

Subleading Isgur-Wise form factors and $O(1/m_Q)$ corrections to the semileptonic decays $B \rightarrow D_1 l \bar{\nu}$ and $B \rightarrow D_2^* l \bar{\nu}$

Ming-Qiu Huang

CCAST (World Laboratory), P.O. Box 8730, Beijing, 100080

and Department of Applied Physics, Changsha Institute of Technology, Hunan 410073, China

Yuan-Ben Dai

Institute of Theoretical Physics, Academia Sinica, P.O. Box 2735, Beijing 100080, China

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The exclusive semileptonic decays $B \rightarrow D_1(2420)l\bar{\nu}$ and $B \rightarrow D_2^*(2460)l\bar{\nu}$ are studied at the subleading order of heavy quark expansion. The subleading Isgur-Wise functions resulting from the kinetic energy and chromomagnetic corrections to the HQET Lagrangian are calculated by QCD sum rules in the framework of heavy quark effective theory. The decay rates and branching ratios are computed with the inclusion of the order of $1/m_Q$ corrections. It is found that the $1/m_Q$ correction to the decay rate is not large for $B \rightarrow D_2^*$ but is very large for $B \rightarrow D_1$.

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I. INTRODUCTION

In recent years there has been continuous interest in the investigation of semileptonic decays of the B meson into excited charmed mesons. This interest arises for several reasons. The current experimental data show that the exclusive B transitions to the ground state s -wave D and D^* mesons make up only approximately 60% of the inclusive semileptonic decay rate; thus a sizable part of semileptonic B decays should go to excited D meson states. Indeed, these decays have been observed and more experimental data are collected with an increasing accuracy [1–4]. Theoretically, the semileptonic B decays into excited charmed meson states can provide an additional source of information for determining the Cabibbo-Kobayashi-Maskawa (CKM) matrix element V_{cb} as well as exploring the internal dynamics of systems containing heavy-light quarks.

The heavy quark symmetry [5,6] has important consequences on the spectroscopy and weak decay matrix elements of mesons containing a single heavy quark Q . In the infinite mass limit, the spin and parity of the heavy quark and that of the light degrees of freedom are separately conserved. This means that the hadronic states can be classified in degenerate doublets by the total angular momentum j and the angular momentum of the light degrees of freedom j_l . In the case of $\bar{q}Q$ mesons, coupling j_l with the spin of heavy quark $s_Q = 1/2$ yields a doublet with total spin $j = j_l \pm 1/2$. The ground state mesons with $j_l^P = \frac{1}{2}^-$ are the doublet (D, D^*) for $Q = c$ and (B, B^*) for $Q = b$. The excited heavy mesons with $j_l^P = 1/2^+$ and $3/2^+$ can be classified in two doublets of spin symmetry $(0^+, 1^+)$ and $(1^+, 2^+)$, which are identified as (D'_0, D'_1) and (D_1, D_2^*) for charmed mesons, respectively. The other important application of heavy quark symmetries has been the study of semileptonic transitions between two heavy hadrons. The hadronic matrix elements of weak currents between members of the doublets identified by j_l and j_l' can be expressed in terms of universal form factors which are functions of the dot product, $y = v \cdot v'$, of the initial and

final hadron four velocities. A well-known result is that the semileptonic B decays to ground state $D^{(*)}$ mesons, in the $m_Q \rightarrow \infty$ limit, can be described in terms of a single universal function, the Isgur-Wise function $\xi(y)$. The decays to P -wave excited D mesons with $j_l^P = 1/2^+$ and $3/2^+$ require two independent functions, $\zeta(y)$ and $\tau(y)$ [7,8], respectively, in the limit $m_Q \rightarrow \infty$.

There are Λ_{QCD}/m_Q corrections to the weak matrix elements parameterized by form factors at the $m_Q \rightarrow \infty$ limit. The Λ_{QCD}/m_Q corrections to the leading term can be analyzed in a systematic way in heavy quark effective theory (HQET) in terms of a reduced number of universal parameters. The Λ_{QCD}/m_Q corrections may play an important role for B decay modes into excited charmed states since the corresponding transition matrix elements in the infinite mass limit vanish at the zero recoil point because of heavy quark spin symmetry, while Λ_{QCD}/m_Q corrections to these decay matrix elements can give nonzero contributions at zero recoil [8]. The kinematically allowed range for these decays mostly occurs near the zero recoil point, thus the magnitude of Λ_{QCD}/m_Q corrections might be comparable with the leading order result.

The universal functions must be estimated in some non-perturbative approaches. A viable approach is the QCD sum rules [9] formulated in the framework of HQET [10]. This method allows us to relate hadronic observables to QCD parameters via the operator product expansion (OPE) of the correlator. A fruitful application of QCD sum rules has been the determination of the Isgur-Wise functions parameterizing the $B \rightarrow D^{(*)}$ semileptonic transitions up to the Λ_{QCD}/m_Q corrections [11–14]. The QCD sum rule analysis for the semileptonic B decays to excited D mesons involves the determination of the universal form factors. At leading order in the $1/m_Q$ expansion, the two independent universal form factors $\zeta(y)$ and $\tau(y)$, that parametrize transitions $B \rightarrow D^{**}$ (D^{**} being the generic $L = 1$ charmed state), have been calculated with QCD sum rules [15,16]. Moreover, perturbative corrections to $O(\alpha_s)$ have been included in the QCD sum

rule for $\zeta(y)$ in Ref. [17]. The other approaches include various versions of the constituent quark model [18–24] and relativistic Bethe-Salpeter equations [25]. The analysis of Λ_{QCD}/m_Q corrections is an important issue for the semileptonic B decays to excited D mesons. Such corrections have been investigated in terms of meson mass splittings in Ref. [8] and by employing the relativistic quark model in Ref. [26]. The corrections have also been included by a variant approach in HQEFT in Ref. [27]. At the order $1/m_Q$, the corrections for matrix elements of $B \rightarrow D^{**}$ include contributions from higher-dimensional operators in the effective currents and in the effective Lagrangian. For the semileptonic transitions $B \rightarrow D_1 l \bar{\nu}$ and $B \rightarrow D_2^* l \bar{\nu}$, the former give rise to two independent universal functions, denoted by $\tau_1(y)$ and $\tau_2(y)$ [8]. In the framework of QCD sum rules, these two independent form factors have been investigated in our previous work in Ref. [28]. Here we shall focus on the second type of corrections, which originate from higher-order HQET effective Lagrangian.

The remainder of this paper is organized as follows. In Sec. II we review the formulas for the matrix elements of the weak currents including the structure of the Λ_{QCD}/m_Q corrections in the heavy quark effective theory. The QCD sum rule analysis for the subleading Isgur-Wise functions related to the corrections from the insertions of the kinetic energy and chromomagnetic operators is presented in Sec. III. Section IV is devoted to numerical results. Concluding remarks are given in Sec. V.

II. THE HEAVY-QUARK EXPANSION AND THE SUBLEADING ISGUR-WISE FORM FACTORS

The theoretical description of semileptonic decays involves the matrix elements of vector and axial vector currents ($V^\mu = \bar{c} \gamma^\mu b$ and $A^\mu = \bar{c} \gamma^\mu \gamma_5 b$) between B mesons and excited D mesons. For the processes $B \rightarrow D_1 l \bar{\nu}$ and $B \rightarrow D_2^* l \bar{\nu}$, these matrix elements can be parametrized as

$$\langle D_1(v', \epsilon) | V^\mu | B(v) \rangle = f_{V_1} \epsilon^{*\mu} + (f_{V_2} v^\mu + f_{V_3} v'^\mu) \epsilon^* \cdot v, \quad (1a)$$

$$\langle D_1(v', \epsilon) | A^\mu | B(v) \rangle = i f_{A_1} \epsilon^{\mu\alpha\beta\gamma} \epsilon_{\alpha}^* v_\beta v'_\gamma, \quad (1b)$$

$$\begin{aligned} \langle D_2^*(v', \epsilon) | A^\mu | B(v) \rangle &= k_{A_1} \epsilon^{*\mu\alpha} v_\alpha + (k_{A_2} v^\mu + k_{A_3} v'^\mu) \\ &\quad \times \epsilon_{\alpha}^* v^\alpha v'^\beta, \end{aligned} \quad (1c)$$

$$\langle D_2^*(v', \epsilon) | V^\mu | B(v) \rangle = i k_{V_2} \epsilon^{\mu\alpha\beta\gamma} \epsilon_{\alpha\sigma}^* v^\sigma v_\beta v'_\gamma. \quad (1d)$$

Here form factors f_i and k_i are dimensionless functions of y . In the above equations we have used the mass-independent normalization $\langle M(v') | M(v) \rangle = (2\pi)^3 2p^0/m_M \delta^3(p-p')$ for the heavy meson states of momentum $p = m_M v$. Therefore there is a different factor from the corresponding equations in Ref. [8]. The differential decay rates expressed in terms of the form factors are given by (taking the mass of the final lepton to zero)

$$\begin{aligned} \frac{d\Gamma_{D_1}}{dy} &= \frac{G_F^2 |V_{cb}|^2 m_B^5}{48\pi^3} r_1^3 \sqrt{y^2 - 1} \{ 2(1 - 2yr_1 + r_1^2) \\ &\quad \times [f_{V_1}^2 + (y^2 - 1)f_{A_1}^2] + [(y - r_1)f_{V_1} \\ &\quad + (y^2 - 1)(f_{V_3} + r_1 f_{V_2})]^2 \}, \end{aligned} \quad (2)$$

$$\begin{aligned} \frac{d\Gamma_{D_2^*}}{dy} &= \frac{G_F^2 |V_{cb}|^2 m_B^5}{144\pi^3} r_2^3 (y^2 - 1)^{3/2} \{ 3(1 - 2yr_2 + r_2^2) \\ &\quad \times [k_{A_1}^2 + (y^2 - 1)k_{V_2}^2] + 2[(y - r_2)k_{A_1} \\ &\quad + (y^2 - 1)(k_{A_3} + r_2 k_{A_2})]^2 \}, \end{aligned} \quad (3)$$

where $r_1 = m_{D_1}/m_B$ and $r_2 = m_{D_2^*}/m_B$.

The form factors f_i and k_i can be expressed by a set of Isgur-Wise functions at each order in $\Lambda_{\text{QCD}}/m_{c,b}$. This is achieved by evaluating the matrix elements of the effective current operators arising from the HQET expansion of the weak currents. A convenient way to evaluate hadronic matrix elements is by using the trace formalism developed in Ref. [29] to parametrize the matrix elements in Eqs. (1). Following this method, one introduces the matrix representations

$$H_v = \frac{1 + \not{v}}{2} [P_v^{*\mu} \gamma_\mu - P_v \gamma_5], \quad (4a)$$

$$F_v^\mu = \frac{1 + \not{v}}{2} \left\{ P_v^{*\mu\nu} \gamma_\nu - \sqrt{\frac{3}{2}} P_v^\nu \gamma_5 \left[g_\nu^\mu - \frac{1}{3} \gamma_\nu (\gamma^\mu - v^\mu) \right] \right\}, \quad (4b)$$

where P_v , $P_v^{*\mu}$, and P_v^ν , $P_v^{*\mu\nu}$ are annihilation operators for members of the $j_l^P = 1/2^-$ and $3/2^+$ doublets with four-velocity v in HQET. The matrices H and F satisfy $\not{v} H_v = H_v \not{v} = -H_v \not{v}$, $\not{v} F_v^\mu = F_v^\mu \not{v} = -F_v^\mu \not{v}$, $F_v^\mu \gamma_\mu = 0$, and $v_\mu F_v^\mu = 0$.

At the leading order of the heavy quark expansion the hadronic matrix elements of weak current between the states annihilated by the fields in H_v and F_v^σ are written as

$$\bar{h}_v^{(c)} \Gamma h_v^{(b)} = \tau \text{Tr} \{ v_\sigma \bar{F}_v^\sigma \Gamma H_v \}, \quad (5)$$

where $h_v^{(Q)}$ is the heavy quark field in the effective theory and τ is a universal Isgur-Wise function of y .

At the order Λ_{QCD}/m_Q there are contributions to the decay matrix elements originating from corrections to the HQET Lagrangian of the same order,

$$\delta\mathcal{L} = \frac{1}{2m_Q} [O_{\text{kin},v}^{(Q)} + O_{\text{mag},v}^{(Q)}], \quad (6)$$

where

$$O_{\text{kin},v}^{(Q)} = \bar{h}_v^{(Q)} (iD)^2 h_v^{(Q)}, \quad O_{\text{mag},v}^{(Q)} = \bar{h}_v^{(Q)} \frac{g_s}{2} \sigma_{\alpha\beta} G^{\alpha\beta} h_v^{(Q)}.$$

The matrix elements of Λ_{QCD}/m_Q corrections from the insertions of the kinetic energy operator O_{kin} and chromomagnetic operator O_{mag} can be parametrized as

$$i \int d^4x T\{O_{\text{kin},v}^{(c)}(x)[\bar{h}_v^{(c)}\Gamma h_v^{(b)}](0)\} = \eta_{\text{ke}}^{(c)} \text{Tr}\{v_\sigma \bar{F}_v^\sigma \Gamma H_v\}, \quad (7)$$

$$i \int d^4x T\{O_{\text{kin},v}^{(b)}(x)[\bar{h}_v^{(c)}\Gamma h_v^{(b)}](0)\} = \eta_{\text{ke}}^{(b)} \text{Tr}\{v_\sigma \bar{F}_v^\sigma \Gamma H_v\}; \quad (8)$$

$$i \int d^4x T\{O_{\text{mag},v}^{(c)}(x)[\bar{h}_v^{(c)}\Gamma h_v^{(b)}](0)\} = \text{Tr}\left\{\mathcal{R}_{\sigma\alpha\beta}^{(c)} \bar{F}_v^\sigma i\sigma^{\alpha\beta} \frac{1+\not{v}}{2} \Gamma H_v\right\}, \quad (9)$$

$$i \int d^4x T\{O_{\text{mag},v}^{(b)}(x)[\bar{h}_v^{(c)}\Gamma h_v^{(b)}](0)\} = \text{Tr}\left\{\mathcal{R}_{\sigma\alpha\beta}^{(b)} \bar{F}_v^\sigma i\sigma^{\alpha\beta} \frac{1+\not{v}}{2} \Gamma H_v\right\}. \quad (10)$$

The functions $\eta_{\text{ke}}^{(c,b)}(y)$ have mass dimension and effectively correct the leading order Isgur-Wise function $\tau(y)$ since the kinetic energy operator does not violate heavy quark spin symmetry. The most general decomposition for $\mathcal{R}^{(c,b)}$ are

$$\begin{aligned} \mathcal{R}_{\sigma\alpha\beta}^{(c)} &= \eta_1^{(c)} v_\sigma \gamma_\alpha \gamma_\beta + \eta_2^{(c)} v_\sigma v_\alpha \gamma_\beta + \eta_3^{(c)} g_{\sigma\alpha} v_\beta, \\ \mathcal{R}_{\sigma\alpha\beta}^{(b)} &= \eta_1^{(b)} v_\sigma \gamma_\alpha \gamma_\beta + \eta_2^{(b)} v_\sigma v'_\alpha \gamma_\beta + \eta_3^{(b)} g_{\sigma\alpha} v'_\beta, \end{aligned} \quad (11)$$

where η_i are function of y , and have mass dimension 1.

There are also order $\Lambda_{\text{QCD}}/m_{c,b}$ corrections originating from the matching of the $b \rightarrow c$ flavor changing current onto the effective theory, they can be parameterized in terms of two independent Isgur-Wise functions, τ_1 and τ_2 [8].

Summing up all the contributions up to order $\Lambda_{\text{QCD}}/m_{c,b}$, it is straightforward to express the form factors f_i and k_i parametrizing $B \rightarrow D_1 l \bar{\nu}$ and $B \rightarrow D_2^* l \bar{\nu}_e$ semileptonic decays in terms of Isgur-Wise functions. The explicit expressions for f_i and k_i are as follows [8]:

$$\begin{aligned} \sqrt{6}f_A &= -(y+1)\tau - \varepsilon_b\{(y-1)[(\bar{\Lambda}' + \bar{\Lambda})\tau \\ &\quad - (2y+1)\tau_1 - \tau_2] + (y+1)\eta_b\} - \varepsilon_c[4(y\bar{\Lambda}' - \bar{\Lambda})\tau \\ &\quad - 3(y-1)(\tau_1 - \tau_2) + (y+1)(\eta_{\text{ke}} - 2\eta_1 - 3\eta_3)], \end{aligned}$$

$$\begin{aligned} \sqrt{6}f_{V_1} &= (1-y^2)\tau - \varepsilon_b(y^2-1)[(\bar{\Lambda}' + \bar{\Lambda})\tau \\ &\quad - (2y+1)\tau_1 - \tau_2 + \eta_b] - \varepsilon_c[4(y+1)(y\bar{\Lambda}' - \bar{\Lambda})\tau \\ &\quad - (y^2-1)(3\tau_1 - 3\tau_2 - \eta_{\text{ke}} + 2\eta_1 + 3\eta_3)], \end{aligned}$$

$$\begin{aligned} \sqrt{6}f_{V_2} &= -3\tau - 3\varepsilon_b[(\bar{\Lambda}' + \bar{\Lambda})\tau - (2y+1)\tau_1 - \tau_2 + \eta_b] \\ &\quad - \varepsilon_c[(4y-1)\tau_1 + 5\tau_2 + 3\eta_{\text{ke}} + 10\eta_1 + 4(y-1)\eta_2 \\ &\quad - 5\eta_3], \end{aligned}$$

$$\begin{aligned} \sqrt{6}f_{V_3} &= (y-2)\tau + \varepsilon_b \\ &\quad \times \{(2+y)[(\bar{\Lambda}' + \bar{\Lambda})\tau - (2y+1)\tau_1 - \tau_2] \\ &\quad - (2-y)\eta_b\} + \varepsilon_c[4(y\bar{\Lambda}' - \bar{\Lambda})\tau + (2+y)\tau_1 \\ &\quad + (2+3y)\tau_2 + (y-2)\eta_{\text{ke}} - 2(6+y)\eta_1 \\ &\quad - 4(y-1)\eta_2 - (3y-2)\eta_3], \end{aligned}$$

$$\begin{aligned} k_V &= -\tau - \varepsilon_b[(\bar{\Lambda}' + \bar{\Lambda})\tau - (2y+1)\tau_1 - \tau_2 + \eta_b] \\ &\quad - \varepsilon_c(\tau_1 - \tau_2 + \eta_{\text{ke}} - 2\eta_1 + \eta_3), \end{aligned}$$

$$\begin{aligned} k_{A_1} &= -(1+y)\tau - \varepsilon_b \\ &\quad \times \{(y-1)[(\bar{\Lambda}' + \bar{\Lambda})\tau \\ &\quad - (2y+1)\tau_1 - \tau_2] + (1+y)\eta_b\} \\ &\quad - \varepsilon_c[(y-1)(\tau_1 - \tau_2) + (y+1)(\eta_{\text{ke}} - 2\eta_1 + \eta_3)], \end{aligned}$$

$$k_{A_2} = -2\varepsilon_c(\tau_1 + \eta_2),$$

$$\begin{aligned} k_{A_3} &= \tau + \varepsilon_b[(\bar{\Lambda}' + \bar{\Lambda})\tau - (2y+1)\tau_1 - \tau_2 + \eta_b] \\ &\quad - \varepsilon_c(\tau_1 + \tau_2 - \eta_{\text{ke}} + 2\eta_1 - 2\eta_2 - \eta_3), \end{aligned} \quad (12)$$

where $\varepsilon_Q = 1/(2m_Q)$, $\eta_{ke} = \eta_{ke}^c$, and $\eta_b = \eta_{\text{ke}}^{(b)} + 6\eta_1^{(b)} - 2(y-1)\eta_2^{(b)} + \eta_3^{(b)}$, $\bar{\Lambda}(\bar{\Lambda}')$ is mass parameter of ground state (excited) mesons in HQET and the superscript on $\tau_i^{(c)}$ and $\eta_i^{(c)}$ are dropped.

The form factors τ and $\tau_i (i=1,2)$ in HQET, that occur in Eq. (12) have been investigated by using QCD sum rules in our previous work [16,28]. In the following sections we shall extend the QCD sum rule analysis to the calculation of the subleading Isgur-Wise functions, $\eta_{\text{ke}}(y)$ and $\eta_i(y)$, associated with the insertions of kinetic energy and chromomagnetic operators of the HQET Lagrangian, $\delta\mathcal{L}$ in Eq. (6).

III. QCD SUM RULES FOR ISGUR-WISE FUNCTIONS η_{KE} AND η_i

A basic element in the application of QCD sum rules to problems involving excited heavy mesons is to choose a set of appropriate interpolating currents in terms of quark fields each of which creates (annihilates) an excited state of the heavy meson with definite quantum numbers j, P, j_l . The proper interpolating current $J_{j,P,j_l}^{\alpha_1 \dots \alpha_j}$ for the state with arbitrary quantum number j, P, j_l in HQET was given in Ref. [30]. These currents have nice properties. They were proven to satisfy the following conditions:

$$\langle 0 | J_{j,P,j_l}^{\alpha_1 \cdots \alpha_j}(0) | j', P', j_l' \rangle = f_{P,j_l} \delta_{jj'} \delta_{PP'} \delta_{j_l j_l'} \eta^{\alpha_1 \cdots \alpha_j}, \quad (13)$$

$$\begin{aligned} i \langle 0 | T(J_{j,P,j_l}^{\alpha_1 \cdots \alpha_j}(x) J_{j',P',j_l'}^{\dagger \beta_1 \cdots \beta_{j'}}(0)) | 0 \rangle \\ = \delta_{jj'} \delta_{PP'} \delta_{j_l j_l'} (-1)^j S g_t^{\alpha_1 \beta_1} \cdots g_t^{\alpha_j \beta_j} \\ \times \int dt \delta(x - vt) \Pi_{P,j_l}(x) \end{aligned} \quad (14)$$

in the $m_Q \rightarrow \infty$ limit, where $\eta^{\alpha_1 \cdots \alpha_j}$ is the polarization tensor for the spin j state, v is the velocity of the heavy quark, $g_t^{\alpha\beta} = g^{\alpha\beta} - v^\alpha v^\beta$ is the transverse metric tensor, S denotes symmetrizing the indices and subtracting the trace terms separately in the sets $(\alpha_1 \cdots \alpha_j)$ and $(\beta_1 \cdots \beta_j)$, f_{P,j_l} and Π_{P,j_l} are a constant and a function of x , respectively, which depend only on P and j_l . Because of Eqs. (13) and (14), the sum rules in HQET for decay amplitudes derived from a correlator containing such currents receive a contribution only from one of the two states with the same spin parity (j, P) but different j_l in the $m_Q \rightarrow \infty$. Starting from the calculations in the leading order, the decay amplitudes for finite m_Q can be calculated unambiguously order by order in the $1/m_Q$ expansion in HQET.

Following Ref. [30] the local interpolating current for creating 0^- pseudoscalar B mesons is taken as

$$J_{0,-,1/2}^\dagger = \sqrt{\frac{1}{2}} \bar{h}_v \gamma_5 q, \quad (15)$$

and the local interpolating currents for creating 1^+ and 2^+ (D_1, D_2^*) mesons in the doublet (D_1, D_2^*) are taken as

$$J_{1,+,3/2}^\dagger = \sqrt{\frac{3}{4}} \bar{h}_v \gamma_5 (-i) \left(\mathcal{D}_t^\alpha - \frac{1}{3} \gamma_t^\alpha \mathcal{D}_t \right) q, \quad (16)$$

$$J_{2,+,3/2}^\dagger = \sqrt{\frac{1}{2}} \bar{h}_v \frac{(-i)}{2} \left(\gamma_t^{\alpha_1} \mathcal{D}_t^{\alpha_2} + \gamma_t^{\alpha_2} \mathcal{D}_t^{\alpha_1} - \frac{2}{3} g_t^{\alpha_1 \alpha_2} \mathcal{D}_t \right) q, \quad (17)$$

where \mathcal{D} is the covariant derivative and $\gamma_t^\mu = \gamma^\mu - v^\mu$. Note that, without the last term in the bracket in Eq. (16) the current would couple also to the 1^+ state in the doublet $(0^+, 1^+)$ even in the limit of infinite m_Q .

The QCD sum rule calculations for the correlators of two heavy-light currents give [14,30]

$$\begin{aligned} f_{-,1/2}^2 e^{-2\bar{\Lambda}_{-,1/2}/T} = \frac{3}{16\pi^2} \int_0^{\omega_{c1}} \omega^2 e^{-\omega/T} d\omega \\ - \frac{1}{2} \langle \bar{q}q \rangle \left(1 - \frac{m_0^2}{4T^2} \right), \end{aligned} \quad (18)$$

$$\begin{aligned} f_{+,3/2}^2 e^{-2\bar{\Lambda}_{+,3/2}/T} = \frac{1}{2^6 \pi^2} \int_0^{\omega_{c2}} \omega^4 e^{-\omega/T} d\omega \\ - \frac{1}{12} m_0^2 \langle \bar{q}q \rangle - \frac{1}{2^5} \left\langle \frac{\alpha_s}{\pi} G^2 \right\rangle T, \end{aligned} \quad (19)$$

where $m_0^2 \langle \bar{q}q \rangle = \langle \bar{q}g\sigma_{\mu\nu}G^{\mu\nu}q \rangle$.

The QCD sum rule analysis for the subleading form factors proceeds along the same lines as that for the leading order Isgur-Wise function. For the determination of the form factor η_{ke} , which relates to the insertion of Λ_{QCD}/m_c kinetic operator of the HQET Lagrangian, one studies the analytic properties of the three-point correlators:

$$\begin{aligned} i^2 \int d^4x d^4x' d^4z e^{i(k' \cdot x' - k \cdot x)} \langle 0 | T \\ \times [J_{1,+,3/2}^\nu(x') O_{\text{kin},v}^{(c)}(z) \mathcal{J}_{V,A}^{\mu(v,v')}(0) J_{0,-,1/2}^\dagger(x)] | 0 \rangle \\ = \Xi(\omega, \omega', y) \mathcal{L}_{V,A}^{\mu\nu}, \end{aligned} \quad (20a)$$

$$\begin{aligned} i^2 \int d^4x d^4x' d^4z e^{i(k' \cdot x' - k \cdot x)} \langle 0 | T \\ \times [J_{2,+,3/2}^{\alpha\beta}(x') O_{\text{kin},v}^{(c)}(z) \mathcal{J}_{V,A}^{\mu(v,v')}(0) J_{0,-,1/2}^\dagger(x)] | 0 \rangle \\ = \Xi(\omega, \omega', y) \mathcal{L}_{V,A}^{\mu\alpha\beta}, \end{aligned} \quad (20b)$$

where $\mathcal{J}_V^{\mu(v,v')} = \bar{h}(v') \gamma^\mu h(v)$, $\mathcal{J}_A^{\mu(v,v')} = \bar{h}(v') \gamma^\mu \gamma_5 h(v)$. The variables k, k' denote residual ‘‘off-shell’’ momenta which are related to the momenta P of the heavy quark in the initial state and P' in the final state by $k = P - m_Q v$, $k' = P' - m_Q v'$, respectively. For heavy quarks in bound states they are typically of order Λ_{QCD} and remain finite in the heavy quark limit. $\mathcal{L}_{V,A}$ are Lorentz structures associated with the vector and axial vector currents (see the Appendix).

The coefficient $\Xi(\omega, \omega', y)$ in Eq. (20) is an analytic function in the ‘‘off-shell energies’’ $\omega = 2v \cdot k$ and $\omega' = 2v' \cdot k'$ with discontinuities for positive values of these variables. It furthermore depends on the velocity transfer $y = v \cdot v'$, which is fixed in its physical region for the process under consideration. By saturating Eq. (20) with physical intermediate states in HQET, one can isolate the contribution of interest as the one having poles at $\omega = 2\bar{\Lambda}_{-,1/2}$, $\omega' = 2\bar{\Lambda}_{+,3/2}$. Notice that the insertions of the kinetic operator not only renormalize the leading Isgur-Wise function, but also the meson coupling constants and the physical masses of the heavy mesons which define the position of the poles. The correct hadronic representation of the correlator is

$$\begin{aligned} \Xi_{\text{hadro}}(\omega, \omega', y) \\ = \frac{f_{-,1/2} f_{+,3/2}}{(2\bar{\Lambda}_{-,1/2} - \omega - i\epsilon)(2\bar{\Lambda}_{+,3/2} - \omega' - i\epsilon)} \\ \times \left[\eta_{ke}(y) + \left(G_{+,3/2}^K + \frac{K_{+,3/2}}{2\bar{\Lambda}_{+,3/2} - \omega' - i\epsilon} \right) \tau(y) \right] \\ + \text{higher resonance}, \end{aligned} \quad (21)$$

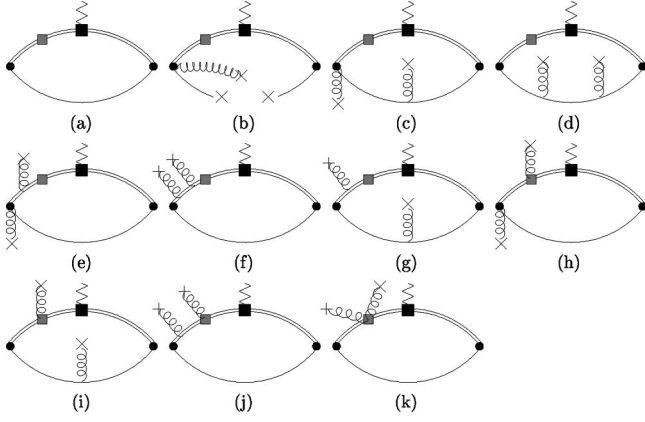


FIG. 1. Feynman diagrams contributing to the sum rules for the Isgur-Wise form factor in the coordinate gauge. The gray square corresponds to the insertion of the kinetic energy operator at $\mathcal{O}(1/m_Q)$ in the HQET Lagrangian.

where f_{P,j_l} are constants defined in Eq. (13), $\bar{\Lambda}_{P,j_l} = m_{P,j_l} - m_Q$, and K_{P,j_l} and G_{P,j_l}^K are defined by [30,32]

$$\langle j, P, j_l | \mathcal{O}_{\text{kin},v}^{(Q)} | j, P, j_l \rangle = K_{P,j_l}, \quad (22a)$$

$$\langle 0 | i \int d^4x \mathcal{O}_{\text{kin},v}^{(Q)}(x) J_{j,P,j_l}^{\alpha_1 \dots \alpha_j}(0) | j, P, j_l \rangle = f_{P,j_l} G_{P,j_l}^K \eta^{\alpha_1 \dots \alpha_j}. \quad (22b)$$

Furthermore, as the result of Eq. (13), only one state with $j^P = 1^+$ or $j^P = 2^+$ contributes to Eq. (21), the other resonance with the same quantum number j^P and different j_l does not contribute. This would not be true for $j^P = 1^+$ if the last term in Eq. (16) is absent.

Following the standard QCD sum rule procedure the calculations of $\Xi(\omega, \omega', y)$ are straightforward. In doing this, for simplicity, the residual momentum k is chosen to be parallel to v such that $k_\mu = (k \cdot v) v_\mu$ (and similar for k'). Confining us to the leading order of perturbation and the operators with dimension $D \leq 5$ in OPE, the relevant Feynman diagrams are shown in Fig. 1. The perturbative part of the spectral density is

$$\begin{aligned} \rho_{\text{pert}}(\tilde{\omega}, \tilde{\omega}', y) &= \frac{3}{2^8 \pi^2} \frac{1}{(1+y)(y^2-1)^{5/2}} \\ &\times [(2y-3)\tilde{\omega}^2 + (8y^2+12y-1)\tilde{\omega}'^2 \\ &+ (6y^2+18y-3)\tilde{\omega}^2 \tilde{\omega}' \\ &- (12y^3+18y^2+9)\tilde{\omega} \tilde{\omega}'^2] \\ &\times \Theta(\tilde{\omega}) \Theta(\tilde{\omega}') \Theta(2y\tilde{\omega} \tilde{\omega}' - \tilde{\omega}^2 - \tilde{\omega}'^2). \end{aligned} \quad (23)$$

The QCD sum rule is obtained by equating the phenomenological and theoretical expressions for Ξ . In doing this the quark-hadron duality needs to be assumed to model the contributions of higher resonance part of Eq. (21). Generally

speaking, the duality is to simulate the resonance contribution by the perturbative part above some threshold energies. In the QCD sum rule analysis for B semileptonic decays into ground state D mesons, it is argued by Neubert, Block, and Shifman in Refs. [6,12] and [14] that the perturbative and the hadronic spectral densities cannot be locally dual to each other, the necessary way to restore duality is to integrate the spectral densities over the “off-diagonal” variable $\tilde{\omega}_- = (\tilde{\omega} - \tilde{\omega}')/2$, keeping the “diagonal” variable $\tilde{\omega}_+ = (\tilde{\omega} + \tilde{\omega}')/2$ fixed. It is in $\tilde{\omega}_+$ that the quark-hadron duality is assumed for the integrated spectral densities. We shall use the same prescription in the case of B semileptonic decays into excited state D mesons.

The Θ functions in Eq. (23) imply that in terms of $\tilde{\omega}_+$ and $\tilde{\omega}_-$ the double discontinuities of the correlator are confined to the region $-\sqrt{y^2-1}/(1+y)\tilde{\omega}_+ \leq \tilde{\omega}_- \leq \sqrt{y^2-1}/(1+y)\tilde{\omega}_+$ and $\tilde{\omega}_+ \geq 0$. According to our prescription an isosceles triangle with the base $\tilde{\omega}_+ = \tilde{\omega}_c$ is retained in the integration domain of the perturbative term in the sum rule.

In view of the asymmetry of the problem at hand with respect to the initial and final states one may attempt to use an asymmetric triangle in the perturbative integral. However, in that case the factor $(y^2-1)^{5/2}$ in the denominator of Eq. (23) is not canceled after the integration so that the Isgur-Wise function or its derivative will be divergent at $y=1$. A similar situation occurs for the sum rule of the Isgur-Wise function for transition between ground states if a different domain is taken in the perturbative integral [14].

In order to suppress the contributions of higher resonance states a double Borel transformation in ω and ω' is performed to both sides of the sum rule, which introduces two Borel parameters T_1 and T_2 . For simplicity we shall take the two Borel parameters equal: $T_1 = T_2 = 2T$.

The nonperturbative power corrections to the correlators are computed from the diagrams involving the quark and gluon condensates in Figs. 1(b)–1(k) in the Fock-Schwinger gauge $x_\mu A^\mu(x) = 0$. We find that the only nonvanishing contribution is the gluon condensate. Note, in particular, the vanishing of the mixed quark-gluon condensate ($D=5$) resulting from the explicit calculation of the diagram shown in Fig. 1(b). After adding the nonperturbative part and making the double Borel transformation one obtains the sum rule for η_{ke} as follows:

$$\begin{aligned} \eta_{\text{ke}}(y) f_{-,1/2} f_{+,3/2} e^{-(\bar{\Lambda}_{-,1/2} + \bar{\Lambda}_{+,3/2})/T} \\ = - \left(G_{+,3/2}^K + \frac{K_{+,3/2}}{2T} \right) \tau(y) f_{-,1/2} f_{+,3/2} \\ \times e^{-(\bar{\Lambda}_{-,1/2} + \bar{\Lambda}_{+,3/2})/T} - \frac{3}{8\pi^2} \frac{y+2}{(y+1)^4} \\ \times \int_0^{\omega_c} d\omega_+ \omega_+^4 e^{-\omega_+/T} + \frac{1}{96} \left\langle \frac{\alpha_s}{\pi} GG \right\rangle \frac{15y-1}{(y+1)^3} T, \end{aligned} \quad (24)$$

in which the sum rule for $\tau(y)$ has been derived from the study of the three-point correlator in Ref. [16] as

$$\begin{aligned} & \tau(y) f_{-,1/2} f_{+,3/2} e^{-(\bar{\Lambda}_{-,1/2} + \bar{\Lambda}_{+,3/2})/T} \\ &= \frac{1}{2\pi^2} \frac{1}{(y+1)^3} \int_0^{\omega_c} d\omega_+ \omega_+^3 e^{-\omega_+/T} - \frac{1}{12} m_0^2 \frac{\langle \bar{q}q \rangle}{T} \\ & \quad - \frac{1}{3 \times 2^5} \left\langle \frac{\alpha_s}{\pi} GG \right\rangle \frac{y+5}{(y+1)^2}. \end{aligned} \quad (25)$$

From the consideration of symmetry, the sum rule for $\eta_{\text{ke}}^{(b)}$ that originates from the insertion of Λ_{QCD}/m_b kinetic operator of the HQET Lagrangian is of the same form as in Eq. (24), but with the HQET parameters $G_{+,3/2}^K$ and $K_{+,3/2}$ replaced by $G_{-,1/2}^K$ and $K_{-,1/2}$, respectively. The definitions of $G_{-,1/2}^K$ and $K_{-,1/2}$ can be found in Eq. (22).

It is worth noting that the QCD $O(\alpha_s)$ corrections have not been included in the sum rule calculations. However, the Isgur-Wise function obtained from the QCD sum rule actually is a ratio of the three-point correlator to the two-point correlator results. While both of these correlators are subject to large perturbative QCD corrections, it is expected that their ratio is not much affected by these corrections because of cancellation. This has been proven to be true in the analysis of Ref. [17].

We now turn to the QCD sum rule calculations of the functions parameterizing the time-ordered products of the chromomagnetic term in the HQET Lagrangian with the leading order currents, η_i ($i=1,2,3$). To obtain QCD sum rules for these universal functions one starts from three-point correlators,

$$\begin{aligned} & i^2 \int d^4x d^4x' d^4z e^{i(k' \cdot x' - k \cdot x)} \langle 0 | T \\ & \quad \times [J_{1,+}^\nu(x') O_{\text{mag},v'}^{(c)}(z) \mathcal{J}_{V,A}^{\mu(v,v')}(0) J_{0,-}^\dagger(x)] | 0 \rangle \\ &= \Xi_{V,A}^{\mu\nu}(\omega, \omega', y), \end{aligned} \quad (26a)$$

$$\begin{aligned} & i^2 \int d^4x d^4x' d^4z e^{i(k' \cdot x' - k \cdot x)} \langle 0 | T \\ & \quad \times [J_{2,+}^{\alpha\beta}(x') O_{\text{mag},v'}^{(c)}(z) \mathcal{J}_{V,A}^{\mu(v,v')}(0) J_{0,-}^\dagger(x)] | 0 \rangle \\ &= \Xi_{V,A}^{\mu\alpha\beta}(\omega, \omega', y). \end{aligned} \quad (26b)$$

By saturating the double dispersion integral for the three-point functions in Eq. (26) with hadronic states, one can isolate the contributions from the double pole at $\omega = 2\bar{\Lambda}_{-,1/2}$, $\omega' = 2\bar{\Lambda}_{+,3/2}$. Similarly, the insertions of the chromomagnetic operator result in the corrections to the leading Isgur-Wise function as well as to the couplings of the heavy mesons to the interpolating currents and to the physical meson masses. It follows from Eq. (26) that

$$\Xi_V^{\mu\nu\text{pole}}(\omega, \omega', y)$$

$$\begin{aligned} &= \frac{f_{-,1/2} f_{+,3/2}}{(2\bar{\Lambda}_{-,1/2} - \omega - i\epsilon)(2\bar{\Lambda}_{+,3/2} - \omega' - i\epsilon)} \\ & \quad \times \left[-\xi_1 \mathcal{L}_V^{\mu\nu} + \xi_2 \mathcal{L}_{V\xi}^{\mu\nu} \right. \\ & \quad \left. + \left(G_{+,3/2}^\Sigma + \frac{\Sigma_{+,3/2}}{2\bar{\Lambda}_{+,3/2} - \omega' - i\epsilon} \right) d_{1,3/2} \tau(y) \mathcal{L}_V^{\mu\nu} \right], \end{aligned} \quad (27)$$

$$\begin{aligned} & \Xi_A^{\mu\nu\text{pole}}(\omega, \omega', y) \\ &= \frac{f_{-,1/2} f_{+,3/2}}{(2\bar{\Lambda}_{-,1/2} - \omega - i\epsilon)(2\bar{\Lambda}_{+,3/2} - \omega' - i\epsilon)} \\ & \quad \times \left[-\xi_1 + \left(G_{+,3/2}^\Sigma + \frac{\Sigma_{+,3/2}}{2\bar{\Lambda}_{+,3/2} - \omega' - i\epsilon} \right) d_{1,3/2} \tau(y) \right] \\ & \quad \times \mathcal{L}_A^{\mu\nu}, \end{aligned} \quad (28)$$

$$\begin{aligned} & \Xi_V^{\mu\alpha\beta\text{pole}}(\omega, \omega', y) \\ &= \frac{f_{-,1/2} f_{+,3/2}}{(2\bar{\Lambda}_{-,1/2} - \omega - i\epsilon)(2\bar{\Lambda}_{+,3/2} - \omega' - i\epsilon)} \\ & \quad \times \left[-\zeta + \left(G_{+,3/2}^\Sigma + \frac{\Sigma_{+,3/2}}{2\bar{\Lambda}_{+,3/2} - \omega' - i\epsilon} \right) d_{2,3/2} \tau(y) \right] \\ & \quad \times \mathcal{L}_V^{\mu\alpha\beta}, \end{aligned} \quad (29)$$

$$\begin{aligned} & \Xi_A^{\mu\alpha\beta\text{pole}}(\omega, \omega', y) \\ &= \frac{f_{-,1/2} f_{+,3/2}}{(2\bar{\Lambda}_{-,1/2} - \omega - i\epsilon)(2\bar{\Lambda}_{+,3/2} - \omega' - i\epsilon)} \\ & \quad \times \left[-\zeta \mathcal{L}_A^{\mu\alpha\beta} + \eta_2 \mathcal{L}_{A\eta_2}^{\mu\alpha\beta} \right. \\ & \quad \left. \times \left(G_{+,3/2}^\Sigma + \frac{\Sigma_{+,3/2}}{2\bar{\Lambda}_{+,3/2} - \omega' - i\epsilon} \right) d_{2,3/2} \tau(y) \mathcal{L}_A^{\mu\alpha\beta} \right], \end{aligned} \quad (30)$$

where $\xi_1 = 2\eta_1 + 3\eta_3$, $\xi_2 = -16\eta_1 - 4(y-1)\eta_2 - 4\eta_3$, $\zeta = 2\eta_1 - \eta_3$, the quantities $\Sigma_{P,j_l}^{(c)}$ and $G_{\Sigma,P,j_l}^{(Q)}$ are defined by [30,32]

$$\langle j, P, j_l | O_{\text{mag},v}^{(Q)} | j, P, j_l \rangle = d_m \Sigma_{P,j_l}, \quad (31a)$$

$$\begin{aligned} & \langle 0 | i \int d^4x O_{\text{mag},v}^{(Q)}(x) J_{j,P,j_l}^{\alpha_1 \dots \alpha_j}(0) | j, P, j_l \rangle \\ &= d_m f_{P,j_l} G_{P,j_l}^\Sigma \eta^{\alpha_1 \dots \alpha_j}, \end{aligned} \quad (31b)$$

$$d_m = d_{j,j_l}, \quad d_{j_l-1/2,j_l} = 2j_l + 2, \quad d_{j_l+1/2,j_l} = -2j_l,$$

$\mathcal{L}_V^{\mu\nu}$, $\mathcal{L}_{V\xi}^{\mu\nu}$, $\mathcal{L}_V^{\mu\alpha\beta}$, $\mathcal{L}_A^{\mu\nu}$, $\mathcal{L}_A^{\mu\alpha\beta}$, and $\mathcal{L}_{A\eta_2}^{\mu\alpha\beta}$ are defined in the Appendix. The three-point correlators (26) can be expressed in QCD in terms of a perturbative part and nonperturbative contributions, which are related to the theoretical calculation in HQET. When we do not consider radiative corrections, the insertions of the chromomagnetic operator only contribute to diagrams involving gluon condensates and do not contribute to the perturbative diagrams since there is no way to contract the gluon contained in O_{mag} . That is, the leading nonperturbative contributions are proportional to the gluon condensates, while the leading perturbative contributions are of order α_s and come from the two-loop radiative corrections to the quark loop. In general, the calculation of the two-loop diagrams is rather cumbersome. In this paper we shall neglect the perturbative term of order α_s and only include the nonperturbative gluon condensates without radiative corrections in the sum rules.

Within this approximation one can perform the calculation conveniently by using the Fock-Schwinger gauge. After making the double Borel transformation, the sum rules for $\eta_i(y)$ are obtained as follows:

$$\begin{aligned} & f_{-,1/2} f_{+,3/2} \eta_1(y) e^{-(\bar{\Lambda}_{-,1/2} + \bar{\Lambda}_{+,3/2})/T} \\ &= -\frac{1}{2} \left(G_{+,3/2}^{\Sigma} + \frac{\Sigma_{+,3/2}}{2T} \right) \tau(y) \\ & \times f_{-,1/2} f_{+,3/2} e^{-(\bar{\Lambda}_{-,1/2} + \bar{\Lambda}_{+,3/2})/T} \\ &+ \frac{1}{480} \left\langle \frac{\alpha_s}{\pi} GG \right\rangle \frac{3y+2}{(y+1)^2} T, \end{aligned} \quad (32a)$$

$$\eta_2(y) = 0, \quad (32b)$$

$$\begin{aligned} & f_{-,1/2} f_{+,3/2} \eta_3(y) e^{-(\bar{\Lambda}_{-,1/2} + \bar{\Lambda}_{+,3/2})/T} \\ &= 2 \left(G_{+,3/2}^{\Sigma} + \frac{\Sigma_{+,3/2}}{2T} \right) \tau(y) \\ & \times f_{-,1/2} f_{+,3/2} e^{-(\bar{\Lambda}_{-,1/2} + \bar{\Lambda}_{+,3/2})/T} \\ &- \frac{1}{240} \left\langle \frac{\alpha_s}{\pi} GG \right\rangle \frac{14+y}{(y+1)^2} T. \end{aligned} \quad (32c)$$

$\eta_{1,2,3}$ are expected to be small compared to Λ_{QCD} since the mass splitting between D_2^* and D_1 is very small. This is supported by the fact that the QCD sum rule calculations indicate that the analogous functions parameterizing the contributions of the chromomagnetic operator for $B \rightarrow D^{(*)} e \bar{\nu}_e$ decays are small [14]. The Λ_{QCD}/m_b correction associated with the insertion of chromomagnetic operator of the HQET Lagrangian can be investigated in a similar way.

IV. NUMERICAL RESULTS AND IMPLICATIONS FOR B DECAYS

We now turn to the numerical evaluation of these sum rules and the phenomenological implications. For the QCD

parameters entering the theoretical expressions, we take the standard values [9,10]

$$\langle \bar{q}q \rangle = -(0.23 \pm 0.02)^3 \text{ GeV}^3,$$

$$\left\langle \frac{\alpha_s}{\pi} GG \right\rangle = (0.012 \pm 0.004) \text{ GeV}^4,$$

$$m_0^2 = (0.8 \pm 0.2) \text{ GeV}^2. \quad (33)$$

In order to obtain numerical results for $\eta_{\text{ke}}(y)$, $\eta_{\text{ke}}^b(y)$, and $\eta_i(y)$ ($i=1,2,3$) from the sum rules which are independent of specific input values of f 's, $\bar{\Lambda}$'s, and τ , we adopt the strategy to evaluate the sum rules by eliminating the explicit dependence on these quantities by using the sum rules for them. Substituting the sum rules (18) and (19) into the left side and the sum rule (25) into the right side of the sum rules (24) and (32) for the three-point correlators, we obtain expressions for the η_{ke} , η_{ke}^b , and η_i ($i=1,2,3$) as functions of the Borel parameter T and the continuum thresholds. This procedure may help to reduce the uncertainties in the calculation. For other HQET parameters we use the following values obtained by QCD sum rules [30–33]:

$$K_{+,3/2} = -(2.0 \pm 0.4) \text{ GeV}^2, \quad G_{+,3/2}^K = -(1.0 \pm 0.45) \text{ GeV}$$

$$K_{-,1/2} = -(1.2 \pm 0.20) \text{ GeV}^2, \quad G_{-,1/2}^K = -(1.6 \pm 0.6) \text{ GeV},$$

$$\Sigma_{+,3/2} = (0.020 \pm 0.003) \text{ GeV}^2,$$

$$G_{+,3/2}^{\Sigma} = (0.013 \pm 0.007) \text{ GeV},$$

$$\Sigma_{-,1/2} = (0.23 \pm 0.07) \text{ GeV}^2,$$

$$G_{-,1/2}^{\Sigma} = (0.042 \pm 0.034 \pm 0.053) \text{ GeV}. \quad (34)$$

Let us evaluate numerically the sum rules for $\eta_{\text{ke}}(y)$ and $\eta_{\text{ke}}^b(y)$ at first. The continuum thresholds ω_{c1} and ω_{c2} in Eqs. (18) and (19) are determined by requiring stability of these sum rules. One finds that $1.7 < \omega_{c1} < 2.2 \text{ GeV}$ and $2.7 < \omega_{c2} < 3.2 \text{ GeV}$ [14,30]. Imposing the usual criterion on the ratio of contribution of the higher-order power corrections and that of the continuum, we find that for the central values of the condensates and HQET parameters given in Eqs. (33) and (34), if the threshold parameter ω_c lies in the range $1.9 < \omega_c < 2.5 \text{ GeV}$, there is an acceptable “stability window” $T = 0.8\text{--}1.0 \text{ GeV}$ in which the calculation results do not change appreciably. This window overlaps largely with those of the sum rules (18) and (19). Therefore our procedure of calculation is justified. For estimating the errors induced by the uncertainties of parameters for the condensates and HQET (33) and (34) we take the maxima deviations from the central values of the condensates and HQET parameters and find that for the existence of stability windows in the two extreme cases the continuum thresholds shift to the range $2.5 < \omega_c < 2.9$ and $1.5 < \omega_c < 1.9 \text{ GeV}$, respectively. The corresponding windows for Borel parameters are

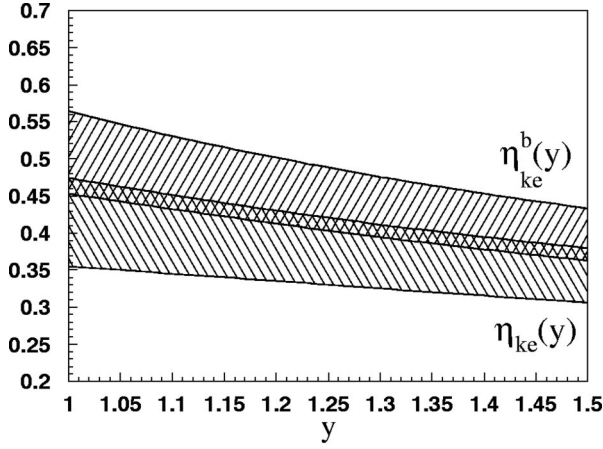


FIG. 2. Results of the numerical evaluation for the sum rules: Isgur-Wise form factors $\eta_{ke}(y)$ and $\eta_{ke}^b(y)$ with $T=0.9$ GeV.

$1.0 < T < 1.2$ GeV and $0.6 < T < 0.8$ GeV, respectively. These are still compatible with the stability windows for the sum rules (18) and (19).

The numerical results of the form factors $\eta_{ke}(y)$ and $\eta_{ke}^b(y)$ are shown in Fig. 2, where the curves refer to various choices for the continuum thresholds and to the central values of the condensates and HQET parameters.

The numerical analysis shows that $\eta_{ke}(y)$ is a slowly varying function in the allowed kinematic range for $B \rightarrow D_1 l \bar{\nu}$ and $B \rightarrow D_2^* l \bar{\nu}$ decays. The resulting curve for $\eta_{ke}(y)$ may be well parametrized by the linear approximation

$$\begin{aligned} \eta_{ke}(y) &= \eta_{ke}(1)[1 - \rho_{\eta}^2(y-1)], \\ \eta_{ke}(1) &= 0.38 \pm 0.17 \text{ GeV}, \quad \rho_{\eta}^2 = 0.8 \pm 0.1, \\ \eta_{ke}^b(y) &= \eta_{ke}^b(1)[1 - \rho_{\eta^b}^2(y-1)], \\ \eta_{ke}^b(1) &= 0.48 \pm 0.21 \text{ GeV}, \quad \rho_{\eta^b}^2 = 1.0 \pm 0.1. \end{aligned} \quad (35)$$

The final sum rules for $\eta_i(y)$ can be obtained by substituting Eq. (25) into Eqs. (32a) and (32c). The numerical evaluation for these sum rules proceeds along the same lines as that for $\eta_{ke}(y)$. Note that we have not included the perturbative term of order α_s , which is the leading perturbative contribution for $\eta_i(y)$. The sum rules for $\eta_i(y)$ are not quantitatively reliable. Nevertheless, they are of correct order of magnitude. The values of the form factors $\eta_1(y)$ and $\eta_3(y)$ at zero recoil as functions of the Borel parameter are shown in Fig. 3, for three different values of the continuum threshold ω_c . The numerical results for $\eta_1(y)$ and $\eta_3(y)$ at zero recoil in the working regions read

$$\eta_1(1) = -0.95 \times 10^{-2}, \quad \eta_3(1) = 3.5 \times 10^{-2}. \quad (36)$$

This result is in agreement with the expectation based on HQET that the spin-symmetry violating corrections described by $\eta_i(y)$ are negligibly small.

Using the forms of linear approximations for $\eta_{ke}(y)$ together with $\tau(y)$ and $\tau_{1,2}(y)$ given in Ref. [16,28],

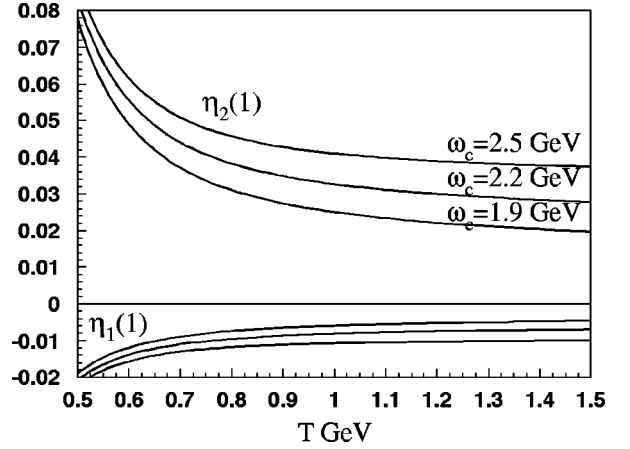


FIG. 3. Dependence of $\eta_1(1)$ and $\eta_2(1)$ on the Borel parameter T for different values of the continuum threshold ω_c .

$$\tau(y) = 0.74[1 - 0.9(y-1)],$$

$$\tau_1(y) = -0.4[1 - 1.4(y-1)], \quad \tau_2 = 0.28[1 - 0.5(y-1)] \quad (37)$$

and neglecting the contribution of chromomagnetic correction, we can calculate the total semileptonic rates and decay branching ratios by integrating Eqs. (2) and (3). We use the physical masses, $m_B = 5.279$, $m_{D_1} = 2.422$ and $m_{D_2^*} = 2.459$ [34], for B , D_1 , and D_2^* mesons. The maximal values of y in the present case are $y_{max}^{D_1} = (1 + r_1^2)/2r_1 \approx 1.32$ and $y_{max}^{D_2^*} = (1 + r_2^2)/2r_2 \approx 1.31$. The quark masses are taken to be $m_b = 4.8$ GeV, $m_c = 1.5$ GeV. In Table I we present our results for decay rates both in the infinitely heavy quark limit and taking account of the first order $1/m_Q$ corrections as well as their ratio

$$R_\infty = \frac{\text{Br}(B \rightarrow D^{**} l \nu)_{\text{with } 1/m_Q}}{\text{Br}(B \rightarrow D^{**} l \nu)_{m_Q \rightarrow \infty}}. \quad (38)$$

From Table I we see that the $B \rightarrow D_1 l \bar{\nu}$ decay rate receives large $1/m_Q$ contributions and gets a sharp increase, while the $B \rightarrow D_2^* l \bar{\nu}$ decay rate is only moderately increased by subleading $1/m_Q$ corrections. The reason for this is as follows. From Eqs. (2), (3), and (12) we see that the $(y - r_1)^2 f_{V_1}^2$ term dominates the differential width for decay to D_1 near $y = 1$. f_{V_1} vanishes at the leading order and receives nonvanishing contributions from first order heavy quark mass corrections:

$$\sqrt{6} f_{V_1}(1) = -8 \varepsilon_c (\bar{\Lambda}' - \bar{\Lambda}) \tau(1). \quad (39)$$

Since the allowed kinematic range for $B \rightarrow D_1 l \bar{\nu}$ is fairly small, the contribution to the decay rate of the $1/m_Q$ corrections is substantially increased. On the other hand, the matrix elements (1c) and (1d) of the $B \rightarrow D_2^* l \bar{\nu}$ decay vanish at zero recoil without using the heavy mass limit. The term $(y - r_2)^2 k_{A_1}^2$ dominates the $B \rightarrow D_2^* l \bar{\nu}$ decay rate, but k_{A_1} does

TABLE I. Decay rates Γ (in 10^{-15} GeV) for $|V_{cb}|=0.04$ and branching ratios BR (in percent and taking $\tau_B=1.6$ ps) for $B \rightarrow D^{**} l \nu$ decays in the infinitely heavy quark mass limit and taking account of first order $1/m_Q$ corrections.

		$B \rightarrow D_1 e \nu$	$B \rightarrow D_2^* e \nu$
$m_Q \rightarrow \infty$	Γ	1.4	2.1
	Br	0.34	0.52
With $1/m_Q$	Γ	5.3	2.4
	Br	1.3	0.59
R_∞		3.78	1.15
Experiment	Br (CLEO) [1]	$0.56 \pm 0.13 \pm 0.08 \pm 0.04$	< 0.8
	Br (ALEPH) [2]	0.74 ± 0.16	< 0.2
	Br (OPAL) [3]	$2.0 \pm 0.6 \pm 0.5$	$0.88 \pm 0.35 \pm 0.17$
	Br (DELPHI) [4]	1.5 ± 0.55	< 6.25

not vanish at the leading order recoil. Therefore this process is not much affected by next-to-leading corrections. Note that although the correction to the rate for decay to D_1 is very large it comes mainly from the effect of the different masses of B and D_1 . The values of η_{ke} and η_{ke}^b in Eq. (23) are of the order Λ_{QCD} and perfectly normal.

In Table I the available experimental data for semileptonic B decay to excited D^{**} mesons are presented. As for the $B \rightarrow D_2^* l \bar{\nu}$ branching ratio there are only upper limits from these experimental groups except the data from OPAL. In comparison with the experimental data our result for the branching ratio of the $B \rightarrow D_1 l \bar{\nu}$ decay with the inclusion of $1/m_Q$ corrections is larger than the CLEO and ALEPH measurements but is consistent with OPAL and DELPHI data. On the other hand, our branching ratio for the $B \rightarrow D_2^* l \bar{\nu}$ decay disagrees with the ALEPH data but is consistent with results from other groups.

V. CONCLUSION

In this work we have presented the investigation for semileptonic B decays into excited charmed mesons. Within the framework of HQET we have applied the QCD sum rules to calculate the universal Isgur-Wise functions up to the subleading order of the heavy quark expansion. The Isgur-Wise functions η_{ke} and η_{ke}^b related to the insertions of kinetic energy operators of the HQET Lagrangian are found of normal values of the order Λ_{QCD} , while the form factors η_i , parametrizing the time-ordered products of the chromomagnetic operator in the HQET Lagrangian with the leading order currents, are negligibly small. These results are in agreement with the HQET-based expectations.

We have computed, for the decays $B \rightarrow D_1 l \bar{\nu}$ and $B \rightarrow D_2^* l \bar{\nu}$, the differential decay widths and the branching ratios with the inclusion of the order of $1/m_Q$ corrections. Our numerical results show that the first order $1/m_Q$ correction is not large for the decay rate of $B \rightarrow D_2^* l \bar{\nu}$ process, but is very large for the $B \rightarrow D_1 l \bar{\nu}$ process. We have explained the reason for this result.

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APPENDIX

We list here the Lorentz structures used in the paper:

$$\mathcal{L}_V^{\mu\nu} = \frac{1}{\sqrt{6}} \{ (y^2 - 1) g_t^{\mu\nu} + [3v^\mu + (2-y)v'^\mu] v_t^\nu \}, \quad (\text{A1})$$

$$\mathcal{L}_A^{\mu\nu} = i \frac{1}{\sqrt{6}} (1+y) \epsilon^{\mu\nu\rho\alpha\beta} g_t^{\rho\nu} v_\alpha v'_\beta, \quad (\text{A2})$$

$$\mathcal{L}_V^{\mu\alpha\beta} = -\frac{i}{2} \epsilon^{\mu\nu\sigma\rho} (v_t^\alpha g_t^{\beta\nu} + g_t^{\alpha\nu} v_t^\beta) v_\sigma v'_\rho, \quad (\text{A3})$$

$$\begin{aligned} \mathcal{L}_A^{\mu\alpha\beta} = & -\frac{1}{2} (1+y) \left(v_t^\alpha g_t^{\mu\beta} + v_t^\beta g_t^{\mu\alpha} - \frac{2}{3} v_t^\mu g_t^{\alpha\beta} \right) \\ & + v'^\mu \left(v_t^\alpha v_t^\beta - \frac{1}{3} (1-y^2) g_t^{\alpha\beta} \right), \end{aligned} \quad (\text{A4})$$

$$\mathcal{L}_{A\xi}^{\mu\nu} = -\frac{1}{\sqrt{6}} (v^\mu + v'^\mu) v_t^\nu, \quad (\text{A5})$$

$$\mathcal{L}_{A\eta_2}^{\mu\alpha\beta} = (-v^\mu + v'^\mu) \left(2v_t^\alpha v_t^\beta - \frac{2}{3} (1-y^2) g_t^{\alpha\beta} \right), \quad (\text{A6})$$

where $g_t^{\alpha\beta} = g^{\alpha\beta} - v'^\alpha v'^\beta$ and $v_t^\alpha = v^\alpha - y v'^\alpha$.

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