Polarization properties of the charmed baryon Λ_c^+ in the $\Lambda_c^+ \to p + K^- + \pi^+ + \pi^0$ decay

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The polarization properties of the charmed Λ_c^+ baryon are investigated in the weak nonleptonic four-body $\Lambda_c^+ \to p + K^- + \pi^+ + \pi^0$ decay. The probability of this decay and the angular distribution of the probability are calculated in the effective quark model with chiral $U(3)\times U(3)$ symmetry incorporating heavy quark effective theory and the extended Nambu–Jona-Lasinio model with a linear realization of chiral $U(3)\times U(3)$ symmetry. The theoretical value of the probability of the decay $\Lambda_c^+ \to p + K^- + \pi^+ + \pi^0$ relative to the probability of the decay $\Lambda_c^+ \to p + K^- + \pi^+$ does not contain free parameters and fits experimental data well. The application of the obtained results to the analysis of the polarization of the Λ_c^+ produced in the processes of photo- and hadroproduction is discussed.

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I. INTRODUCTION

It is known that in reactions of photo- and hadroproduction the charmed baryon Λ_c^+ is produced polarized [1]. The analysis of the Λ_c^+ polarization via the investigation of the decay products should give an understanding of the mechanism of the charmed baryon production at high energies.

Recently [2] we have given a theoretical analysis of the polarization properties of the Λ_c^+ in the mode $\Lambda_c^+ \to p + K^- + \pi^+$. This is the most favorable mode of the Λ_c^+ decays from the experimental point of view. From the theoretical point of view this mode is the most difficult case of the analysis of the weak nonleptonic decays of the Λ_c^+ baryon [1,2]. Indeed, for the calculation of the matrix element of the transition $\Lambda_c^+ \to p + K^- + \pi^+$ the baryonic and mesonic degrees of freedom cannot be fully factorized.

In spite of these theoretical difficulties the problem of the theoretical analysis of the decay $\Lambda_c^+ \to p + K^- + \pi^+$ has been successfully solved within the effective quark model with chiral $U(3)\times U(3)$ symmetry incorporating heavy quark effective theory (HQET) [3,4] and the extended Nambu–Jona-Lasinio (ENJL) model with a linear realization of chiral $U(3)\times U(3)$ symmetry [5–7].[1] Such an effective quark model with chiral $U(3)\times U(3)$ symmetry motivated by the low-energy effective QCD with a linearly rising interquark potential responsible for a quark confinement [9] describes well low-energy properties of light and heavy mesons [5,6] as well as the octet and decuplet of light baryons [7].

In the effective quark model with chiral $U(3)\times U(3)$ symmetry (i) baryons are the three-quark states [10] and do not contain any bound diquark states, then (ii) the spinorial structure of the three-quark currents is defined as the product of the axial-vector diquark densities $[\overline{q^c}_i(x)\gamma^\mu q_j(x)]$ and a quark field $q_k(x)$ transforming under $SU(3)_f\times SU(3)_c$ group like $(6_f,\tilde{3}_c)$ and $(3_f,3_c)$ multiplets, respectively, where i, j, and k are the color indices running through $i=1,2,3$ and $q=u$, d, or s quark field. This agrees with the structure of the three-quark currents used for the investigation of the properties of baryons within QCD sum rules approach [11]. As has been shown in Ref. [9] this is caused by the dynamics of strong low-energy interactions imposed by a linearly rising interquark potential. The fixed structure of the three-quark currents allows us to describe all varieties of low-energy interactions of baryon octet and decuplet in terms of the phenomenological coupling constant $g_{\rm B}$. The coupling constants $g_{\pi\rm NN}$, $g_{\pi\rm N\Delta}$, and $g_{\gamma\rm N\Delta}$ interactions, and the $\sigma_{\pi\rm N}$ term of the low-energy πN scattering have been calculated in good agreement with the experimental data and other phenomenological approaches based on QCD [7,12].

In this paper we apply the effective quark model with chiral $U(3)\times U(3)$ symmetry [2,5–7] to the investigation of the polarization properties of the Λ_c^+ baryon in weak nonleptonic four-body decays and treat the most experimentally favorable four-body mode $\Lambda_c^+ \to p + K^- + \pi^+ + \pi^0$. The experimental value of the probability of this decay is equal to [13]

$$B(\Lambda_c^+ \to pK^-\pi^+\pi^0)_{\rm exp} = (3.4\pm 1.0)\%. \tag{1}$$

Relative to the decay $\Lambda_c^+ \to p + K^- + \pi^+$ the experimental probability of which is $B(\Lambda_c^+ \to pK^-\pi^+) = 0.050\pm 0.013$ [13] the probability of the decay $\Lambda_c^+ \to p + K^- + \pi^+ + \pi^0$ reads

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[1]All results obtained below are valid for the linear sigma model (LσM) [8] supplemented by HQET as well.

$$B(\Lambda_c^+ \to pK^-\pi^+\pi^0/\Lambda_c^+ \to pK^-\pi^+)_{\rm exp} = (0.68\pm 0.27). \tag{2}$$

We would like to emphasize that the weak nonleptonic four-body mode $\Lambda_c^+ \to p+K^-+\pi^++\pi^0$ as well as the mode $\Lambda_c^+ \to p+K^-+\pi^+$ is rather difficult for the theoretical analysis [1,2], since baryonic and mesonic degrees of freedom cannot be fully factorized.

For the theoretical analysis of the weak nonleptonic decays of the Λ_c^+ baryon we would use the effective low-energy Lagrangian [2] (see also Refs. [12] and [14])

$$\mathcal{L}_{\rm eff}(x) = -\frac{G_F}{\sqrt{2}} V_{cs}^* V_{ud}\{C_1(\Lambda_\chi)[\overline{s(x)}\gamma^\mu(1-\gamma^5)c(x)] \times[\overline{u(x)}\gamma_\mu(1-\gamma^5)d(x)] + C_2(\Lambda_\chi)[\overline{u(x)}\gamma^\mu \times(1-\gamma^5)c(x)][\overline{s(x)}\gamma_\mu(1-\gamma^5)d(x)]\}, \tag{3}$$

where $G_F = 1.166\times 10^{-5}$ GeV^{-2} is the Fermi weak constant, V_{cs}^* and V_{ud} are the elements of the Cabibbo-Kobayashi-Maskawa- (CKM-)mixing matrix, $C_i(\Lambda_\chi)$ $(i=1,2)$ are the Wilson coefficients caused by the strong quark-gluon interactions at scales $p > \Lambda_\chi$ (short-distance contributions), where $\Lambda_\chi = 940$ MeV is the scale of spontaneous breaking of chiral symmetry (SBχS) [2,5–7]. The numerical values of the coefficients $C_1(\Lambda_\chi) = 1.24$ and $C_2(\Lambda_\chi) = -0.47$ have been calculated in Ref. [2].

Following Ref. [2] for the calculation of the probability of the decay $\Lambda_c^+ \to p+K^-+\pi^++\pi^0$ we suggest the use of the effective Lagrangian Eq. (3) reduced to the form

$$\mathcal{L}_{\rm eff}(x) = -\frac{G_F}{\sqrt{2}} V_{cs}^* V_{ud}\bar{C}_1(\Lambda_\chi)[\overline{s(x)}\gamma_\mu(1-\gamma^5)c(x)] \times[\overline{u(x)}\gamma^\mu(1-\gamma^5)d(x)] \tag{4}$$

by means of a Fierz transformation [2], where $\bar{C}_1(\Lambda_\chi) = C_1(\Lambda_\chi) + C_2(\Lambda_\chi)/N$ with $N=3$, the number of quark color degrees of freedom.[2]

The paper is organized as follows. In Sec. II we calculate the amplitude of the decay mode $\Lambda_c^+ \to p+K^-+\pi^++\pi^0$. In Sec. III we calculate the angular distribution of the probability and the probability of the decay $\Lambda_c^+ \to p+K^-+\pi^+ +\pi^0$ relative to the probability of the $\Lambda_c^+ \to p+K^-+\pi^+$. In Sec. IV we analyze the polarization properties of the charmed baryon Λ_c^+. In the Conclusion we discuss the obtained results. In the Appendix by example of the $\Lambda_c^+ \to p +\bar{K}^0$ decay we estimate the contribution of exchanges breaking the factorization of baryonic and mesonic degrees of freedom and find the value of order of magnitude less compared with the experimental one fit well by the vacuum intermediate state contribution. This can serve as a justification of a dominance of vacuum intermediate state approximation for the calculation of matrix elements of weak decays of the Λ_c^* [2].

[2]We would like to accentuate that our approach to nonleptonic decays of charmed baryons agrees in principle with the current-algebra analysis of nonleptonic decays of light and charmed baryons based on $(V-A)\times(V-A)$ effective coupling developed by Scadron *et al.* in Refs. [15].

II. AMPLITUDE OF THE $\Lambda_c^+ \to p+K^-+\pi^++\pi^0$ DECAY

The amplitude of the decay $\Lambda_c^+ \to p+K^-+\pi^++\pi^0$ decay we define in the usual way [2,12]

$$\frac{\mathcal{M}[\Lambda_c^+(Q)\to p(q)K^-(q_1)\pi^+(q_2)\pi^0(q_3)]}{\sqrt{2E_{\Lambda_c^+}V2E_pV2E_{K^-}V2E_{\pi^+}V2E_{\pi^0}V}} = \langle p(q)K^-(q_1)\pi^+(q_2)\pi^0(q_3)|\mathcal{L}_{\rm eff}(0)|\Lambda_c^+(Q)\rangle, \tag{5}$$

where $E_i(i=\Lambda_c^+, p, K^-, \pi^+, \pi^0)$ are the energies of the Λ_c^+, the proton, and mesons, respectively.

Since experimentally the probability of the decay mode $\Lambda_c^+ \to p+K^-+\pi^++\pi^0$ is measured relative to the probability of the $\Lambda_c^+ \to p+K^-+\pi^+$ decay, so that we would treat it with respect to the probability of the decay $\Lambda_c^+ \to p+K^- +\pi^+$ the partial width of which has been calculated in Ref. [2] and reads

$$\Gamma(\Lambda_c^+ \to pK^-\pi^+) = |G_F V_{cs}^* V_{ud}\bar{C}_1(\Lambda_\chi)|^2\left[g_{\pi NN}\frac{4}{5}\frac{g_C}{g_B}\frac{F_\pi\Lambda_\chi}{m^2}\right]^2 \times\left[\frac{5M_{\Lambda_c^+}^5}{512\pi^3}\right]\times f(\xi). \tag{6}$$

The function $f(\xi)$ is determined by the integral [2]

$$f(\xi) = \int_\xi^{1+\xi^2/4}\left(1-\frac{3}{5}x+\frac{2}{15}x^2+\frac{7}{60}\xi^2-\frac{2}{5}\frac{\xi^2}{x}\right)x\sqrt{x^2-\xi^2}\,dx = 0.065, \tag{7}$$

where $\xi = 2M_p/M_{\Lambda_c^+}$. The numerical value has been obtained at $M_{\Lambda_c^+} = 2285$ and $M_p = 938$ MeV, the mass of the Λ_c^+ baryon and the proton, respectively, and in the chiral limit, i.e., at zero masses of daughter mesons. The coupling constants g_B and g_C determine the interactions of the proton and the Λ_c^+ baryon with the three-quark currents $\eta_N(x) = -\varepsilon^{ijk}[\overline{u_i^c}(x)\gamma^\mu u_j(x)]\gamma_\mu\gamma^5 d_k(x)$ and $\overline{\eta}_{\Lambda_c^+}(x) = \varepsilon^{ijk}\overline{c_i}(x)\gamma_\mu\gamma^5[\overline{d_j}(x)\gamma^\mu u_k^c(x)]$, respectively [2,7]:

$$\mathcal{L}_{\rm int}(x) = \frac{g_B}{\sqrt{2}}\bar{\psi}_p(x)\eta_N(x) + \frac{g_C}{\sqrt{2}}\bar{\eta}_{\Lambda_c^+}(x)\psi_{\Lambda_c^+}(x) + \text{H.c.} \tag{8}$$

Here $\psi_p(x)$ and $\psi_{\Lambda_c^+}(x)$ are the interpolating fields of the proton and the Λ_c^+ baryon. The coupling constant $g_{\rm B}$ has been related in Ref. [7] to the quark condensate $\langle\overline{q}(0)q(0)\rangle = -255$ MeV,[3] the constituent quark mass $m = 330$ MeV calculated in the chiral limit,[3] the leptonic coupling constant $F_\pi = 92.4$ MeV of pions calculated in the chiral limit, the πNN coupling constant $g_{\pi \rm NN} = 13.4$ and as well as the mass of the proton $M_{\rm p}$:

$$g_{\pi\rm NN} = g_{\rm B}^2 \frac{2m}{3F_\pi}\frac{\langle\overline{q}(0)q(0)\rangle^2}{M_p^2}. \tag{9}$$

Numerically $g_{\rm B}$ is equal to $g_{\rm B} = 1.34\times 10^{-4}$MeV [7]. The coupling constant $g_{\rm C}$ has been fixed in Ref. [2] through the experimental value of the partial width of the decay $\Lambda_c^+ \to p + K^- + \pi^+$. The coupling constant $g_{\rm C}$ appears in all partial widths of the decay modes of the Λ_c^+ baryon and cancels itself in the ratio

$$B(\Lambda_c^+ \to pK^-\pi^+\pi^0/\Lambda_c^+ \to pK^-\pi^+) = \frac{\Gamma(\Lambda_c^+ \to pK^-\pi^+\pi^0)}{\Gamma(\Lambda_c^+ \to pK^-\pi^+)}. \tag{10}$$

The amplitude of the decay $\Lambda_c^+ \to p + K^- + \pi^+ + \pi^0$ we calculate in the three-meson approximation and in the chiral limit [2]

$$\begin{aligned}&\frac{\mathcal{M}[\Lambda_c^+(Q)\to p(q)K^-(q_1)\pi^+(q_2)\pi^0(q_3)]}{\sqrt{2E_{\Lambda_c^+}V2E_pV2E_{K^-}V2E_{\pi^+}V2E_{\pi^0}V}}\\
&= \langle p(q)K^-(q_1)\pi^+(q_2)\pi^0(q_3)|\mathcal{L}_{\rm eff}(0)|\Lambda_c^+(Q)\rangle\\
&= -\frac{G_F}{\sqrt{2}}V_{cs}^*V_{ud}\overline{C}_1(\Lambda_\chi)\\
&\quad\times\langle p(q)K^-(q_1)|\overline{s}(0)\gamma_\mu(1-\gamma^5)c(0)|\Lambda_c^+(Q)\rangle\\
&\quad\times\langle\pi^+(q_2)\pi^0(q_3)|\overline{u}(0)\gamma^\mu(1-\gamma^5)d(0)|0\rangle.\end{aligned} \tag{11}$$

The matrix element of the transition $\Lambda_c^+ \to p + K^-$ has been calculated in Ref. [2] and reads

$$\begin{aligned}&\sqrt{2E_{\Lambda_c^+}V2E_pV2E_{K^-}V}\langle p(q)K^-(q_-)|\overline{s}\\
&\quad\times(0)\gamma_\mu(1-\gamma^5)c(0)|\Lambda_c^+(Q)\rangle\\
&= ig_{\pi\rm NN}\frac{4}{5}\frac{g_{\rm C}}{g_{\rm B}}\frac{\Lambda_\chi}{m^2}\overline{u}_p(q,\sigma')[2v_\mu(1-\gamma^5)\\
&\quad+\gamma_\mu(1+\gamma^5)]u_{\Lambda_c^+}(Q,\sigma)\\
&= ig_{\pi\rm NN}\frac{4}{5}\frac{g_{\rm C}}{g_{\rm B}}\frac{\Lambda_\chi}{m^2}\overline{u}_p(q,\sigma')\\
&\quad\times(1-\gamma^5)(2v_\mu+\gamma_\mu)u_{\Lambda_c^+}(Q,\sigma),\end{aligned} \tag{12}$$

where $\overline{u}_p(q,\sigma')$ and $u_{\Lambda_c^+}(Q,\sigma)$ are the Dirac bispinors of the proton and the Λ_c^+ baryon, v^μ is a four-velocity of the Λ_c^+ baryon defined by $Q^\mu = M_{\Lambda_c^+}v^\mu$.

The matrix element of the transition $0\to\pi^+ + \pi^0$ has been calculated in Ref. [5] and reads

$$\begin{aligned}&\sqrt{2E_{\pi^+}V2E_{\pi^0}V}\langle\pi^+(q_2)\pi^0(q_3)|\overline{u}(0)\gamma^\mu(1-\gamma^5)d(0)|0\rangle\\
&= -\sqrt{2}(q_2-q_3)^\mu.\end{aligned} \tag{13}$$

Hence the amplitude of the decay $\Lambda_c^+ \to p + K^- + \pi^+ + \pi^0$ is given by

$$\begin{aligned}&\mathcal{M}[\Lambda_c^+(Q)\to p(q)K^-(q_1)\pi^+(q_2)\pi^0(q_3)]\\
&= iG_FV_{cs}^*V_{ud}\overline{C}_1(\Lambda_\chi)\frac{4}{5}\frac{g_{\pi\rm NN}}{M_{\Lambda_c^+}}\left[\frac{g_{\rm C}}{g_{\rm B}}\frac{\Lambda_\chi}{m^2}\right]\overline{u}_p(q,\sigma')\\
&\quad\times(1-\gamma^5)[2Q\cdot(q_2-q_3)+M_{\Lambda_c^+}\\
&\quad\times(\hat{q}_2-\hat{q}_3)]u_{\Lambda_c^+}(Q,\sigma).\end{aligned} \tag{14}$$

Now we can proceed to the evaluation of the probability of the $\Lambda_c^+ \to p + K^- + \pi^+ + \pi^0$ decay.

III. PROBABILITY AND ANGULAR DISTRIBUTION OF THE DECAY $\Lambda_c^+ \to p + K^- + \pi^+ + \pi^0$

The differential partial width of the $\Lambda_c^+ \to p + K^- + \pi^+ + \pi^0$ decay is determined by

$$\begin{aligned}&d\Gamma(\Lambda_c^+ \to pK^-\pi^+\pi^0)\\
&= \frac{1}{2M_{\Lambda_c^+}}\overline{|\mathcal{M}[\Lambda_c^+(Q)\to p(q)K^-(q_1)\pi^+(q_2)\pi^0(q_3)]|^2}\\
&\quad\times(2\pi)^4\delta^{(4)}(Q-q-q_1-q_2-q_3)\frac{d^3q}{(2\pi)^32E_p}\\
&\quad\times\frac{d^3q_1}{(2\pi)^32E_{K^-}}\frac{d^3q_2}{(2\pi)^32E_{\pi^+}}\frac{d^3q_1}{(2\pi)^32E_{\pi^0}}.\end{aligned} \tag{15}$$

We calculate the quantity

$$\overline{|\mathcal{M}[\Lambda_c^+(Q)\to p(q)K^-(q_1)\pi^+(q_2)\pi^0(q_3)]|^2}$$

for the polarized Λ_c^+ and unpolarized proton

[3]This agrees with the results obtained by Elias and Scadron [16].

$$\overline{|\mathcal{M}(\Lambda_c^+(Q)\to p(q)K^-(q_1)\pi^+(q_2)\pi^0(q_3))|^2}$$
$$=|G_F V_{cs}^* V_{ud}\bar{C}_1(\Lambda_\chi)|^2\left[\frac{4}{5}\frac{g_{\pi\mathrm{NN}}}{M_{\Lambda_c^+}}\frac{g_{\mathrm{C}}}{g_{\mathrm{B}}}\frac{\Lambda_\chi}{m^2}\right]^2\frac{1}{2}\mathrm{tr}$$
$$\times\{(M_{\Lambda_c^+}+\hat{Q})(1+\gamma^5\hat{\omega}_{\Lambda_c^+})[2Q\cdot(q_2-q_3)$$
$$+M_{\Lambda_c^+}(\hat{q}_2-\hat{q}_3)](1+\gamma^5)(M_p+\hat{q})(1-\gamma^5)$$
$$\times[2Q\cdot(q_2-q_3)+M_{\Lambda_c^+}(\hat{q}_2-\hat{q}_3)]\}, \tag{16}$$

where $\omega^\mu_{\Lambda_c^+}$ is a spacelike unit vector, $\omega^2_{\Lambda_c^+}=-1$, orthogonal to the four-momentum of the Λ_c^+, $Q\cdot\omega_{\Lambda_c^+}=0$. It is related to the direction of the Λ_c^+ spin defined by

$$\omega^\mu_{\Lambda_c^+}=\left(\frac{\vec{Q}\cdot\vec{\omega}_{\Lambda_c^+}}{M_{\Lambda_c^+}},\vec{\omega}_{\Lambda_c^+}+\frac{\vec{Q}(\vec{Q}\cdot\vec{\omega}_{\Lambda_c^+})}{M_{\Lambda_c^+}(E_{\Lambda_c^+}+M_{\Lambda_c^+})}\right), \tag{17}$$

where $\vec{\omega}^2_{\Lambda_c^+}=1$. At the rest frame of the Λ_c^+ we have $\omega^\mu_{\Lambda_c^+}=(0,\vec{\omega}_{\Lambda_c^+})$.

For the differential branching ratio $B(\Lambda_c^+\to pK^-\pi^+\pi^0/\Lambda_c^+\to pK^-\pi^+)$ defined by Eq. (10) we get

$$dB(\Lambda_c^+\to pK^-\pi^+\pi^0/\Lambda_c^+\to pK^-\pi^+)=\frac{1024\pi^3}{1.3M^8_{\Lambda_c^+}}\frac{1}{F_\pi^2}\frac{1}{2}\mathrm{tr}\{(M_{\Lambda_c^+}+\hat{Q})(1+\gamma^5\hat{\omega}_{\Lambda_c^+})[2Q\cdot(q_2-q_3)+M_{\Lambda_c^+}(\hat{q}_2-\hat{q}_3)](1+\gamma^5)$$
$$\times(M_p+\hat{q})(1-\gamma^5)[2Q\cdot(q_2-q_3)+M_{\Lambda_c^+}(\hat{q}_2-\hat{q}_3)]\}(2\pi)^4$$
$$\times\delta^{(4)}(Q-q-q_1-q_2-q_3)\frac{d^3q}{(2\pi)^3 2E_p}\frac{d^3q_1}{(2\pi)^3 2E_{K^-}}\frac{d^3q_2}{(2\pi)^3 2E_{\pi^+}}\frac{d^3q_3}{(2\pi)^3 2E_{\pi^0}}. \tag{18}$$

The trace amounts to

$$\frac{1}{2}\mathrm{tr}\{\cdots\}=16Q\cdot q[Q\cdot(q_2-q_3)]^2+M_{\Lambda_c^+}\{16Q\cdot qQ\cdot(q_2-q_3)(q_2-q_3)\cdot\omega_{\Lambda_c^+}-32[Q\cdot(q_2-q_3)]^2q\cdot\omega_{\Lambda_c^+}\}$$
$$+M^2_{\Lambda_c^+}[24Q\cdot(q_2-q_3)q\cdot(q_2-q_3)-4Q\cdot q(q_2-q_3)^2]+M^3_{\Lambda_c^+}[8q\cdot(q_2-q_3)(q_2-q_3)\cdot\omega_{\Lambda_c^+}-4(q_2-q_3)^2q\cdot\omega_{\Lambda_c^+}]. \tag{19}$$

For the integration over the momenta of π mesons it is useful to apply the formula [2]

$$\int(q_2-q_3)_\alpha(q_2-q_3)_\beta\delta^{(4)}(P-q_2-q_3)\frac{d^3q_2}{2E_{\pi^+}}\frac{d^3q_3}{2E_{\pi^0}}=\frac{\pi}{6}(-P^2g_{\alpha\beta}+P_\alpha P_\beta), \tag{20}$$

where $P=Q-q-q_1$. Integrating over the momenta of pions we arrive at the following expression for the differential branching ratio $B(\Lambda_c^+\to pK^-\pi^+\pi^0/\Lambda_c^+\to pK^-\pi^+)$:

$$dB(\Lambda_c^+\to pK^-\pi^+\pi^0/\Lambda_c^+\to pK^-\pi^+)=\frac{2.1}{4\pi^4}\frac{1}{M^8_{\Lambda_c^+}}\frac{1}{F_\pi^2}(4Q\cdot q[(Q\cdot P)^2-Q^2P^2]+M_{\Lambda_c^+}\{4Q\cdot qQ\cdot PP\cdot\omega_{\Lambda_c^+}$$
$$-8[(Q\cdot P)^2-Q^2P^2]P\cdot\omega_{\Lambda_c^+}\}+M^2_{\Lambda_c^+}(-3Q\cdot qP^2+6Q\cdot Pq\cdot P)$$
$$+M^3_{\Lambda_c^+}(P^2q\cdot\omega_{\Lambda_c^+}+2q\cdot PP\cdot\omega_{\Lambda_c^+}))\frac{d^3q}{E_p}\frac{d^3q_1}{E_{K^-}}. \tag{21}$$

After the integration over the momenta of the K^- meson and the energies of the proton we obtain the angular distribution of the probability of the decay mode $\Lambda_c^+ \to p+K^- + \pi^+ + \pi^0$ relative to the probability of the decay $\Lambda_c^+ \to p+K^- + \pi^+$ in the rest frame of the Λ_c^+ baryon:

$$4\pi \frac{dB}{d\Omega_{\vec{n}_p}}(\Lambda_c^+ \to pK^-\pi^+\pi^0/\Lambda_c^+ \to pK^-\pi^+) = 0.87(1-0.09\vec{n}_p \cdot \vec{\omega}_{\Lambda_c^+}), \quad (22)$$

where $\vec{n}_p = \vec{q}/|\vec{q}|$ is a unit vector directed along the momentum of the proton and $\Omega_{\vec{n}_p}$ is the solid angle of the unit vector $\vec{n}_p$.

Integrating the angular distribution Eq. (22) over the solid angle $\Omega_{\vec{n}_p}$ we obtain the total branching ratio

$$B(\Lambda_c^+ \to pK^-\pi^+\pi^0/\Lambda_c^+ \to pK^-\pi^+) = 0.87. \quad (23)$$

The theoretical value fits well the experimental data Eq. (2): $B(\Lambda_c^+ \to pK^-\pi^+\pi^0/\Lambda_c^+ \to pK^-\pi^+)_{\rm exp} = (0.68 \pm 0.27)$.

IV. POLARIZATION OF THE CHARMED BARYON Λ_c^+

The formula Eq. (22) describes the polarization of the charmed Λ_c^+ baryon relative to the momentum of the proton in the decay mode $\Lambda_c^+ \to p+K^- + \pi^+ + \pi^0$. If the spin of the Λ_c^+ is parallel to the momentum of the proton, the right-handed (R) polarization, the scalar product $\vec{\omega}_{\Lambda_c^+} \cdot \vec{n}_p$ amounts to $\vec{\omega}_{\Lambda_c^+} \cdot \vec{n}_p = \cos\vartheta$. The angular distribution of the probability reads

$$4\pi \frac{dB}{d\Omega_{\vec{n}_p}}(\Lambda_c^+ \to pK^-\pi^+\pi^0/\Lambda_c^+ \to pK^-\pi^+)_{\rm (R)} = 0.87(1-0.09\cos\vartheta). \quad (24)$$

In turn, for the left-handed (L) polarization of the Λ_c^+, the spin of the Λ_c^+ is antiparallel to the momentum of the proton, the scalar product reads $(\vec{\omega}_{\Lambda_c^+} \cdot \vec{n}_p) = -\cos\vartheta$ and the angular distribution becomes equal to

$$4\pi \frac{dB}{d\Omega_{\vec{n}_p}}(\Lambda_c^+ \to pK^-\pi^+\pi^0/\Lambda_c^+ \to pK^-\pi^+)_{\rm (L)} = 0.87(1+0.09\cos\vartheta). \quad (25)$$

Since the coefficient in front of $\cos\vartheta$ is rather small, so that the angular distribution of the probability of the decays is practically isotropic. Therefore one can conclude that in the four-body mode $\Lambda_c^+ \to p+K^- + \pi^+ + \pi^0$ the charmed baryon Λ_c^+ seems to be practically unpolarized.

V. CONCLUSION

We have considered the four-body mode of the weak nonleptonic decay of the charmed Λ_c^+ baryon: $\Lambda_c^+ \to p+K^+ + \pi^+ + \pi^0$. Experimentally this is the most favorable mode among the four-body modes of the Λ_c^+ decays. From the theoretical point of view this mode is rather difficult for the calculation, since baryonic and mesonic degrees of freedom are not fully factorized. However, as has been shown in Ref. [2] this problem has been overcome for the three-body mode $\Lambda_c^+ \to p+K^- + \pi^+$ within the effective quark model with chiral $U(3)\times U(3)$ symmetry incorporating heavy quark effective theory (HQET) and the ENJL model [2].

Following Ref. [2] we have calculated in the chiral limit the probability and angular distribution of the probability of the mode $\Lambda_c^+ \to p+K^+ + \pi^+ + \pi^0$ in the rest frame of the Λ_c^+ baryon and relative to the momentum of the daughter proton. The probability of the mode $\Lambda_c^+ \to p+K^+ + \pi^+ + \pi^0$ is obtained with respect to the probability of the mode $\Lambda_c^+ \to p+K^+ + \pi^+$. The theoretical prediction $B(\Lambda_c^+ \to pK^-\pi^+\pi^0/\Lambda_c^+ \to pK^-\pi^+) = 0.87$ fits well the experimental data $B(\Lambda_c^+ \to pK^-\pi^+\pi^0/\Lambda_c^+ \to pK^-\pi^+)_{\rm exp} = (0.68 \pm 0.27)$. We would like to accentuate that in our approach the probability $B(\Lambda_c^+ \to pK^-\pi^+\pi^0/\Lambda_c^+ \to pK^-\pi^+)$ does not contain free parameters. Hence such an agreement with experimental data testifies a correct description of low-energy dynamics of strong interactions in our approach.

The theoretical angular distribution of the probability of the decay mode $\Lambda_c^+ \to p+K^- + \pi^+ + \pi^0$ predicts a rather weak polarization of the charmed baryon Λ_c^+. This means that for the experimental analysis of the polarization properties of the Λ_c^+ produced in reactions of photo- and hadroproduction the three-body decay mode $\Lambda_c^+ \to p+K^- + \pi^+$ seems to be preferable with respect to the four-body Λ_c^+

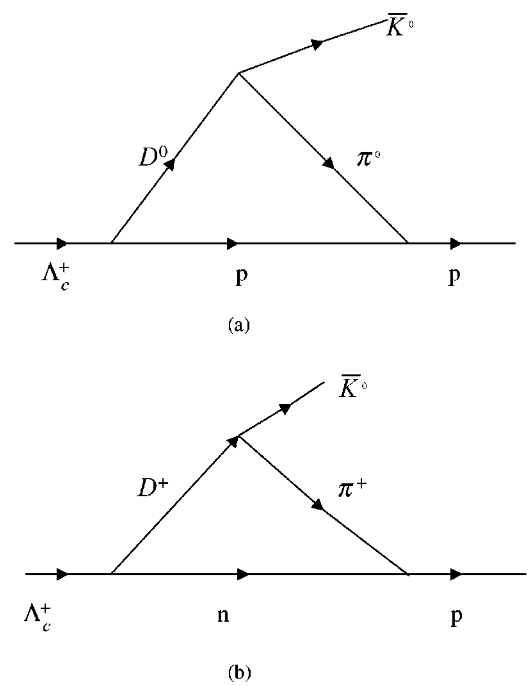

FIG. 1. Feynman diagrams describing the contribution of exchanges breaking the factorization of baryonic and mesonic degrees of freedom for the calculation of the matrix element of the $\Lambda_c^+ \to p + \overline{K^0}$ decay.

$\to p+K^-+\pi^++\pi^0$. Nevertheless, the theoretical analysis of polarization properties of the charmed baryon Λ_c^+ in the weak nonleptonic four-body modes like (1) $\Lambda_c^+\to\Lambda+\pi^++\pi^++\pi^-$, (2) $\Lambda_c^+\to\Sigma^0+\pi^++\pi^++\pi^-$, and (3) $\Lambda_c^+\to p+\bar{K}^0+\pi^++\pi^-$ with branching ratios [13]

$$B(\Lambda_c^+\to\Lambda\pi^+\pi^+\pi^-)_{\rm exp}=(3.3\pm1.0)\%,$$

$$B(\Lambda_c^+\to\Sigma^0\pi^+\pi^+\pi^-)_{\rm exp}=(1.1\pm0.4)\%,$$

$$B(\Lambda_c^+\to p\bar{K}^0\pi^+\pi^-)_{\rm exp}=(2.6\pm0.7)\%$$

comeasurable with the branching ratio of the mode $\Lambda_c^+\to p+K^-+\pi^++\pi^0$ is rather actual and would be carried out in our forthcoming publications.

In the Appendix by example of the decay $\Lambda_c^+\to p+\bar{K}^0$ we have estimated the contribution of exchanges leading to breaking of the factorization of baryonic and mesonic degrees of freedom. We show that the contribution of such exchanges is about an order of magnitude less compared with the contribution of vacuum intermediate state fitting the experimental data well. This justifies a dominance of the vacuum intermediate state approximation for calculations of matrix elements of weak decays of the Λ_c^+ baryon.

APPENDIX: THE UNFACTORIZED CONTRIBUTIONS

The main point of our approach to the calculation of amplitudes of weak transitions of the Λ_c^+ baryon is in the dominance of the vacuum intermediate state approximation or differently the vacuum saturation approximation factorizing either partly, for example, for the decay $\Lambda_c^+\to p+K^-+\pi^+$ or fully, for the decay $\Lambda_c^+\to p+\bar{K}^0$ and so, mesonic and baryonic degrees of freedom. In order to justify the validity of this approach one should estimate the contributions coming from meson-baryon loop exchanges and breaking fully the factorization of mesonic and baryonic degrees of freedom. Below we demonstrate the validity to neglect the contribution of exchanges breaking fully a factorization mesonic and baryonic degrees of freedom. For simplicity we treat the decay $\Lambda_c^+\to p+\bar{K}^0$. The probability of this decay has been calculated in Ref. [2] and earlier by Karlson and Scadron [17]. The Feynman diagrams of the exchanges breaking the factorization of mesonic and baryonic degrees of freedom in the decay $\Lambda_c^+\to p+\bar{K}^0$ are depicted in Fig. 1. The analytical expressions of these diagrams read

$$\begin{aligned}\mathcal{M}[\Lambda_c^+(Q)\to p(q)\bar{K}^0(q')]_{\rm unfact.}=&-G_FV^*_{cs}V_{ud}\bar{C}_2(\Lambda_\chi)F_Kg_{\rm D^0\bar{K}^0\pi^0}\frac{g^2_{\pi\rm NN}}{8\pi^2}\bar{u}_p(q,\sigma')\int\frac{d^4k}{\pi^2 i}\gamma^5\frac{1}{M_{\rm p}-\hat{q}-\hat{k}}\gamma^5\\&\times\frac{q'\cdot(2k+q')}{M^2_{\rm D^0}-(k+q')^2}\frac{1}{M^2_{\pi^0}-k^2}u_{\Lambda_c^+}(Q,\sigma)-G_FV^*_{cs}V_{ud}\bar{C}_2(\Lambda_\chi)F_Kg_{\rm D^+\bar{K}^0\pi^+}\frac{g^2_{\pi\rm NN}}{8\pi^2}\bar{u}_p\\&\times(q,\sigma')\int\frac{d^4k}{\pi^2 i}\gamma^5\frac{1}{M_{\rm n}-\hat{q}-\hat{k}}\gamma^5\frac{q'\cdot(2k+q')}{M^2_{\rm D^+}-(k+q')^2}\frac{1}{M^2_{\pi^+}-k^2}u_{\Lambda_c^+}(Q,\sigma),\end{aligned}\tag{A1}$$

where for simplicity we have set $g_{\Lambda_c^+{\rm D^0p}}=g_{\Lambda_c^+{\rm D^+n}}=\sqrt{2}g_{\pi\rm NN}$. Then, $F_K=1.23F_\pi$MeV [13] is a leptonic coupling constant of K mesons, $M_{\rm n}=M_{\rm p}=938$ MeV, $\bar{C}_2(\Lambda_\chi)=-0.057$ [2], $g_{\rm D^0\bar{K}^0\pi^0}=(7.25\pm0.37)$, and $g_{\rm D^+\bar{K}^0\pi^+}=(4.10\pm0.07)$ are effective coupling constants describing contributions of strong interactions to the amplitudes of weak decays $D^0\to\bar{K}^0+\pi^0$ and $D^+\to\bar{K}^0+\pi^+$, respectively, estimated from the experimental data [13]. The amplitudes of the decays $D^0\to\bar{K}^0+\pi^0$ and $D^+\to\bar{K}^0+\pi^+$ we define as

$$\begin{aligned}&\mathcal{M}[D^0(p)\to\bar{K}^0(p_K)\pi^0(p_{\pi^0})]\\&\quad=-G_FV^*_{cs}V_{ud}\bar{C}_2(\Lambda_\chi)F_Kg_{\rm D^0\bar{K}^0\pi^0}(p+p_{\pi^0})\cdot p_K,\\&\mathcal{M}[D^+(p)\to\bar{K}^0(p_K)\pi^+(p_{\pi^+})]\\&\quad=-G_FV^*_{cs}V_{ud}\bar{C}_2(\Lambda_\chi)F_Kg_{\rm D^+\bar{K}^0\pi^+}(p+p_{\pi^+})\cdot p_K.\end{aligned}\tag{A2}$$

On mass shell of interacting particles we have

$$\begin{aligned}&\mathcal{M}[D^0(p)\to\bar{K}^0(p_K)\pi^0(p_{\pi^0})]\\&\quad=-G_FV^*_{cs}V_{ud}\bar{C}_2(\Lambda_\chi)F_Kg_{\rm D^0\bar{K}^0\pi^0}(M^2_{D^0}-M^2_{\pi^0}),\\&\mathcal{M}[D^+(p)\to\bar{K}^0(p_K)\pi^+(p_{\pi^+})]\\&\quad=-G_FV^*_{cs}V_{ud}\bar{C}_2(\Lambda_\chi)F_Kg_{\rm D^+\bar{K}^0\pi^+}(M^2_{D^0}-M^2_{\pi^+}).\end{aligned}\tag{A3}$$

In the chiral limit $q'\to0$ and $M_\pi\to0$ the amplitude Eq. (A1) is defined by

$$\mathcal{M}[\Lambda_c^+(Q)\to p(q)\overline{K}^0(q')]_{\rm unfact.}$$
$$=G_F V_{cs}^{*}V_{ud}\overline{C}_2(\Lambda_\chi)F_K g_{\rm D^0\overline{K}^0\pi^0}\frac{g_{\pi \rm NN}^2}{4\pi^2}\overline{u}_p(q,\sigma')$$
$$\times\int\frac{d^4k}{\pi^2 i}\gamma^5\frac{1}{M_{\rm p}-\hat{q}-\hat{k}}\gamma^5\frac{q'\cdot k}{M_{\rm D^0}^2-k^2}\frac{1}{k^2}u_{\Lambda_c^+}$$
$$\times(Q,\sigma)+G_F V_{cs}^{*}V_{ud}\overline{C}_2(\Lambda_\chi)F_K g_{\rm D^+\overline{K}^0\pi^+}$$
$$\times\frac{g_{\pi \rm NN}^2}{4\pi^2}\overline{u}_p(q,\sigma')\int\frac{d^4k}{\pi^2 i}\gamma^5\frac{1}{M_{\rm p}-\hat{q}-\hat{k}}\gamma^5$$
$$\times\frac{q'\cdot k}{M_{\rm D^+}^2-k^2}\frac{1}{k^2}u_{\Lambda_c^+}(Q,\sigma). \quad (A4)$$

According to our prescription [2,6] we should keep only divergent contributions. This yields

$$\mathcal{M}[\Lambda_c^+(Q)\to p(q)\overline{K}^0(q')]_{\rm unfact.}$$
$$=-G_F V_{cs}^{*}V_{ud}\overline{C}_2(\Lambda_\chi)F_K(g_{\rm D^0\overline{K}^0\pi^0}$$
$$+g_{\rm D^+\overline{K}^0\pi^+})\frac{g_{\pi \rm NN}^2}{16\pi^2}(M_{\Lambda_c^+}-M_p)$$
$$\times\int\frac{d^4k}{\pi^2 i}\frac{1}{M_{\rm p}^2-k^2}\frac{1}{M_{\rm D^+}^2-k^2}$$
$$\times[\overline{u}_p(q,\sigma')u_{\Lambda_c^+}(Q,\sigma)]. \quad (A5)$$

Integrating over k_0 we obtain the expression

$$\mathcal{M}[\Lambda_c^+(Q)\to p(q)\overline{K}^0(q')]_{\rm unfact.}=-G_F V_{cs}^{*}V_{ud}\overline{C}_2(\Lambda_\chi)F_K(g_{\rm D^0\overline{K}^0\pi^0}+g_{\rm D^+\overline{K}^0\pi^+})\frac{g_{\pi \rm NN}^2}{14\pi^2}\frac{M_{\Lambda_c^+}-M_p}{M_D^2-M_p^2}$$
$$\times\int_0^\Lambda d|\vec{k}|\left(\frac{\vec{k}^2}{\sqrt{M_p^2+\vec{k}^2}}-\frac{\vec{k}^2}{\sqrt{M_D^2+\vec{k}^2}}\right)[\overline{u}_p(q,\sigma')u_{\Lambda_c^+}(Q,\sigma)]$$
$$=G_F V_{cs}^{*}V_{ud}\overline{C}_2(\Lambda_\chi)F_K(g_{\rm D^0\overline{K}^0\pi^0}+g_{\rm D^+\overline{K}^0\pi^+})\frac{g_{\pi \rm NN}^2}{8\pi^2}(M_{\Lambda_c^+}-M_p)[\overline{u}_p(q,\sigma')u_{\Lambda_c^+}(Q,\sigma)]$$
$$\times\left[\Lambda\sqrt{\Lambda^2+M_D^2}-\Lambda\sqrt{\Lambda^2+M_p^2}-M_D^2\ln\left(\frac{\Lambda}{M_D}+\sqrt{1+\frac{\Lambda^2}{M_D^2}}\right)\right.$$
$$\left.+M_p^2\ln\left(\frac{\Lambda}{M_p}+\sqrt{1+\frac{\Lambda^2}{M_p^2}}\right)\right], \quad (A6)$$

where we have set $M_{D^0}=M_{D^+}=M_D=1870$ MeV and the cutoff $\Lambda=\Lambda_\chi/\sqrt{2}=670$ MeV restricts from above three-momenta of virtual particles [6].

The partial width of the decay $\Lambda_c^+\to p+\overline{K}^0$ caused by unfactorized exchanges is equal to

$$\Gamma(\Lambda_c^+\to\overline{K}^0 p)_{\rm unfact.}=\left\{G_F V_{cs}^{*}V_{ud}\overline{C}_2(\Lambda_\chi)F_K(g_{\rm D^0\overline{K}^0\pi^0}+g_{\rm D^+\overline{K}^0\pi^+})\frac{g_{\pi \rm NN}^2}{8\pi^2}\frac{M_{\Lambda_c^+}-M_p}{M_D^2-M_p^2}\right.$$
$$\left.\times\left[\Lambda\sqrt{\Lambda^2+M_D^2}-\Lambda\sqrt{\Lambda^2+M_p^2}-M_D^2\ln\left(\frac{\Lambda}{M_D}+\sqrt{1+\frac{\Lambda^2}{M_D^2}}\right)+M_p^2\ln\left(\frac{\Lambda}{M_p}+\sqrt{1+\frac{\Lambda^2}{M_p^2}}\right)\right]\right\}^2\frac{M_{\Lambda_c^+}}{16\pi}$$
$$\times\left[\left(1+\frac{M_{\rm p}}{M_{\Lambda_c^+}}\right)^2-\frac{M_{\overline{K}^0}^2}{M_{\Lambda_c^+}^2}\right]\sqrt{\left[1-\frac{(M_{\rm p}+M_{\overline{K}^0})^2}{M_{\Lambda_c^+}^2}\right]\left[1-\frac{(M_{\rm p}-M_{\overline{K}^0})^2}{M_{\Lambda_c^+}^2}\right]}. \quad (A7)$$

The numerical value of $\Gamma(\Lambda_c^+ \to \overline{K}^0 p)_{\rm unfact.}$ calculated at $M_{\overline{K}^0} = 498$ MeV [13] amounts to

$$\Gamma(\Lambda_c^+ \to \overline{K}^0 p)_{\rm unfact.} = (4.60 \pm 0.50) \times 10^{-13} \text{ MeV}. \tag{A8}$$

The experimental value of the partial width of the decay $\Lambda_c^+ \to \overline{K}^0 p$ is [13]

$$\Gamma(\Lambda_c^+ \to \overline{K}^0 p)_{\rm exp} = (7.36 \pm 1.97) \times 10^{-11} \text{ MeV}. \tag{A9}$$

Thus the unfactorized exchanges make the contribution to the amplitude of the decay $\Lambda_c^+ \to p + \overline{K}^0$ of order of magnitude less compared with the experimental one. Hence such a contribution does not exceed the experimental error of the partial width and can be neglected without damage for the description.

Since the contribution of a vacuum intermediate state to the amplitude of the decay $\Lambda_c^+ \to p + \overline{K}^0$ fits well the experimental value of the partial width [2], this estimate can serve as a justification of the dominance of vacuum intermediate state approximation for the calculation of matrix elements of weak decays of the Λ_c^+.

[1] J. D. Bjorken, Phys. Rev. D **40**, 1513 (1989), and references therein.

[2] Ya. A. Berdnikov, A. N. Ivanov, V. F. Kosmach, and N. I. Troitskaya, Phys. Rev. C **60**, 015201 (1999).

[3] E. Eichten and F. L. Feinberg, Phys. Rev. D **23**, 2724 (1981); E. Eichten, Nucl. Phys. B (Proc. Suppl.) **4**, 70 (1988); M. B. Voloshin and M. A. Shifman, Sov. J. Nucl. Phys. **45**, 292 (1987); H. D. Politzer and M. Wise, Phys. Lett. B **206**, 681 (1988); **208**, 504 (1988); H. Georgi, *ibid.* **240**, 447 (1990).

[4] M. Neubert, Phys. Rep. **245**, 259 (1994); M. Neubert, Heavy Quark Effective Theory, CERN-TH/96-292, 1996, invited talk presented at the 20th Johns Hopkins Workshop on Current Problems in Particle Theory, Heidelberg, Germany, 1996, hep-ph/9610385.

[5] A. N. Ivanov, M. Nagy, and N. I. Troitskaya, Int. J. Mod. Phys. A **7**, 7305 (1992); A. N. Ivanov, Phys. Lett. B **275**, 450 (1992); Int. J. Mod. Phys. A **8**, 853 (1993); A. N. Ivanov, N. I. Troitskaya, and M. Nagy, *ibid.* **8**, 2027 (1993); **8**, 3425 (1993); Phys. Lett. B **308**, 111 (1993); **326**, 312 (1994); Nuovo Cimento A **107**, 1375 (1994); A. N. Ivanov and N. I. Troitskaya, *ibid.* **108**, 555 (1995).

[6] A. N. Ivanov and N. I. Troitskaya, Phys. Lett. B **342**, 323 (1995); **345**, 175 (1995); Nuovo Cimento A **110**, 65 (1997); A. N. Ivanov, N. I. Troitskaya, and M. Nagy, Phys. Lett. B **339**, 167 (1994); F. Hussain, A. N. Ivanov, and N. I. Troitskaya, *ibid.* **329**, 98 (1994); **348**, 609 (1995); **369**, 351 (1996); A. N. Ivanov and N. I. Troitskaya, *ibid.* **390**, 341 (1997); **394**, 195 (1997); **387**, 386 (1996); **388**, 869(E) (1996).

[7] A. N. Ivanov, M. Nagy, and N. I. Troitskaya, Phys. Rev. C **59**, 451 (1999).

[8] T. Hakioglu and M. D. Scadron, Phys. Rev. D **42**, 941 (1990); **43**, 2439 (1991); R. Karlsen and M. D. Scadron, Mod. Phys. Lett. A **6**, 543 (1991); M. D. Scadron, *ibid.* **7**, 669 (1992); Phys. At. Nucl. **56**, 1595 (1993); R. Delbourgo and M. D. Scadron, Mod. Phys. Lett. A **10**, 251 (1995); L. R. Baboukhadia, V. Elias, and M. D. Scadron, J. Phys. G **23**, 1065 (1997); R. Delbourgo and M. D. Scadron, Int. J. Mod. Phys. A **13**, 657 (1998); A. Bramon, Riazuddin, and M. D. Scadron, J. Phys. G **24**, 1 (1998); M. D. Scadron, Phys. Rev. D **57**, 5307 (1998); L. R. Baboukhadia and M. D. Scadron, Eur. Phys. J. C **8**, 527 (1999).

[9] A. N. Ivanov, N. I. Troitskaya, M. Faber, M. Schaler, and M. Nagy, Nuovo Cimento A **107**, 1667 (1994); A. N. Ivanov, N. I. Troitskaya, and M. Faber, *ibid.* **108**, 613 (1995).

[10] M. Gell-Mann, Phys. Rev. Lett. **8**, 214 (1964).

[11] B. L. Ioffe, Nucl. Phys. **B188**, 317 (1981); **B191**, 591(E) (1981); P. Pascual and R. Tarrach, Barcelona, UBFT-FP-5-82, 1982; L. J. Reinders, H. R. Rubinstein, and S. Yazaki, Phys. Lett. **120B**, 209 (1983).

[12] M. D. Scadron, in *Advanced Quantum Theory and its Applications Through Feynman Diagrams* (Springer-Verlag, New York, 1st ed. 1979 and 2nd ed. 1991).

[13] Particle Data Group, D. E. Groom *et al.*, Eur. Phys. J. C **15**, 1 (2000).

[14] B. W. Lee and M. K. Gaillard, Phys. Rev. Lett. **33**, 108 (1974); G. Altarelli, G. Curci, G. Martinelli, and S. Petrarca, Nucl. Phys. **B187**, 461 (1981); A. Buras, J.-M. Gèrard, and R. Ruckl, *ibid.* **B268**, 16 (1986); M. Bauez, B. Stech, and M. Wizbel, Z. Phys. C **34**, 103 (1987).

[15] M. D. Scadron and L. R. Thebaud, Phys. Rev. D **8**, 2190 (1973); R. E. Karlsen and M. D. Scadron, *ibid.* **43**, 1739 (1991); M. D. Scadron and D. Tadić, J. Phys. G **27**, 163 (2001).

[16] V. Elias and M. D. Scadron, Phys. Rev. D **30**, 647 (1984); Phys. Rev. Lett. **53**, 1129 (1984).

[17] R. E. Karlson and M. D. Scadron, Europhys. Lett. **14**, 319 (1991).