

QCD factorization for $B \rightarrow PP$

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In this work, we give a detailed discussion for QCD factorization involved in the complete chirally enhanced power corrections in the heavy quark limit for B decays to two light pseudoscalar mesons, and present some detailed calculations of radiative corrections at the order of α_s . We point out that the infrared finiteness of the vertex corrections in the chirally enhanced power corrections requires twist-3 light-cone distribution amplitudes (LCDAs) of the light pseudoscalar symmetric. However, even in the symmetric condition, there is also a logarithmic divergence from the end points of the twist-3 LCDAs in the hard spectator scattering. We point out that the decay amplitudes of $B \rightarrow PP$ predicted by QCD factorization are really free of the renormalization scale dependence, at least at the order of α_s . At last, we briefly compare the QCD factorization with the generalized factorization and PQCD method.

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I. INTRODUCTION

The study of B decays plays an important role in understanding the origin of CP violation and the physics of heavy flavor. We expect that the parameters of the Cabibbo-Kobayashi-Maskawa (CKM) matrix in the standard model, for instance, the three angles α , β , and γ in the unitary triangle, can be well determined from B decays, especially from the charmless nonleptonic two-body B decays. Experimentally, many B experiment projects have been running (CLEO, BaBar, Belle, etc.), or will run in forthcoming years (BTeV, CERN LHCb, DESY HeraB, etc.). With the accumulation of the data, the theorists will be urged to gain a deeper sight into B decays, and to reduce the theoretical errors in determining the CKM parameters from the experimental data.

In the theoretical frame, the standard approach to deal with such decays is based on the low-energy effective Hamiltonian which is obtained by the Wilson operator product expansion method (OPE). In this effective Hamiltonian, the short-distance contributions from the scale above $\mu \simeq m_b$ have been absorbed into the Wilson coefficients with the perturbative theory and renormalization group method. The Wilson coefficients have been evaluated to next-to-leading order. Then the main task in studying nonleptonic two-body B decays is to calculate the hadronic matrix elements of the effective operators. However, we do not have a reliable approach to evaluate them from the first principles of QCD dynamics up to now.

Generally, we must resort to the factorization assumption to calculate the hadronic matrix elements for nonleptonic B

decays, in which the hadronic matrix element of the effective operator (in general, which is in the form of current-current four-quark operator) can be approximated as a product of two single current hadronic matrix elements; then it is parametrized into a meson decay constant and meson-meson transition form factor. The most popular factorization model is the Bauer-Stech-Wirbel (BSW) model [1]. In many cases, the BSW model achieves great success, which can predict the branching ratios of many modes of nonleptonic B decays in correct order of magnitude. This factorization assumption does hold in the limit that the soft interactions in the initial and final states can be ignored. It seems that the argument of color transparency can give reasonable support to the above limit. Because the b quark is heavy, the quarks from b quark decay move so fast that a pair of quarks in a small color-singlet object decouple from the soft interactions. But the shortcomings of this simple model are obvious. First, the renormalization scheme and scale dependence in the hadronic matrix elements of the effective operators are apparently missed. Then the full decay amplitude predicted by the BSW model remains dependent on the renormalization scheme and scale, which are mainly from Wilson coefficients. In past years, many researchers improved the simple factorization scheme and made many remarkable progresses, such as scheme and scale independent effective Wilson coefficients [2,3], effective color number which is introduced to compensate the ‘‘nonfactorizable’’ contributions, etc. Furthermore, some progresses in nonperturbative methods, such as lattice QCD, QCD sum rule, etc. [4–6], allow us to compute many nonperturbative parameters in B decays, such as the meson decay constants and meson-meson transition form factors. Every improvement allows us to have a closer look at the B nonleptonic decays.

Except for the factorization approximation, another important approach has been applied to study many B exclusive hadronic decay channels, such as $B \rightarrow D\pi$, $\pi\pi$, πK , etc. This is the PQCD method [7–9]. In this method, people as-

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sume that B exclusive hadronic decay is dominated by hard gluon exchange. It is analogous to the framework of perturbative factorization for exclusive processes in QCD at large momentum transfer, such as the calculation of the electromagnetic form factor of the pion [10]. The decay amplitude for B decay can be written as a convolution of a hard-scattering kernel with light-cone wave functions of the participating mesons. Furthermore, in Refs. [8,9] the Sudakov suppression has been taken into account.

Two years ago, Beneke, Buchalla, Neubert, and Sachrajda (BBNS) gave a QCD factorization formula in the heavy quark limit for the decays $B \rightarrow \pi\pi$ [11]. They pointed out that the radiative corrections from hard gluon exchange can be calculated by use of the perturbative QCD method if one neglects the power contributions of Λ_{QCD}/m_b . This factorization formula can be justified in the case that the ejected meson from the b quark decay is a light meson or an onium, no matter whether the other recoiling meson which absorbs the spectator quark in B meson is light or heavy. But for the case that the ejected meson is in an extremely asymmetric configuration, such as D meson, this factorization formula does not hold. The contribution from the hard scattering with the spectator quark in B meson is also involved in their formula. This kind of contribution cannot be contained in the naive factorization. But it appears in the order of α_s . So they said that the naive factorization can be recovered if one neglects the radiative corrections and power Λ_{QCD}/m_b suppressed contributions in the QCD factorization, and the ‘‘nonfactorizable’’ contributions in the naive factorization can be calculated perturbatively, then we do not need a phenomenological parameter N_c^{eff} to compensate the ‘‘nonfactorizable’’ effects any more [12–14].

This QCD factorization (BBNS approach) has been applied to study many B meson decay modes, such as $B \rightarrow D^{(*)}\pi^-$ [15,16], $\pi\pi$, πK [17–19] and other interesting channels [20–23]. Some theoretical generalizations of the BBNS approach have also been made, such as the chirally enhanced power corrections [18,19,24,25] from the twist-3 light-cone distribution amplitudes of the light pseudoscalar mesons. In this work, we will take a closer look at this issue. This work is organized as follows: Sec. II is devoted to a sketch of the low energy effective Hamiltonian; in Sec. III, we will give a detailed overview of QCD factorization, in which some elaborate calculations are shown, especially for the chirally enhanced power corrections; Sec. IV is for some detailed discussions and comparison of the BBNS approach to the generalized factorization and the PQCD method; we conclude in Sec. V with a summary.

II. EFFECTIVE HAMILTONIAN—FIRST STEP FACTORIZATION

B decays involve three characteristic scales which are strongly ordered: $m_W \gg m_b \gg \Lambda_{QCD}$. How to separate or factorize these three scales is the most essential question in B hadronic decays.

With the operator product expansion method (OPE), the relevant $|\Delta B|=1$ effective Hamiltonian is given by [26]

$$\begin{aligned} \mathcal{H}_{eff} = & \frac{G_F}{\sqrt{2}} \left[\sum_{q=u,c} v_q \left(C_1(\mu) Q_1^q(\mu) + C_2(\mu) Q_2^q(\mu) \right. \right. \\ & \left. \left. + \sum_{k=3}^{10} C_k(\mu) Q_k(\mu) \right) - v_t (C_{7\gamma}(\mu) Q_{7\gamma}(\mu) \right. \\ & \left. \left. + C_{8G}(\mu) Q_{8G}(\mu) \right) \right] + \text{H.c.}, \end{aligned} \quad (1)$$

where $v_q = V_{qb} V_{qd}^*$ (for $b \rightarrow d$ transition) or $v_q = V_{qb} V_{qs}^*$ (for $b \rightarrow s$ transition) and $C_i(\mu)$ are the Wilson coefficients which have been evaluated to next-to-leading order approximation with the perturbative theory and renormalization group method.

In Eq. (1), the four-quark operators Q_i are given by

$$\begin{aligned} Q_1^u &= (\bar{u}_\alpha b_\beta)_{V-A} (\bar{q}_\beta u_\alpha)_{V-A}, & Q_1^c &= (\bar{c}_\alpha b_\beta)_{V-A} (\bar{q}_\beta c_\alpha)_{V-A}, \\ Q_2^u &= (\bar{u}_\alpha b_\beta)_{V-A} (\bar{q}_\beta u_\alpha)_{V-A}, & Q_2^c &= (\bar{c}_\alpha b_\beta)_{V-A} (\bar{q}_\beta c_\alpha)_{V-A}, \\ Q_3 &= (\bar{q}_\alpha b_\alpha)_{V-A} \sum_{q'} (\bar{q}'_\beta q'_\beta)_{V-A}, \\ Q_4 &= (\bar{q}_\beta b_\alpha)_{V-A} \sum_{q'} (\bar{q}'_\alpha q'_\alpha)_{V-A}, \\ Q_5 &= (\bar{q}_\alpha b_\alpha)_{V-A} \sum_{q'} (\bar{q}'_\beta q'_\beta)_{V+A}, \\ Q_6 &= (\bar{q}_\beta b_\alpha)_{V-A} \sum_{q'} (\bar{q}'_\alpha q'_\alpha)_{V+A}, \\ Q_7 &= \frac{3}{2} (\bar{q}_\alpha b_\alpha)_{V-A} \sum_{q'} e_{q'} (\bar{q}'_\beta q'_\beta)_{V+A}, \\ Q_8 &= \frac{3}{2} (\bar{q}_\beta b_\alpha)_{V-A} \sum_{q'} e_{q'} (\bar{q}'_\alpha q'_\alpha)_{V+A}, \\ Q_9 &= \frac{3}{2} (\bar{q}_\alpha b_\alpha)_{V-A} \sum_{q'} e_{q'} (\bar{q}'_\beta q'_\beta)_{V-A}, \\ Q_{10} &= \frac{3}{2} (\bar{q}_\beta b_\alpha)_{V-A} \sum_{q'} e_{q'} (\bar{q}'_\alpha q'_\alpha)_{V-A}, \end{aligned} \quad (2)$$

and

$$Q_{7\gamma} = \frac{e}{8\pi^2} m_b \bar{q}_\alpha \sigma^{\mu\nu} (1 + \gamma_5) b_\alpha F_{\mu\nu},$$

$$Q_{8G} = \frac{g}{8\pi^2} m_b \bar{q}_\alpha \sigma^{\mu\nu} (1 + \gamma_5) t_{\alpha\beta}^a b_\beta G_{\mu\nu}^a \quad (q = d \text{ or } s) \quad (3)$$

with Q_1^q and Q_2^q being the tree operators, $Q_3 - Q_6$ the QCD penguin operators, $Q_7 - Q_{10}$ the electroweak penguin operators, and $Q_{7\gamma}$, Q_{8G} the magnetic-penguin operators.

In this effective Hamiltonian for B decays, the contributions from large virtual momenta of the loop corrections from scale $\mu = \mathcal{O}(m_b)$ to M_W are attributed to the Wilson coefficients, and the low energy contributions are fully incorporated into the matrix elements of the operators [26]. So the derivation of the effective Hamiltonian can be called “the first step factorization.”

To evaluate the Wilson coefficients, we must extract them at a large renormalization scale [for example $\mu = \mathcal{O}(M_W)$ in the standard model] by matching the amplitude of the effective Hamiltonian (A_{eff}) to that of the full theory (A_{full}), then evolve them by the renormalization group equations from the scale $\mu = \mathcal{O}(m_W)$ to the scale $\mu = \mathcal{O}(m_b)$. It should be noted that the extraction of the Wilson coefficients C_i by matching does not depend on the choice of the external states, if we regularize the infrared (and mass) singularities properly [26]. All dependence on the choice of external states only appears in the matrix elements $\langle Q_i \rangle$, and is not contained in C_i . So C_i only contains the short-distance contributions from the region where the perturbative theory can be applied. But for the matrix elements $\langle Q_i \rangle$, the long-distance contributions appear, and are process-dependent.

Several years ago, the perturbative corrections to the Wilson coefficients in SM have been evaluated to next-to-leading order with the renormalization group method [26]. As we know, the Wilson coefficients are generally renormalization scheme and scale dependent. So, in order to cancel such dependence, we must calculate the hadronic matrix elements of the effective operators to the corresponding perturbative order with the same renormalization scheme and at the same scale, then we can obtain a complete decay amplitude which is free from those unphysical dependences.

III. QCD FACTORIZATION FOR $B \rightarrow PP$

After “the first step factorization,” the decay amplitude for $B \rightarrow h_1 h_2$ can be written as

$$A(B \rightarrow h_1 h_2) = \sum_i v_i C_i(\mu) \langle h_1 h_2 | Q_i(\mu) | B \rangle, \quad (4)$$

in which, as mentioned in the previous section, the contributions from the large scale m_W down to m_b has been separated into the Wilson coefficients $C_i(\mu)$. The remaining task is to calculate the hadronic matrix elements of the effective operators. But for the complexity of QCD dynamics, it is difficult to calculate these matrix elements reliably from first principles. The most popular approximation is factorization hypothesis, in which the matrix element of the current-current operator is approximated to a product of two matrix elements of a single current operator:

$$\langle h_1 h_2 | Q_i | B \rangle \simeq \langle h_2 | J_2 | 0 \rangle \langle h_1 | J_1 | B \rangle. \quad (5)$$

Obviously, under this approximation, the original hadronic matrix element $\langle Q_i(\mu) \rangle$ misses the dependence of the renormalization scheme and scale which should be used to cancel the corresponding dependence in the Wilson coefficients $C_i(\mu)$. A plausible solution to recover this scale and scheme

dependence of $\langle Q_i \rangle$ is to calculate the radiative corrections. In the one-loop level, they can be written as [13,14,27]

$$\langle \mathbf{Q} \rangle = \left[\hat{\mathbf{1}} + \frac{\alpha_s}{4\pi} \hat{\mathbf{m}}_s + \frac{\alpha_{em}}{4\pi} \hat{\mathbf{m}}_e \right] \cdot \langle \mathbf{Q} \rangle_{\text{Tree}}. \quad (6)$$

Here $\hat{\mathbf{m}}_s$ and $\hat{\mathbf{m}}_e$ represent the one loop corrections of QCD and QED, respectively. Then one takes

$$\langle h_1 h_2 | Q_i | B \rangle_{\text{Tree}} \simeq \langle h_2 | J_2 | 0 \rangle \langle h_1 | J_1 | B \rangle. \quad (7)$$

Therefore, the scheme and scale dependence of $\langle Q_i \rangle$ which are expressed in the form of $\hat{\mathbf{m}}_s$ and $\hat{\mathbf{m}}_e$ is recovered. But in quark level, $\hat{\mathbf{m}}_s$ and $\hat{\mathbf{m}}_e$ usually contain infrared divergences if we take the external quarks on-shell [28]. To remove or regularize the infrared divergence, the conventional treatment is to assume that external quarks are off-shell by $-p^2$. But this introduction of the infrared cutoff $-p^2$ results in a gauge dependence of one-loop corrections. So how to factorize the infrared part of the matrix elements is a very subtle question. But maybe this question would get an important simplification in the case that the final states of B meson decay are two light mesons.

Two years ago, Beneke, Buchalla, Neubert, and Sachrajda proposed a promising QCD factorization method for $B \rightarrow \pi\pi$. They pointed out that in the heavy quark limit $m_b \gg \Lambda_{QCD}$, the hadronic matrix elements for $B \rightarrow \pi\pi$ can be written in the form

$$\langle \pi\pi | Q | B \rangle = \langle \pi | J_2 | 0 \rangle \langle \pi | J_1 | B \rangle \cdot \left[1 + \sum r_n \alpha_s^n + \mathcal{O}(\Lambda_{QCD}/m_b) \right]. \quad (8)$$

Obviously, the above formula reduces to the naive factorization if we neglect the power corrections in Λ_{QCD}/m_b and the radiative corrections in α_s . They find that the radiative corrections, which are dominated by hard gluon exchange, can be calculated systematically with the perturbative theory in the limit $m_b \rightarrow \infty$, in terms of the convolution of the hard scattering kernel and the light-cone distribution amplitudes of the mesons. This is also similar to the framework of perturbative factorization for exclusive processes in QCD at large momentum transfer, such as the calculation of the electromagnetic form factor of the pion [10]. Then a factorization formula for $B \rightarrow \pi\pi$ can be written as [11]

$$\begin{aligned} & \langle \pi(p') \pi(q) | Q_i | B(p) \rangle \\ &= F^{B \rightarrow \pi}(q^2) \int_0^1 dx T_i^l(x) \Phi_\pi(x) \\ &+ \int_0^1 d\xi dx dy T^H(\xi, x, y) \Phi_B(\xi) \Phi_\pi(x) \Phi_\pi(y). \end{aligned} \quad (9)$$

We call this factorization formalism QCD factorization or the BBNS approach. In the above formula, $\Phi_B(\xi)$ and

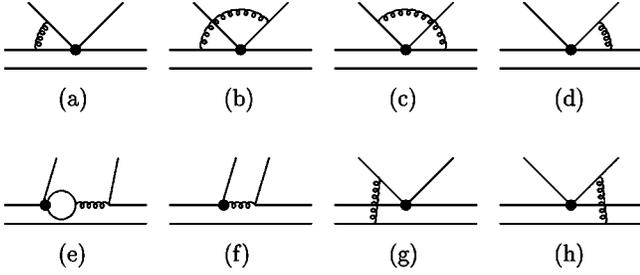


FIG. 1. Order of α_s corrections to hard-scattering kernels T^I and T^{II} . The upward quark lines represent the ejected quark pairs from b quark weak decays.

$\Phi_\pi(x)$ are the leading-twist wave functions of B and pion mesons, respectively, and the $T_i^{I,II}$ denote hard-scattering kernels which are calculable in perturbative theory. At the order of α_s , the hard kernels $T_i^{I,II}$ can be depicted by Fig. 1. Figures 1(a)–1(d) represent vertex corrections, Figs. 1(e) and 1(f) penguin corrections, and Figs. 1(g) and 1(h) hard spectator scattering.

In the heavy quark limit, both pions are energetic. The pion ejected from b quark decay moves so fast that it can be described as its leading-twist light-cone distribution amplitude. The $q\bar{q}$ pair in the ejected pion is produced as a small-size color dipole. Consequently, the ejected pion decouples from the soft gluons at leading order of Λ_{QCD}/m_b . Of course, only the cancellation of soft gluons is not enough to make the factorization hold; it is necessary that the $q\bar{q}$ pair also decouples from the collinear gluons. Both the cancellations of soft gluons and collinear gluons guarantee that the hard kernel T_i^I is of infrared finiteness. Contrast to the pion ejected from b quark weak decay, the recoiling pion which picks up the spectator in B meson cannot be described by its leading-twist light-cone distribution amplitude (LCDA), because the spectator is transferred to the recoiling pion as a soft quark. Here Beneke *et al.* take the point of view that the form factor $F^{B \rightarrow \pi}$ cannot be calculated perturbatively. If we attempt to calculate the form factor within the perturbative framework, by the naive power counting, we find that the leading twist LCDA of pion does not fall fast enough to suppress the singularity at the end point where the quark from b decay carries almost all momentum of the pion. It indicates that the contributions to form factor are dominated by the soft gluon exchange [16]. This point of view can be justified also from the calculation of the form factor $F^{B \rightarrow \pi}$ by using the light-cone sum rule (LCSR) [5,6], in which the dominated contribution to $F^{B \rightarrow \pi}$ comes from the region where the spectator quark is transferred as a soft quark to the pion. So the transition form factor survives in the factorization formula as a nonperturbative parameter. However, when

the spectator quark in B meson interacts with a hard gluon from the ejected pion, the recoiling pion can also be described by its light-cone distribution amplitude. This hard spectator scattering is missed in the naive factorization, but calculable in the perturbative QCD at leading power in Λ_{QCD}/m_b . So with this factorization formula, the remaining hard part of the hadronic matrix element $\langle Q_i \rangle$ from the scale about m_b has been factorized into the hard scattering kernel, and the long distance contributions are absorbed into the transition form factors and the light-cone wave functions of the participating mesons. Thus this is the ‘‘final factorization’’ for the two-body nonleptonic charmless B decays.

An explicitly technical demonstration of the above argument has been presented in one-loop level in Refs. [11,16]. For $B \rightarrow D\pi$, this QCD factorization has been proved to two-loop order [16]. In the literature, the ejected pion is represented by its leading twist light-cone distribution amplitude (LCDA). However, since the mass of b quark is not asymptotically large, in particular, some power corrections might be enhanced by certain factors, such as the scale of chiral symmetry breaking $\mu_\pi = m_\pi^2/(m_u + m_d) \sim 1.5$ GeV, and have significant effects in studying B two-body nonleptonic charmless decays. So, in this manner, the chirally enhanced power corrections must be taken into account. Accordingly, describing the ejected pion not by its leading twist LCDA is not enough; the two-particle twist-3 LCDAs must be taken into account. Below, we will show the elaborate results of QCD factorization in these two cases. For illustration, we take $\bar{B}_d^0 \rightarrow \pi^+ \pi^-$ as an example, but the result is easily generalized to the cases that the final states are the other light pseudoscalars.

A. Leading-twist distribution amplitude insertion

When inserting leading-twist LCDA of the light pseudo-scalar, in the heavy quark limit, the quark constituents of the ejected pion can be treated as a pair of collinear massless quark and antiquark with the momentum uq and $\bar{u}q$, respectively (q is the momentum of the ejected pion and we take q as a hard light-cone momentum in calculation, $\bar{u} = 1 - u$), because the contributions from the transverse momenta of the quarks in ejected pion are power suppressed [16].

1. Vertex corrections

Now we move on to the explicit one-loop calculation of the diagram Figs. 1(a)–1(d) for $B \rightarrow \pi\pi$. For illustration, we write down the one-gluon exchange contribution to the $\bar{B}_d^0 \rightarrow \pi^+ \pi^-$ matrix element of the operator $Q_2^u = (\bar{u}_\alpha b_\beta)_{V-A} (\bar{d}_\beta u_\alpha)_{V-A} = (\bar{d}_\alpha b_\alpha)_{V-A} (\bar{u}_\beta u_\beta)_{V-A}$

$$\begin{aligned} \langle Q_2^u \rangle_{(a)} = & -g_s^2 \frac{f_\pi}{4} \frac{C_F}{N} \int_0^1 du \phi(u) \int \frac{d^4 k}{(2\pi)^4} \frac{1}{k^2(uq-k)^2[(p-k)^2-m_b^2]} \\ & \times \langle \pi^+ | \bar{u}_i \gamma^\mu (1-\gamma_5) q \gamma_5 \gamma^\alpha (uq-k) \gamma_\mu (1-\gamma_5) (\not{p}-\not{k}+m_b) \gamma_\alpha b_i | \bar{B}_d^0 \rangle, \end{aligned} \quad (10)$$

$$\begin{aligned} \langle Q_2^u \rangle_{(b)} &= g_s^2 \frac{f_\pi}{4} \frac{C_F}{N} \int_0^1 du \phi(u) \int \frac{d^4 k}{(2\pi)^4} \frac{1}{k^2 (\bar{u}q - k)^2 [(p - k)^2 - m_b^2]} \\ &\quad \times \langle \pi^+ | \bar{u}_i \gamma^\mu (1 - \gamma_5) (\bar{u}q - \mathbf{k}) \gamma^\alpha \not{q} \gamma_5 \gamma_\mu (1 - \gamma_5) (\not{p} - \mathbf{k} + m_b) \gamma_\alpha b_i | \bar{B}_d^0 \rangle, \end{aligned} \quad (11)$$

$$\begin{aligned} \langle Q_2^u \rangle_{(c)} &= -g_s^2 \frac{f_\pi}{4} \frac{C_F}{N} \int_0^1 du \phi(u) \int \frac{d^4 k}{(2\pi)^4} \frac{1}{k^2 (uq + k)^2 (p - q - k)^2} \\ &\quad \times \langle \pi^+ | \bar{u}_i \gamma_\alpha (\not{p} - \not{q} - \mathbf{k}) \gamma^\mu (1 - \gamma_5) \not{q} \gamma_5 \gamma_\alpha (uq + \mathbf{k}) \gamma_\mu (1 - \gamma_5) b_i | \bar{B}_d^0 \rangle, \end{aligned} \quad (12)$$

$$\begin{aligned} \langle Q_2^u \rangle_{(d)} &= g_s^2 \frac{f_\pi}{4} \frac{C_F}{N} \int_0^1 du \phi(u) \int \frac{d^4 k}{(2\pi)^4} \frac{1}{k^2 (\bar{u}q + k)^2 (p - q - k)^2} \\ &\quad \times \langle \pi^+ | \bar{u}_i \gamma_\alpha (\not{p} - \not{q} - \mathbf{k}) \gamma^\mu (1 - \gamma_5) (\bar{u}q + \mathbf{k}) \gamma_\alpha \not{q} \gamma_5 \gamma_\mu (1 - \gamma_5) b_i | \bar{B}_d^0 \rangle. \end{aligned} \quad (13)$$

When we calculate the vertex corrections in the leading power of Λ_{QCD}/m_b , not only ultraviolet divergence emerges but infrared divergence does also. Infrared divergence arises from two regions where the virtuality of the loop k is soft or collinear to the momentum of the pions. In Ref. [16], the authors gave an explicit cancellation of soft and collinear divergence in vertex corrections for $B \rightarrow D\pi$ in eikonal approximation. Figures 1(a),1(b) and 1(c),1(d) cancel the soft divergence; 1(a),1(c) and 1(b),1(d) cancel the collinear divergence. For $B \rightarrow \pi\pi$, the cancellation is similar except that the collinear divergence also arises from the region where k is collinear to the momentum of the recoiling pion. So Figs. 1(c),1(d) cancel not only part of the soft divergence but also part of the collinear divergence. Below, we give an explicit calculation of the Feynman diagrams Figs. 1(a)–1(d) to show the cancellation of the infrared divergences. In order to regularize the infrared divergence, there are two choices for us. One is the dimensional regularization (DR) scheme, in which the infrared divergence can be regularized into the pole terms $1/(d-4)$. In contrast to the dimensional regularization of ultraviolet divergence, the infrared divergence arises when $d \leq 4$, instead of $d \geq 4$ in the case of the ultra-

violet divergence. So the dimension d in regularization for infrared divergence must be set to be greater than 4. This is a subtle point, but it will not cause any ambiguity in our calculation because the infrared part and ultraviolet part can be safely separated. The other method to regularize the infrared divergence is the well-known massive gluon (MG) scheme, in which the infrared divergence is handled by replacing $1/k^2$ by $1/(k^2 - m_g^2)$ in the gluon propagator. A similar scheme has been applied in earlier computation of the radiative corrections for $\mu^- \rightarrow e^- \bar{\nu}_e \nu_\mu$, in which the massless photon is replaced by a massive photon. In addition, in our latter calculation, there are also several schemes in treating γ_5 , the most popular two are the naive dimensional regularization (NDR) scheme and the 't Hooft–Veltman renormalization (HV) scheme. Both have been applied to calculate the Wilson coefficients [26]. In this work, if there is no specification, the NDR scheme is always applied in our calculations for its simplicity.¹

After a straightforward calculation in DR scheme and using the corresponding Feynman parameter integrals listed in Appendix C, we obtain

$$\begin{aligned} \langle Q_2^u \rangle_{(a)} &= \frac{\alpha_s}{4\pi} \frac{C_F}{N} \langle \pi^- | \bar{d}_\alpha \gamma^\mu (1 - \gamma_5) u_\alpha | 0 \rangle \langle \pi^+ | \bar{u}_\beta \gamma_\mu (1 - \gamma_5) b_\beta | \bar{B}_d^0 \rangle \\ &\quad \times \int_0^1 du \phi(u) \left\{ \left[\frac{1}{\epsilon} - \gamma_E + \ln 4\pi + 2 \ln \frac{\mu}{m_b} + 1 + \frac{u}{1-u} \ln u \right] \right. \\ &\quad \left. - \frac{\Gamma(1-a)}{(4\pi)^a} \left(\frac{m_b}{\mu} \right)^{2a} \left[\frac{1}{a^2} + \frac{2(\ln u - 1)}{a} + \ln^2 u - 2\text{Li}_2 \left(1 - \frac{1}{u} \right) - 4 \ln u + 5 + \frac{2 \ln u}{1-u} \right] \right\}, \end{aligned} \quad (14)$$

¹Such a choice does cause a scheme dependence in the matrix elements. However, when we choose the Wilson coefficients in the same scheme as for the matrix elements, the final full decay amplitude is free of scheme dependence.

$$\begin{aligned}
\langle Q_2^u \rangle_{(b)} &= \frac{\alpha_s}{4\pi} \frac{C_F}{N} \langle \pi^- | \bar{d}_\alpha \gamma^\mu (1 - \gamma_5) u_\alpha | 0 \rangle \langle \pi^+ | \bar{u}_\beta \gamma_\mu (1 - \gamma_5) b_\beta | \bar{B}_d^0 \rangle \\
&\times \int_0^1 du \phi(u) \left\{ -4 \left[\frac{1}{\epsilon} - \gamma_E + \ln 4\pi + 2 \ln \frac{\mu}{m_b} + \frac{11}{4} + \frac{\bar{u}}{1 - \bar{u}} \ln \bar{u} \right] \right. \\
&\left. + \frac{\Gamma(1-a)}{(4\pi)^a} \left(\frac{m_b}{\mu} \right)^{2a} \left[\frac{1}{a^2} + \frac{2(\ln \bar{u} - 1)}{a} + \ln^2 \bar{u} - 2\text{Li}_2 \left(1 - \frac{1}{\bar{u}} \right) - 4 \ln \bar{u} + 6 + \frac{2 \ln \bar{u}}{1 - \bar{u}} \right] \right\}, \quad (15)
\end{aligned}$$

$$\begin{aligned}
\langle Q_2^u \rangle_{(c)} &= \frac{\alpha_s}{4\pi} \frac{C_F}{N} \langle \pi^- | \bar{d}_\alpha \gamma^\mu (1 - \gamma_5) u_\alpha | 0 \rangle \langle \pi^+ | \bar{u}_\beta \gamma_\mu (1 - \gamma_5) b_\beta | \bar{B}_d^0 \rangle \\
&\times \int_0^1 du \phi(u) \left\{ -4 \left[\frac{1}{\epsilon} - \gamma_E + \ln 4\pi + 2 \ln \frac{\mu}{m_b} + \frac{11}{4} - \ln(-u) \right] \right. \\
&\left. + \frac{\Gamma(1-a)}{(4\pi)^a} \left(\frac{m_b}{\mu} \right)^{2a} \left[\frac{2}{a^2} + \frac{2(\ln(-u) - 2)}{a} + 10 - \frac{\pi^2}{3} - 4 \ln(-u) + \ln^2(-u) \right] \right\}, \quad (16)
\end{aligned}$$

$$\begin{aligned}
\langle Q_2^u \rangle_{(d)} &= \frac{\alpha_s}{4\pi} \frac{C_F}{N} \langle \pi^- | \bar{d}_\alpha \gamma^\mu (1 - \gamma_5) u_\alpha | 0 \rangle \langle \pi^+ | \bar{u}_\beta \gamma_\mu (1 - \gamma_5) b_\beta | \bar{B}_d^0 \rangle \\
&\times \int_0^1 du \phi(u) \left\{ \left[\frac{1}{\epsilon} - \gamma_E + \ln 4\pi + 2 \ln \frac{\mu}{m_b} + 1 - \ln(-\bar{u}) \right] \right. \\
&\left. - \frac{\Gamma(1-a)}{(4\pi)^a} \left(\frac{m_b}{\mu} \right)^{2a} \left[\frac{2}{a^2} + \frac{2(\ln(-\bar{u}) - 2)}{a} + 10 - \frac{\pi^2}{3} - 4 \ln(-\bar{u}) + \ln^2(-\bar{u}) \right] \right\}. \quad (17)
\end{aligned}$$

In the above, we have set $d=4+2a$ ($a>0$) in regularizing the infrared divergence. Then, after summing over all four diagrams, we find that all pole terms in $1/a$ are really canceled before we integrate over the momentum fraction variable u . So after modified minimal subtraction (MS), we get

$$\begin{aligned}
\overline{\langle Q_2^u \rangle}_{(a)+(b)+(c)+(d)} &= \frac{\alpha_s}{4\pi} \frac{C_F}{N} \langle \pi^- | \bar{d}_\alpha \gamma^\mu (1 - \gamma_5) u_\alpha | 0 \rangle \langle \pi^+ | \bar{u}_\beta \gamma_\mu (1 - \gamma_5) b_\beta | \bar{B}_d^0 \rangle \\
&\times \int_0^1 du \phi(u) \left[-18 - 12 \ln \frac{\mu}{m_b} + \frac{u}{1-u} \ln u - \frac{4\bar{u}}{1-\bar{u}} \ln \bar{u} + 4 \ln(-u) - \ln(-\bar{u}) - (\ln^2 u - \ln^2 \bar{u}) \right. \\
&\left. + 2 \left(\text{Li}_2 \left(1 - \frac{1}{u} \right) - \text{Li}_2 \left(1 - \frac{1}{\bar{u}} \right) \right) - \left(\frac{2 \ln u}{1-u} - \frac{2 \ln \bar{u}}{1-\bar{u}} \right) + (\ln^2(-u) - \ln^2(-\bar{u})) \right]. \quad (18)
\end{aligned}$$

Assuming that the light-cone distribution amplitude $\phi(u)$ is symmetric, then the above equation can be simplified as follows:

$$\begin{aligned}
\overline{\langle Q_2^u \rangle}_{(a)+(b)+(c)+(d)} &= \frac{\alpha_s}{4\pi} \frac{C_F}{N} \langle \pi^- | \bar{d}_\alpha \gamma^\mu (1 - \gamma_5) u_\alpha | 0 \rangle \langle \pi^+ | \bar{u}_\beta \gamma_\mu (1 - \gamma_5) b_\beta | \bar{B}_d^0 \rangle \\
&\times \int_0^1 du \phi(u) \left[-18 - 12 \ln \frac{\mu}{m_b} + 3 \frac{1-2u}{1-u} \ln u - 3i\pi \right]. \quad (19)
\end{aligned}$$

It is easy to check that the above equations are consistent with the results in previous works. Actually, with the MG scheme, we get the same results as that by using the DR scheme.

With Eqs. (18),(19), we can compute the vertex corrections no matter whether the LCDA $\phi(u)$ is symmetric or asymmetric. This is very important in principle. For instance, when kaon is ejected from b quark decay, we must take the contributions from the asymmetric part of LCDA of kaon into account, although the contributions from the asymmetric part are very small numerically [19].

2. Penguin corrections

There are two kinds of penguin corrections. One is the four quark operators insertion [Fig. 1(e)]; the other is the magnetic penguin insertion [Fig. 1(f)]. The first kind is generally called BSS mechanism. In generalized factorization, BSS mechanism plays a very important role in CP violation because it is the unique source of strong phases. But in generalized factorization, the virtuality of the gluon or photon is ambiguous; usually one varies k^2 around $m_B^2/2$. This variation does not have an important effect on the branching ratios, but it does for CP asymmetries. In QCD factorization, this ambiguity is rendered by taking the virtuality of the gluon as $k^2 = (p - uq)^2 = \bar{u}m_b^2$. When treating penguin contractions, one should be careful that Fig. 1(e) contains two kinds of topology, which is depicted in Fig. 2. They are equivalent in 4 dimensions according to Fierz rearrangement. However, since penguin corrections contain ultraviolet divergence, we must do calculations in d dimensions where these two kinds of topology are not equivalent [29]. Below we give an explicit calculation of Q_4 or Q_6 penguin insertions for $\bar{B}_d^0 \rightarrow \pi^+ \pi^-$ which belong to the second topology, Fig. 2(b):

$$\begin{aligned}
\langle Q_{4,6} \rangle_{(e)}^{\text{twist-2}} &= \frac{f_\pi}{4} g_s^2 \mu^2 \epsilon \frac{C_F}{N} \int_0^1 du \phi(u) \langle \pi^+ | \bar{u}_i \gamma_\alpha \not{q} \gamma_5 \gamma_\mu (1 - \gamma_5) b_i | \bar{B}_d^0 \rangle \\
&\quad \times \sum_q \int \frac{d^d k}{(2\pi)^d} \frac{-\text{Tr}[(l - \mathbf{k} - m_q) \gamma^\alpha (\mathbf{k} + m_q) \gamma^\mu (1 \mp \gamma_5)]}{[(l - k)^2 - m_q^2][k^2 - m_q^2]l^2} \Big|_{l=p-uq} \\
&= -2if_\pi \frac{\alpha_s}{4\pi} \frac{C_F}{N} \int_0^1 du \phi(u) \langle \pi^+ | \bar{u}_i \gamma_\alpha \not{q} \gamma_5 \gamma_\mu (1 - \gamma_5) b_i | \bar{B}_d^0 \rangle \left[\frac{l_\alpha l_\mu}{l^2} - g_{\alpha\mu} \right] \\
&\quad \times \sum_q \left[\frac{1}{6} \left(\frac{1}{\epsilon} - \gamma_E + \ln 4\pi \right) + \int_0^1 dt t(1-t) \ln \frac{\mu^2}{m_q^2 - t(1-t)l^2 - i\epsilon} \right] \Big|_{l=p-uq}. \tag{20}
\end{aligned}$$

After $\overline{\text{MS}}$ subtraction and using the equations of motions, we get the finite result

$$\begin{aligned}
\overline{\langle Q_{4,6} \rangle}_{(e)}^{\text{twist-2}} &= -\frac{\alpha_s}{4\pi} \frac{C_F}{N} \langle \pi^- | \bar{d}_i \gamma^\mu (1 - \gamma_5) u_i | 0 \rangle \langle \pi^+ | \bar{u}_j \gamma_\mu (1 - \gamma_5) b_j | \bar{B}_d^0 \rangle \\
&\quad \times \sum_q \left[\frac{4}{3} \ln \frac{\mu}{m_b} - 4 \int_0^1 du \phi(u) \int_0^1 dt t(1-t) \ln(s_q - t(1-t)\bar{u} - i\epsilon) \right], \tag{21}
\end{aligned}$$

where $s_q = m_q^2/m_b^2$. The first topology, Fig. 2(a), for example, Q_1^c penguin insertion for $\bar{B}_d^0 \rightarrow \pi^+ \pi^-$, is similar to the results of the second topology, Fig. 2(b), except that there is an extra factor $-2/3$:

$$\begin{aligned}
\overline{\langle Q_1^c \rangle}_{(e)}^{\text{twist-2}} &= -\frac{\alpha_s}{4\pi} \frac{C_F}{N} \langle \pi^- | \bar{d}_i \gamma^\mu (1 - \gamma_5) u_i | 0 \rangle \langle \pi^+ | \bar{u}_j \gamma_\mu (1 - \gamma_5) b_j | \bar{B}_d^0 \rangle \\
&\quad \times \left[-\frac{2}{3} + \frac{4}{3} \ln \frac{\mu}{m_b} - 4 \int_0^1 du \phi(u) \int_0^1 dt t(1-t) \ln(s_c - t(1-t)\bar{u} - i\epsilon) \right]. \tag{22}
\end{aligned}$$

For the magnetic penguin insertion, it is the easiest calculation of the radiative corrections. The result of Q_{8G} insertion for $\bar{B}_d^0 \rightarrow \pi^+ \pi^-$ is

$$\begin{aligned}
\langle Q_{8G} \rangle_{(f)}^{\text{twist-2}} &= -\frac{\alpha_s}{4\pi} \frac{C_F}{N} f_\pi m_b \int_0^1 du \phi(u) \frac{1}{k^2} \langle \pi^+ | \bar{u}_i \gamma^\alpha \not{q} \gamma_5 \sigma_{\beta\alpha} k^\beta (1 + \gamma_5) b_i | \bar{B}_d^0 \rangle \Big|_{k=p-uq} \\
&= -\frac{\alpha_s}{4\pi} \frac{C_F}{N} \int_0^1 du \frac{2\phi(u)}{\bar{u}} \langle \pi^- | \bar{d}_i \gamma^\mu (1 - \gamma_5) | 0 \rangle \langle \pi^+ | \bar{u}_i \gamma_\mu (1 - \gamma_5) | \bar{B}_d^0 \rangle. \tag{23}
\end{aligned}$$

3. Hard scattering with the spectator

Hard spectator scattering [Figs. 1(g) and 1(h)] is completely missing in the naive factorization. But in QCD factorization, it can be calculated in the perturbative QCD, and expressed by a convolution of the hard kernel T^H and the LCDAs of mesons.

At the leading power of Λ_{QCD}/m_b , both of the light pseudoscalars from the B meson decay can be represented by their leading twist LCDAs. So after a straightforward calculation, we obtain this contribution for $\bar{B}_d^0 \rightarrow \pi^+ \pi^-$ from the operator Q_2^u insertion,

$$\begin{aligned} \langle Q_2^u \rangle_{(g)+(h)}^{\text{twist-2}} &= \frac{-if_B f_\pi^2 C_F}{64} \frac{C_F}{N^2} g_s^2 \int_0^1 d\xi \, du \, dv \, \phi_B(\xi) \phi(u) \phi(v) \\ &\times \left\{ \frac{\text{Tr}[(\not{p} + m_B) \gamma_5 \gamma^\alpha \not{q}_1 \gamma_5 \gamma_\rho (1 - \gamma_5) \not{q}_2 \gamma_5 \gamma_\alpha \not{q}_d \gamma_\rho (1 - \gamma_5)]}{k^2 l_d^2} \Bigg|_{l_d = uq_2 - k}^{k = \xi p - \bar{v}q_1} \right. \\ &\left. + \frac{\text{Tr}[(\not{p} + m_B) \gamma_5 \gamma^\alpha \not{q}_1 \gamma_5 \gamma_\rho (1 - \gamma_5) \not{l}_u \gamma_\alpha \not{q}_2 \gamma_5 \gamma_\rho (1 - \gamma_5)]}{k^2 l_u^2} \Bigg|_{l_u = k - \bar{u}q_2}^{k = \xi p - \bar{v}q_1} \right\} \\ &= i\pi \alpha_s f_B f_\pi^2 \frac{C_F}{N^2} \int_0^1 d\xi \frac{\phi_B(\xi)}{\xi} \int_0^1 du \frac{\phi(u)}{u} \int_0^1 dv \frac{\phi(v)}{\bar{v}}. \end{aligned} \quad (24)$$

B. Chirally enhanced corrections—twist-3 LCDAs insertion

It has been observed that QCD factorization is demonstrated only in the strict heavy quark limit. This means that any generalization of QCD factorization to include or partly include power corrections of Λ_{QCD}/m_b should redemonstrate that factorization still holds. There are a variety of sources which may contribute to power corrections in $1/m_b$; examples are higher twist distribution amplitudes, transverse momenta of quarks in the light meson, annihilation diagrams, etc. Unfortunately, there is no known systematic way to evaluate these power corrections for exclusive decays. Moreover, factorization might break down when these power corrections, for instance, transverse momenta effects, are considered. This indicates that one might have to give up such an ambitious plan that all power corrections could be, at least in principle, incorporated into QCD factorization order by order. One might argue that power corrections in B decays are numerically unimportant because these corrections are expanded in order of a small number $\Lambda_{QCD}/m_b \simeq 1/15$. But this is not true. For instance, the contributions of operator Q_6 to decay amplitudes would formally vanish in the strict heavy quark limit. However, it is numerically very important in penguin-dominated B rare decays, such as interesting channels $B \rightarrow \pi K$, etc. This is because Q_6 is always multiplied by a formally power suppressed but chirally enhanced factor $r_\chi = 2m_p^2/m_b(m_1 + m_2) \sim \mathcal{O}(1)$, where m_1 and m_2 are current quark masses. So power suppression might probably fail at least in this case. Therefore, phenomenological applicability of QCD factorization in B rare decays requires at least a consistent inclusion of chirally enhanced corrections.

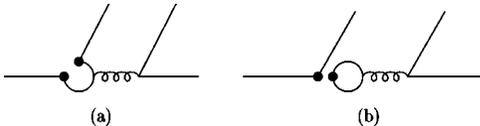


FIG. 2. Two kinds of topology for penguin contractions.

Chirally enhanced corrections arise from the two particle twist-3 light-cone distribution amplitudes, generally called $\phi_p(x)$ and $\phi_\sigma(x)$. So when chirally enhanced corrections are concerned, the final light mesons should be described by leading twist and twist-3 distribution amplitudes. Then it is crucial to show that factorization really holds when considering twist-3 distribution amplitudes. The most difficult part is to demonstrate the infrared finiteness of the hard scattering kernels T_i^j . In addition, possible chirally enhanced power corrections can also appear in the hard spectator scattering. So, for consistency, we must involve these corrections.

1. Vertex corrections

When we calculate the chirally enhanced power corrections, contrast to the leading-twist light-cone wave function insertion, the cancellation of the infrared divergences in the vertex corrections to $(V-A)(V+A)$ operator (here it is Q_5 or Q_7) cannot be shown simply by the eikonal approximation similar to what has been done at the leading power of Λ_{QCD}/m_b , because the Dirac structure or spin structure of twist-3 light-cone wave functions of the light pseudoscalar makes the ‘‘on-shell’’ condition for the external quarks invalid. Thus, to justify the cancellation of the infrared divergence in $(V-A)(V+A)$ vertex corrections, we must give the explicit calculation. As mentioned in the previous subsections, we have two choices to regularize the infrared divergence in one-loop calculation. One is the DR scheme; the other is MG scheme. Generally, these two schemes are equivalent, for instance, similar to what has been done in $(V-A)(V-A)$ vertex corrections. However, in the DR scheme, it is difficult to extrapolate the twist-3 wave functions of the light pseudoscalar to d dimensions properly, although they are well-defined in 4 dimensions. Therefore, we prefer to use the MG scheme in our calculation for chirally enhanced corrections to avoid the above possible problems.

In addition, generally we calculate the Feynman diagrams in the momentum space, so the correct projection of the

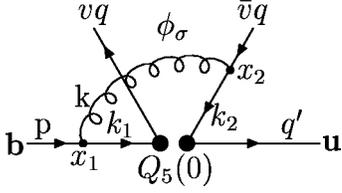


FIG. 3. An example of the vertex corrections for $Q_5(0)$ in the case of ϕ_σ insertion.

light-cone wave functions of the light pseudoscalar in the momentum space is necessary. From Appendix B, we find that it is easy to find the proper momentum space projection of the leading twist and ϕ_p type twist-3 wave function, but for ϕ_σ , the projection is not very clear. Note that the coordinate x^μ in the definition of ϕ_σ by the nonlocal matrix

element must be transformed into a partial derivative of a certain momentum in the projection of momentum space. However, it is difficult to find the derivative which makes the projection only depend on the structure of the light pseudoscalar itself. Generally, the momentum which the partial derivative acts on is dependent on the hard kernel. Therefore, we prefer to compute the Feynman diagrams of the twist-3 wave functions insertion, especially ϕ_σ insertion in the coordinate space. We think that such calculation can avoid the ambiguity about how to project the coordinate x^μ into the momentum space. We recalculate the leading twist insertion by using the same method, and obtain the same results as those in the previous sections. Below, we will show how to perform this trick in calculation of ϕ_σ insertion. For example, let us consider Fig. 3. In coordinate space, we have

$$\begin{aligned}
 \text{Fig. 3} &= \frac{f_P \mu_P C_F}{4 N} g_s^2 \int du \frac{\phi_\sigma(u)}{6} \int d^4 x_1 d^4 x_2 \int \frac{d^4 k}{(2\pi)^4} \frac{d^4 l_u^-}{(2\pi)^4} \frac{d^4 l_b}{(2\pi)^4} \frac{e^{i(\bar{u}q - k + l_u^-) \cdot x_2} e^{i(k + l_b - p) \cdot x_1}}{(k^2 - m_g^2) l_u^2 (l_b^2 - m_b^2)} \\
 &\quad \times \langle \pi^+ | \bar{u}_i \gamma^\rho (1 + \gamma_5) t_u^- \gamma^\alpha \gamma_5 \sigma_{\mu\nu} q^\mu x_2^\nu \gamma_\rho (1 - \gamma_5) (t_b + m_b) \gamma_\alpha b_i | \bar{B}_d^0 \rangle \\
 &= \frac{f_P \mu_P C_F}{4 N} g_s^2 \int du \frac{\phi_\sigma(u)}{6} \int \frac{d^4 k}{(2\pi)^4} \frac{d^4 l_u^- d^4 x_2}{(2\pi)^4} \frac{e^{i(\bar{u}q - k + l_u^-) \cdot x_2} q^\mu x_2^\nu}{(k^2 - m_g^2) l_u^2 (l_b^2 - m_b^2)} \\
 &\quad \times \langle \pi^+ | \bar{u}_i \gamma^\rho (1 + \gamma_5) t_u^- \gamma^\alpha \gamma_5 \sigma_{\mu\nu} \gamma_\rho (1 - \gamma_5) (t_b + m_b) \gamma_\alpha b_i | \bar{B}_d^0 \rangle \Bigg|_{l_b = p - k} \\
 &= \frac{if_P \mu_P C_F}{4 N} g_s^2 \int du \frac{\phi_\sigma(u)}{6} \int \frac{d^4 k}{(2\pi)^4} \frac{q^\mu}{(k^2 - m_g^2) (l_b^2 - m_b^2)} \\
 &\quad \times \frac{\partial}{\partial l_{u\nu}^-} \left\{ \langle \pi^+ | \bar{u}_i \gamma^\rho (1 + \gamma_5) \frac{t_u^-}{l_u^-} \gamma^\alpha \gamma_5 \sigma_{\mu\nu} \gamma_\rho (1 - \gamma_5) (t_b + m_b) \gamma_\alpha b_i | \bar{B}_d^0 \rangle \right\} \Bigg|_{l_b = p - k}^{l_u^- = k - \bar{u}q}. \tag{25}
 \end{aligned}$$

The above trick has been applied in the calculation of the proper correlation function to extract the transition form factor $F^{B \rightarrow \pi}$ within the frame of the light-cone sum rule [6]. So within the same frame, we obtain

$$\begin{aligned}
 \langle Q_5 \rangle_{(a)} &= g_s^2 \frac{f_\pi \mu_\pi C_F}{4 N} \int_0^1 du \int \frac{d^4 k}{(2\pi)^4} \frac{1}{(k^2 - m_g^2) l_d^2 (l_b^2 - m_b^2)} \left\{ \phi_p(u) \langle \pi^+ | \bar{u}_i \gamma^\mu (1 + \gamma_5) \gamma_5 \gamma^\alpha t_d \gamma_\mu (1 - \gamma_5) (t_b + m_b) \gamma_\alpha b_i | \bar{B}_d^0 \rangle \right. \\
 &\quad \left. + i \frac{\phi_\sigma(u)}{6} \phi_\sigma(u) \langle \pi^+ | \bar{u}_i \gamma^\rho (1 + \gamma_5) \gamma_5 \sigma_{\mu\nu} q^\mu \gamma^\alpha \left(\gamma^\nu - \frac{2l_d^\nu t_d}{l_d^2} \right) \gamma_\rho (1 - \gamma_5) (t_b + m_b) \gamma_\alpha b_i | \bar{B}_d^0 \rangle \right\} \Bigg|_{l_b = p - k}^{l_d = uq - k}, \tag{26}
 \end{aligned}$$

$$\begin{aligned}
 \langle Q_5 \rangle_{(b)} &= g_s^2 \frac{f_\pi \mu_\pi C_F}{4 N} \int_0^1 du \int \frac{d^4 k}{(2\pi)^4} \frac{1}{(k^2 - m_g^2) l_u^2 (l_b^2 - m_b^2)} \\
 &\quad \times \left\{ \phi_p(u) \langle \pi^+ | \bar{u}_i \gamma^\mu (1 + \gamma_5) t_u^- \gamma^\alpha \gamma_5 \gamma_\mu (1 - \gamma_5) (t_b + m_b) \gamma_\alpha b_i | \bar{B}_d^0 \rangle \right. \\
 &\quad \left. + i \frac{\phi_\sigma(u)}{6} \phi_\sigma(u) \langle \pi^+ | \bar{u}_i \gamma^\rho (1 + \gamma_5) \left(\gamma^\nu - \frac{2l_u^\nu t_u^-}{l_u^2} \right) \gamma^\alpha \gamma_5 \sigma_{\mu\nu} q^\mu \gamma_\rho (1 - \gamma_5) (t_b + m_b) \gamma_\alpha b_i | \bar{B}_d^0 \rangle \right\} \Bigg|_{l_b = p - k}^{l_u^- = k - \bar{u}q}, \tag{27}
 \end{aligned}$$

$$\begin{aligned}
\langle Q_5 \rangle_{(c)} = & g_s^2 \frac{f_\pi \mu_\pi}{4} \frac{C_F}{N} \int_0^1 du \int \frac{d^4 k}{(2\pi)^4} \frac{1}{(k^2 - m_g^2) l_d^2 l_u^2} \left\{ \phi_p(u) \langle \pi^+ | \bar{u}_i \gamma_\alpha t_u \gamma^\mu (1 + \gamma_5) \gamma_5 \gamma_\alpha t_d \gamma_\mu (1 - \gamma_5) b_i | \bar{B}_d^0 \rangle \right. \\
& \left. + i \frac{\phi_\sigma(u)}{6} \langle \pi^+ | \bar{u}_i \gamma^\alpha t_u \gamma^\mu (1 + \gamma_5) \gamma_5 \sigma_{\mu\nu} q^\mu \gamma_\alpha \left(\gamma^\nu - \frac{2l_d^\nu t_d}{l_d^2} \right) \gamma_\rho (1 - \gamma_5) b_i | \bar{B}_d^0 \rangle \right\} \Bigg|_{l_u = p - q + k}^{l_d = uq - k}, \quad (28)
\end{aligned}$$

$$\begin{aligned}
\langle Q_5 \rangle_{(d)} = & g_s^2 \frac{f_\pi \mu_\pi}{4} \frac{C_F}{N} \int_0^1 du \int \frac{d^4 k}{(2\pi)^4} \frac{1}{(k^2 - m_g^2) l_u^2 l_u^2} \left\{ \phi_p(u) \langle \pi^+ | \bar{u}_i \gamma^\alpha t_u \gamma^\mu (1 + \gamma_5) t_u^- \gamma_\alpha \gamma_5 \gamma_\mu (1 - \gamma_5) b_i | \bar{B}_d^0 \rangle \right. \\
& \left. + i \frac{\phi_\sigma(u)}{6} \langle \pi^+ | \bar{u}_i \gamma^\alpha t_u \gamma^\rho (1 + \gamma_5) \left(\gamma^\nu - \frac{2l_u^\nu t_u^-}{l_u^2} \right) \gamma_\alpha \gamma_5 \sigma_{\mu\nu} q^\mu \gamma_\rho (1 - \gamma_5) b_i | \bar{B}_d^0 \rangle \right\} \Bigg|_{l_u = p - q + k}^{l_u^- = k - \bar{u}q}. \quad (29)
\end{aligned}$$

Perform the one-loop integrations:

$$\begin{aligned}
\langle Q_5 \rangle_{(a)} = & 2 \frac{\alpha_s}{4\pi} \frac{C_F}{N} \langle \pi^- | \bar{d}_i (1 + \gamma_5) u_i | 0 \rangle \langle \pi^+ | \bar{u}_j (1 - \gamma_5) b_j | \bar{B}_d^0 \rangle \int_0^1 du \left\{ \phi_p(u) \left[- \left(\frac{1}{\epsilon} - \gamma_E + \ln 4\pi + 2 \ln \frac{\mu}{m_b} \right) + \frac{1}{4} \ln^2 \lambda \right. \right. \\
& + \ln(-u) \ln \lambda - 2 \ln u \ln \lambda + \frac{1}{2} \ln^2 u - \text{Li}_2 \left(1 - \frac{1}{u} \right) + \frac{1}{2} + \frac{5}{4} \pi^2 \left. \right] + \frac{\phi_\sigma(u)}{6u} \left[\frac{1}{2} \ln^2 \lambda + 2 \ln(-u) \ln \lambda - 4 \ln u \ln \lambda \right. \\
& \left. \left. + \ln \lambda + 4 \ln^2 u - \ln^2(-u) - 2 \ln u \ln(1-u) + 2 \text{Li}_2 \left(\frac{1}{u} \right) + \frac{7}{6} \pi^2 - \ln u - \frac{\ln u}{1-u} \right] \right\}, \quad (30)
\end{aligned}$$

$$\begin{aligned}
\langle Q_5 \rangle_{(b)} = & -2 \frac{\alpha_s}{4\pi} \frac{C_F}{N} \langle \pi^- | \bar{d}_i (1 + \gamma_5) u_i | 0 \rangle \langle \pi^+ | \bar{u}_j (1 - \gamma_5) b_j | \bar{B}_d^0 \rangle \int_0^1 du \left\{ \phi_p(u) \left[- \left(\frac{1}{\epsilon} - \gamma_E + \ln 4\pi + 2 \ln \frac{\mu}{m_b} \right) + \frac{1}{4} \ln^2 \lambda \right. \right. \\
& + \ln(-\bar{u}) \ln \lambda - 2 \ln \bar{u} \ln \lambda + \frac{1}{2} \ln^2 \bar{u} - \text{Li}_2 \left(1 - \frac{1}{\bar{u}} \right) - \frac{5}{2} + \frac{5}{4} \pi^2 \left. \right] + \frac{\phi_\sigma(u)}{6\bar{u}} \left[\frac{1}{2} \ln^2 \lambda + 2 \ln(-\bar{u}) \ln \lambda - 4 \ln \bar{u} \ln \lambda \right. \\
& \left. \left. + \ln \lambda + 4 \ln^2 \bar{u} - \ln^2(-\bar{u}) - 2 \ln \bar{u} \ln(1-\bar{u}) + 2 \text{Li}_2 \left(\frac{1}{\bar{u}} \right) + \frac{7}{6} \pi^2 - \ln \bar{u} - \frac{\ln \bar{u}}{1-\bar{u}} \right] \right\}, \quad (31)
\end{aligned}$$

$$\begin{aligned}
\langle Q_5 \rangle_{(c)} = & -2 \frac{\alpha_s}{4\pi} \frac{C_F}{N} \langle \pi^- | \bar{d}_i (1 + \gamma_5) u_i | 0 \rangle \langle \pi^+ | \bar{u}_j (1 - \gamma_5) b_j | \bar{B}_d^0 \rangle \int_0^1 du \left\{ \phi_p(u) \left[- \left(\frac{1}{\epsilon} - \gamma_E + \ln 4\pi + 2 \ln \frac{\mu}{m_b} \right) \right. \right. \\
& + \frac{1}{2} \ln^2 \lambda - [\ln(-u) - 2] \ln \lambda + \frac{1}{2} \ln^2(-u) - \ln(-u) - \frac{3}{2} + \frac{\pi^2}{3} \left. \right] + \frac{\phi_\sigma(u)}{6u} \left[\ln^2 \lambda - [2 \ln(-u) - 3] \ln \lambda \right. \\
& \left. \left. + 2 \ln(-u) \ln u - \ln^2 u - 3 \ln(-u) + 3 - \frac{\pi^2}{3} \right] \right\}, \quad (32)
\end{aligned}$$

$$\begin{aligned}
\langle Q_5 \rangle_{(d)} = & 2 \frac{\alpha_s}{4\pi} \frac{C_F}{N} \langle \pi^- | \bar{d}_i (1 + \gamma_5) u_i | 0 \rangle \langle \pi^+ | \bar{u}_j (1 - \gamma_5) b_j | \bar{B}_d^0 \rangle \int_0^1 du \left\{ \phi_p(u) \left[- \left(\frac{1}{\epsilon} - \gamma_E + \ln 4\pi + 2 \ln \frac{\mu}{m_b} \right) \right. \right. \\
& + \frac{1}{2} \ln^2 \lambda - [\ln(-\bar{u}) - 2] \ln \lambda + \frac{1}{2} \ln^2(-\bar{u}) - \ln(-\bar{u}) + \frac{3}{2} + \frac{\pi^2}{3} \left. \right] \\
& \left. + \frac{\phi_\sigma(u)}{6\bar{u}} \left[\ln^2 \lambda - [2 \ln(-\bar{u}) - 3] \ln \lambda + 2 \ln(-\bar{u}) \ln \bar{u} - \ln^2 \bar{u} - 3 \ln(-\bar{u}) + 3 - \frac{\pi^2}{3} \right] \right\}. \quad (33)
\end{aligned}$$

Here $\lambda = m_g^2/m_b^2$. From the above equations, it is observed that, in the case of ϕ_σ distribution amplitudes, the terms with infrared divergence in vertex correction diagrams cannot cancel unless $\phi_\sigma(u)$ is a symmetric function: $\phi_\sigma(u) = \phi_\sigma(\bar{u})$. This is an unexpected result, which means QCD factorization is violated for asymmetric twist-3 light-cone distribution amplitudes.

This indicates that chirally enhanced corrections can be included consistently in the framework of QCD factorization only when twist-3 light-cone distribution amplitudes are symmetric. Therefore, in the following, we will implicitly assume a symmetric twist-3 light-cone distribution amplitude for light pseudoscalar mesons. It is then straightforward to show that vertex corrections of $(V-A)(V+A)$ operator are completely cancelled after summing over four diagrams in the case of ϕ_σ distribution amplitude. The final result of $(V-A)(V+A)$ vertex corrections, in the condition that the twist-3 LCDA is symmetric, is

$$\langle Q_5 \rangle_{(a)+(b)+(c)+(d)} = 12 \frac{\alpha_s}{4\pi} \frac{C_F}{N} \langle \pi^- | \bar{d}_i (1 + \gamma_5) u_i | 0 \rangle \langle \pi^+ | \bar{u}_j (1 - \gamma_5) b_j | \bar{B}_d^0 \rangle. \quad (34)$$

2. Penguin corrections

In quark level, usually one decomposes the basic QCD vertex $-ig_s \gamma^\mu T_{ij}^a$ in penguin insertion into the two chiral current couplings $-i\frac{1}{2}g_s \gamma^\mu T_{ij}^a (1 + \gamma_5)$ and $-i\frac{1}{2}g_s \gamma^\mu T_{ij}^a (1 - \gamma_5)$; then the penguin insertions contribute the same magnitude to the $(V-A)(V-A)$ and $(V-A)(V+A)$ vertex. But in hadron level, this point of view must be examined in elaborate calculation.

For illustration, we give the results of Q_4 or Q_6 penguin corrections to $\bar{B}_d^0 \rightarrow \pi^+ \pi^-$, which belong to the second penguin topology Fig. 2(b), when $\phi_p(u)$ is inserted:

$$\begin{aligned} \langle Q_{4,6} \rangle_{(e)}^{\phi_p} &= -\frac{f_{\pi} \mu_{\pi}}{4} g_s^2 \mu^2 \epsilon \frac{C_F}{N} \int_0^1 du \phi_p(u) \langle \pi^+ | \bar{u}_i \gamma_\alpha \gamma_5 \gamma_\mu (1 - \gamma_5) b_i | \bar{B}_d^0 \rangle \\ &\times \sum_q \int \frac{d^d k}{(2\pi)^d} \frac{-\text{Tr}[(\not{p} - u\not{q} - \not{k} - m_q) \gamma^\alpha (\not{k} + m_q) \gamma^\mu (1 \mp \gamma_5)]}{[(p - uq - k)^2 - m_q^2][k^2 - m_q^2](p - uq)^2} \\ &= 2if_{\pi} \mu_{\pi} \frac{\alpha_s}{4\pi} \frac{C_F}{N} \int_0^1 du \phi_p(u) \langle \pi^+ | \bar{u}_i \gamma_\alpha \gamma_5 \gamma_\mu (1 - \gamma_5) b_i | \bar{B}_d^0 \rangle \left[\frac{l_{\alpha} l_{\mu}}{l^2} - g_{\alpha\mu} \right] \\ &\times \sum_q \left[\frac{1}{6} \left(\frac{1}{\epsilon} - \gamma_E + \ln 4\pi \right) + \int_0^1 dt t(1-t) \ln \frac{\mu^2}{m_q^2 - t(1-t)l^2 - i\epsilon} \right]_{l=p-ug}. \end{aligned} \quad (35)$$

After $\overline{\text{MS}}$ subtraction, we obtain

$$\begin{aligned} \overline{\langle Q_{4,6} \rangle}_{(e)}^{\phi_p} &= 2 \frac{\alpha_s}{4\pi} \frac{C_F}{N} \langle \pi^- | \bar{d}_i (1 + \gamma_5) u_i | 0 \rangle \langle \pi^+ | \bar{u}_j (1 - \gamma_5) b_j | \bar{B}_d^0 \rangle \\ &\times \sum_q \left[\ln \frac{\mu}{m_b} - 3 \int_0^1 du \phi_p(u) \int_0^1 dt t(1-t) \ln [s_q - t(1-t)\bar{u} - i\epsilon] \right]. \end{aligned} \quad (36)$$

For the first penguin insertion topology, Fig. 2(a), the result is

$$\begin{aligned} \overline{\langle Q_1 \rangle}_{(e)}^{\phi_p} &= 2 \frac{\alpha_s}{4\pi} \frac{C_F}{N} \langle \pi^- | \bar{d}_i (1 + \gamma_5) u_i | 0 \rangle \langle \pi^+ | \bar{u}_j (1 - \gamma_5) b_j | \bar{B}_d^0 \rangle \\ &\times \left[\ln \frac{\mu}{m_b} - \frac{1}{2} - 3 \int_0^1 du \phi_p(u) \int_0^1 dt t(1-t) \ln [s_c - t(1-t)\bar{u} - i\epsilon] \right]. \end{aligned} \quad (37)$$

Similarly, when $\phi_\sigma(u)$ is inserted, by using the method in the previous subsection, we have

$$\begin{aligned} \langle Q_{4,6} \rangle_{(e)}^{\phi_\sigma} &= \frac{if_{\pi} \mu_{\pi}}{4} g_s^2 \mu^2 \epsilon \frac{C_F}{N} \int_0^1 du \frac{\phi_\sigma(u)}{6} \langle \pi^+ | \bar{u}_i (1 - \gamma_5) \gamma_\alpha \sigma_{\mu\nu} q^\mu \gamma_\rho b_i | \bar{B}_d^0 \rangle \\ &\times \sum_q \left[\frac{\partial}{\partial k_\nu} \int \frac{d^d l_q}{(2\pi)^d} \frac{\text{Tr}[(\not{l}_q - \not{k} + m_q) \gamma^\alpha (\not{l}_q + m_q) \gamma^\rho (1 \mp \gamma_5)]}{[(l_q - k)^2 - m_q^2][l_q^2 - m_q^2]k^2} \right]_{k=p-ug}. \end{aligned} \quad (38)$$

After integration and subtraction,

$$\begin{aligned} \overline{\langle Q_{4,6} \rangle}_{(e)}^{\phi_\sigma} &= 2 \frac{\alpha_s}{4\pi} \frac{C_F}{N} \langle \pi^- | \bar{d}_i (1 + \gamma_5) u_i | 0 \rangle \langle \pi^+ | \bar{u}_j (1 - \gamma_5) b_j | \bar{B}_d^0 \rangle \\ &\times \sum_q \int_0^1 du \frac{\phi_\sigma}{6\bar{u}} \left[\frac{2}{3} \ln \frac{\mu}{m_b} - \int_0^1 dt \left(2t(1-t) \ln [s_q - t(1-t)\bar{u} - i\epsilon] + \frac{t^2(1-t)^2\bar{u}}{s_q - t(1-t)\bar{u} - i\epsilon} \right) \right]. \end{aligned} \quad (39)$$

The magnetic penguin insertion is easier; we write the result of Q_{8G} insertion as follows:

$$\begin{aligned} \langle Q_{8G} \rangle_{(f)}^{\text{twist-3}} &= \frac{\alpha_s}{4\pi} \frac{C_F}{N} f_\pi \mu_\pi m_b \int_0^1 du \left\{ \phi_p(u) \frac{1}{k^2} \langle \pi^+ | \bar{u}_i \gamma^\alpha \gamma_5 \sigma_{\beta\alpha} k^\beta (1 + \gamma_5) b_i | \bar{B}_d^0 \rangle \right. \\ &\quad + i \frac{\phi_\sigma(u)}{6} \frac{1}{4k^2} \left[\langle \pi^+ | \bar{u}_i \gamma^\alpha \gamma_5 [\not{q}, \gamma_\sigma] [\not{\gamma}^\sigma, \gamma_\alpha] (1 + \gamma_5) b_i | \bar{B}_d^0 \rangle \right. \\ &\quad \left. \left. - \frac{2}{k^2} \langle \pi^+ | \bar{u}_i \gamma^\alpha \gamma_5 [\not{q}, \not{k}] [\not{k}, \gamma_\alpha] (1 + \gamma_5) b_i | \bar{B}_d^0 \rangle \right] \right\} \Big|_{k=p-uv} \\ &= 2 \frac{\alpha_s}{4\pi} \frac{C_F}{N} \left(\frac{3}{2} + \int_0^1 du \frac{\phi_\sigma(u)}{6\bar{u}} \right) \langle \pi^- | \bar{d}_i (1 + \gamma_5) u_i | 0 \rangle \langle \pi^+ | \bar{u}_j (1 - \gamma_5) b_j | \bar{B}_d^0 \rangle. \end{aligned} \quad (40)$$

3. Hard scattering with the spectator

The chirally enhanced power corrections in hard spectator scattering not only occurs in the case of $(V-A)(V+A)$ vertex insertion, but also in the case of $(V-A)(V-A)$ insertion. But in the case of $(V-A)(V+A)$ insertion, after a straightforward calculation, we find that there will be serious linear divergence at the end points of the LCDAs if the twist-3 LCDAs are not symmetric. Because infrared finiteness of the vertex corrections requires that the twist-3 LCDAs, especially $\phi_\sigma(u)$, must be symmetric, we shall implicitly assume this symmetric condition for the LCDAs in latter computation. So, in this symmetric condition, the hard scattering with the spectator vanishes when $(V-A)(V+A)$ vertex is inserted. However, even in this strict symmetric condition, there is still a logarithmic divergence from the end point of the recoiling pion in hard spectator scattering when $(V-A)(V-A)$ vertex is inserted. For example,

$$\begin{aligned} \langle Q_2 \rangle_{(g)+(h)}^u &= i \pi \alpha_s f_B f_\pi^2 \frac{C_F}{N^2} \int_0^1 d\xi \frac{\phi_B(\xi)}{\xi} \int_0^1 du \frac{\phi(u)}{u} \int_0^1 dv \\ &\times \left[\frac{\phi(v)}{\bar{v}} + \frac{2\mu_\pi}{m_B} \frac{\phi_\sigma(v)}{6\bar{v}^2} \right]. \end{aligned} \quad (41)$$

This means that QCD factorization is broken down. But we can still give a phenomenological treatment for this hard spectator scattering. By using the asymptotic form of $\phi_\sigma(u)$, we find that there is a divergent integral over v : $\int_0^1 dv (1/\bar{v})$. In Refs. [19,25], the authors prefer to introduce a phenomenological parametrization for this logarithmic divergence.

They take $\int_0^1 dv (1/\bar{v}) = \ln(m_B/\Lambda_B) + r e^{i\theta}$, where r is taken from 3 (realistic) to 6 (conservative), and the phase θ from $-\pi$ to π . We shall take similar phenomenological treatment in the numerical computation below.

We notice that the above approach of evaluating hard spectator contribution is naive. For instance, the scale of hard spectator contribution should be different from the vertex correction contribution. While it seems reasonable to take the scale $\mu \sim \mathcal{O}(m_b)$ for the vertex correction diagrams to avoid large logarithm $\alpha_s \log(\mu/m_b)$, a natural choice of the scale of hard spectator contribution may be around $\mathcal{O}(1 \text{ GeV})$ because the average momentum squared of the exchanged gluon is about 1 GeV^2 . Another disturbing feature of hard spectator contribution is that, as pointed out in Refs. [19,25], when including the contribution of ϕ_σ , there would appear a divergent integral $\int_0^1 dv (1/\bar{v})$ even if the symmetric distribution amplitude is applied. This divergent integral implies that the dominant contribution comes from the end-point region, or, in other words, it is dominated by soft gluon exchange. However, the transverse momentum may not be omitted in the end-point region [30]; if so, the corresponding divergent integral would then be changed to

$$\int dv \frac{1}{\bar{v}} \rightarrow \int dv d^2 k_T \frac{\Psi(v, k_T)}{\bar{v} \xi m_b^2 + k_T^2}. \quad (42)$$

As an illustration, we do not consider the k_T dependence of wave functions (though it is certainly not a good approximation); then the above integral is proportional to

$$\int \frac{dv dk_T^2}{\bar{v} \xi m_b^2 + k_T^2} \alpha \int \frac{dx dy}{x+y}. \quad (43)$$

The above integration converges now; furthermore it is not dominated by end-point contribution. This illustrates that the treatment of hard spectator diagrams may need further discussion.

There exists ‘‘annihilation topology’’ contributions which may belong to chirally enhanced corrections. In Ref. [25], the authors have discussed this topic and find that a divergent integral $[\int (dx/x)]^2$ will appear. We suspect that this divergence may disappear, similar to the hard spectator term, if the effect of transverse momenta can be included. It is also possible that ‘‘annihilation topology’’ contributions are really dominated by soft interactions and thus violate factorization. Due to its complexity, we do not include ‘‘annihilation topology’’ contributions in this work.

C. Final formulas

With these effective operators, $B \rightarrow P_1 P_2$ decay amplitudes in QCD factorization can be written as

$$A(B \rightarrow P_1 P_2) = \frac{G_F}{\sqrt{2}} \sum_{p=u,c} \sum_{i=1,10} v_p a_i^p \langle P_1 P_2 | Q_i | B \rangle_F, \quad (44)$$

where v_p is the CKM factor, $\langle P_1 P_2 | Q_i | B \rangle_F$ is the factorized matrix element and is the same as the definition of the BSW Lagrangian [1]. Then as an illustration, the explicit expressions of a_i^p ($i=1$ to 10) for $B \rightarrow \pi\pi$ (using symmetric LCDAs of the pion) are obtained. But it is easy to generalize these formulas to the case that the final states are other light pseudoscalars. Furthermore, we take only part of QED corrections into account in our final formula, in particular the QED penguin insertions. Now a_i^p for $B \rightarrow \pi\pi$ in NDR γ_5 scheme is listed as follows:²

$$a_1^u = C_1 + \frac{C_2}{N} + \frac{\alpha_s}{4\pi} \frac{C_F}{N} C_2 F, \quad (45)$$

$$a_2^u = C_2 + \frac{C_1}{N} + \frac{\alpha_s}{4\pi} \frac{C_F}{N} C_1 F, \quad (46)$$

$$a_3 = C_3 + \frac{C_4}{N} + \frac{\alpha_s}{4\pi} \frac{C_F}{N} C_4 F, \quad (47)$$

$$a_4^p = C_4 + \frac{C_3}{N} + \frac{\alpha_s}{4\pi} \frac{C_F}{N} C_3 F - \frac{\alpha_s}{4\pi} \frac{C_F}{N} \left\{ C_1 \left(\frac{4}{3} \log \frac{\mu}{m_b} + G(s_p) - \frac{2}{3} \right) + \left(C_3 - \frac{C_9}{2} \right) \left(\frac{8}{3} \log \frac{\mu}{m_b} + G(0) + G(1) - \frac{4}{3} \right) \right. \\ \left. + \sum_{q=u,d,s,c,b} \left(C_4 + C_6 + \frac{3}{2} e_q C_8 + \frac{3}{2} e_q C_{10} \right) \left(\frac{4}{3} \log \frac{\mu}{m_b} + G(s_q) \right) + G_8 C_{8G} \right\}, \quad (48)$$

$$a_5 = C_5 + \frac{C_6}{N} + \frac{\alpha_s}{4\pi} \frac{C_F}{N} C_6 (-F - 12), \quad (49)$$

$$a_6^p = C_6 + \frac{C_5}{N} - \frac{\alpha_s}{4\pi} \frac{C_F}{N} 6C_5 - \frac{\alpha_s}{4\pi} \frac{C_F}{N} \left\{ C_1 \left[\left(1 + \frac{2}{3} A_\sigma \right) \log \frac{\mu}{m_b} - \frac{1}{2} - \frac{1}{3} A_\sigma + G'(s_p) + G^\sigma(s_p) \right] \right. \\ \left. + \sum_{q=d,b} \left(C_3 - \frac{C_9}{2} \right) \left[\left(1 + \frac{2}{3} A_\sigma \right) \log \frac{\mu}{m_b} - \frac{1}{2} - \frac{1}{3} A_\sigma + G'(s_q) + G^\sigma(s_q) \right] \right. \\ \left. + \sum_{q=u,d,s,c,b} \left(C_4 + C_6 + \frac{3}{2} e_q C_8 + \frac{3}{2} e_q C_{10} \right) \left[\left(1 + \frac{2}{3} A_\sigma \right) \log \frac{\mu}{m_b} + G'(s_q) + G^\sigma(s_q) \right] + \left(\frac{3}{2} + A_\sigma \right) C_{8G} \right\}, \quad (50)$$

$$a_7 = C_7 + \frac{C_8}{N} + \frac{\alpha_s}{4\pi} \frac{C_F}{N} C_8 (-F - 12), \quad (51)$$

²Because of the tedium, we do not calculate the radiative corrections in the HV scheme. However, generally, the results in the NDR scheme and HV scheme can be related by a constant matrix $\Delta \hat{r}_s = \hat{r}_{s,HV} - \hat{r}_{s,NDR}$ [29] which is free from the gauge dependence and infrared structure of the theory. Thus, in principle, we can obtain the results in the HV scheme just by using $\Delta \hat{r}_s$. In [23], the constant matrix has been applied to obtain the results in the NDR and HV scheme for the coefficients a_i which only contain the current-current vertex corrections. But whether we can obtain the expression of a_6 or a_8 in HV scheme in a similar way needs further discussion.

$$\begin{aligned}
a_8^p &= C_8 + \frac{C_7}{N} - \frac{\alpha_s}{4\pi} \frac{C_F}{N} 6C_7 - \frac{\alpha_{em}}{9\pi} \left\{ \left(C_2 + \frac{C_1}{N} \right) \left[\left(1 + \frac{2}{3} A_\sigma \right) \log \frac{\mu}{m_b} - \frac{1}{2} - \frac{1}{3} A_\sigma + G'(s_p) + G^\sigma(s_p) \right] \right. \\
&\quad + \left(C_4 + \frac{C_3}{N} \right) \sum_{q=d,b} \frac{3}{2} e_q \left[\left(1 + \frac{2}{3} A_\sigma \right) \log \frac{\mu}{m_b} - \frac{1}{2} - \frac{1}{3} A_\sigma + G'(s_q) + G^\sigma(s_q) \right] \\
&\quad \left. + \left(C_3 + \frac{C_4}{N} + C_5 + \frac{C_6}{N} \right) \sum_{q=u,d,s,c,b} \frac{3}{2} e_q \left[\left(1 + \frac{2}{3} A_\sigma \right) \log \frac{\mu}{m_b} + G'(s_q) + G^\sigma(s_q) \right] + \left(\frac{3}{4} + \frac{1}{2} A_\sigma \right) C_{7\gamma} \right\}, \quad (52)
\end{aligned}$$

$$a_9 = C_9 + \frac{C_{10}}{N} + \frac{\alpha_s}{4\pi} \frac{C_F}{N} C_{10} F, \quad (53)$$

$$\begin{aligned}
a_{10}^p &= C_{10} + \frac{C_9}{N} + \frac{\alpha_s}{4\pi} \frac{C_F}{N} C_9 F - \frac{\alpha_{em}}{9\pi} \left\{ \left(C_2 + \frac{C_1}{N} \right) \left(\frac{4}{3} \log \frac{\mu}{m_b} + G(s_p) - \frac{2}{3} \right) + \left(C_4 + \frac{C_3}{N} \right) \sum_{q=d,b} \frac{3}{2} e_q \left(\frac{4}{3} \log \frac{\mu}{m_b} + G(s_q) - \frac{2}{3} \right) \right. \\
&\quad \left. + \left(C_3 + \frac{C_4}{N} + C_5 + \frac{C_6}{N} \right) \sum_{q=u,d,s,c,b} \frac{3}{2} e_q \left(\frac{4}{3} \log \frac{\mu}{m_b} + G(s_q) \right) + \frac{1}{2} G_8 C_{7\gamma} \right\}. \quad (54)
\end{aligned}$$

Here $N=3$ is the number of color, $C_F=(N^2-1)/2N$ is the factor of color, $s_q=m_q^2/m_b^2$ and we define the other symbols in the above expressions as

$$F = -12 \ln \frac{\mu}{m_b} - 18 + f^I + f^{II}, \quad (55)$$

$$f^I = \int_0^1 dx g(x) \phi(x), \quad G_8 = \int_0^1 dx G_8(x) \phi(x), \quad (56)$$

$$G(s) = \int_0^1 dx G(s,x) \phi(x), \quad (57)$$

$$G'(s) = \int_0^1 dx G'(s,x) \phi_p(x), \quad (58)$$

$$G^\sigma(s) = \int_0^1 dx G^\sigma(s,x) \frac{\phi_\sigma(x)}{6(1-x)}, \quad A_\sigma = \int_0^1 dx \frac{\phi_\sigma(x)}{6(1-x)}, \quad (59)$$

where $\phi(x)[\phi_p(x), \phi_\sigma(x)]$ is leading twist (twist-3) LCDA of the ejected pion, and the hard-scattering functions are

$$g(x) = 3 \frac{1-2x}{1-x} \ln x - 3i\pi, \quad G_8(x) = \frac{2}{1-x}, \quad (60)$$

$$G(s,x) = -4 \int_0^1 du u(1-u) \ln[s-u(1-u)(1-x)-i\epsilon], \quad (61)$$

$$G'(s,x) = -3 \int_0^1 du u(1-u) \ln[s-u(1-u)(1-x)-i\epsilon], \quad (62)$$

$$\begin{aligned}
G^\sigma(s,x) &= -2 \int_0^1 du u(1-u) \ln[s-u(1-u)(1-x)-i\epsilon] \\
&\quad + \int_0^1 du \frac{u^2(1-u)^2(1-x)}{s-u(1-u)(1-x)-i\epsilon}. \quad (63)
\end{aligned}$$

The contributions from the hard spectator scattering [Figs. 1(g),1(h)] are reduced to the factor f^{II} :

$$\begin{aligned}
f^{II} &= \frac{4\pi^2}{N} \frac{f_\pi f_B}{F_{+}^{B \rightarrow \pi}(0) m_B^2} \int_0^1 d\xi \frac{\Phi_B(\xi)}{\xi} \int_0^1 dx \frac{\phi(x)}{x} \\
&\quad \times \int_0^1 dy \left[\frac{\phi(y)}{1-y} + \frac{2\mu_\pi}{M_B} \frac{\phi_\sigma(y)}{6(1-y)^2} \right]. \quad (64)
\end{aligned}$$

There contains a divergent integral in f^{II} . Here we simply assume that $f(dy/y) \sim \ln(m_b/\Lambda_{QCD})$ (similar to what has been done in Refs. [19,25], though our assumption here is certainly an oversimplification). We thus illustrate numerically the scale dependence of $a_i^p(\pi\pi)$ in Table I. Here we use the asymptotic form of the LCDAs of the light pseudo-scalar meson which are listed in Appendix A, and the other input parameters are taken as follows [13]: $F^{B\pi}(0)=0.33$, $f_B=0.2$ GeV, $f_\pi=133$ MeV, the pole masses $m_b=4.8$ GeV, $m_c=1.4$ GeV, the $\overline{\text{MS}}$ masses $\bar{m}_t(\bar{m}_t)=170$ GeV, $\bar{m}_b(\bar{m}_b)=4.4$ GeV, $\bar{m}_u(2 \text{ GeV})=4.2$ MeV, $\bar{m}_d(2 \text{ GeV})=7.6$ MeV and $\Lambda_{QCD}^{(5)}=225$ MeV.

IV. DISCUSSIONS AND GENERAL REMARKS

A. Color transparency and factorization

Color transparency gives a clear physics picture of QCD factorization. In the argument of the color transparency, a fast-moving small color singlet formed by a pair of $q\bar{q}$ decouples from the surrounding soft gluons. Of course, as mentioned in the previous section, only the decoupling with the

TABLE I. The QCD coefficients $a_i^p(\pi\pi)$ at NLO and LO for the renormalization scales at $\mu=5$ GeV and $\mu=2.5$ GeV, where $r_\chi = 2m_\pi^2/m_b(m_u + m_d)$.

| QCD Coefficients | $\mu=5.0$ GeV | | $\mu=2.5$ GeV | |
|----------------------------|---------------|--------|---------------|--------|
| | NLO | LO | NLO | LO |
| a_1^u | 1.024+0.012i | 1.017 | 1.034+0.024i | 1.037 |
| a_2^u | 0.144-0.076i | 0.188 | 0.123-0.100i | 0.109 |
| a_3 | 0.003+0.002i | 0.002 | 0.004+0.004i | 0.004 |
| a_4^u | -0.027-0.014i | -0.029 | -0.029-0.017i | -0.040 |
| a_4^c | -0.033-0.007i | -0.029 | -0.036-0.007i | -0.040 |
| a_5 | -0.003-0.003i | -0.005 | -0.002-0.005i | -0.010 |
| $r_\chi a_6^u$ | -0.036-0.012i | -0.033 | -0.037-0.011i | -0.040 |
| $r_\chi a_6^c$ | -0.039-0.005i | -0.033 | -0.040-0.004i | -0.040 |
| $a_7 \times 10^5$ | 11.9+2.8i | 13.8 | 0.0+5.4i | 7.6 |
| $r_\chi a_8^u \times 10^5$ | 36.8-10.9i | 36.8 | 45.0-5.2i | 39.8 |
| $r_\chi a_8^c \times 10^5$ | 35.0-6.2i | 36.8 | 44.2+3.1i | 39.8 |
| $a_9 \times 10^5$ | -936.1-13.4i | -928.4 | -953.9-24.5i | -957.3 |
| $a_{10}^u \times 10^5$ | -81.8+58.8i | -141.4 | -58.3+86.1i | -74.0 |
| $a_{10}^c \times 10^5$ | -85.2+63.5i | -141.4 | -60.3+88.8i | -74.0 |

soft gluons is not enough for a factorization formula, the decoupling from the collinear divergence is also necessary. We really find that both of the requirements can be satisfied in the one-loop calculations. So the QCD factorization is guaranteed. Therefore, the calculations in the above sections seem to be a one-loop technical manifestation (or demonstration) of the color transparency. On the other hand, at the leading power of Λ_{QCD}/m_b , the soft or collinear gluons only “see” the direction of the light meson, but are “blind” to the spins of the quark constituents. So the soft or collinear gluon cannot distinguish whether the ejected meson from b quark decay is a light pseudoscalar or a light longitudinally polarized vector meson. As a consequence, the cancellation of the infrared divergence is universal for B decays to two light mesons, no matter whether the meson is a pseudoscalar meson or a vector meson. Therefore, the QCD factorization formula for $B \rightarrow PP$ at the leading power of Λ_{QCD}/m_b is easily generalized to $B \rightarrow PV$ and VV .

Similarly, the color transparency argument cannot only be applied to the strong interactions, but also generalized to the electromagnetic interactions. When the ejected meson is electric neutral, the soft photons also decouple from the fast moving small electric dipole. So QED vertex corrections are also of infrared finiteness. But for the case that the ejected meson is charged, QED corrections are infrared divergent, and the infrared divergence must be cancelled by the soft photon emission mechanism, which is common in the calculation of the radiative corrections for $\mu^- \rightarrow e^- \nu_\mu \bar{\nu}_e$. About this, it is easily covered in the calculation in the previous section, just replacing the QCD vertex by a QED vertex. This can be called a one-loop demonstration for “charge” transparency.

It should be noted that the above arguments must be based on the condition that the ejected meson is in a very compact configuration, then it, as a small color dipole, is disentangled

with the soft gluons. Otherwise, if its size is too large, it is difficult to decouple from the soft gluons. For example, the spectator quark in D meson is very soft, and runs around c quark like a soft quark cloud, which has a large overlap with the B meson spectator system [16]. As a consequence, the process in which D meson is ejected from b decay is dominated by the soft gluon exchange.

B. The scale dependence

From the expressions of the QCD coefficients a_i^p obtained in previous sections, it is found that the renormalization scale dependence of the hadronic matrix elements of the effective operators is recovered. Apparently, we expect this recovered dependence can cancel the scale dependence of the Wilson coefficients C_i .

With the renormalization group equations for the Wilson coefficients $C_i(\mu)$ at leading order logarithm approximation [26],

$$\mu \frac{d}{d\mu} \mathbf{C}(\mu) = \frac{\alpha_s}{4\pi} \hat{\gamma}^{(0)T} \mathbf{C}(\mu) \quad (65)$$

we do find

$$\mu \frac{d}{d\mu} a_i^p = 0 \quad (\text{for } i=1-5 \text{ and } 7, 9, 10) \quad (66)$$

when we neglect the contributions from higher order of α_s . But for a_6 or a_8 , some scale dependence at the order of α_s still remains. Note that other QCD coefficients ($a_{i=1,2,3,4,5,7,9,10}$) are multiplied by the product of the matrix elements of the conserved currents which are independent of the renormalization scale; whereas the coefficient a_6 or a_8 is multiplied by a product of the two matrix elements of scalar and pseudoscalar current

$$-2 \langle P_1 | \bar{d}(1 + \gamma_5)q | 0 \rangle \langle P_2 | \bar{q}(1 - \gamma_5)b | B \rangle,$$

which is still of scale dependence. This scale dependence is generally represented by the factor

$$r_\chi(\mu) = \frac{2m_{P_1}^2}{\bar{m}_b(\mu)[\bar{m}_1(\mu) + \bar{m}_2(\mu)]}$$

after we apply the equations of motion to transform the $(S+P)(S-P)$ matrix elements into the type of $(V-A)(V-A)$. Here m_1 and m_2 are the current masses of the valence quarks in meson P_1 . With the renormalization group equations for the running mass of the current quark

$$\mu \frac{d}{d\mu} \bar{m}(\mu) = -6 \frac{\alpha_s}{4\pi} C_F \bar{m}(\mu), \quad (67)$$

we have

$$\mu \frac{d}{d\mu} r_\chi(\mu) = 12 \frac{\alpha_s}{4\pi} C_F r_\chi(\mu). \quad (68)$$

Consequently, we find

$$\mu \frac{d}{d\mu} (a_{6,s}^p r_\chi) = 0 \quad (69)$$

with the asymptotic form of $\phi_\sigma(u) = 6u(1-u)$. Then, as we expect, the decay amplitude for B decays to two light pseudoscalars predicted by QCD factorization is really independent of the renormalization scale within the constraint $A_\sigma = 1/2$. This also can be obviously seen from the numerical results of $a_i^p(\pi\pi)$ listed in Table I. In particular, if we think that the results of QCD factorization are reliable and really independent of the renormalization scale, maybe $A_\sigma = 1/2$ is a strict constraint for the form of $\phi_\sigma(u)$.

It should be noted that the imaginary part of QCD coefficients a_i only arises at the order of α_s , and depends on the renormalization scale. This dependence would bring some uncertainties in determining the CP asymmetries in B decays. Maybe this scale dependence of the imaginary part could be canceled by the results on higher order of α_s .

C. Comparison to the generalized factorization and PQCD method

Comparing the QCD factorization approach with the generalized factorization [13,14] and PQCD method [31], some interesting comments are in order.

(i) At the zeroth order of α_s , both QCD factorization (BBNS approach) and generalized factorization can reproduce the results of ‘‘naive factorization,’’ at the higher order of α_s , the renormalization scheme and scale dependence for the hadronic matrix elements can be recovered from the hard-scattering kernels T_i^f in BBNS approach and $\hat{\mathbf{m}}_s$ in generalized factorization. However, in generalized factorization, $\hat{\mathbf{m}}_s$ is from the one-loop calculations of quark-level matrix elements. According to Buras *et al.*, quark-level matrix elements are accompanied with infrared divergences. To avoid these divergences, one may assume that external quarks are off-shell. Unfortunately, it will introduce gauge dependence which is also unphysical. But in the BBNS approach, it is different because the external states are all physical and can be approximated as on-shell quarks in the leading order of Λ_{QCD}/m_b . As a consequence, the unphysical gauge dependence does not appear. In the PQCD method, Li *et al.* claim that their method is based on a six-quark system in which the external quarks are on-shell, so the gauge invariance of PQCD predictions is guaranteed. The scale dependence in PQCD prediction is removed by evolving the Wilson coefficients down to the proper hard scale. So no explicit scale dependence is left.

(ii) In generalized factorization and BBNS approach, the hadronic transition form factors are not calculable, and they are dominated by soft gluon exchange and determined only by experiments or some nonperturbative approaches such as sum rule, lattice QCD, etc. In particular, the above assumption can be also justified in the BBNS approach by naive power counting [11,16]. However, in the PQCD method, this naive power counting rule may be invalid when the transverse momenta of the quark constituents and Sudakov suppression are taken into account. So, Li *et al.* thought that the

hadronic transition form factors can be calculated in the PQCD method because b quark is heavy enough and the soft gluon exchange is suppressed by Sudakov form factors. This is the essential difference between the BBNS approach and the PQCD method.

(iii) The generalized factorization considers ‘‘nonfactorizable’’ contributions as intractable. Therefore, one may introduce one or more effective color numbers N_c^{eff} to phenomenologically represent ‘‘nonfactorizable’’ contributions [12–14]. Furthermore, N_c^{eff} is assumed to be universal to maintain predictive power. However, ‘‘nonfactorizable’’ contribution is really process-dependent. In the BBNS approach and the PQCD method, such ‘‘nonfactorizable’’ contributions are indeed calculable in perturbative theory. In consequence, N_c^{eff} need not be introduced.

(iv) As mentioned in the above sections, the strong phases predicted by generalized factorization are only from the BSS mechanism which is represented by the penguin insertion. However, the virtuality of the gluon or photon k^2 in the penguin insertion is ambiguous in generalized factorization, and usually it is approximated around $m_B^2/2$. This brings significant uncertainties for predicting the CP asymmetries for B decays. A particularly interesting result of the BBNS approach is that strong phases are not only from the BSS mechanism but also from the hard scattering, and there are no uncertainties in determining k^2 of penguin insertion. However, compared with the real part of the decay amplitude, the imaginary part is $\mathcal{O}(\alpha_s)$ or power Λ_{QCD}/m_b suppressed and cannot lead to large CP asymmetries, since they come solely from hard scattering processes which are only calculable in the heavy quark limit. In the PQCD method, there is no such α_s suppression in the imaginary part of the decay amplitude. Thus CP asymmetries predicted by the PQCD method are usually greater than the prediction of BBNS approach and generalized factorization. So maybe these differences of prediction for CP asymmetries can be an experimental test for the BBNS and PQCD approaches.

(v) Hard spectator contributions [Figs. 1(g) and 1(h)], which are leading power effects in QCD factorization, miss out in ‘‘naive factorization’’ and ‘‘generalized factorization.’’ They are, however, $\mathcal{O}(\alpha_s)$ suppressed compared with the leading factorized contributions (the hadronic transition form factors). But, in the PQCD method, they are of the same order as the form factors.

(vi) In the PQCD method, penguin contributions receive a dynamical enhancement called ‘‘Fat Penguin’’ [8]. But in generalized and QCD factorization, they are missed. This enhancement in the PQCD method arises from the strong scale dependence of the penguin Wilson coefficients $C_{4,6}$, etc.

(vii) Final state interactions (FSI) do not appear in the three methods. In QCD factorization, Beneke *et al.* point out that the cancellation of the infrared divergences implies that the long distance FSI is power suppressed due to the quark-hadron duality. However, this point of view is controversial [32], but can be examined by the experimental measurements of $B \rightarrow KK$ [33].

D. Limitation of QCD factorization

QCD factorization formula only holds in the heavy quark limit $m_b \rightarrow \infty$. In the real world m_b is only about 4.8 GeV, the validity of the power suppression may be questionable. In particular, for several cases, the power suppressed corrections can be numerically large [16], because the perturbative expansion is in order of α_s which is not small at the realistic scale $\mathcal{O}(m_b)$ compared to Λ_{QCD}/m_b .

(i) The hard ‘‘nonfactorizable’’ contributions computed by QCD factorization are generally small compared to the leading ‘‘factorizable’’ contribution. But when the leading ‘‘factorizable’’ contributions are color suppressed, the ‘‘nonfactorizable’’ contribution may be larger than the leading results. At the same time, the potentially soft contribution, which is formally power suppressed, may be important. For example, in $\bar{B}_d^0 \rightarrow \pi^0 \pi^0$, any perturbative and soft power suppressed contributions can have a significant effect on predicting the branching ratio and CP asymmetry. Furthermore, this problem also arises when the entire leading power contribution is suppressed by small Wilson coefficients, for example, in $B \rightarrow KK$; or when the leading power contribution is suppressed by the small CKM elements.

(ii) An important power suppressed contribution is from the higher twist light-cone wave functions of the light mesons. The chirally enhanced power correction from the two-particle twist-3 wave functions is the most important, and has been partly involved in this work except for the annihilation topologies. Other contribution from multiparticle nonvalence Fock state has been proved to be also power suppressed [16]. However, there is no systematic way to evaluate it. The author of Ref. [34] proposed a way to evaluate the soft gluon exchange contribution from higher twist $q\bar{q}g$ wave functions within the frame of the light-cone sum rule (LCSR). But the accuracy of LCSR is limited due to the quark-hadron duality approximation. On the other hand, power correction from transverse momenta needs a subtle treatment in the future. In Ref. [16], the authors point out that the contribution from the transverse momenta might be considered when we evaluate the hadronic matrix elements to two-loop order. In this case, it is possible that Sudakov suppression might be taken into account as well.

In summary, up to now, we do not have a systematic way to evaluate many kinds of power suppressed corrections for exclusive processes. How to evaluate such corrections in a consistent way within the frame of QCD factorization is a potentially interesting work.

V. SUMMARY

In this work, we give a detailed discussion for QCD factorization involving the complete chirally enhanced power corrections in the heavy quark limit for B decays to two light pseudoscalar mesons, and present some elaborate calculations of radiative corrections at the order of α_s . We point out that the infrared finiteness of the vertex corrections in the chirally enhanced power corrections requires twist-3 light-cone distribution amplitudes (LCDAs) of the light pseudoscalar symmetric. However, even in the symmetric condi-

tion, there is also infrared divergence from the end point of the LCDAs in the hard spectator scattering and annihilation topology. So the transverse momenta and Sudakov suppression should be taken into account. We also point out that the decay amplitude of $B \rightarrow PP$ predicted by QCD factorization is really independent of the renormalization scale, at least at the order of α_s . At last, we briefly compare the QCD factorization to the generalized factorization and PQCD method which are generally used in studying B exclusive hadronic decays.

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APPENDIX A: TWIST-2 AND -3 LCDAS OF LIGHT PSEUDOSCALAR MESON

Two particle twist-2 and twist-3 light-cone distribution amplitudes of light pseudoscalar mesons are defined by the following nonlocal matrix elements [35]:

$$\begin{aligned} \langle P(p') | \bar{q}(y) \gamma_\mu \gamma_5 q(x) | 0 \rangle \\ = -if_P p'_\mu \int_0^1 du e^{iup' \cdot y + \bar{u}p' \cdot x} \phi(u), \end{aligned} \quad (A1)$$

$$\begin{aligned} \langle P(p') | \bar{q}(y) \gamma_5 q(x) | 0 \rangle \\ = -if_P \mu_P \int_0^1 du e^{iup' \cdot y + \bar{u}p' \cdot x} \phi_p(u), \end{aligned} \quad (A2)$$

$$\begin{aligned} \langle P(p') | \bar{q}(y) \sigma_{\mu\nu} \gamma_5 q(x) | 0 \rangle \\ = if_P \mu_P (p'_\mu z_\nu - p'_\nu z_\mu) \int_0^1 du e^{iup' \cdot y + \bar{u}p' \cdot x} \frac{\phi_\sigma(u)}{6}, \end{aligned} \quad (A3)$$

with f_P being the decay constant of the light pseudoscalar, $\mu_P = M_P^2/(m_1 + m_2)$ (m_1 and m_2 are the masses of the constituent quarks in the pseudoscalar), and $z = y - x$. Here $\phi(u)$ is the twist-2 light-cone distribution amplitude; $\phi_p(u)$ and $\phi_\sigma(u)$ are two-particle twist-3 distribution amplitudes. The above definitions can be combined into the below nonlocal matrix element:

$$\begin{aligned} \langle P(p') | \bar{q}_\alpha(y) q_\beta(x) | 0 \rangle \\ = \frac{if_P}{4} \int_0^1 e^{iup' \cdot y + \bar{u}p' \cdot x} \left\{ p'_\mu \gamma_5 \phi(u) - \mu_P \gamma_5 \right. \\ \left. \times \left(\phi_p(u) - \sigma_{\mu\nu} p'^\mu z^\nu \frac{\phi_\sigma(u)}{6} \right) \right\}_{\beta\alpha}. \end{aligned} \quad (A4)$$

Neglecting the three-particle twist-3 light-cone wave function, the asymptotic forms of the above distribution amplitudes are given as

$$\phi(u) = 6u(1-u), \quad (\text{A5})$$

$$\phi_p(u) = 1, \quad (\text{A6})$$

$$\phi_\sigma(u) = 6u(1-u). \quad (\text{A7})$$

APPENDIX B: THE EVOLUTION OF $C_i(\mu)$

The renormalization group equation for the Wilson coefficients $C_i(\mu)$ is written as follows [26]:

$$\mu \frac{d}{d\mu} \mathbf{C}(\mu) = \hat{\gamma}^T \mathbf{C}(\mu). \quad (\text{B1})$$

Here γ is the anomalous dimension matrix, which can be calculated by the perturbative theory and expanded in order of the coupling constants α_s and α_{em} :

$$\hat{\gamma} = \frac{\alpha_s}{4\pi} \hat{\gamma}_s^{(0)} + \left(\frac{\alpha_s}{4\pi} \right)^2 \hat{\gamma}_s^{(1)} + \frac{\alpha_{em}}{4\pi} \hat{\gamma}_e^{(0)} + \frac{\alpha_s \alpha_{em}}{(4\pi)^2} \hat{\gamma}_{se}^{(1)} + \dots \quad (\text{B2})$$

The LO anomalous dimension matrix $\gamma_s^{(0)}$ of the above equations has the explicit form

$$\hat{\gamma}_s^{(0)} = \begin{pmatrix} -\frac{6}{N} & 6 & -\frac{2}{3N} & \frac{2}{3} & -\frac{2}{3N} & \frac{2}{3} & 0 & 0 & 0 & 0 \\ 6 & -\frac{6}{N} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -\frac{22}{3N} & \frac{22}{3} & -\frac{4}{3N} & \frac{4}{3} & 0 & 0 & 0 & 0 \\ 0 & 0 & 6 - \frac{2f}{3N} & -\frac{6}{N} + \frac{2f}{3} & -\frac{2f}{3N} & \frac{2f}{3} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{6}{N} & -6 & 0 & 0 & 0 & 0 \\ 0 & 0 & -\frac{2f}{3N} & \frac{2f}{3} & -\frac{2f}{3N} & -\frac{6(N^2-1)}{N} + \frac{2f}{3} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{6}{N} & -6 & 0 & 0 \\ 0 & 0 & -\frac{2(u-d/2)}{3N} & \frac{2(u-d/2)}{3} & -\frac{2(u-d/2)}{3N} & \frac{2(u-d/2)}{3} & 0 & -\frac{6(N^2-1)}{N} & 0 & 0 \\ 0 & 0 & \frac{2}{3N} & -\frac{2}{3} & \frac{2}{3N} & -\frac{2}{3} & 0 & 0 & -\frac{6}{N} & 6 \\ 0 & 0 & -\frac{2(u-d/2)}{3N} & \frac{2(u-d/2)}{3} & -\frac{2(u-d/2)}{3N} & \frac{2(u-d/2)}{3} & 0 & 0 & 6 & -\frac{6}{N} \end{pmatrix}, \quad (\text{B3})$$

where N is the color number, f is the active flavor number, and u and d denote the number of the active up- and down-type flavors, respectively.

APPENDIX C: SOME USEFUL FEYNMAN PARAMETER INTEGRALS

In calculation of the perturbative diagrams shown in Fig. 1, one might encounter some Feynman parameter integrals which involve nontrivial infrared divergence. To deal with the infrared divergence, as mentioned in preceding sections, the dimensional regularization (DR) and massive gluon (MG) scheme are applied. Below, we give the explicit calculation of some useful Feynman parameter integrals in the above two regularization schemes.

First, we deal with the integrals in the DR scheme. In the DR scheme (here we take $d=4+2a$ and $a>0$), the integrals involving the infrared divergence are written as follows:

$$\int_0^1 dt_1 \int_0^{1-t_1} dt_2 \frac{1}{[t_1(t_1+t_2u)]^{1-a}} = \frac{1}{u} \left[\frac{1}{2a^2} + \frac{2 \ln u}{a} + \frac{1}{2} \ln^2 u - \text{Li}_2 \left(1 - \frac{1}{u} \right) \right], \quad (\text{C1})$$

$$\int_0^1 dt_1 \int_0^{1-t_1} dt_2 \frac{t_2}{[t_1(t_1+t_2u)]^{1-a}} = \frac{1}{u} \left[\frac{1}{a} - 2 + \ln u + \frac{\ln u}{1-u} \right], \quad (\text{C2})$$

$$\int_0^1 dt_1 \int_0^{1-t_1} dt_2 \frac{(1-t_1)(1-t_2)}{(-t_1 t_2 u)^{1-a}} = -\frac{1}{u} \left[\frac{1}{a^2} + \frac{\ln(-u)-2}{a} + \frac{27-\pi^2}{6} - 2 \ln(-u) + \frac{1}{2} \ln^2(-u) \right]. \quad (\text{C3})$$

Here $\text{Li}_2(x)$ is the dilogarithm function. It is defined by

$$\text{Li}_2(x) = -\int_0^x \frac{\ln(1-t)}{t} dt. \quad (\text{C4})$$

The Feynman parameter integrals in the MG scheme are listed as follows:

$$\int_0^1 dt_1 \int_0^{1-t_1} dt_2 \frac{1}{[t_1(t_1+t_2u) + (1-t_1-t_2)\lambda]} = \frac{1}{u} \left[\frac{1}{4} \ln^2 \lambda + \ln(-u) \ln \lambda - 2 \ln u \ln \lambda + \frac{1}{2} \ln^2 u - \text{Li}_2\left(1 - \frac{1}{u}\right) + \frac{5}{4} \pi^2 \right], \quad (\text{C5})$$

$$\int_0^1 dt_1 \int_0^{1-t_1} dt_2 \frac{t_2}{[t_1(t_1+t_2\omega) + (1-t_1-t_2)\lambda]} = \frac{1}{u} \left[-\ln \lambda - 1 + \ln u + \frac{\ln u}{1-u} \right], \quad (\text{C6})$$

$$\int_0^1 dt_1 \int_0^{1-t_1} dt_2 \frac{(1-t_1)(1-t_2)}{-t_1 t_2 u + (1-t_1-t_2)\lambda} = -\frac{1}{u} \left[\frac{1}{2} \ln^2 \lambda - [\ln(-u)-2] \ln \lambda - 2 \ln(-u) + \frac{1}{2} \ln^2(-u) + \frac{5}{2} + \frac{\pi^2}{3} \right]. \quad (\text{C7})$$

When we calculate the above integrals in the MG scheme, the following equations about dilogarithm function may be useful:

$$\text{Li}_2(-x) + \text{Li}_2\left(-\frac{1}{x}\right) = -\frac{\pi^2}{6} - \frac{1}{2} \ln^2 x \quad (x > 0), \quad (\text{C8})$$

$$\text{Li}_2(x) + \text{Li}_2\left(\frac{1}{x}\right) = \frac{\pi^2}{3} - \frac{1}{2} \ln^2 x - i\pi \ln x \quad (x > 1), \quad (\text{C9})$$

$$\text{Li}_2(ix) + \text{Li}_2\left(-\frac{i}{x}\right) = -\frac{\pi^2}{24} - \frac{1}{2} \ln^2 x + \frac{i}{2} \pi \ln x \quad (x > 0), \quad (\text{C10})$$

$$\text{Li}_2(-ix) + \text{Li}_2\left(\frac{i}{x}\right) = -\frac{\pi^2}{24} - \frac{1}{2} \ln^2 x - \frac{i}{2} \pi \ln x \quad (x > 0), \quad (\text{C11})$$

$$\text{Li}_2(x) + \text{Li}_2(1-x) = \frac{\pi^2}{6} - \ln x \ln(1-x). \quad (\text{C12})$$

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