lambda_\chi^2}U^\dagger\mathcal{M}U^\dagger + r\frac{\kappa_2}{\Lambda_\chi^2}\mathcal{M}^\dagger\right)_{ij}. \tag{7}$$

Similarly one can obtain the right-handed currents and densities. With these definitions, all the current$\times$current and density $\times$density four quark operators can be reexpressed in terms of the chiral fields, we may call such chiral representations of four quark operators Q_i as chiral operators denoted by Q_i^χ correspondingly. At p^2 order, Q_i^χ can be written as the following form:

$$\begin{aligned}
Q_1^\chi + \text{H.c.} &= -f^4\,\text{tr}(\lambda_6 U^\dagger\partial_\mu U)\text{tr}(\lambda^{(1)}U^\dagger\partial^\mu U) + O(1/\Lambda_\chi^2),\\
Q_2^\chi + \text{H.c.} &= -f^4\,\text{tr}(\lambda_6 U^\dagger\partial_\mu U\lambda^{(1)}U^\dagger\partial^\mu U) + O(1/\Lambda_\chi^2),\\
Q_3^\chi + \text{H.c.} &= -f^4\,\text{tr}(\lambda_6 U^\dagger\partial_\mu U)\text{tr}(U^\dagger\partial^\mu U) + O(1/\Lambda_\chi^2)\\
Q_4^\chi + \text{H.c.} &= -f^4\,\text{tr}(\lambda_6\partial_\mu U^\dagger\partial^\mu U) + O(1/\Lambda_\chi^2),\\
Q_5^\chi + \text{H.c.} &= -f^4\,\text{tr}(\lambda_6 U^\dagger\partial_\mu U)\text{tr}(U\partial^\mu U^\dagger) + O(1/\Lambda_\chi^2),\\
Q_6^\chi + \text{H.c.} &= +f^4\left(\frac{r^2\chi_5}{\Lambda_\chi^2}\right)\text{tr}(\lambda_6\partial_\mu U^\dagger\partial^\mu U) + O(1/\Lambda_\chi^4),\\
Q_7^\chi + \text{H.c.} &= -\frac{1}{2}Q_5^\chi - \frac{3}{2}f^4\,\text{tr}(\lambda_6 U^\dagger\partial_\mu U)\text{tr}(\lambda^{(1)}U\partial^\mu U^\dagger) + O(1/\Lambda_\chi^2),\\
Q_8^\chi + \text{H.c.} &= -\frac{1}{2}Q_6^\chi + f^4 r^2\frac{3}{4}\text{tr}(\lambda_6 U^\dagger\lambda^{(1)}U) + f^4 r^2\frac{3}{4}\frac{\chi_5}{\Lambda_\chi^2}\text{tr}\,\lambda_6(U^\dagger\lambda^{(1)}U\partial_\mu U^\dagger\partial^\mu U + \partial_\mu U^\dagger\partial^\mu U U^\dagger\lambda^{(1)}U)\\
&\quad + f^4 r^2\frac{3}{4}\frac{\chi_8}{\Lambda_\chi^2}2r\,\text{tr}\,\lambda_6(U^\dagger\lambda^{(1)}U\mathcal{M}^\dagger U + U^\dagger\mathcal{M}U^\dagger\lambda^{(1)}U) + O(1/\Lambda_\chi^4),
\end{aligned} \tag{8}$$

with the matrix $\lambda^{(1)} = \text{diag}(1,0,0)$. Thus loop contributions of the chiral operators Q_i^χ can be systematically calculated by using ChPT with functional cutoff momentum.

For $K\rightarrow\pi\pi$ decay amplitudes and direct CP-violating parameter ε'/ε, the most important chiral operators are Q_1^χ, Q_2^χ, Q_6^χ, and Q_8^χ. In fact, the chiral operators Q_3^χ and Q_5^χ decouples from the loop evaluations at the p^2 order [9], i.e.,

$$Q_5^\chi = Q_3^\chi = 0 \tag{9}$$

which can explicitly be seen from the above chiral representations due to the traceless factor $\text{tr}(U\partial^\mu U^\dagger)=0$ when ignoring the singlet $U(1)$ nonet term which is irrelevant to the kaon decays. Here $U\partial^\mu U^\dagger = A_\mu^a\lambda^a$ may be regarded as a pure gauge. This feature may also be understood as the fact that Q_3 and Q_5 operators are generated from strong penguin diagrams and suppressed by $1/N_c$ factor in comparison with the operators Q_4 and Q_6, thus in the large N_c limit, they decouple automatically. As a consequence, it implies that at the lowest order of p^2, we arrive at two additional algebraic chiral relations

$$Q_4^\chi = Q_2^\chi - Q_1^\chi = -f^4\,\text{tr}(\lambda_6\partial_\mu U^\dagger\partial^\mu U) + O(1/\Lambda_\chi^2) \tag{10}$$

and

$$Q_6^\chi = -\left(\frac{r^2\chi_5}{\Lambda_\chi^2}\right)(Q_2^\chi - Q_1^\chi) = \left(\frac{r^2\chi_5}{\Lambda_\chi^2}\right)f^4\,\text{tr}(\lambda_6\partial_\mu U^\dagger\partial^\mu U). \tag{11}$$

Notice that the mass parameter r is at the same order of the energy scale Λ_χ, and χ_5 is at order of unit, thus the leading nonzero contribution of Q_6^χ is at the same order of Q_2^χ and Q_1^χ.

The above algebraic chiral relations were first derived in Ref. [9], they have also been checked from an explicit calculation up to the chiral one-loop level by using the usual cutoff regularization [19]. If naively identifying the cutoff momentum M to the QCD running scale μ, the above algebraic chiral Wu relations, as commented by Buras, Jamin, and Lautenbacher [24], seem to hold only at one point when matching to QCD. Thus two questions have arised: From which energy scales and up to which order of chiral loop corrections do the algebraic chiral Wu relations hold? How can the algebraic chiral Wu-relations survive when matching the ultraviolet cutoff momentum of ChPT to the infrared cut-off momentum of perturbative QCD? Let us briefly address the first question and leave the second question to the next section. The answer to the first question is manifest, the algebraic chiral Wu-relations hold starting from the energy scale where the low-energy dynamics of QCD is considered to be described by the ChPT with functional cutoff momentum. They even survive when we include chiral-loop corrections generated from the lowest p^2 order terms as Wu-relations are the algebraic chiral operator relations, they should not be modified by the chiral loops of the strong interactions. Note that the coupling constants must also be replaced by the renormalized ones at the same order of $1/N_c$ or $1/\Lambda_F^2$. The reason is simple as the nontrivial structures of Q_6^χ and $(Q_2^\chi - Q_1^\chi)$ at the order of p^2 and p^2/N_c (or p^2/Λ_F^2) are unique. The order of p^4 terms are suppressed by the factors m_K^2/Λ_χ^2 and m_π^2/Λ_χ^2. Therefore, in the chiral limit, namely, $m_K^2, m_\pi^2 \ll \Lambda_\chi^2$, the above algebraic chiral operator relations should hold up to the order of p^2/N_c and p^4. This may be understood in an analogous way to QCD, where relations of the quark operators survive from all order of QCD corrections. The reason is simply due to that QCD is a renormalizable theory. For the ChPT, though it is an effective theory and not a renormalizable one in the usual sense, but it can be constructed to be a consistent theory order by order in the expansion of momentum and quark mass as well as $1/N_c$. Thus, to a given order of expansion, ChPT may be regarded as a renormalizable one in the more general sense [25]. Therefore, the algebraic chiral operator relations must survive, at least, up to the one-loop corrections, which has actually been checked from our explicit calculations. It was based on this observation, we came to our early conclusion that the direct CP-violating parameter ε'/ε can be large enough to be measured and its favorable numerical value is likely to be around $\varepsilon'/\varepsilon \sim (10-30)\times 10^{-4}$ [9,10]. As the matching to QCD was not completely considered, our previous results [9,10] strongly depend on the strange quark mass.

IV. MATCHING BETWEEN QCD AND CHPT

Let us begin with the short-distance operator evolution from perturbative QCD. When the energy scale μ is high, $m_W > \mu > m_b$, there are eleven independent operators Q_i ($i = 1, \ldots, 11$). When the energy scale μ runs down to below the bottom quark mass m_b and above the charm quark mass m_c, i.e., $m_b > \mu > m_c$, the operator Q_{11} decouples and operator Q_{10} is given by the linear combination $Q_{10} = -2Q_1 + 2Q_2 + Q_3 - Q_4$. Once the energy scale μ goes down to below m_c but above the confining scale or the energy scale Λ_χ, i.e., $m_c > \mu > \Lambda_\chi$, two operators Q_9 and Q_4 become no longer independent and are given by the linear combination $Q_9 = Q_2 + Q_1$ and $Q_4 = Q_3 + Q_2 - Q_1$. Thus there are only seven independent operators below m_c and above Λ_χ. In order to match to the long-distance evolution of the operators, let us present one-loop QCD corrections of the quark operators at the energy scale just above the energy scale Λ_χ:

$$Q_1(\mu_Q) = Q_1(\mu) - 3\frac{\alpha_s}{4\pi}\ln\left(\frac{\mu_Q^2}{\mu^2}\right)Q_2(\mu) + O(1/N_c), \tag{12}$$

$$Q_2(\mu_Q) = Q_2(\mu) - 3\frac{\alpha_s}{4\pi}\ln\left(\frac{\mu_Q^2}{\mu^2}\right)Q_1(\mu) - \frac{1}{3}\frac{\alpha_s}{4\pi}\ln\left(\frac{\mu_Q^2}{\mu^2}\right)Q_4(\mu) - \frac{1}{3}\frac{\alpha_s}{4\pi}\ln\left(\frac{\mu_Q^2}{\mu^2}\right)Q_6(\mu) + O(1/N_c), \tag{13}$$

$$Q_4(\mu_Q) = Q_4(\mu) - 3\frac{\alpha_s}{4\pi}\ln\left(\frac{\mu_Q^2}{\mu^2}\right)Q_3(\mu) - \frac{\alpha_s}{4\pi}\ln\left(\frac{\mu_Q^2}{\mu^2}\right)Q_4(\mu) - \frac{\alpha_s}{4\pi}\ln\left(\frac{\mu_Q^2}{\mu^2}\right)Q_6(\mu) + O(1/N_c), \tag{14}$$

$$Q_6(\mu_Q) = Q_6(\mu) - \frac{\alpha_s}{4\pi}\ln\left(\frac{\mu_Q^2}{\mu^2}\right)Q_4(\mu) + [3(N_c - 1/N_c) - 1]\frac{\alpha_s}{4\pi}\ln\left(\frac{\mu_Q^2}{\mu^2}\right)Q_6(\mu) + O(1/N_c), \tag{15}$$

$$Q_8(\mu_Q) = Q_8(\mu) + [3(N_c - 1/N_c) - 1]\frac{\alpha_s}{4\pi}\ln\left(\frac{\mu_Q^2}{\mu^2}\right)Q_8(\mu), \tag{16}$$

and

$$Q_3(\mu_Q) = Q_3(\mu) - \frac{11}{3}\frac{\alpha_s}{4\pi}\ln\left(\frac{\mu_Q^2}{\mu^2}\right)Q_4(\mu) - \frac{2}{3}\frac{\alpha_s}{4\pi}\ln\left(\frac{\mu_Q^2}{\mu^2}\right)Q_6(\mu) + O(1/N_c), \tag{17}$$

$$Q_5(\mu_Q) = Q_5(\mu) + 3\frac{\alpha_s}{4\pi}\ln\left(\frac{\mu_Q^2}{\mu^2}\right)Q_6(\mu) + O(1/N_c), \tag{18}$$

$$Q_7(\mu_Q) = Q_7(\mu) + 3\frac{\alpha_s}{4\pi}\ln\left(\frac{\mu_Q^2}{\mu^2}\right)Q_8(\mu) + O(1/N_c). \tag{19}$$

From the above results, we come to the following observations: (i) In the large N_c limit, Q_1, Q_2, Q_4, and Q_6 form a complete set of operators under QCD corrections; (ii) the evolution of Q_8 is independent of other operators and only caused by loop corrections of the density; (iii) the operator Q_3 is given by the linear combination $Q_3 = Q_4 - (Q_2 - Q_1)$. The operator Q_5 is driven by the operator Q_6, and the operator Q_7 is driven by the operator Q_8.

When the energy scale μ approaches to the confining scale, or $\mu < \Lambda_\chi \sim \Lambda_F \sim 1$ GeV, as we have discussed in the above sections, long-distance effects have to be considered. The evolution of the operators $Q_i(\mu)$ when $\mu < \Lambda_\chi$ is supposed to be carried out by the one of the chiral operators $Q_i^\chi[M(\mu)]$ in the framework of the functional cutoff ChPT truncated to the pseudoscalars. To be treated at the same approximations made in the short-distance operator evolution of QCD, we should only keep the leading terms (i.e., quadratic terms of functional cutoff momentum) and take the chiral limit, i.e., $m_K^2, m_\pi^2 \ll \Lambda_F^2$. In such a leading $1/N_c$ approximation and chiral limit, we find that the evolution of the operators Q_1^χ and Q_2^χ can be simply given by the following forms when the functional cutoff momentum runs from $M(\mu)$ down to $M(\mu')$:

$$Q_1(\mu) \to Q_1^\chi[M(\mu)] = Q_1^\chi[M(\mu')] - \frac{2[M^2(\mu) - M^2(\mu')]}{\Lambda_F^2} Q_2^\chi[M(\mu')], \tag{20}$$

$$Q_2(\mu) \to Q_2^\chi[M(\mu)] = Q_2^\chi[M(\mu')] - \frac{2[M^2(\mu) - M^2(\mu')]}{\Lambda_F^2} Q_1^\chi[M(\mu')] + \frac{M^2(\mu) - M^2(\mu')}{\Lambda_F^2}(Q_2^\chi - Q_1^\chi)[M(\mu')], \tag{21}$$

where $\Lambda_F = 4\pi F = 1.16$ GeV with F the renormalized one of f. Notice that the operators Q_3^χ and Q_5^χ decouple from the evolution, namely $Q_3^\chi = 0$ and $Q_5^\chi = 0$. The results for the operators Q_i^χ $(i = 4,6,8)$ can be written as follows:

$$Q_4(\mu) \to Q_4^\chi[M(\mu)] = (Q_2^\chi - Q_1^\chi)[M(\mu)], \tag{22}$$

$$Q_6(\mu) \to Q_6^\chi[\mu, M(\mu)] = \left[1 + 3(N_c - 1/N_c)\frac{\alpha_s}{4\pi}\ln\left(\frac{\mu^2}{\mu_\chi^2}\right)\right] \times Q_6^\chi[\mu_\chi, M(\mu)], \tag{23}$$

$$Q_8(\mu) \to Q_8^\chi[\mu, M(\mu)] = \left[1 + 3(N_c - 1/N_c)\frac{\alpha_s}{4\pi}\ln\left(\frac{\mu^2}{\mu_\chi^2}\right)\right] \times Q_8^\chi[\mu_\chi, M(\mu)], \tag{24}$$

where the explicit μ dependence of the operators $Q_6^\chi[\mu, M(\mu)]$ and $Q_8^\chi[\mu, M(\mu)]$ arise from the running quark mass and behaviors such as $1/[m_s(\mu) + \hat{m}(\mu)]^2$. Notice that the independent operators are reduced once more in the long-distance operator evolution when $\mu < \Lambda_\chi$ due to the algebraic chiral operator relations. Let us now compare and match the loop results evaluated from QCD with the ones from the ChPT with functional cutoff momentum at the energy scale Λ_χ. Substituting Eqs. (20) and (21) into Eq. (12), keeping the leading $1/N_c$ terms, we obtain, from the requirement of μ-independence in the large N_c limit, i.e., $(\partial/\partial\mu) Q_1(\mu_Q) = 0$, the first matching condition

$$\mu \frac{\partial}{\partial \mu}\left(\frac{2M^2(\mu)}{\Lambda_F^2}\right) = \frac{3\alpha_s}{2\pi}, \tag{25}$$

which can in general be obtained by requiring the matching between the anomalous dimensions of quark operators $Q_i(\mu)$ in QCD and the ones of the corresponding chiral operators $Q_i^\chi[M(\mu)]$ in ChPT in the large N_c limit, i.e.,

$$\gamma_i^{\text{meson}} \equiv \mu \frac{\partial}{\partial \mu} Q_i^\chi[M(\mu)] = \gamma_i^{\text{quark}} \equiv \mu \frac{\partial}{\partial \mu} Q_i(\mu). \tag{26}$$

Analogously, substituting Eqs. (20)–(23) into Eq. (13), keeping the leading $1/N_c$ terms and adopting the above first matching condition, we arrive at the second matching condition

$$Q_6^\chi[\mu_\chi, M(\mu)] = -\frac{11}{2}(Q_2^\chi - Q_1^\chi)[M(\mu)], \quad \mu < \Lambda_\chi. \tag{27}$$

Note that such a matching condition holds for the whole energy scale $\mu < \Lambda_\chi$. At the special point $M(\mu) = 0$, it covers the condition first presented in Ref. [20]. In fact, the above two matching conditions may simply be seen by comparing Eqs. (20) and (21) with Eqs. (12) and (13). On the other hand, from the chiral representation of operators and their chiral loop corrections, we have the following chiral relation in the leading $1/N_c$ approximation and chiral limit:

$$Q_6^\chi[\mu_\chi, M(\mu)] \simeq \left(-\frac{R_\chi^2 \chi_5^r}{\Lambda_\chi^2}\right)(Q_2^\chi - Q_1^\chi)[M(\mu)], \quad \mu < \Lambda_\chi, \tag{28}$$

$$R_\chi \equiv R(\mu \simeq \mu_\chi) \simeq m_\pi^2/\hat{m}(\mu_\chi) \simeq 2m_K^2/(m_s + \hat{m})(\mu_\chi), \tag{29}$$

where we have simply replaced the coupling constants χ_5 and r by the corresponding renormalized ones χ_5^r and $R(\mu)$ as their loop corrections are at the subleading order. When combining the second matching condition, it allows us to fix the strange quark mass

$$\frac{R_\chi^2 \chi_5^r}{\Lambda_\chi^2} = \frac{11}{2} \to m_s(\mu_\chi) \simeq 196 \text{ MeV}. \tag{30}$$

Here we have used the result $\Lambda_\chi = 1.03\sqrt{\chi_5^r}$ GeV which is fixed from the ratio of the kaon and pion decay constants.

The first matching condition can be rewritten as follows after integration:

$$\frac{2M^2(\mu)}{\Lambda_F^2} = \frac{2M_0^2}{\Lambda_F^2} + \frac{3\alpha_s}{4\pi}\ln\left(\frac{\mu^2}{\mu_0^2}\right), \tag{31}$$

where μ_0 and $M_0 \equiv M(\mu = \mu_0)$ are two integral constants. It is seen that the μ dependence of the functional cutoff momentum $M(\mu)$ is now logarithmic. Noticing the approximation $\ln(\mu^2/\mu_0^2) \simeq \mu^2/\mu_0^2 - 1$ when $\mu^2 \sim \mu_0^2$, namely, the functional cutoff momentum $M(\mu)$ is approximately proportional to μ when μ runs down and approaches to the low-energy scale μ_0 which is expected to be slightly above the QCD scale $\Lambda_{\rm QCD}$, we then have $M_0^2 = [3\alpha_s(\mu_0)/8\pi]\Lambda_F^2$. Thus the μ dependence of the functional cutoff momentum $M(\mu)$ can be written as

$$\frac{2M^2(\mu)}{\Lambda_F^2} \simeq \frac{3\alpha_s}{4\pi} + \frac{3\alpha_s}{4\pi}\ln\left(\frac{\mu^2}{\mu_0^2}\right), \tag{32}$$

which shows that after imposing the matching condition for the anomalous dimensions between quark operators $Q_i(\mu)$ in QCD and the corresponding chiral operators $Q_i^\chi[M(\mu)]$ in the ChPT with functional cutoff momentum, the dimensionless ratio M^2/Λ_F^2 is only related to the strong coupling constant α_s and becomes scheme independent, which implies that the long-distance operator evolution in ChPT with functional cutoff momentum can be carried out by using any approach. For instance, with and without separating factorized and nonfactorized contributions, we should obtain the same results after appropriately considering the matching between QCD and ChPT with functional cutoff momentum.

In general, we have $\mu_0 > \Lambda_{\rm QCD}$. To fix the value of μ_0, we use $M_0 \simeq \mu_0$. Thus μ_0 [or $\alpha_s(\mu_0)$] is determined via

$$\mu_0 \simeq \Lambda_F\sqrt{3\alpha_s(\mu_0)/8\pi}. \tag{33}$$

Using the definition $\alpha_s(\mu) = 6\pi/[(33-2n_f)\ln(\mu^2/\Lambda_{\rm QCD}^2)]$ with $n_f = 3$, the initial low-energy scale μ_0 is found, for $\Lambda_{QCD} = 325 \pm 80$ MeV, to be

$$\mu_0 \simeq 435 \pm 70 \text{ MeV or } \alpha_s(\mu_0)/2\pi \simeq 0.19^{+0.06}_{-0.05}. \tag{34}$$

With such an initial value of μ_0, the functional cutoff momentum $M(\mu)$ at $\mu = \Lambda_\chi$ yields the following corresponding value:

$$M_\chi \equiv M(\mu = \Lambda_\chi \simeq 1 \text{ GeV}) \simeq 0.71^{+0.11}_{-0.12} \text{ GeV}, \tag{35}$$

which provides the possible allowed range of the energy scale where the ChPT with functional cutoff momentum can be used to describe the low energy behavior of QCD at large N_c limit.

V. EVOLUTION OF LONG-DISTANCE CHIRAL OPERATORS

From the above analyses, the $\Delta S = 1$ low-energy ($\mu < \Lambda_\chi$) effective Hamiltonian for calculating $K \to \pi\pi$ decay amplitudes may be written as

$$\mathcal{H}_{\rm eff}^{\Delta S=1} = \frac{G_F}{\sqrt{2}}\lambda_u \sum_{i=1,2,4,6,8} c_i(\mu) Q_i^\chi[M(\mu)] \quad (\mu < \Lambda_\chi). \tag{36}$$

We may now adopt the matching conditions and algebraic chiral operator relations to investigate the evolution of the chiral operators $Q_i^\chi[M(\Lambda_\chi)]$. The first matching condition enables us to sum over all the leading terms via renormalization group equation down to the energy scale μ_0, and the second matching condition together with the algebraic chiral operator relations allows us to evaluate the penguin operators $Q_4^\chi(M)$ and $Q_6^\chi(M)$ from the operators $Q_1^\chi(M)$ and $Q_2^\chi(M)$. So that the operators $Q_1^\chi(M)$ and $Q_2^\chi(M)$ form a complete set for the operator evolution below the energy scale $\mu \simeq \Lambda_\chi \simeq 1$ GeV, or correspondingly, below the functional cutoff momentum $M(\mu \simeq \Lambda_\chi) \simeq 0.71^{+0.11}_{-0.12}$ GeV for $\Lambda_{\rm QCD} = 325 \pm 80$ MeV. It is convenient to choose a new operator basis $Q_\pm^\chi[M(\mu)] = Q_2^\chi[M(\mu)] \pm Q_2^\chi[M(\mu)]$. The anomalous dimension matrix for the basis (Q_-^χ, Q_+^χ) is found to be

$$\gamma = \frac{\alpha_s}{2\pi}\begin{pmatrix} -9/2 & 0 \\ -3/2 & 3 \end{pmatrix}. \tag{37}$$

Following the standard procedure of the renormalization group evolution with the initial conditions for the Wilson coefficient functions $c_-(\Lambda_\chi) = c_2(\Lambda_\chi) - c_1(\Lambda_\chi)$ and $c_+(\Lambda_\chi) = c_2(\Lambda_\chi) + c_1(\Lambda_\chi)$, we find in the leading logarithmic approximation that

$$Q_-^\chi[M(\Lambda_\chi)] = \eta_\chi^{-1/2} Q_-^\chi(\mu_0), \tag{38}$$

$$Q_+^\chi[M(\Lambda_\chi)] = \eta_\chi^{1/3} Q_+^\chi(\mu_0) + \frac{1}{5}(\eta_\chi^{-1/2} - \eta_\chi^{1/3}) Q_-^\chi(\mu_0), \tag{39}$$

with $\eta_\chi = \alpha_s(\Lambda_\chi)/\alpha_s(\mu_0)$, and

$$Q_-^\chi(\mu_0) = Q_-^\chi(0) + \frac{9\alpha_s(\mu_0)}{8\pi} Q_-^\chi(0), \tag{40}$$

$$Q_+^\chi(\mu_0) = Q_+^\chi(0) - \frac{3\alpha_s(\mu_0)}{4\pi} Q_+^\chi(0) + \frac{3\alpha_s(\mu_0)}{8\pi} Q_-^\chi(0). \tag{41}$$

In the above analyses, we have taken the chiral limit $m_K^2, m_\pi^2 \ll \Lambda_F^2$. From the chiral one loop results, the finite meson mass contributions can be approximately included by modifying the above results into the following form:

$$Q_-^\chi[M(\Lambda_\chi)] = \eta_\chi^{-1/2} \eta_-(M_\chi) Q_-^\chi(\mu_0), \tag{42}$$

$$Q_+^\chi[M(\Lambda_\chi)] = \eta_\chi^{1/3}\eta_1(M_\chi)Q_+^\chi(\mu_0) + \frac{1}{5}(\eta_\chi^{-1/2} - \eta_\chi^{1/3})\eta_2(M_\chi)Q_-^\chi(\mu_0), \tag{43}$$

with

$$\eta_-(M_\chi) \simeq 1 + \frac{\frac{3}{4}m_K^2 - \frac{9}{2}m_\pi^2}{\Lambda_F^2}\ln\left(1 + \frac{M^2(\mu)}{\tilde{m}^2}\right),$$

$$\eta_1(M_\chi) \simeq 1 + \frac{\frac{1}{4}m_K^2 + 3m_\pi^2}{\Lambda_F^2}\ln\left(1 + \frac{M^2(\mu)}{\tilde{m}^2}\right),$$

$$\eta_2(M_\chi) \simeq 1 + \frac{m_K^2 - \frac{3}{2}m_\pi^2}{M_\chi^2}\ln\left(1 + \frac{M^2(\mu)}{\tilde{m}^2}\right). \tag{44}$$

Numerically, we use $\tilde{m} \simeq 300$ MeV, $m_K = 0.495$ GeV, and $m_\pi = 0.137$ GeV. When the QCD scale takes the value $\Lambda_{\mathrm{QCD}} = 325 \pm 80$ MeV with the corresponding low energy cutoff momentum $\mu_0 \simeq 435 \pm 70$ MeV, we have

$$Q_-^\chi[M(\Lambda_\chi)] = (3.17^{+0.66}_{-0.43})Q_-^\chi(0) = Q_4^\chi[M(\Lambda_\chi)], \tag{45}$$

$$Q_+^\chi[M(\Lambda_\chi)] = (0.55^{-0.09}_{+0.06})Q_+^\chi(0) + (0.8^{+0.11}_{-0.05})Q_-^\chi(0), \tag{46}$$

$$Q_6^\chi[\mu_\chi, M(\Lambda_\chi)] = -\frac{11}{2}Q_-^\chi[M(\Lambda_\chi)] = -(17.44^{+3.62}_{-2.37})Q_-^\chi(0), \tag{47}$$

$$Q_8^\chi[\mu_\chi, M(\Lambda_\chi)] = \frac{33}{8}\frac{\Lambda_\chi^2}{\chi_5^r(m_K^2 - m_\pi^2)}(Q_+^\chi + Q_-^\chi)(0) = 19.18(Q_+^\chi + Q_-^\chi)(0), \tag{48}$$

which shows that the isospin $I=2$ amplitude A_2 is suppressed by a factor of about 2 as it only receives contributions from the operator $Q_+^\chi[M(\Lambda_\chi)]$, while the isospin $I=0$ amplitude A_0 is enhanced by a large factor as it mainly gets contributions from the operator $Q_-^\chi[M(\Lambda_\chi)]$. On the other hand, the direct CP-violating parameter ε'/ε is expected to be large since the significant enhancement of $Q_6^\chi[\mu_\chi, M(\Lambda_\chi)]$ relative to $Q_8^\chi[\mu_\chi, M(\Lambda_\chi)]$ is seen to result from the algebraic chiral operator relation and matching condition. We are going to present our numerical predictions for the isospin amplitudes and the direct CP-violating parameter ε'/ε in the next section.

VI. PREDICTIONS FOR ε'/ε AND THE $\Delta I = 1/2$ RULE

We are now in the position to calculate the $K \to \pi\pi$ decay amplitudes A_I with isospin I

$$A_I \cos\delta_I = \langle \pi\pi | \mathcal{H}_{\mathrm{eff}}^{\Delta S=1} | K \rangle \equiv \frac{G_F}{\sqrt{2}}\lambda_u \sum_{i=1,2,4,6,8} c_i(\Lambda_\chi)\mathrm{Re}\langle Q_i^\chi[M(\Lambda_\chi)]\rangle_I. \tag{49}$$

The CP-conserving amplitudes are given by

$$\begin{aligned}\mathrm{Re}\,A_0\cos\delta_0 &= \frac{G_F}{\sqrt{2}}\mathrm{Re}\,\lambda_u \sum_{i=1,2,4,6,8} z_i(\Lambda_\chi)\mathrm{Re}\langle Q_i^\chi[M(\Lambda_\chi)]\rangle_0 \\ &\simeq \frac{G_F}{\sqrt{2}}\mathrm{Re}\,\lambda_u\Big[\frac{1}{2}z_-(\Lambda_\chi)\mathrm{Re}\langle Q_-^\chi[M(\Lambda_\chi)]\rangle_0 \\ &\quad + \frac{1}{2}z_+(\Lambda_\chi)\mathrm{Re}\langle Q_+^\chi[M(\Lambda_\chi)]\rangle_0 \\ &\quad + z_4(\Lambda_\chi)\mathrm{Re}\langle Q_4^\chi[M(\Lambda_\chi)]\rangle_0 \\ &\quad + z_6(\Lambda_\chi)\mathrm{Re}\langle Q_6^\chi[M(\Lambda_\chi)]\rangle_0\Big],\end{aligned} \tag{50}$$

$$\begin{aligned}\mathrm{Re}\,A_2\cos\delta_2 &= \frac{G_F}{\sqrt{2}}\mathrm{Re}\,\lambda_u \sum_{i=1,2,8} z_i(\Lambda_\chi)\mathrm{Re}\langle Q_i^\chi[M(\Lambda_\chi)]\rangle_2 \\ &\simeq \frac{G_F}{\sqrt{2}}\mathrm{Re}\,\lambda_u\Big[\frac{1}{2}z_-(\Lambda_\chi)\mathrm{Re}\langle Q_-^\chi[M(\Lambda_\chi)]\rangle_2 \\ &\quad + \frac{1}{2}z_+(\Lambda_\chi)\mathrm{Re}\langle Q_+^\chi[M(\Lambda_\chi)]\rangle_2\Big],\end{aligned} \tag{51}$$

and the CP-violating amplitudes are dominated by $\langle Q_6^\chi[M(\Lambda_\chi)]\rangle_0$ and $\langle Q_8^\chi[M(\Lambda_\chi)]\rangle_2$

$$\begin{aligned}\mathrm{Im}\,A_0\cos\delta_0 &= -\frac{G_F}{\sqrt{2}}\mathrm{Im}\,\lambda_t \sum_{i=1,2,4,6,8} y_i(\Lambda_\chi)\mathrm{Re}\langle Q_i^\chi[M(\Lambda_\chi)]\rangle_0 \\ &\simeq -\frac{G_F}{\sqrt{2}}\mathrm{Im}\,\lambda_t[y_6(\Lambda_\chi)\mathrm{Re}\langle Q_6^\chi[M(\Lambda_\chi)]\rangle_0],\end{aligned} \tag{52}$$

$$\begin{aligned}\mathrm{Im}\,A_2\cos\delta_2 &= -\frac{G_F}{\sqrt{2}}\mathrm{Im}\,\lambda_t \sum_{i=1,2,8} y_i(\Lambda_\chi)\mathrm{Re}\langle Q_i^\chi[M(\Lambda_\chi)]\rangle_2 \\ &\simeq \frac{G_F}{\sqrt{2}}\mathrm{Im}\,\lambda_t[y_8(\Lambda_\chi)\mathrm{Re}\langle Q_8^\chi[M(\Lambda_\chi)]\rangle_2].\end{aligned} \tag{53}$$

From the definition of direct CP-violating parameter ε'/ε

$$\frac{\varepsilon'}{\varepsilon} = \frac{\omega}{\sqrt{2}|\varepsilon|}\left(\frac{\mathrm{Im}\,A_2}{\mathrm{Re}\,A_2} - \frac{\mathrm{Im}\,A_0}{\mathrm{Re}\,A_0}\right), \tag{54}$$

with $\omega = \mathrm{Re}\,A_2/\mathrm{Re}\,A_0 = 1/22.2$, we arrive at the following general expression:

$$\frac{\varepsilon'}{\varepsilon} = \frac{G_F}{2}\frac{\omega}{|\varepsilon|\mathrm{Re}\,A_0}\mathrm{Im}\,\lambda_t(h_0 - h_2/\omega). \tag{55}$$

Here h_0 and h_2 are given by the isospin $I=0$ and $I=2$ hadronic matrix elements of relevant operators

$$h_0 = (\cos\delta_0)^{-1}\sum_{i=1,2,4,6,8} y_i(\Lambda_\chi)\mathrm{Re}\langle Q_i^\chi[M(\Lambda_\chi)]\rangle_0(1-\Omega_{IB})$$
$$\simeq (\cos\delta_0)^{-1} y_6(\Lambda_\chi)\mathrm{Re}\langle Q_6^\chi[M(\Lambda_\chi)]\rangle_0(1-\Omega_{IB}), \tag{56}$$

$$h_2 = (\cos\delta_2)^{-1}\sum_{i=1,2,8} y_i(\Lambda_\chi)\mathrm{Re}\langle Q_i^\chi[M(\Lambda_\chi)]\rangle_2$$
$$\simeq (\cos\delta_2)^{-1} y_8(\Lambda_\chi)\mathrm{Re}\langle Q_8^\chi[M(\Lambda_\chi)]\rangle_2, \tag{57}$$

where we have taken into account the possible isospin breaking effect Ω_{IB}, its previously estimated value was $\Omega_{IB} \simeq 0.25 \pm 0.1$ [26]. The most recent refined calculation in Ref. [27] gives a smaller value $\Omega_{IB} \simeq 0.16 \pm 0.03$ but with a large error [28]. In our present numerical calculations, we use $\Omega_{IB} \simeq 0.16$. The Cabibbo-Kobayashi-Maskawa (CKM) factors $\mathrm{Re}\,\lambda_u$ and $\mathrm{Im}\,\lambda_t$ are given in the Wolfenstein parametrization [29] as follows:

$$\mathrm{Re}\,\lambda_u = \mathrm{Re}(V_{us}^* V_{ud}) = \lambda, \quad \mathrm{Im}\,\lambda_t = \mathrm{Im}(V_{ts}^* V_{ta}) = A^2\lambda^5\eta. \tag{58}$$

To evaluate the numerical results, we are going to take the following reliable values for all relevant parameters. For the involved energy scales, we have $\Lambda_{\mathrm{QCD}} = 325 \pm 80$ MeV, $\mu_0 = 435 \pm 70$ MeV, $\Lambda_\chi = 1.0$ GeV, and $\Lambda_F = 1.16$ GeV. For the Wilson coefficient functions, we only use the leading order results at one-loop level for a consistent analysis since the chiral operators have only been evaluated up to the leading order at the chiral one-loop level, namely, at order $1/N_c \sim M^2/\Lambda_F^2 \sim \alpha_s$ in the large N_c approach. Their values can be read following Refs. [13,14]. The numerical values at $\mu = \Lambda_\chi$ are regarded as the ''initial conditions'' for the chiral operator evolution and read for $\Lambda_{\mathrm{QCD}} = 325 \pm 80$ MeV: $z_-(\Lambda_\chi) = (z_2 - z_1)(\Lambda_\chi) = 2.181^{+0.197}_{-0.177}$, $z_+(\Lambda_\chi) = (z_2 + z_1)(\Lambda_\chi) = 0.685 \mp 0.029$, $z_4(\Lambda_\chi) = -(0.012 \pm 0.003)$, and $z_6(\Lambda_\chi) = -(0.013 \pm 0.003)$, as well as $y_6(\Lambda_\chi) = -(0.113^{+0.024}_{-0.021})$ and $y_8(\Lambda_\chi)/\alpha = 0.158^{+0.040}_{-0.033}$. The hadronic matrix elements of chiral operators at cutoff momentum $M=0$ take their values at the tree-level: $\langle Q_-^\chi(0)\rangle_0 = 36.9 \times 10^6\ \mathrm{MeV}^3$, $\langle Q_+^\chi(0)\rangle_0 = 12.3 \times 10^6\ \mathrm{MeV}^3$, $\langle Q_+^\chi(0)\rangle_2 = 34.8 \times 10^6\ \mathrm{MeV}^3$, and $\langle Q_8^\chi(\Lambda_\chi, 0)\rangle_2 = 328.8 \times 10^6\ \mathrm{MeV}^3$. For the CKM matrix elements, there remain big uncertainties arising from the single CP-violating phase, two matrix elements V_{ub} and V_{cb}, or the corresponding Wolfenstein parameters η, ρ, and A. For a numerical estimate, we take $\mathrm{Re}\,\lambda_u = 0.22$ and $\mathrm{Im}\,\lambda_t = 1.2 \times 10^{-4}$ as the central values. With these input values, we obtain, in the leading $1/N_c$ approximation, the isospin amplitudes

$$\mathrm{Re}\,A_0 = (2.56^{+0.78}_{-0.37}) \times 10^{-4}(\cos\delta_0)^{-1}\ \mathrm{MeV}$$
$$= (3.10^{+0.94}_{-0.61}) \times 10^{-4}\ \mathrm{MeV}, \tag{59}$$

$$\mathrm{Re}\,A_2 = (0.12 \mp 0.02) \times 10^{-4}(\cos\delta_2)^{-1}\ \mathrm{MeV}$$
$$= (0.12 \mp 0.02) \times 10^{-4}\ \mathrm{MeV}, \tag{60}$$

which agree well with the experimental data: $\mathrm{Re}\,A_0 = 3.33 \times 10^{-4}$ MeV and $\mathrm{Re}\,A_2 = 0.15 \times 10^{-4}$ MeV. Here we have used the final state interaction phases, $\delta_0 = (34.2 \pm 2.2)°$ and $\delta_2 = (-6.9 \pm 0.2)°$ [30]. Consistently, the direct CP-violating parameter ε'/ε is found, in the leading $1/N_c$ approximation with $\Lambda_{\mathrm{QCD}} = 325 \pm 80$ MeV, to be

$$\frac{\varepsilon'}{\varepsilon} = (23.6^{+12.4}_{-7.8}) \times 10^{-4} \tag{61}$$

for the central value of $\mathrm{Im}\,\lambda_t = 1.2 \times 10^{-4}$ resulted from fitting the indirect CP-violating parameter ε, $|V_{ub}|$, B^0-$\overline{B}^0$, and B_s^0-$\overline{B}_s^0$ mixings. It is of interest to note that this central value also agrees with the one predicted from ten useful relations among the quark masses and mixing angles obtained in the supersymmetric (SUSY) grand unified theory (GUT) model [31]. When considering the possible allowed range for the CKM matrix elements extracted from fitting the present experimental data, we have

$$(13.8^{+7.2}_{-4.5}) \times 10^{-4} \leqslant \frac{\varepsilon'}{\varepsilon} = (23.6^{+12.4}_{-7.8}) \times 10^{-4}\left(\frac{\mathrm{Im}\,\lambda_t}{1.2 \times 10^{-4}}\right)$$
$$\leqslant (33.6^{+17.6}_{-11.1}) \times 10^{-4} \tag{62}$$

for the possible allowed range $0.7 \times 10^{-4} < \mathrm{Im}\,\lambda_t < 1.7 \times 10^{-4}$. It is noticed that the present new predictions for the isospin amplitudes and direct CP-violating parameter ε'/ε further confirm our early conclusions [9,10]. Our new predictions are consistent with the world average [8]

$$\mathrm{Re}(\varepsilon'/\varepsilon) = (19.2 \pm 2.4) \times 10^{-4} \quad \text{(world average 2000)} \tag{63}$$

which was obtained by taking into account the results from four collaboration groups. They contain two published results reported earlier by the NA31 Collaboration and E731 Collaboration:

$$\mathrm{Re}(\varepsilon'/\varepsilon) = (23 \pm 7) \times 10^{-4} \quad \text{(1993 NA31) [4]},$$
$$\mathrm{Re}(\varepsilon'/\varepsilon) = (7.4 \pm 5.9) \times 10^{-4} \quad \text{(1993 E731) [5]} \tag{64}$$

and the recent new results reported by the KTeV Collaboration at Fermilab and the NA48 Collaboration at CERN:

$$\mathrm{Re}(\varepsilon'/\varepsilon) = (28.0 \pm 3.0 \pm 2.8) \times 10^{-4} \quad \text{(1999 KTeV) [6]},$$
$$\mathrm{Re}(\varepsilon'/\varepsilon) = (18.5 \pm 4.5 \pm 5.8) \times 10^{-4} \quad \text{(1999 NA48) [7]},$$
$$\mathrm{Re}(\varepsilon'/\varepsilon) = (14.4 \pm 4.3) \times 10^{-4} \quad \text{(2000 NA48) [8]}. \tag{65}$$

Before drawing our conclusions, we would like to address the following points.

(1) The main uncertainties for the predictions arise from the QCD scale $\Lambda_{\rm QCD}$ (or the low-energy scale μ_0) and the combined CKM factor $\mathrm{Im}\,\lambda_t$. Nevertheless, the uncertainties from the energy scale may be reduced from comparing the predicted isospin amplitudes A_0 and A_2 with the well measured ones. As a consequence, it is seen from Eqs. (59) and (60) that the results corresponding to the large values of $\Lambda_{\rm QCD} > 325$ MeV appear not so favorable.

(2) From the above point of view, it is seen that from the isospin amplitude A_2, the ratio ε'/ε favors the low values

$$\frac{\varepsilon'}{\varepsilon} = (16 \pm 7) \times 10^{-4} \tag{66}$$

while from the isospin amplitude A_0, it favors the high values

$$\frac{\varepsilon'}{\varepsilon} = (24 \pm 10) \times 10^{-4}. \tag{67}$$

From the ratio of the two amplitudes $\mathrm{Re}\,A_0/\mathrm{Re}\,A_2$, i.e., the $\Delta I = 1/2$ rule, the ratio ε'/ε favors the middle values

$$\frac{\varepsilon'}{\varepsilon} = (20 \pm 9) \times 10^{-4} \tag{68}$$

which is consistent with the most recent results reported by the NA48 Collaboration at CERN [8,7] and the KTeV Collaboration at Fermilab [6]. In fact, it is very close to the average value from NA48 and KTeV. While the central values from two experimental groups differ from each other at 3.5-σ level.

(3) The above results are renormalization scheme independent as the consistent matching between QCD and ChPT considered in the present paper is at the leading one-loop order of $1/N_c \sim \alpha_s \sim 1/\Lambda_F^2$ around the scale Λ_χ. The renormalization scheme dependence arises from the next-to-leading order of perturbative QCD, which could become substantial for some of the Wilson coefficient functions when the renormalization scale μ runs down to around the scale $\Lambda_\chi = 1$ GeV. In our present approach, the scheme for the long-distance evolution is fixed by the ChPT with functional cutoff momentum. For matching to this scheme, it is useful to introduce a scheme independent basis for the perturbative QCD calculation of short-distance physics. Then applying our above procedure to find out the matching conditions at the next-to-leading order $1/N_c^2 \sim \alpha_s^2 \sim 1/\Lambda_F^4$. To work out the scheme independent basis, it may be helpful to adopt the method discussed in Ref. [32] and use the cutoff momentum basis. The study of scheme independent basis in perturbative QCD is beyond the purposes of the present paper. Some effort is being made [33] though it is not yet fully understood.

VII. CONCLUSIONS

We have simultaneously analyzed the direct CP-violating parameter ε'/ε and $\Delta I = 1/2$ rule in kaon decays by considering a consistent matching scheme between QCD and ChPT. Our main points may be summarized as follows. (i) Starting from the chiral Lagrangian obtained in terms of the momentum and quark mass expansion with low-energy coupling constants given by the leading terms of the $1/N_c$ expansion. The N_c behavior in the concerned chiral Lagrangian has explicitly been characterized by the scale factor $\Lambda_f^2 \sim N_c$. (ii) The four quark operators for weak kaon decays at low energies have been assumed to be represented by the chiral operators in the large N_c limit. It has also been shown that there is simplification in ChPT which leads to useful algebraic chiral operator relations. Those relations survive even when including loop corrections. (iii) We have adopted the usual cutoff regularization scheme [19] for all the diagrams with a single cutoff momentum for a systematical analysis, and given up the scheme of separating factorized and nonfactorized parts with two cutoff scales. (iv) The cutoff momentum M has been considered to be the function of the QCD running scale μ, i.e., $M \equiv M(\mu)$, instead of naively identifying it to the perturbative QCD running scale μ. The form of the functional cutoff momentum $M(\mu)$ has been determined via the matching requirement, so that the chiral loop results become scheme independent. As a consequence, two useful matching conditions have been obtained, which has allowed us to evaluate the long-distance chiral operators and sum over the leading nonperturbative contributions. In particular, the $\Delta I = 1/2$ rule can consistently be understood and the resulting direct CP-violating parameter ε'/ε become large enough to be measured, which also confirms our early conclusions [9,10]. Taking into account the simultaneous consistent analysis for the isospin amplitudes A_0 and A_2, the numerical result for the ratio ε'/ε is in favor of the values

$$\frac{\varepsilon'}{\varepsilon} = (20 \pm 9) \times 10^{-4} \tag{69}$$

which may be regarded as the favorable prediction in our present analyses. The prediction is also consistent, within the theoretical and experimental uncertainties, with the present data [8,4–7]. Finally, we would like to remark that we have neglected in our present analyses the subleading contributions, their effects are in general small and will be investigated elsewhere in detail.

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