# Branching ratios and $C P$-violating asymmetries of $B_{s} \rightarrow h_{1} h_{2}$ decays in the general two-Higgs-doublet models 

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#### Abstract

Based on the low-energy effective Hamiltonian with generalized factorization, we calculate the new physics contributions to branching ratios and $C P$-violating asymmetries of the charmless hadronic decays $B_{s} \rightarrow h_{1} h_{2}$ in the standard model and the general two-Higgs-doublet models (models I, II, and III). Within the considered parameter space, we find the following. (a) In models I and II, the new physics corrections are always small in size and will be masked by other larger known theoretical uncertainties. (b) In model III, the new physics corrections to the branching ratios of those QCD penguin-dominated decays $\bar{B}_{s} \rightarrow K^{0} \eta^{\left({ }^{\prime}\right)}, K^{+} K^{-*}$, etc., are large in size and insensitive to the variations of $M_{H^{+}}$and $N_{c}^{\text {eff }}$. For tree- or electroweak penguin-dominated decay modes, however, the new physics corrections are very small in size. (c) For $\bar{B}_{s} \rightarrow K^{+} K^{-*}$ and the other seven decay modes, the branching ratios are at the level of $(1-3) \times 10^{-5}$ and will be measurable at future hadron colliders with large $b$ production. (d) Among the studied $39 B_{s}$ meson decay modes, seven of them can have a $C P$-violating asymmetry $\mathcal{A}_{C P}$ larger than $20 \%$ in magnitude. The new physics corrections are small or moderate in magnitude. (e) Because of its large and $N_{c}^{\text {eff }}$ stable branching ratio and $C P$-violating asymmetry, the decay $\bar{B}_{s} \rightarrow K^{+} K^{-*}$ seems to be the "best" channel to find $C P$ violation of $B_{s}$ system through studies of two-body charmless decays of the $B_{s}$ meson.


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## I. INTRODUCTION

In $B$ experiments, new physics beyond the standard model (SM) may manifest itself, for example, in the following two ways $[1,2]:$ (a) decays which are expected to be rare in the SM are found to have large branching ratios; (b) $C P$-violating asymmetries which are expected to vanish or be very small in the SM are found to be significantly large or with a very different pattern than what is predicted in the SM. These potential deviations may be induced by the virtual effects of new physics through loop diagrams.

The observation of many two-body charmless hadronic $B_{u, d}$ meson decays by CLEO, BaBar, and Belle [3-7], the successful start of the asymmetric $B$ factories at SLAC and KEK, and the expectation for a large number of events of $B_{u, d}$ meson decays to be accumulated at $B$ factories and other hadron colliders stimulated intensive investigations of various $B$ decay channels. The two-body charmless hadronic decays $B_{u, d} \rightarrow h_{1} h_{2}$ [where $h_{1}$ and $h_{2}$ are the light pseudoscalar $(P)$ and/or vector ( $V$ ) mesons] have been studied, for example, in Refs. [8-13].

It is well known that the low-energy effective Hamiltonian is the basic tool to calculate the branching ratios and $\mathcal{A}_{C P}$ of $B$ meson decays. The short-distance QCD corrected Lagrangian at next leading order (NLO) level is available now $[14,15]$, but we do not know how to calculate the hadronic matrix element from first principles. One conventionally can resort to the factorization approximation [16]. However, we also know that the nonfactorizable contribution really exists and cannot be neglected numerically for most hadronic $B$ decay channels. To remedy the naive factoriza-

[^0]tion hypothesis, some authors [17,10,11] introduced a phenomenological parameter $N_{c}^{\mathrm{eff}}$ (i.e., the effective number of color) to model the nonfactorizable contribution to the hadronic matrix element, which is commonly called the generalized factorization. Very recently, Cheng et al. [18] studied and resolved the controversies on the gauge dependence and infrared singularity of the effective Wilson coefficients $C_{i}^{e f f}$ [19] by using the perturbative QCD factorization theorem.

Unlike the $B_{u, d}$ meson, the heavier $B_{s}$ meson cannot be produced by the Cornell Electron Storage Ring (CESR), KEKB, and PEP-II SLAC $e^{+} e^{-}$storage ring. Only upper limits on the decay rates of several charmless hadronic $B_{s}$ decays are currently available from the CERN $e^{+} e^{-}$collider LEP collaborations [20,21], such as $B_{s} \rightarrow K^{+} K^{-}, K^{+} \pi^{-}$, $\pi^{0} \eta$, and $B_{s} \rightarrow \eta \eta$, while most of them are far beyond the theoretical predictions. However, it is expected that many $B_{s}$ decays can be seen at future hadron colliders with large $b$ production. Recent theoretical studies and experimental measurements about the mixing of $B_{s}^{0}-\bar{B}_{s}^{0}$ can be found in Refs. [22,23]. Early studies of two-body charmless hadronic decays of $B_{s}$ mesons can be found in Refs. [24,25]. Based on the framework of generalized factorization, Tseng [26] analyzed the exclusive charmless $B_{s}$ decays involving $\eta^{(\prime)}$, while Chen, Cheng, and Tseng [12] calculated the branching ratios of 39 charmless two-body decays of $B_{s}$ mesons. It is found that the branching ratios of $\eta \eta^{\left({ }^{\prime}\right)}$ and several other decay modes can be as large as $10^{-5}$ and measurable at future experiments.

In a recent work [27], we made a systematic study of the new physics contributions to the branching ratios of 76 $B_{u, d} \rightarrow h_{1} h_{2}$ decay channels in the framework of general two-Higgs-doublet models (2HDMs). In this paper we extend the work to the case of $B_{s}$ mesons. In addition to the
branching ratios, we here also calculate the new physics contributions to the $C P$-violating asymmetries $A_{C P}$ of charmless hadronic decays $B_{s} \rightarrow h_{1} h_{2}$ induced by the new gluonic and electroweak charged-Higgs-boson penguin diagrams in the general 2HDMs (models I, II, and III). Using the effective Hamiltonian with improved generalized factorization [18], we evaluate analytically all new strong and electroweak penguin diagrams induced by exchanges of charged Higgs bosons in the quark level processes $b \rightarrow q V^{*}$ with $q \in\{d, s\}$ and $V \in\{$ gluon, $\gamma, Z\}$, and then combine the new physics contributions with their SM counterparts and finally calculate the branching ratios and $C P$-violating asymmetries for all 39 exclusive $B_{s} \rightarrow h_{1} h_{2}$ decay modes.

This paper is organized as follows. In Sec. II, we describe the basic structures of the 2 HDMs and examine the allowed parameter space of the general 2HDMs from currently available data. In Sec. III, we evaluate analytically the new penguin diagrams and find the effective Wilson coefficients $C_{i}^{e f f}$ with the inclusion of new physics contributions, and present the formulas needed to calculate the branching ratios $\mathcal{B}(B$ $\rightarrow h_{1} h_{2}$ ). In Secs. IV and V, we calculate and show numerical results of branching ratios and $C P$-violating asymmetries for $39 B_{s}$ decay modes, respectively. We focus on those decay modes with large branching ratios and large $C P$-violating asymmetries. The conclusions and discussions are included in the final section.

## II. GENERAL 2HDMs AND EXPERIMENTAL CONSTRAINTS

The simplest extension of the SM is the so-called two-Higgs-doublet models [28]. In such models, tree level flavor changing neutral current (FCNCs) is absent if one introduces a discrete symmetry to constrain the 2 HDM scalar potential and Yukawa Lagrangian. Let us consider a Yukawa Lagrangian of the form [29]

$$
\begin{align*}
\mathcal{L}_{Y}= & \eta_{i j}^{U} \bar{Q}_{i, L} \widetilde{\phi_{1}} U_{j, R}+\eta_{i j}^{D} \bar{Q}_{i, L} \phi_{1} D_{j, R}+\xi_{i j}^{U} \bar{Q}_{i, L} \widetilde{\phi_{2}} U_{j, R} \\
& +\xi_{i j}^{D} \bar{Q}_{i, L} \phi_{2} D_{j, R}+\text { H.c. }, \tag{1}
\end{align*}
$$

where $\phi_{i}(i=1,2)$ are the two Higgs doublets of a two-Higgs-doublet model, $\widetilde{\phi}_{1,2}=i \tau_{2} \phi_{1,2}^{*}, Q_{i, L}\left(U_{j, R}\right)$ with $i$ $=(1,2,3)$ are the left-handed isodoublet quarks (right-handed up-type quarks), and $D_{j, R}$ are the right-handed isosinglet down-type quarks, while $\eta_{i, j}^{U, D}$ and $\xi_{i, j}^{U, D}(i, j=1,2,3$ are family index) are generally the nondiagonal matrices of the Yukawa coupling. By imposing the discrete symmetry $\phi_{1}$ $\rightarrow-\phi_{1}, \phi_{2} \rightarrow \phi_{2}, D_{i} \rightarrow-D_{i}$, and $U_{i} \rightarrow \mp U_{i}$, one obtains the so-called model I and model II.

During recent years, models I and II have been studied extensively in the literature and tested experimentally, and model II has been very popular since it is the building block of the minimal supersymmetric standard model. In this paper, we focus on the third type of the two-Higgs-doublet model [30], usually known as model III [29,30]. In model III, no discrete symmetry is imposed and both up- and downtype quarks then may have diagonal and/or flavor changing couplings with $\phi_{1}$ and $\phi_{2}$. As described in [29], one can
choose a suitable basis ( $H^{0}, H^{1}, H^{2}, H^{ \pm}$) to express two Higgs doublets. The $H^{ \pm}$are the physical charged Higgs boson, $H^{0}$ and $h^{0}$ are the physical $C P$-even neutral Higgs boson, and $A^{0}$ is the physical $C P$-odd neutral Higgs boson. After rotation of the quark fields, the Yukawa Lagrangian of quarks are of the form [29],

$$
\begin{align*}
\mathcal{L}_{Y}^{I I I}= & \eta_{i j}^{U} \bar{Q}_{i, L} \widetilde{\phi_{1}} U_{j, R}+\eta_{i j}^{D} \bar{Q}_{i, L} \phi_{1} D_{j, R}+\hat{\xi}_{i j}^{U} \bar{Q}_{i, L} \widetilde{\phi_{2}} U_{j, R} \\
& +\hat{\xi}_{i j}^{D} \bar{Q}_{i, L} \phi_{2} D_{j, R}+\text { H.c. }, \tag{2}
\end{align*}
$$

where $\eta_{i j}^{U, D}$ correspond to the diagonal mass matrices of upand down-type quarks, while the neutral and charged flavor changing couplings will be [29]. We make the same ansatz on the $\xi_{i j}^{U, D}$ couplings as Ref. [29]:

$$
\begin{gather*}
\xi_{i j}^{U, D}=\frac{\sqrt{m_{i} m_{j}}}{v} \lambda_{i j}, \quad \hat{\xi}_{\text {neutral }}^{U, D}=\xi^{U, D}, \\
\hat{\xi}_{\text {charged }}^{U}=\xi^{U} V_{C K M}, \quad \hat{\xi}_{\text {charged }}^{D}=V_{C K M} \xi^{D}, \tag{3}
\end{gather*}
$$

where $V_{C K M}$ is the Cabibbo-Kobayashi-Maskawa mixing matrix [31], and $i, j=(1,2,3)$ are the generation index. The coupling constants $\lambda_{i j}$ are free parameters to be determined by experiments, and they may also be complex.

In model II and setting $1 \leqslant \tan \beta=v_{2} / v_{1} \leqslant 50$ favored by experimental measurements [20], the constraint on the mass of charged Higgs boson due to the CLEO data of $b \rightarrow s \gamma$ is $M_{H^{+}} \geqslant 200 \mathrm{GeV}$ at the NLO level [32]. For model I, however, the limit can be much weaker due to the possible destructive interference with the SM amplitude. For model III, the situation is not as clear as model II because there are more free parameters here [29,33]. In a recent paper [34], Chao et al. studied the decay $b \rightarrow s \gamma$ by assuming that only the couplings $\lambda_{t t}=\left|\lambda_{t t}\right| e^{i \theta_{t}}$ and $\lambda_{b b}=\left|\lambda_{b b}\right| e^{i \theta_{b}}$ are nonzero. They found that the constraint on $M_{H^{+}}$imposed by the CLEO data of $b \rightarrow s \gamma$ can be greatly relaxed by considering the phase effects of $\lambda_{t t}$ and $\lambda_{b b}$. From the studies of Refs. [34,35], we know that for model III the parameter space

$$
\begin{align*}
\lambda_{i j} & =0, \quad \text { for } \quad i j \neq t t \quad \text { or } \quad b b, \\
\left|\lambda_{t t}\right| & =0.3, \quad\left|\lambda_{b b}\right|=35, \quad \theta=\left(0^{\circ}-30^{\circ}\right), \\
M_{H^{+}} & =(200 \pm 100) \mathrm{GeV}, \tag{4}
\end{align*}
$$

are allowed by the available data, where $\theta=\theta_{b b}-\theta_{t t}$.
From the CERN $e^{+} e^{-}$collider (LEP) and the Fermilab Tevatron searches for charged Higgs bosons [36], the new combined constraint in the $\left(M_{H^{+}}-\tan \beta\right)$ plane has been given, for example, in Ref. [20]: the direct lower limit is $M_{H^{+}}>77 \mathrm{GeV}$, while $0.5 \leqslant \tan \beta \leqslant 60$ for a relatively light charged Higgs boson with $M_{H^{+}} \sim 100 \mathrm{GeV}$. Combining the direct and indirect limits together, we here conservatively consider the range of $100 \mathrm{GeV} \leqslant M_{H^{+}} \leqslant 300 \mathrm{GeV}$, while take $M_{H^{+}}=200 \mathrm{GeV}$ as the typical value for models I, II, and III. For models I and II we consider the range of 1 $\leqslant \tan \beta \leqslant 50$, while take $\tan \beta=2$ as the typical value.

## III. EFFECTIVE HAMILTONIAN IN THE SM AND 2HDMs

The standard theoretical frame to calculate the inclusive three-body decays $b \rightarrow s \bar{q} q^{1}$ is based on the effective Hamiltonian [15,11,13]

$$
\begin{align*}
\mathcal{H}_{e f f}(\Delta B=1)= & \frac{G_{F}}{\sqrt{2}}\left\{\sum_{j=1}^{2} C_{j}\left(V_{u b} V_{u s}^{*} Q_{j}^{u}+V_{c b} V_{c s}^{*} Q_{j}^{c}\right)\right. \\
& \left.-V_{t b} V_{t s}^{*}\left[\sum_{j=3}^{10} C_{j} Q_{j}+C_{g} Q_{g}\right]\right\} \tag{5}
\end{align*}
$$

Here the first ten operators $Q_{1}-Q_{10}$ can be found, for example, in Refs. [11,13,27], while the chromomagnetic operator reads

$$
\begin{equation*}
Q_{g}=\frac{g_{s}}{8 \pi^{2}} m_{b} \bar{s}_{\alpha} \sigma^{\mu \nu}\left(1+\gamma_{5}\right) T_{\alpha \beta}^{a} b_{\beta} G_{\mu \nu}^{a} \tag{6}
\end{equation*}
$$

where $\alpha$ and $\beta$ are the $S U(3)$ color indices, and $T_{\alpha \beta}^{a}(a$ $=1, \ldots, 8)$ are the Gell-Mann matrices. Following Ref. [12], we do not consider the effect of weak annihilation and exchange diagrams.

The coefficients $C_{i}$ in Eq. (5) are the well-known Wilson coefficient. Within the SM and at scale $M_{W}$, the Wilson coefficients $C_{1}\left(M_{W}\right), \ldots, C_{10}\left(M_{W}\right)$ and $C_{g}\left(M_{W}\right)$ have been given, for example, in Refs. [14,15]. By using QCD renormalization group equations, it is straightforward to run Wilson coefficients $C_{i}\left(M_{W}\right)$ from the scale $\mu=0\left(M_{W}\right)$ down to the lower scale $\mu=O\left(m_{b}\right)$. Working consistently to NLO precision, the Wilson coefficients $C_{i}$ for $i=1, \ldots, 10$ are needed in NLO precision, while it is sufficient to use the leading logarithmic value for $C_{g}$.

## A. New strong and electroweak penguins

For the charmless hadronic decays of $B$ mesons under consideration, the new physics will manifest itself by modifying the corresponding Inami-Lim functions $C_{0}(x), D_{0}(x), E_{0}(x)$, and $E_{0}^{\prime}(x)$ which determine the coefficients $C_{3}\left(M_{W}\right), \ldots, C_{10}\left(M_{W}\right)$ and $C_{g}\left(M_{W}\right)$. These modifications, in turn, will change the SM predictions of the branching ratios and $C P$-violating asymmetries for the decays $B_{s} \rightarrow h_{1} h_{2}$ under study.

The new strong and electroweak penguin diagrams can be obtained from the corresponding penguin diagrams in the SM by replacing the internal $W^{ \pm}$lines with the charged-Higgs-boson $H^{+}$lines. In Ref. [27], we calculated analytically the new $Z^{0}-, \gamma$ - and gluon-penguin diagrams induced by the exchanges of charged Higgs boson $H^{+}$, and found the new $C_{0}, D_{0}, E_{0}$, and $E_{0}^{\prime}$ functions which describe the new physics contributions to the Wilson coefficients through the new penguin diagrams

[^1]\[

$$
\begin{align*}
C_{0}^{I I I} & =\frac{-x_{t}}{16}\left[\frac{y_{t}}{1-y_{t}}+\frac{y_{t}}{\left(1-y_{t}\right)^{2}} \ln \left[y_{t}\right]\right]\left|\lambda_{t t}\right|^{2},  \tag{7}\\
D_{0}^{I I I} & =-\frac{1}{3} H\left(y_{t}\right)\left|\lambda_{t t}\right|^{2},  \tag{8}\\
E_{0}^{I I I} & =-\frac{1}{2} I\left(y_{t}\right)\left|\lambda_{t t}\right|^{2},  \tag{9}\\
E_{0}^{\prime I I I} & =\frac{1}{6} J\left(y_{t}\right)\left|\lambda_{t t}\right|^{2}-K\left(y_{t}\right)\left|\lambda_{t t} \lambda_{b b}\right| e^{i \theta}, \tag{10}
\end{align*}
$$
\]

with

$$
\begin{align*}
& H(y)=\frac{38 y-79 y^{2}+47 y^{3}}{72(1-y)^{3}}+\frac{4 y-6 y^{2}+3 y^{4}}{12(1-y)^{4}} \ln [y]  \tag{11}\\
& I(y)=\frac{16 y-29 y^{2}+7 y^{3}}{36(1-y)^{3}}+\frac{2 y-3 y^{2}}{6(1-y)^{4}} \ln [y]  \tag{12}\\
& J(y)=\frac{2 y+5 y^{2}-y^{3}}{4(1-y)^{3}}+\frac{3 y^{2}}{2(1-y)^{4}} \ln [y]  \tag{13}\\
& K(y)=\frac{-3 y+y^{2}}{4(1-y)^{2}}-\frac{y}{2(1-y)^{3}} \ln [y] \tag{14}
\end{align*}
$$

where $x_{t}=m_{t}^{2} / M_{W}^{2}, y_{t}=m_{t}^{2} / M_{H^{+}}^{2}$, and the small terms proportional to $m_{b}^{2} / m_{t}^{2}$ have been neglected. In models I and II, one can find the corresponding functions $C_{0}, D_{0}, E_{0}$, and $E_{0}^{\prime}$ by evaluating the new strong and electroweak penguin diagrams in the same way as that in model III:

$$
\begin{align*}
& C_{0}^{I}=C_{0}^{I I}=\frac{-x_{t}}{8 \tan ^{2} \beta}\left[\frac{y_{t}}{1-y_{t}}+\frac{y_{t}}{\left(1-y_{t}\right)^{2}} \ln \left[y_{t}\right]\right],  \tag{15}\\
& D_{0}^{I}=D_{0}^{I I}=-\frac{2}{3 \tan ^{2} \beta} H\left(y_{t}\right),  \tag{16}\\
& E_{0}^{I}=E_{0}^{I I}=-\frac{1}{\tan ^{2} \beta} I\left(y_{t}\right),  \tag{17}\\
& E_{0}^{\prime I}=\frac{1}{3 \tan ^{2} \beta}\left[J\left(y_{t}\right)-6 K\left(y_{t}\right)\right],  \tag{18}\\
& E_{0}^{\prime I I}=\frac{1}{3 \tan ^{2} \beta} J\left(y_{t}\right)+2 K\left(y_{t}\right), \tag{19}
\end{align*}
$$

where $y_{t}=m_{t}^{2} / M_{H^{+}}^{2}, \tan \beta=v_{2} / v_{1}$, and where $v_{1}$ and $v_{2}$ are the vacuum expectation values of the Higgs doublet $\phi_{1}$ and $\phi_{2}$ as defined before.

Combining the SM part and the new physics part together, the NLO Wilson coefficients $C_{i}\left(M_{W}\right)$ and $C_{g}\left(M_{W}\right)$ can be written as

$$
\begin{align*}
& C_{1}\left(M_{W}\right)= 1-\frac{11}{6} \frac{\alpha_{s}\left(M_{W}\right)}{4 \pi}-\frac{35}{18} \frac{\alpha_{e m}}{4 \pi},  \tag{20}\\
& C_{2}\left(M_{W}\right)= \frac{11}{2} \frac{\alpha_{s}\left(M_{W}\right)}{4 \pi},  \tag{21}\\
& C_{3}\left(M_{W}\right)=-\frac{\alpha_{s}\left(M_{W}\right)}{24 \pi}\left[E_{0}\left(x_{t}\right)+E_{0}^{N P}-\frac{2}{3}\right] \\
&+\frac{\alpha_{e m}}{6 \pi} \frac{1}{\sin ^{2} \theta_{W}}\left[2 B_{0}\left(x_{t}\right)+C_{0}\left(x_{t}\right)+C_{0}^{N P}\right],  \tag{22}\\
& C_{4}\left(M_{W}\right)= \frac{\alpha_{s}\left(M_{W}\right)}{8 \pi}\left[E_{0}\left(x_{t}\right)+E_{0}^{N P}-\frac{2}{3}\right],  \tag{23}\\
& C_{5}\left(M_{W}\right)=-\frac{\alpha_{s}\left(M_{W}\right)}{24 \pi}\left[E_{0}\left(x_{t}\right)+E_{0}^{N P}-\frac{2}{3}\right],  \tag{24}\\
& C_{6}\left(M_{W}\right)= \frac{\alpha_{s}\left(M_{W}\right)}{8 \pi}\left[E_{0}\left(x_{t}\right)+E_{0}^{N P}-\frac{2}{3}\right],  \tag{25}\\
& C_{7}\left(M_{W}\right)= \frac{\alpha_{e m}}{6 \pi}\left[4 C_{0}\left(x_{t}\right)+4 C_{0}^{N P}+D_{0}\left(x_{t}\right)+D_{0}^{N P}-\frac{4}{9}\right],  \tag{26}\\
& C_{8}\left(M_{W}\right)= C_{10}\left(M_{W}\right)=0,  \tag{27}\\
& C_{g}\left(M_{W}\right)=-\frac{1}{2}\left(E_{0}^{\prime}\left(x_{t}\right)+E_{0}^{\prime N P}\right), \\
& C_{9}\left(M_{W}\right)= \frac{\alpha_{e m}}{6 \pi}\left\{4 C_{0}\left(x_{t}\right)+4 C_{0}^{N P}+D_{0}\left(x_{t}\right)+D_{0}^{N P}-\frac{4}{9}\right.  \tag{28}\\
& {\left.\left[10 B_{0}\left(x_{t}\right)-4 C_{0}\left(x_{t}\right)+4 C_{0}^{N P}\right]\right\}, }  \tag{29}\\
& \\
& \sin _{W} \\
& \hline
\end{align*}
$$

where $x_{t}=m_{t}^{2} / M_{W}^{2}$, and the functions $B_{0}(x), C_{0}(x), D_{0}(x)$, $E_{0}(x)$, and $E_{0}^{\prime}$ are the familiar Inami-Lim functions [37] in the SM and can be found easily, for example, in Refs. [14,38].

Since the heavy new particles appearing in the 2HDMs have been integrated out at the scale $M_{W}$, the QCD running of the Wilson coefficients $C_{i}\left(M_{W}\right)$ down to the scale $\mu$ $=O\left(m_{b}\right)$ after including the new physics contributions will be the same as in the SM:

$$
\begin{align*}
\mathbf{C}(\mu) & =U\left(\mu, M_{W}\right) \mathbf{C}\left(M_{W}\right),  \tag{30}\\
C_{g}(\mu) & =\eta^{14 / 23} C_{g}\left(M_{W}\right)+\sum_{i=1}^{8} \bar{h}_{i} \eta^{a_{i}} \tag{31}
\end{align*}
$$

where $\mathbf{C}\left(M_{W}\right)=\left[C_{1}\left(M_{W}\right), \ldots, C_{10}\left(M_{W}\right)\right]^{T}, U\left(\mu, M_{W}\right)$ is the five-flavor $10 \times 10$ evolution matrix at NLO level as de-
fined in Ref. [14], $\eta=\alpha_{s}\left(M_{W}\right) / \alpha_{s}(\mu)$, and the constants $\bar{h}_{i}$ and $a_{i}$ can also be found in Ref. [14].

In the naive dimensional reduction (NDR) scheme and for $S U(3)_{C}$, the effective Wilson coefficients ${ }^{2}$ can be written as [13]

$$
\begin{align*}
C_{i}^{e f f}= & {\left[1+\frac{\alpha_{s}}{4 \pi}\left(\hat{r}_{V}^{T}+\gamma_{V}^{T} \log \frac{m_{b}}{\mu}\right)\right]_{i j} C_{j} } \\
& +\frac{\alpha_{s}}{24 \pi} A_{i}^{\prime}\left(C_{t}+C_{p}+C_{g}\right)+\frac{\alpha_{e w}}{8 \pi} B_{i}^{\prime} C_{e} \tag{32}
\end{align*}
$$

where

$$
\begin{gathered}
A_{i}^{\prime}=(0,0,-1,3,-1,3,0,0,0,0)^{T}, \\
B_{i}^{\prime}=(0,0,0,0,0,0,1,0,1,0)^{T}
\end{gathered}
$$

the matrices $\hat{r}_{V}$ and $\gamma_{V}$ contain the process-independent contributions from the vertex diagrams. The matrix $\gamma_{V}$ and $\hat{r}_{V}$ have been given explicitly, for example, in Eqs. (2.17) and (2.18) of Ref. [13]. Note that the correct value of the element $\left(\hat{r}_{N D R}\right)_{66}$ and $\left(\hat{r}_{N D R}\right)_{88}$ should be 17 instead of 1 as pointed out in Ref. [39].

The functions $C_{t}, C_{p}$, and $C_{g}$ describe the contributions arising from the penguin diagrams of the current-current $Q_{1,2}$ and the QCD operators $Q_{3}-Q_{6}$ and the tree-level diagram of the magnetic dipole operator $Q_{8 G}$, respectively. We here also follow the procedure of Ref. [10] to include the contribution of magnetic gluon penguin. The functions $C_{t}, C_{p}$, and $C_{g}$ are given in the NDR scheme by $[11,13]$

$$
\begin{align*}
C_{t}= & {\left[\frac{2}{3}+\frac{\lambda_{u}}{\lambda_{t}} G\left(m_{u}\right)+\frac{\lambda_{c}}{\lambda_{t}} G\left(m_{c}\right)\right] C_{1}, }  \tag{33}\\
C_{p}= & {\left[\frac{4}{3}-G\left(m_{q}\right)-G\left(m_{b}\right)\right] C_{3} } \\
& +\left[\frac{10}{3}-\sum_{i=u, d, s, c, b} G\left(m_{i}\right)\right]\left(C_{4}+C_{6}\right),  \tag{34}\\
C_{e}= & \frac{8}{9}\left[\frac{2}{3}+\frac{\lambda_{u}}{\lambda_{t}} G\left(m_{u}\right)+\frac{\lambda_{c}}{\lambda_{t}} G\left(m_{c}\right)\right]\left(C_{1}+3 C_{2}\right),  \tag{35}\\
C_{g}= & -\frac{2 m_{b}}{\sqrt{\left\langle k^{2}\right\rangle}} C_{g}^{\mathrm{eff}} \tag{36}
\end{align*}
$$

with $\lambda_{q^{\prime}} \equiv V_{q^{\prime} b} V_{q^{\prime} q}^{*}$ and $C_{g}^{e f f}=C_{g}(\mu)+C_{5}$. The function $G(m)$ can be found, for example, in Refs. [13,27]. For the two-body exclusive $B$ meson decays any information on $k^{2}$ is lost in the factorization assumption; one usually uses the

[^2]TABLE I. Numerical values of $a_{i}$ for the transitions $b \rightarrow d[\bar{b} \rightarrow \bar{d}]$. The first, second and third entries for $a_{3}, \ldots, a_{10}$ refer to the values of $a_{i}$ in the SM and models II and III, respectively. All entries for $a_{3}, \ldots, a_{10}$ should be multiplied by $10^{-4}$.

|  | $N_{c}^{\text {eff }}=2$ | $N_{c}^{\text {eff }}=3$ | $N_{c}^{\text {eff }}=\infty$ |
| :--- | :---: | :---: | :---: |
| $a_{1}$ | $0.985[0.985]$ | $1.046[1.046]$ | $1.169[1.169]$ |
| $a_{2}$ | $0.216[0.216]$ | $0.021[0.021]$ | $-0.369[-0.369]$ |
| $a_{3}$ | $-10.4-19.1 i[-11.5-25.7 i]$ | $66.1[66.1]$ | $219+38.1 i[221+51.4 i]$ |
|  | $-33.0-19.1 i[-34.1-25.7 i]$ | $66.2[66.2]$ | $265+38.1 i[267+51.4 i]$ |
| $a_{4}$ | $-349-95.3 i[-354-129 i]$ | $-386-102 i[-392-137 i]$ | $-459-114 i[-466-154 i]$ |
|  | $-463-95.3 i[-469-129 i]$ | $-507-102 i[-513-137 i]$ | $-596-114 i[-602-154 i]$ |
| $a_{5}$ | $-163-19.1 i[-164-25.7 i]$ | $-61.5[-61.5]$ | $142+38.1 i[144+51.4 i]$ |
|  | $-186-19.1 i[-187-25.7 i]$ | $-61.4[-61.4]$ | $187+38.1 i[189+51 i .4]$ |
| $a_{6}$ | $-538-95.3 i[-544-129 i]$ | $-562-102 i[-568-137 i]$ | $-609-114 i[-616-154 i]$ |
|  | $-652-95.3 i[-657-129 i]$ | $-683-102 i[-689-137 i]$ | $-746-114 i[-752-154 i]$ |
| $a_{7}$ | $5.2-2.5 i[5.1-3.1 i]$ | $4.1-2.5 i[4.0-3.1 i]$ | $2.1-2.5 i[2.0-3.1 i]$ |
|  | $5.4-2.5 i[5.3-3.1 i]$ | $4.3-2.5 i[4.2-3.1 i]$ | $2.2-2.5 i[2.1-3.1 i]$ |
| $a_{8}$ | $7.2-1.3 i[7.2-1.6 i]$ | $6.9-0.8 i[6.8-1.0 i]$ | $6.2[6.2]$ |
|  | $7.4-1.3 i[7.3-1.6 i]$ | $7.0-0.8 i[7.0-1.0 i]$ | $6.3[6.3]$ |
| $a_{9}$ | $-85.8-2.5 i[-85.9-3.1 i]$ | $-91.7-2.5 i[-91.8-3.1 i]$ | $-103-2.5 i[-104-3.1 i]$ |
|  | $-86.4-2.5 i[-86.5-3.1 i]$ | $-92.3-2.5 i[-92.4-3.1 i]$ | $-104.1-2.5 i[-104-3.1 i]$ |
| $a_{10}$ | $-16.5-1.3 i[-16.6-1.6 i]$ | $0.7-0.8 i[0.7-1.0 i]$ | $35.2[35.2]$ |
|  | $-16.6-1.3 i[-16.7-1.6 i]$ | $0.7-0.8 i[0.7-1.0 i]$ | $35.4[35.4]$ |

"physical" range for $k^{2}$ [11-13]: $m_{b}^{2} / 4 \leq k^{2} \leq m_{b}^{2} / 2$. Following Refs. [11-13] we take $k^{2}=m_{b}^{2} / 2$ in the numerical calculation.

## B. Decay amplitudes in the BSW model

Following Ref. [12], the possible effects of final state interactions (FSIs) and contributions from annihilation channels will be neglected although they may play a significant
role for some decay modes. The new physics effects on the $B$ decays under study will be included by using the modified effective coefficients $a_{i}(i=3, \ldots, 10)$ as given in the second entries of Table I and Table II for model III. The effective coefficients $a_{i}$ in models I and II are not shown explicitly in Table I and Table II. In the numerical calculations the input parameters as given in the Appendix and Eq. (4) will be used implicitly.

TABLE II. Same as Table I but for $b \rightarrow s[\bar{b} \rightarrow \bar{s}]$ transitions.

|  | $N_{c}^{\text {eff }}=2$ | $N_{c}^{\text {eff }}=3$ | $N_{c}^{\text {eff }}=\infty$ |
| :--- | :---: | :---: | :---: |
| $a_{1}$ | $0.985[0.985]$ | $1.046[1.046]$ | $1.169[1.169]$ |
| $a_{2}$ | $0.216[0.216]$ | $0.021[0.021]$ | $-0.369[-0.369]$ |
| $a_{3}$ | $-10.9-21.7 i[-9.8-22.1 i]$ | $66.1[66.1]$ | $220+43.3 i[218+44.3 i]$ |
|  | $-33.6-22.7 i[-32.5-22.2 i]$ | $66.2[66.2]$ | $266+43.3 i[264+44.3 i]$ |
| $a_{4}$ | $-352-108 i[-346-111 i]$ | $-389-116 i[-383-118 i]$ | $-462-130 i[-455-133 i]$ |
|  | $-467-108 i[-460-111 i]$ | $-510-116 i[-504-118 i]$ | $-599-130 i[-592-133 i]$ |
| $a_{5}$ | $-164-22.7 i[-162-22.2 i]$ | $-61.5[-61.5]$ | $143+43.3 i[140+44.3 i]$ |
|  | $-186-21.7 i[-185-22.2 i]$ | $-61.4[-61.4]$ | $188+43.3 i[186+44.3 i]$ |
| $a_{6}$ | $-541-108 i[-535-111 i]$ | $-565-116 i[-559-118 i]$ | $-612-130 i[-606-133 i]$ |
|  | $-654-108 i[-649-111 i]$ | $-686-116 i[-680-118 i]$ | $-749-130 i[-742-133 i]$ |
| $a_{7}$ | $5.1-2.8 i[5.2-2.8 i]$ | $4.1-2.8 i[4.2-2.8 i]$ | $2.0-2.8 i[2.1-2.8 i]$ |
|  | $5.3-2.8 i[5.4-2.8 i]$ | $4.3-2.8 i[4.4-2.8 i]$ | $2.2-2.8 i[2.3-2.8 i]$ |
| $a_{8}$ | $7.2-1.4 i[7.2-1.4 i]$ | $6.9-0.9 i[6.9-0.9 i]$ | $6.2[6.2]$ |
|  | $7.4-1.4 i[7.4-1.4 i]$ | $7.0-0.9 i[7.0-0.9 i]$ | $6.3[6.3]$ |
| $a_{9}$ | $-85.9-2.8 i[-85.8-2.8 i]$ | $-91.7-2.8 i[-91.6-2.8 i]$ | $-104-2.8 i[-103-2.8 i]$ |
|  | $-86.5-2.8 i[-86.4-2.8 i]$ | $-92.4-2.8 i[-92.3-2.8 i]$ | $-104-2.8 i[-104-2.8 i]$ |
| $a_{10}$ | $-16.6-1.4 i[-16.5-1.4 i]$ | $0.7-0.9 i[0.7-0.9 i]$ | $35.2[35.2]$ |
|  | $-16.7-1.4 i[-16.6-1.4 i]$ | $0.7-0.9 i[0.7-0.9 i]$ | $35.4[35.4]$ |

With the factorization ansatz [16], the three-hadron matrix elements or the decay amplitude $\langle X Y| H_{e f f}\left|B_{s}\right\rangle$ can be factorized into a sum of products of two current matrix elements $\langle X| J_{1}^{\mu}|0\rangle$ and $\langle Y| J_{2 \mu}\left|B_{s}\right\rangle$ ( or $\langle Y| J_{1}^{\mu}|0\rangle$ and $\langle X| J_{2 \mu}\left|B_{s}\right\rangle$ ). The explicit expressions of matrix elements can be found, for example, in Refs. [16,40].

In the $B$ rest frame, the branching ratios of two-body $B$ meson decays can be written as

$$
\begin{equation*}
\mathcal{B}\left(B_{s} \rightarrow X Y\right)=\tau_{B_{s}} \frac{|p|}{8 \pi M_{B_{s}}^{2}}\left|M\left(B_{s} \rightarrow X Y\right)\right|^{2} \tag{37}
\end{equation*}
$$

for $B_{s} \rightarrow P P$ decays and

$$
\begin{equation*}
\mathcal{B}\left(B_{s} \rightarrow X Y\right)=\tau_{B_{s}} \frac{|p|^{3}}{8 \pi M_{V}^{2}}\left|M\left(B_{s} \rightarrow X Y\right) /\left(\epsilon \cdot p_{B}\right)\right|^{2} \tag{38}
\end{equation*}
$$

for $B_{s} \rightarrow P V$ decays. Here $\tau\left(B_{s}^{0}\right)=1.493 \mathrm{ps}$ [20], $p_{B}$ is the four-momentum of the $B$ meson, $M_{V}$ and $\epsilon$ are the mass and polarization vector of the produced light vector meson, respectively, and $|p|$ is the magnitude of momentum of particle $X$ and $Y$ in the $B$ rest frame:

$$
\begin{equation*}
|p|=\frac{1}{2 M_{B}} \sqrt{\left[M_{B}^{2}-\left(M_{X}+M_{Y}\right)^{2}\right]\left[M_{B}^{2}-\left(M_{X}-M_{Y}\right)^{2}\right]} . \tag{39}
\end{equation*}
$$

For $B_{s} \rightarrow V V$ decays, the situation is more involved. One needs to evaluate the helicity matrix element $H_{\lambda}$ $=\left\langle V_{1}(\lambda) V_{2}(\lambda)\right| H_{e f f}|B\rangle$ with $\lambda=0, \pm 1$. The branching ratio of the decay $B \rightarrow V_{1} V_{2}$ is given in terms of $H_{\lambda}$ by

$$
\begin{equation*}
\mathcal{B}\left(B_{s} \rightarrow V_{1} V_{2}\right)=\tau_{B_{s}} \frac{|p|}{8 \pi M_{B}^{2}}\left(\left|H_{0}\right|^{2}+\left|H_{+1}\right|^{2}+\left|H_{-1}\right|^{2}\right), \tag{40}
\end{equation*}
$$

where $|p|$ has been given in Eq. (39). The three independent helicity amplitudes $H_{0}, H_{+1}$, and $H_{-1}$ can be expressed by three invariant amplitudes $a, b, c$ defined by the decomposition

$$
\begin{align*}
H_{\lambda}= & i \epsilon^{\mu}(\lambda) \eta^{\nu}(\lambda)\left[a g_{\mu \nu}+\frac{b}{M_{1} M_{2}} p_{\mu} p_{\nu}\right. \\
& \left.+\frac{i c}{M_{1} M_{2}} \epsilon_{\mu \nu \alpha \beta} p_{1}^{\alpha} p^{\beta}\right] \tag{41}
\end{align*}
$$

where $p_{1,2}$ and $M_{1,2}$ are the four-momentum and masses of $V_{1,2}$, respectively. $p=p_{1}+p_{2}$ is the four-momentum of the $B$ meson, and

$$
\begin{align*}
H_{ \pm 1} & =a \pm c \sqrt{x^{2}-1}, \quad H_{0}=-a x-b\left(x^{2}-1\right)  \tag{42}\\
x & =\frac{M_{B}^{2}-M_{1}^{2}-M_{2}^{2}}{2 M_{1} M_{2}} \tag{43}
\end{align*}
$$

For individual decay modes, the coefficients $a, b$, and $c$ can be determined by comparing the helicity amplitude $H_{\lambda}$ $=\left\langle V_{1}(\lambda) V_{2}(\lambda)\right| H_{e f f}\left|B_{s}\right\rangle$ with expression (41).

In the generalized factorization approach, the effective Wilson coefficients $C_{i}^{\text {eff }}$ will appear in the decay amplitudes in the combinations

$$
\begin{equation*}
a_{2 i-1} \equiv C_{2 i-1}^{\mathrm{eff}}+\frac{C_{2 i}^{\mathrm{eff}}}{N_{c}^{\mathrm{eff}}}, \quad a_{2 i} \equiv C_{2 i}^{\mathrm{eff}}+\frac{C_{2 i-1}^{\mathrm{eff}}}{N_{c}^{\mathrm{eff}}} \quad(i=1, \ldots, 5) \tag{44}
\end{equation*}
$$

where the effective number of colors, $N_{c}^{\mathrm{eff}}$, is treated as a free parameter varying in the range of $2 \leqslant N_{c}^{\text {eff }} \leqslant \infty$, in order to model the nonfactorizable contribution to the hadronic matrix elements. Although $N_{c}^{\text {eff }}$ can in principle vary from channel to channel, in the energetic two-body hadronic $B$ meson decays, it is expected to be process insensitive as supported by the data [12]. As argued in Ref. [17], $N_{c}^{\text {eff }}(L L)$ induced by the $(V-A)(V-A)$ operators can be rather different from $N_{c}^{\text {eff }}(L R)$ generated by $(V-A)(V+A)$ operators. Since we here focus on the calculation of new physics effects on the studied $B$ meson decays induced by the new penguin diagrams in the two-Higgs-doublet models, we will simply assume that $N_{c}^{\text {eff }}(L L) \equiv N_{c}^{\text {eff }}(L R)=N_{c}^{\text {eff }}$ and consider the variation of $N_{c}^{\mathrm{eff}}$ in the range of $2 \leqslant N_{c}^{\mathrm{eff}} \leqslant \infty$. For more details about the cases of $N_{c}^{\mathrm{eff}}(L L) \neq N_{c}^{\mathrm{eff}}(L R)$, one can see, for example, Ref. [12]. We here will not consider the possible effects of FSIs and the contributions from annihilation channels although they may play a significant role for some decay modes.

Using the input parameters as given in the Appendix and assuming $k^{2}=m_{b}^{2} / 2, M_{H^{+}}=200 \mathrm{GeV}$, the theoretical predictions of effective coefficients $a_{i}$ are calculated and displayed in Table I and Table II for the transitions $b \rightarrow d(\bar{b} \rightarrow \bar{d})$ and $b \rightarrow s(\bar{b} \rightarrow \bar{s})$, respectively. For coefficients $a_{3}, \ldots, a_{10}$, the first and second entries in Tables I and II refer to the values of $a_{i}$ in the SM and model III, respectively.

Compared with Ref. [12], the effective coefficients $a_{i}$ given here have two new features.
(i) The effective Wilson coefficients $C_{i}^{e f f}$ here are not only renormalization scale and scheme independent, but also gauge invariant and infrared safe.
(ii) The contribution due to the chromomagnetic dipole operator $Q_{g}$ has been included here through the function $C_{g}$ as given in Eq. (36). For the penguin-diagram-dominated decay channels, the operator $C_{g}$ will play an important role.
(iii) The coefficients $a_{1}$ and $a_{2}$ remain unchanged in 2HDMs since the new physics considered here does not contribute through tree diagrams.
(iv) The new physics contributions are significant to the coefficients $a_{4}$ and $a_{6}$, but negligibly small to the coefficients $a_{3,5}$ and $a_{7-10}$.

All branching ratios here are the averages of the branching ratios of $B$ and anti- $B$ decays. The ratio $\delta \mathcal{B}$ describes the new physics correction on the decay ratio and is defined as

$$
\begin{equation*}
\delta \mathcal{B}\left(B_{s} \rightarrow X Y\right)=\frac{\mathcal{B}\left(B_{s} \rightarrow X Y\right)^{N P}-\mathcal{B}\left(B_{s} \rightarrow X Y\right)^{S M}}{\mathcal{B}\left(B_{s} \rightarrow X Y\right)^{S M}} . \tag{45}
\end{equation*}
$$

## IV. BRANCHING RATIOS OF $B_{s}$ MESON DECAYS

Using the formulas and input parameters as given in the last section and in the Appendix, it is straightforward to find the branching ratios for the $39 B_{s} \rightarrow P P, P V, V V$ decay channels. In the numerical calculations, we use the decay amplitudes as given in Appendixes A, B, and C of Refs. [12] directly without further discussions about details.

Following Refs. [16,12], the hadronic charmless $B$ meson decays can be classified into six classes: the first and last three classes correspond to the tree-dominated and penguindominated amplitudes, respectively.
(i) Class-I and class-II decays are dominated by the external and internal $W$-emission tree diagrams, respectively. Examples are $\bar{B}_{s} \rightarrow K^{+} \pi^{-}, K^{0} \pi^{0}, \ldots$.
(ii) Class-III decays: the decays involving both external and internal $W$ emissions. But this class does not exist for the $B_{s}$ decays.
(iii) Class-IV and class-V decay modes are governed by effective coefficients $a_{4,6,8,10}$ and $a_{3,5,7,9}$, respectively. Examples are $\bar{B}_{s} \rightarrow K^{+} K^{-}, \pi \eta^{\left({ }^{\prime}\right)}, \ldots$.
(iv) Class-VI decays involve the interference of class-IV and class- V decays.

In Tables III-VI, we present the numerical results of the branching ratios for the $39 B_{s} \rightarrow P P, P V, V V$ decays in the framework of the SM and models I, II, and III. Theoretical predictions are made by using the central values of input parameters as given in Eq. (4) and the Appendix, and assuming $A=0.804, \lambda=0.22, \rho=0.16, \eta=0.34, M_{H^{+}}=200 \mathrm{GeV}$, $\theta=0^{\circ}, 30^{\circ}, \tan \beta=2$, and $N_{c}^{\mathrm{eff}}=2,3, \infty$ in the generalized factorization approach. The $k^{2}$ dependence of the branching ratios is small in the range of $k^{2}=m_{b}^{2} / 2 \pm 2 \mathrm{GeV}^{2}$ and hence the numerical results are given by fixing $k^{2}=m_{b}^{2} / 2$.

The SM predictions for all $B_{s}$ decay modes as listed in Tables III and IV agree well with those given in Ref. [12]. The effect of changing $\hat{r}_{V}$ and including the new contribution from the chromomagnetic operator $Q_{g}$ in the SM is not significant.

For decay modes involving $B_{s} \rightarrow K^{*}$ or $B_{s} \rightarrow \phi$ transitions, we use two different set of form factors: the Bauer-Stech-Wirbel (BSW) form factor [16] and the light-cone sum rule (LCSR) form factor as given explicitly in the Appendix. For the decay modes $B_{s} \rightarrow \pi^{0} \phi, \phi \eta^{\prime}, \rho^{0} \phi, \omega \phi$ and $B_{s}$ $\rightarrow \phi \phi$, the variation of the branching ratios induced by using different set of form factors is about a factor of 2, but small or moderate for all other decay modes.

From numerical results, we see the following general features of new physics corrections.
(i) In model III, the new physics corrections to QCD-penguin-dominated decay modes, such as $B_{s} \rightarrow K^{0} \eta^{\left({ }^{\prime}\right)}$, $\eta^{(\prime)} \eta^{(\prime)}, K^{0} \bar{K}^{0}$, etc., are large in size and insensitive to variations of the mass $M_{H^{+}}$and $N_{c}^{\text {eff }}$ : from $30 \%$ to $130 \%$ with respect to the SM predictions for both cases of $\theta$ $=0^{\circ}, 30^{\circ}$. For tree-dominated or electroweak penguin-
dominated decay modes, however, the new physics corrections are very small in size: $\delta \mathcal{B} \leqslant 5 \%$.
(ii) In models I and II, the new physics corrections to all $B_{s} \rightarrow h_{1} h_{2}$ decay modes are always small in size within the considered parameter space: less than $10 \%$ and $20 \%$ in models I and II, respectively, as shown in Tables V and VI. So small corrections will be masked by other larger known theoretical uncertainties. Variation of $\tan \beta$ in the range of 2 $\leqslant \tan \beta \leqslant 50$ cannot change this feature.
(iii) In model III, the new gluonic penguins will contribute effectively through the mixing of chromomagnetic operator $Q_{g}$ with QCD penguin operators $Q_{3}-Q_{6}$, as shown in Eq. (32). The $C_{g}^{e f f}$ will strongly dominate the new physics contributions to $B_{s}$ meson decays. The branching ratios for all 39 decay modes have a very weak dependence on $\theta$ in the range of $0^{\circ} \leqslant \theta \leqslant 30^{\circ}$.

As pointed in Refs. [12,41], the decays

$$
\begin{equation*}
\bar{B}_{s} \rightarrow \eta \pi, \eta^{\prime} \pi, \eta \rho, \eta^{\prime} \rho, \phi \pi, \phi \rho \tag{46}
\end{equation*}
$$

do not receive any QCD penguin contributions, and are predominately governed by $a_{9}$ and hence are $N_{c}^{\text {eff }}$ insensitive. In 2 HDMs , this remains true because the new physics corrections to the coefficients $a_{7-10}$ are negligibly small as shown in Tables I and II, and therefore, the new physics contributions to these decay modes are also very small: $\leqslant 2 \%$. As suggested in Ref. [12], a measurement of these six decay modes can be utilized to fix the parameter $a_{9}$. It is clear that the inclusion of new physics contributions in the 2 HDMs does not change this picture.

For the decays

$$
\begin{equation*}
\bar{B}_{s} \rightarrow \omega \eta, \omega \eta^{\prime}, \phi \eta^{(\prime)}, K \phi, K^{*} \phi, \phi \pi \tag{47}
\end{equation*}
$$

the SM electroweak penguin corrections are in general as important as QCD penguin effects and very sensitive to $N_{c}^{\text {eff }}$. The new physics corrections to these decay modes in model III also have a strong dependence on the variation of $N_{c}^{\text {eff }}$ : $\delta \mathcal{B}=-20 \%-110 \%$ for $2 \leqslant N_{c}^{\text {eff }} \leqslant \infty$. As illustrated in Fig. 1, for example, the branching ratio of $B_{s} \rightarrow \phi \eta$ decay has a moderate $M_{H^{+}}$dependence, but a strong $N_{c}^{\text {eff }}$ dependence. For Figs. 1(a) and 1(b), we set $N_{c}^{\mathrm{eff}}=3$ and $M_{H^{+}}=200 \mathrm{GeV}$, respectively. The four curves correspond to the theoretical predictions in the SM (dotted curve), model II (dot-dashed curve), model III with $\theta=0^{\circ}$ (solid curve), and $\theta=30^{\circ}$ (short-dashed curve), respectively.

Among the 39 charmless two-body hadronic $B_{s}$ decays, we find that only seven (eight) of them have branching ratios at the level of $10^{-5}$ in the SM (model III):

$$
\begin{equation*}
\bar{B}_{s} \rightarrow K^{+} K^{-}, K^{0} \bar{K}^{0}, \eta \eta, \eta^{\prime} \eta^{(\prime)}, K^{+} \rho, K^{+*} \rho^{-}, \phi \phi \tag{48}
\end{equation*}
$$

Among these eight decay modes, the new physics correction to the class-I decay mode $\bar{B}_{s} \rightarrow K^{+} \rho^{-}$and $K^{+} * \rho^{-}$are very small, from $-2 \%$ to $1 \%$. For the remaining six decay modes, the new physics enhancement is significant-from $\sim 50 \%$ to $\sim 130 \%$-and insensitive to variation of $N_{c}^{\mathrm{eff}}$. These decay modes will be measurable at the future hadron colliders with large $b$ production [12]. In Figs. 2 and 3, we

TABLE III. $\mathcal{B}\left(B_{s} \rightarrow h_{1}, h_{2}\right)$ (in units of $\left.10^{-6}\right)$ in the SM and model III by using the BSW form factors, and assuming $k^{2}=m_{b}^{2} / 2, \rho=0.16, \eta=0.34, M_{H^{+}}=200 \mathrm{GeV}, \theta=0^{\circ}$, and $N_{c}^{\text {eff }}=2,3, \infty$.

| Channel | Class | SM: $\mathcal{B}$ |  |  | Model III: $\mathcal{B}$ and $\delta \mathcal{B}$ [\%] |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | 2 | 3 | $\infty$ | 2 | 3 | $\infty$ | 2 | 3 | $\infty$ |
| $\bar{B}_{s} \rightarrow K^{+} \pi^{-}$ | I | 6.33 | 7.14 | 8.89 | 6.52 | 7.35 | 9.16 | 3.1 | 3.1 | 3.0 |
| $\bar{B}_{s} \rightarrow K^{0} \pi^{0}$ | II | 0.19 | 0.08 | 0.56 | 0.24 | 0.14 | 0.64 | 23.8 | 67.1 | 14.4 |
| $\bar{B}_{s} \rightarrow K^{0} \eta$ | VI | 0.34 | 0.31 | 0.79 | 0.47 | 0.46 | 1.00 | 38.3 | 49.9 | 26.8 |
| $\bar{B}_{s} \rightarrow K^{0} \eta^{\prime}$ | VI | 0.57 | 0.51 | 0.77 | 0.88 | 0.84 | 1.17 | 53.0 | 65.6 | 52.3 |
| $\bar{B}_{s} \rightarrow K^{+} K^{-}$ | IV | 10.7 | 11.7 | 14.0 | 16.7 | 18.5 | 22.3 | 56.5 | 57.6 | 59.4 |
| $\bar{B}_{s} \rightarrow \pi^{0} \eta$ | V | 0.04 | 0.06 | 0.11 | 0.04 | 0.06 | 0.11 | 1.9 | 1.8 | 1.3 |
| $\bar{B}_{s} \rightarrow \pi^{0} \eta^{\prime}$ | V | 0.04 | 0.06 | 0.10 | 0.04 | 0.06 | 0.11 | 1.9 | 1.8 | 1.3 |
| $\bar{B}_{s} \rightarrow \eta \eta^{\prime}$ | VI | 13.8 | 15.9 | 20.5 | 22.5 | 25.9 | 33.4 | 63.8 | 63.3 | 62.6 |
| $\bar{B}_{s} \rightarrow \eta^{\prime} \eta^{\prime}$ | VI | 6.79 | 7.51 | 9.08 | 11.6 | 12.9 | 15.7 | 70.6 | 71.7 | 73.4 |
| $\bar{B}_{s} \rightarrow \eta \eta$ | VI | 6.97 | 8.37 | 11.6 | 10.9 | 13.0 | 17.7 | 56.9 | 55.3 | 52.8 |
| $\bar{B}_{s} \rightarrow K^{0} \bar{K}^{0}$ | IV | 11.4 | 13.2 | 17.3 | 17.6 | 20.4 | 26.4 | 66.2 | 65.6 | 64.5 |
| $\bar{B}_{s} \rightarrow K^{*+} \pi^{-}$ | I | 4.04 | 4.56 | 5.70 | 4.04 | 4.56 | 5.70 | 0.0 | 0.0 | 0.0 |
| $\bar{B}_{s} \rightarrow K^{+} \rho^{-}$ | I | 14.8 | 16.7 | 20.8 | 14.9 | 16.8 | 21.0 | 0.9 | 0.9 | 0.9 |
| $\bar{B}_{s} \rightarrow K^{0 *} \pi^{0}$ | II | 0.10 | 0.003 | 0.29 | 0.10 | 0.002 | 0.29 | -1.7 | -36.3 | 0.1 |
| $\bar{B}_{s} \rightarrow K^{0} \rho^{0}$ | II | 0.35 | 0.04 | 1.11 | 0.37 | 0.07 | 1.17 | 6.8 | 93.8 | 5.3 |
| $\bar{B}_{s} \rightarrow K^{0} \omega$ | II,VI | 1.14 | 0.16 | 1.81 | 1.42 | 0.26 | 1.83 | 24.7 | 56.7 | 1.2 |
| $\bar{B}_{s} \rightarrow \bar{K}^{0 *} \eta$ | II,VI | 0.16 | 0.13 | 0.44 | 0.22 | 0.21 | 0.55 | 38.4 | 58.2 | 24.9 |
| $\bar{B}_{s} \rightarrow \bar{K}^{0 *} \eta^{\prime}$ | II,VI | 0.08 | 0.02 | 0.16 | 0.10 | 0.05 | 0.20 | 33.6 | 131 | 21.7 |
| $\bar{B}_{s} \rightarrow K^{+} K^{-*}$ | IV | 3.05 | 3.39 | 4.12 | 5.03 | 5.61 | 6.86 | 64.7 | 65.3 | 66.4 |
| $\bar{B}_{s} \rightarrow K^{+*} K^{-}$ | IV | 0.89 | 0.97 | 1.15 | 0.90 | 0.99 | 1.18 | 2.2 | 2.3 | 2.5 |
| $\bar{B}_{s} \rightarrow \rho \eta$ | V | 0.08 | 0.11 | 0.25 | 0.08 | 0.12 | 0.25 | 1.0 | 1.0 | 0.8 |
| $\bar{B}_{s} \rightarrow \rho \eta^{\prime}$ | V | 0.08 | 0.11 | 0.24 | 0.08 | 0.11 | 0.24 | 1.0 | 1.0 | 0.8 |
| $\bar{B}_{s} \rightarrow \omega \eta$ | V | 0.85 | 0.01 | 2.60 | 1.29 | 0.01 | 4.15 | 51.5 | -1.4 | 59.9 |
| $\bar{B}_{s} \rightarrow \omega \eta^{\prime}$ | V | 0.84 | 0.01 | 2.56 | 1.28 | 0.01 | 4.09 | 51.5 | -1.4 | 59.9 |
| $\bar{B}_{s} \rightarrow \pi^{0} \phi$ | V | 0.13 | 0.17 | 0.32 | 0.13 | 0.17 | 0.32 | 1.9 | 1.8 | 1.3 |
| $\bar{B}_{s} \rightarrow \phi \eta$ | VI | 1.85 | 0.76 | 0.07 | 3.78 | 1.69 | 0.03 | 104 | 122 | -53.5 |
| $\bar{B}_{s} \rightarrow \phi \eta^{\prime}$ | VI | 0.70 | 0.20 | 1.49 | 1.82 | 0.40 | 1.14 | 161 | 107 | -23.5 |
| $\bar{B}_{s} \rightarrow K^{0} \bar{K}^{0 *}$ | IV | 3.24 | 4.11 | 6.17 | 5.52 | 6.85 | 9.93 | 70.6 | 66.7 | 61.0 |
| $\bar{B}_{s} \rightarrow K^{0 *} \bar{K}^{0}$ | IV | 0.39 | 0.31 | 0.18 | 0.40 | 0.32 | 0.19 | 0.8 | 0.9 | 1.0 |
| $\bar{B}_{s} \rightarrow K^{0} \phi$ | VI | 0.001 | 0.03 | 0.30 | 0.004 | 0.03 | 0.40 | 118 | 1.1 | 38.4 |
| $\bar{B}_{s} \rightarrow K^{+*} \rho^{-}$ | I | 12.5 | 14.1 | 17.5 | 12.6 | 14.2 | 17.7 | 0.9 | 0.9 | 0.9 |
| $\bar{B}_{s} \rightarrow K^{0} * \rho^{0}$ | II | 0.29 | 0.03 | 0.94 | 0.31 | 0.06 | 0.99 | 6.8 | 93.8 | 5.3 |
| $\bar{B}_{s} \rightarrow K^{0 *} \omega$ | II,VI | 0.24 | 0.03 | 0.38 | 0.30 | 0.05 | 0.39 | 24.7 | 56.7 | 1.2 |
| $\bar{B}_{s} \rightarrow K^{+*} K^{-*}$ | IV | 2.72 | 3.02 | 3.68 | 4.48 | 5.00 | 6.12 | 64.7 | 65.3 | 66.4 |
| $\bar{B}_{s} \rightarrow \rho^{0} \phi$ | V | 0.15 | 0.21 | 0.45 | 0.15 | 0.21 | 0.46 | 1.0 | 0.99 | 0.8 |
| $\bar{B}_{s} \rightarrow \omega \phi$ | V | 0.79 | 0.01 | 2.41 | 1.20 | 0.01 | 3.85 | 51.3 | -1.35 | 59.9 |
| $\bar{B}_{s} \rightarrow K^{0 *} \bar{K}^{0 *}$ | IV | 2.14 | 2.71 | 4.07 | 3.65 | 4.53 | 6.56 | 70.7 | 66.8 | 61.1 |
| $\bar{B}_{s} \rightarrow K^{0 *} \phi$ | VI | 0.03 | 0.12 | 0.48 | 0.05 | 0.19 | 0.74 | 68.5 | 58.9 | 54.1 |
| $\bar{B}_{s} \rightarrow \phi \phi$ | VI | 17.5 | 8.99 | 0.42 | 29.9 | 15.8 | 0.98 | 71.1 | 75.8 | 134 |

TABLE IV. $\mathcal{B}\left(B_{s} \rightarrow P V, V V\right)$ (in units of $10^{-6}$ ) in the SM and model III by using the LCSR form factors for $B_{s} \rightarrow K^{*}$ or $B_{s} \rightarrow \phi$ transition, and assuming $k^{2}=m_{b}^{2} / 2, \rho=0.16, \eta=0.34, M_{H^{+}}=200 \mathrm{GeV}, \theta=0^{\circ}$, and $N_{c}^{\mathrm{eff}}=2,3, \infty$.

| Channel | Class | SM: $\mathcal{B}$ |  |  | Model III: $\mathcal{B}$ and $\delta \mathcal{B}$ [\%] |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | 2 | 3 | $\infty$ | 2 | 3 | $\infty$ | 2 | 3 | $\infty$ |
| $\bar{B}_{s} \rightarrow K^{*+} \pi^{-}$ | I | 4.68 | 5.29 | 6.61 | 4.69 | 5.29 | 6.61 | 0.0 | 0.0 | 0.0 |
| $\bar{B}_{s} \rightarrow K^{+} \rho^{-}$ | I | 15.4 | 17.4 | 21.7 | 15.5 | 17.5 | 21.9 | 0.9 | 0.9 | 0.9 |
| $\bar{B}_{s} \rightarrow K^{0} * \pi^{0}$ | II | 0.12 | 0.003 | 0.33 | 0.11 | 0.002 | 0.33 | $-1.7$ | -36.3 | 0.1 |
| $\bar{B}_{s} \rightarrow K^{0} \rho^{0}$ | II | 0.36 | 0.04 | 1.16 | 0.39 | 0.07 | 1.22 | 6.8 | 93.8 | 5.3 |
| $\bar{B}_{s} \rightarrow K^{0} \omega$ | II,VI | 1.19 | 0.17 | 1.89 | 1.49 | 0.27 | 1.91 | 24.7 | 56.7 | 1.2 |
| $\bar{B}_{s} \rightarrow \bar{K}^{0 *} \eta$ | II,VI | 0.18 | 0.14 | 0.50 | 0.24 | 0.22 | 0.62 | 36.7 | 57.8 | 23.6 |
| $\bar{B}_{s} \rightarrow \bar{K}^{0 *} \eta^{\prime}$ | II,VI | 0.09 | 0.02 | 0.19 | 0.11 | 0.05 | 0.23 | 28.7 | 137 | 17.5 |
| $\bar{B}_{s} \rightarrow K^{+} K^{-*}$ | IV | 3.22 | 3.58 | 4.35 | 5.31 | 5.92 | 7.22 | 64.7 | 65.3 | 66.4 |
| $\bar{B}_{s} \rightarrow K^{+*} K^{-}$ | IV | 1.04 | 1.14 | 1.35 | 1.06 | 1.16 | 1.38 | 2.2 | 2.3 | 2.8 |
| $\bar{B}_{s} \rightarrow \rho \eta$ | V | 0.09 | 0.12 | 0.26 | 0.09 | 0.12 | 0.26 | 1.0 | 1.0 | 0.8 |
| $\bar{B}_{s} \rightarrow \rho \eta^{\prime}$ | V | 0.09 | 0.12 | 0.25 | 0.09 | 0.12 | 0.25 | 1.0 | 1.0 | 0.8 |
| $\bar{B}_{s} \rightarrow \omega \eta$ | V | 0.89 | 0.01 | 2.71 | 1.35 | 0.01 | 4.33 | 51.5 | $-1.4$ | 59.9 |
| $\bar{B}_{s} \rightarrow \omega \eta^{\prime}$ | V | 0.88 | 0.01 | 2.67 | 1.33 | 0.01 | 4.27 | 51.5 | $-1.4$ | 59.9 |
| $\bar{B}_{s} \rightarrow \pi^{0} \phi$ | V | 0.26 | 0.33 | 0.63 | 0.26 | 0.34 | 0.64 | 1.9 | 1.8 | 1.3 |
| $\bar{B}_{s} \rightarrow \phi \eta$ | VI | 1.36 | 0.49 | 0.18 | 3.04 | 1.23 | 0.09 | 124 | 151 | $-50.3$ |
| $\bar{B}_{s} \rightarrow \phi \eta^{\prime}$ | VI | 0.38 | 0.53 | 3.43 | 0.87 | 0.21 | 2.91 | 127 | $-60.3$ | $-15.3$ |
| $\bar{B}_{s} \rightarrow K^{0} \bar{K}^{0 *}$ | IV | 3.42 | 4.34 | 6.52 | 5.83 | 7.23 | 10.5 | 70.6 | 66.7 | 61.0 |
| $\bar{B}_{s} \rightarrow K^{0 *} \bar{K}^{0}$ | IV | 0.46 | 0.37 | 0.22 | 0.46 | 0.37 | 0.22 | 0.8 | 0.9 | 1.0 |
| $\bar{B}_{s} \rightarrow K^{0} \phi$ | VI | 0.004 | 0.05 | 0.36 | 0.002 | 0.05 | 0.50 | - 56.3 | 1.1 | 36.0 |
| $\bar{B}_{s} \rightarrow K^{+*} \rho^{-}$ | I | 13.2 | 14.9 | 18.6 | 13.3 | 15.0 | 18.8 | 0.9 | 0.9 | 0.9 |
| $\bar{B}_{s} \rightarrow K^{0} * \rho^{0}$ | II | 0.31 | 0.03 | 0.99 | 0.33 | 0.06 | 1.05 | 6.8 | 93.8 | 5.3 |
| $\bar{B}_{s} \rightarrow K^{0 *} \omega$ | II,VI | 0.26 | 0.04 | 0.40 | 0.32 | 0.06 | 0.41 | 24.7 | 56.7 | 1.2 |
| $\bar{B}_{s} \rightarrow K^{+*} K^{-*}$ | IV | 2.82 | 3.13 | 3.79 | 4.64 | 5.17 | 6.33 | 64.7 | 65.3 | 66.4 |
| $\bar{B}_{s} \rightarrow \rho^{0} \phi$ | V | 0.27 | 0.38 | 0.82 | 0.28 | 0.38 | 0.82 | 1.0 | 1.0 | 0.8 |
| $\bar{B}_{s} \rightarrow \omega \phi$ | V | 1.43 | 0.01 | 4.33 | 2.16 | 0.01 | 6.93 | 51.5 | $-1.4$ | 59.9 |
| $\bar{B}_{s} \rightarrow K^{0} * \bar{K}^{0 *}$ | IV | 2.20 | 2.80 | 4.20 | 3.76 | 4.67 | 6.77 | 70.7 | 66.8 | 61.1 |
| $\bar{B}_{s} \rightarrow K^{0} * \phi$ | VI | 0.07 | 0.20 | 0.66 | 0.12 | 0.32 | 1.03 | 68.9 | 60.5 | 55.1 |
| $\bar{B}_{s} \rightarrow \phi \phi$ | VI | 29.9 | 15.4 | 0.72 | 51.1 | 27.0 | 1.68 | 71.1 | 75.8 | 134 |

plot the mass and $N_{c}^{\text {eff }}$ dependence of the branching ratios of $\bar{B}_{s} \rightarrow K^{+} K^{-}$and $\eta \eta^{\prime}$ decay modes.

After inclusion of new physics contributions in models I, II, and III, the patterns observed in Ref. [12] remain unchanged:

$$
\begin{align*}
& \Gamma\left(\bar{B}_{s} \rightarrow K^{+} K^{-}\right)>\Gamma\left(\bar{B}_{s} \rightarrow K^{+} K^{*-}\right) \\
& \quad \gtrsim \Gamma\left(\bar{B}_{s} \rightarrow K^{*+} K^{*-}\right)>\Gamma\left(\bar{B}_{s} \rightarrow K^{+*} K^{-}\right), \\
& \Gamma\left(\bar{B}_{s} \rightarrow K^{0} \bar{K}^{0}\right)>\Gamma\left(\bar{B}_{s} \rightarrow K^{0} \bar{K}^{* 0}\right) \\
& \quad \gtrsim \Gamma\left(\bar{B}_{s} \rightarrow K^{* 0} \bar{K}^{* 0}\right)>\Gamma\left(\bar{B}_{s} \rightarrow K^{* 0} \bar{K}^{0}\right) . \tag{49}
\end{align*}
$$

Recently, large decay rates for $B_{u}^{+} \rightarrow K^{+} \eta^{\prime}$ and $B_{d} \rightarrow K^{0} \eta^{\prime}$ decays have been reported by the CLEO and BaBar Collaborations [4,5]. The CLEO measurement of $B_{d}^{0} \rightarrow K^{0} \eta^{\prime}$ decay
is $\mathcal{B}\left(B_{d}^{0} \rightarrow K^{0} \eta^{\prime}\right)=\left(89_{-16}^{+18} \pm 9\right) \times 10^{-6}$, which is larger than the branching ratios of $B \rightarrow K \pi$ decays by a factor of 3-5. For $B_{s}$ decays, the decay modes $\bar{B}_{s} \rightarrow \eta \eta^{\prime}$ and $\bar{B}_{s} \eta^{\prime} \eta^{\prime}$ are the analogue of $B_{d} \rightarrow K^{0} \eta^{\prime}$ decay and are expected to have large branching ratios. From Table III, one can see that the SM predictions of the branching ratios $\mathcal{B}\left(B_{s} \rightarrow \eta \eta^{\prime}\right)$ and $\mathcal{B}\left(B_{s} \rightarrow \eta^{\prime} \eta^{\prime}\right)$ are indeed large, but comparable in size with the other six decay modes listed in Eq. (48). The new physics enhancement to these two decay modes is significant in size, $\sim 70 \%$ in model III, as illustrated in Fig. 3. After the inclusion of new physics contributions, we find numerically that

$$
\begin{align*}
& \mathcal{B}\left(\bar{B}_{s} \rightarrow \eta \eta^{\prime}\right) \approx(23-33) \times 10^{-6},  \tag{50}\\
& \mathcal{B}\left(\bar{B}_{s} \rightarrow \eta^{\prime} \eta^{\prime}\right) \approx(12-16) \times 10^{-6} . \tag{51}
\end{align*}
$$

TABLE V. $\mathcal{B}\left(B_{s} \rightarrow h_{1} h_{2}\right)$ (in units of $10^{-6}$ ) in model I, with $k^{2}=m_{b}^{2} / 2, \rho=0.16, \eta=0.34, M_{H^{+}}$ $=200 \mathrm{GeV}, \tan \beta=2$, and $N_{c}^{\mathrm{eff}}=2,3, \infty$.

| Channel | Class | SM: $\mathcal{B}$ |  |  | Model I: $\mathcal{B}$ and $\delta \mathcal{B}$ [\%] |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | 2 | 3 | $\infty$ | 2 | 3 | $\infty$ | 2 | 3 | $\infty$ |
| $\bar{B}_{s} \rightarrow K i^{+} \pi^{-}$ | I | 6.33 | 7.13 | 8.88 | 6.34 | 7.14 | 8.90 | 0.1 | 0.1 | 0.1 |
| $\bar{B}_{s} \rightarrow K^{0} \pi^{0}$ | II | 0.19 | 0.08 | 0.56 | 0.19 | 0.08 | 0.56 | -0.1 | 0.0 | 0.1 |
| $\bar{B}_{s} \rightarrow K^{0} \eta$ | VI | 0.34 | 0.31 | 0.78 | 0.34 | 0.31 | 0.79 | 0.4 | 0.7 | 0.4 |
| $\bar{B}_{s} \rightarrow K^{0} \eta^{\prime}$ | VI | 0.57 | 0.51 | 0.76 | 0.58 | 0.52 | 0.77 | 1.2 | 1.5 | 1.3 |
| $\bar{B}_{s} \rightarrow K^{+} K^{-}$ | IV | 10.6 | 11.7 | 14.0 | 10.8 | 11.9 | 14.1 | 1.4 | 1.3 | 1.3 |
| $\bar{B}_{s} \rightarrow \pi^{0} \eta$ | V | 0.04 | 0.06 | 0.11 | 0.05 | 0.06 | 0.11 | 10.5 | 10.0 | 7.3 |
| $\bar{B}_{s} \rightarrow \pi^{0} \eta^{\prime}$ | V | 0.04 | 0.05 | 0.10 | 0.05 | 0.06 | 0.11 | 10.5 | 10.0 | 7.3 |
| $\bar{B}_{s} \rightarrow \eta \eta^{\prime}$ | VI | 13.7 | 15.8 | 20.5 | 13.9 | 16.1 | 20.8 | 1.1 | 1.2 | 1.3 |
| $\bar{B}_{s} \rightarrow \eta^{\prime} \eta^{\prime}$ | VI | 6.77 | 7.48 | 9.05 | 6.89 | 7.63 | 9.22 | 1.5 | 1.5 | 1.6 |
| $\bar{B}_{s} \rightarrow \eta \eta$ | VI | 6.95 | 8.35 | 11.5 | 7.03 | 8.44 | 11.7 | 0.8 | 0.9 | 1.0 |
| $\bar{B}_{s} \rightarrow K^{0} \bar{K}^{0}$ | IV | 11.4 | 13.2 | 17.2 | 11.6 | 13.4 | 17.5 | 1.8 | 1.9 | 2.0 |
| $\bar{B}_{s} \rightarrow K^{*+} \pi^{-}$ | I | 4.04 | 4.56 | 5.70 | 4.04 | 4.56 | 5.70 | 0.0 | 0.0 | 0.0 |
| $\bar{B}_{s} \rightarrow K^{+} \rho^{-}$ | I | 14.7 | 16.6 | 20.8 | 14.8 | 16.7 | 20.8 | 0.0 | 0.0 | 0.0 |
| $\bar{B}_{s} \rightarrow K^{0 *} \pi^{0}$ | II | 0.10 | 0.003 | 0.29 | 0.10 | 0.003 | 0.29 | 0.3 | 5.1 | 0.0 |
| $\bar{B}_{s} \rightarrow K^{0} \rho^{0}$ | II | 0.35 | 0.04 | 1.11 | 0.35 | 0.04 | 1.11 | 0.0 | 0.5 | 0.0 |
| $\bar{B}_{s} \rightarrow K^{0} \omega$ | II,VI | 1.14 | 0.16 | 1.81 | 1.15 | 0.17 | 1.81 | 0.6 | 1.3 | 0.0 |
| $\bar{B}_{s} \rightarrow \bar{K}^{0 *} \eta$ | II,VI | 0.16 | 0.13 | 0.44 | 0.16 | 0.13 | 0.45 | 0.4 | 0.8 | 0.5 |
| $\bar{B}_{s} \rightarrow \bar{K}^{0 *} \eta^{\prime}$ | II,VI | 0.08 | 0.02 | 0.16 | 0.08 | 0.02 | 0.17 | 0.6 | 2.8 | 0.5 |
| $\bar{B}_{s} \rightarrow K^{+} K^{-*}$ | IV | 3.04 | 3.38 | 4.11 | 3.12 | 3.45 | 4.18 | 2.1 | 1.9 | 1.5 |
| $\bar{B}_{s} \rightarrow K^{+*} K^{-}$ | IV | 0.89 | 0.97 | 1.15 | 0.87 | 0.96 | 1.14 | $-1.5$ | $-1.0$ | -0.2 |
| $\bar{B}_{s} \rightarrow \rho \eta$ | V | 0.08 | 0.11 | 0.25 | 0.09 | 0.12 | 0.26 | 5.4 | 5.6 | 4.6 |
| $\bar{B}_{s} \rightarrow \rho \eta^{\prime}$ | V | 0.08 | 0.11 | 0.24 | 0.09 | 0.12 | 0.25 | 5.4 | 5.6 | 4.6 |
| $\bar{B}_{s} \rightarrow \omega \eta$ | V | 0.85 | 0.01 | 2.59 | 0.86 | 0.01 | 2.65 | 0.8 | -7.4 | 2.0 |
| $\bar{B}_{s} \rightarrow \omega \eta^{\prime}$ | V | 0.84 | 0.01 | 2.55 | 0.85 | 0.01 | 2.61 | 0.8 | -7.4 | 2.0 |
| $\bar{B}_{s} \rightarrow \pi^{0} \phi$ | V | 0.13 | 0.17 | 0.32 | 0.14 | 0.18 | 0.34 | 10.5 | 10.0 | 7.3 |
| $\bar{B}_{s} \rightarrow \phi \eta$ | VI | 1.84 | 0.75 | 0.07 | 1.86 | 0.76 | 0.07 | 0.4 | 0.2 | 2.6 |
| $\bar{B}_{s} \rightarrow \phi \eta^{\prime}$ | VI | 0.69 | 0.20 | 1.49 | 0.71 | 0.20 | 1.51 | 1.0 | 0.0 | 1.2 |
| $\bar{B}_{s} \rightarrow K^{0} \bar{K}^{0 *}$ | IV | 3.22 | 4.09 | 6.15 | 3.30 | 4.19 | 6.31 | 2.0 | 2.1 | 2.3 |
| $\bar{B}_{s} \rightarrow K^{0 *} \bar{K}^{0}$ | IV | 0.39 | 0.31 | 0.18 | 0.39 | 0.31 | 0.18 | -0.4 | $-1.0$ | -2.9 |
| $\bar{B}_{s} \rightarrow K^{0} \phi$ | VI | 0.001 | 0.03 | 0.30 | 0.002 | 0.03 | 0.30 | 0.3 | 2.1 | 1.7 |
| $\bar{B}_{s} \rightarrow K^{+*} \rho^{-}$ | I | 12.4 | 14.1 | 17.5 | 12.5 | 14.1 | 17.5 | 0.0 | 0.0 | 0.0 |
| $\bar{B}_{s} \rightarrow K^{0} * \rho^{0}$ | II | 0.29 | 0.03 | 0.94 | 0.29 | 0.03 | 0.94 | 0.0 | 0.5 | 0.0 |
| $\bar{B}_{s} \rightarrow K^{0} * \omega$ | II,VI | 0.24 | 0.03 | 0.38 | 0.24 | 0.04 | 0.38 | 0.6 | 1.3 | 0.0 |
| $\bar{B}_{s} \rightarrow K^{+*} K^{-*}$ | IV | 2.71 | 3.02 | 3.66 | 2.78 | 3.08 | 3.73 | 2.1 | 1.9 | 1.5 |
| $\bar{B}_{s} \rightarrow \rho^{0} \phi$ | V | 0.15 | 0.21 | 0.45 | 0.16 | 0.22 | 0.47 | 5.4 | 5.6 | 4.6 |
| $\bar{B}_{s} \rightarrow \omega \phi$ | V | 0.79 | 0.01 | 2.40 | 0.80 | 0.01 | 2.46 | 0.7 | -7.4 | 2.0 |
| $\bar{B}_{s} \rightarrow K^{0 *} \bar{K}^{0 *}$ | IV | 2.13 | 2.70 | 4.06 | 2.17 | 2.77 | 4.17 | 2.0 | 2.1 | 2.3 |
| $\bar{B}_{s} \rightarrow K^{0 *} \phi$ | VI | 0.03 | 0.12 | 0.48 | 0.03 | 0.12 | 0.49 | 3.8 | 2.9 | 2.4 |
| $\bar{B}_{s} \rightarrow \phi \phi$ | VI | 17.4 | 8.95 | 0.42 | 17.7 | 9.08 | 0.42 | 1.1 | 1.1 | 0.5 |

These theoretical predictions will be tested by future experimental measurements.

For the decays $\bar{B} \rightarrow K^{+} K^{-*}$ and $\bar{B} \rightarrow K^{+*} K^{-*}$, they have relatively large decay rates and weak $M_{H^{+}}$and $N_{c}^{\text {eff }}$ depen-
dence. In Figs. 4 and 5, we plot the mass and $N_{c}^{\text {eff }}$ dependence of the branching ratios $\mathcal{B}\left(\bar{B}_{s} \rightarrow K^{+} K^{-*}\right)$ and $\mathcal{B}\left(K^{+*} K^{-*}\right)$. It is easy to see that the new physics contributions in the model III to these two class-IV decays are

TABLE VI. $\mathcal{B}\left(B_{s} \rightarrow h_{1} h_{2}\right)$ (in units of $10^{-6}$ ) in model II, with $k^{2}=m_{b}^{2} / 2, \rho=0.16, \eta=0.34, M_{H^{+}}$ $=200 \mathrm{GeV}, \tan \beta=2$, and $N_{c}^{\mathrm{eff}}=2,3, \infty$.

| Channel | Class | SM: $\mathcal{B}$ |  |  | Model II: $\mathcal{B}$ and $\delta \mathcal{B}$ [\%] |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | 2 | 3 | $\infty$ | 2 | 3 | $\infty$ | 2 | 3 | $\infty$ |
| $\bar{B}_{s} \rightarrow K^{+} \pi^{-}$ | I | 6.33 | 7.13 | 8.88 | 6.29 | 7.09 | 8.84 | -0.6 | -0.6 | -0.6 |
| $\bar{B}_{s} \rightarrow K^{0} \pi^{0}$ | II | 0.19 | 0.08 | 0.56 | 0.18 | 0.07 | 0.55 | -4.7 | $-13.4$ | -2.9 |
| $\bar{B}_{s} \rightarrow K^{0} \eta$ | VI | 0.34 | 0.31 | 0.78 | 0.31 | 0.28 | 0.74 | -7.5 | -9.8 | -5.2 |
| $\bar{B}_{s} \rightarrow K^{0} \eta^{\prime}$ | VI | 0.57 | 0.51 | 0.76 | 0.52 | 0.45 | 0.70 | -9.5 | $-11.6$ | -9.1 |
| $\bar{B}_{s} \rightarrow K^{+} K^{-}$ | IV | 10.6 | 11.7 | 14.0 | 9.58 | 10.5 | 12.5 | $-10.2$ | $-10.4$ | $-10.8$ |
| $\bar{B}_{s} \rightarrow \pi^{0} \eta$ | V | 0.04 | 0.06 | 0.11 | 0.05 | 0.06 | 0.11 | 10.5 | 10.0 | 7.3 |
| $\bar{B}_{s} \rightarrow \pi^{0} \eta^{\prime}$ | V | 0.04 | 0.05 | 0.10 | 0.05 | 0.06 | 0.11 | 10.5 | 10.0 | 7.3 |
| $\bar{B}_{s} \rightarrow \eta \eta^{\prime}$ | VI | 13.7 | 15.8 | 20.5 | 12.1 | 14.0 | 18.2 | -11.7 | - 11.6 | -11.4 |
| $\bar{B}_{s} \rightarrow \eta^{\prime} \eta^{\prime}$ | VI | 6.77 | 7.48 | 9.05 | 5.94 | 6.56 | 7.91 | - 12.5 | $-12.7$ | -12.9 |
| $\bar{B}_{s} \rightarrow \eta \eta$ | VI | 6.95 | 8.35 | 11.5 | 6.21 | 7.49 | 10.4 | -10.9 | - 10.6 | -9.9 |
| $\bar{B}_{s} \rightarrow K^{0} \bar{K}^{0}$ | IV | 11.4 | 13.2 | 17.2 | 10.3 | 11.9 | 15.6 | -11.9 | -11.7 | -11.4 |
| $\bar{B}_{s} \rightarrow K^{*+} \pi^{-}$ | I | 4.04 | 4.56 | 5.70 | 4.04 | 4.56 | 5.70 | 0.0 | 0.0 | 0.0 |
| $\bar{B}_{s} \rightarrow K^{+} \rho^{-}$ | I | 14.7 | 16.6 | 20.8 | 14.7 | 16.6 | 20.8 | -0.2 | -0.2 | -0.2 |
| $\bar{B}_{s} \rightarrow K^{0 *} \pi^{0}$ | II | 0.10 | 0.003 | 0.29 | 0.10 | 0.003 | 0.29 | 0.8 | 16.5 | 0.0 |
| $\bar{B}_{s} \rightarrow K^{0} \rho^{0}$ | II | 0.35 | 0.04 | 1.11 | 0.35 | 0.03 | 1.10 | -1.1 | -16.7 | -1.0 |
| $\bar{B}_{s} \rightarrow K^{0} \omega$ | II,VI | 1.14 | 0.16 | 1.81 | 1.09 | 0.15 | 1.80 | -4.5 | -10.3 | -0.2 |
| $\bar{B}_{s} \rightarrow \bar{K}^{0 *} \eta$ | II,VI | 0.16 | 0.13 | 0.44 | 0.15 | 0.12 | 0.42 | -7.3 | $-11.1$ | -4.7 |
| $\bar{B}_{s} \rightarrow \bar{K}^{0 *} \eta^{\prime}$ | II,VI | 0.08 | 0.02 | 0.16 | 0.07 | 0.02 | 0.16 | -4.7 | -17.9 | -2.9 |
| $\bar{B}_{s} \rightarrow K^{+} K^{-*}$ | IV | 3.04 | 3.38 | 4.11 | 2.74 | 3.03 | 3.66 | $-10.3$ | -10.7 | $-11.2$ |
| $\bar{B}_{s} \rightarrow K^{+*} K^{-}$ | IV | 0.89 | 0.97 | 1.15 | 0.87 | 0.95 | 1.14 | -2.0 | -1.6 | -0.8 |
| $\bar{B}_{s} \rightarrow \rho \eta$ | V | 0.08 | 0.11 | 0.25 | 0.09 | 0.12 | 0.26 | 5.4 | 5.6 | 4.6 |
| $\bar{B}_{s} \rightarrow \rho \eta^{\prime}$ | V | 0.08 | 0.11 | 0.24 | 0.09 | 0.12 | 0.25 | 5.4 | 5.6 | 4.6 |
| $\bar{B}_{s} \rightarrow \omega \eta$ | V | 0.85 | 0.01 | 2.59 | 0.77 | 0.01 | 2.33 | -9.9 | -7.4 | -10.3 |
| $\bar{B}_{s} \rightarrow \omega \eta^{\prime}$ | V | 0.84 | 0.01 | 2.55 | 0.76 | 0.01 | 2.30 | -9.9 | -7.4 | -10.3 |
| $\bar{B}_{s} \rightarrow \pi^{0} \phi$ | V | 0.13 | 0.17 | 0.32 | 0.14 | 0.18 | 0.34 | 10.5 | 10.0 | 7.3 |
| $\bar{B}_{s} \rightarrow \phi \eta$ | VI | 1.84 | 0.75 | 0.07 | 1.51 | 0.60 | 0.09 | $-18.3$ | -20.5 | 26.8 |
| $\bar{B}_{s} \rightarrow \phi \eta^{\prime}$ | VI | 0.69 | 0.20 | 1.49 | 0.55 | 0.21 | 1.60 | -21.3 | 7.5 | 7.6 |
| $\bar{B}_{s} \rightarrow K^{0} \bar{K}^{0 *}$ | IV | 3.22 | 4.09 | 6.15 | 2.85 | 3.64 | 5.54 | $-12.0$ | $-11.3$ | -10.1 |
| $\bar{B}_{s} \rightarrow K^{0 *} \bar{K}^{0}$ | IV | 0.39 | 0.31 | 0.18 | 0.39 | 0.31 | 0.18 | -0.6 | -1.3 | -3.2 |
| $\bar{B}_{s} \rightarrow K^{0} \phi$ | VI | 0.001 | 0.03 | 0.30 | 0.002 | 0.03 | 0.28 | 6.6 | 2.0 | -6.6 |
| $\bar{B}_{s} \rightarrow K^{+*} \rho^{-}$ | I | 12.4 | 14.1 | 17.5 | 12.4 | 14.0 | 17.5 | -0.2 | -0.2 | -0.2 |
| $\bar{B}_{s} \rightarrow K^{0} * \rho^{0}$ | II | 0.29 | 0.03 | 0.94 | 0.29 | 0.03 | 0.93 | -1.1 | -16.7 | - 1.0 |
| $\bar{B}_{s} \rightarrow K^{0 *} \omega$ | II,VI | 0.24 | 0.03 | 0.38 | 0.23 | 0.03 | 0.38 | -4.5 | -10.3 | -0.2 |
| $\bar{B}_{s} \rightarrow K^{+*} K^{-*}$ | IV | 2.71 | 3.02 | 3.66 | 2.43 | 2.70 | 3.27 | $-10.3$ | $-10.7$ | -11.2 |
| $\bar{B}_{s} \rightarrow \rho^{0} \phi$ | V | 0.15 | 0.21 | 0.45 | 0.16 | 0.22 | 0.47 | 5.4 | 5.6 | 4.6 |
| $\bar{B}_{s} \rightarrow \omega \phi$ | V | 0.79 | 0.01 | 2.40 | 0.71 | 0.01 | 2.16 | $-10.0$ | -7.4 | -10.3 |
| $\bar{B}_{s} \rightarrow K^{0 *} \bar{K}^{0 *}$ | IV | 2.13 | 2.70 | 4.06 | 1.88 | 2.41 | 3.66 | $-12.1$ | $-11.3$ | -10.2 |
| $\bar{B}_{s} \rightarrow K^{0 *} \phi$ | VI | 0.03 | 0.12 | 0.48 | 0.03 | 0.11 | 0.44 | -10.0 | -9.2 | -8.8 |
| $\bar{B}_{s} \rightarrow \phi \phi$ | VI | 17.4 | 8.95 | 0.42 | 15.2 | 7.75 | 0.33 | - 13.0 | -13.8 | -21.4 |



FIG. 1. Branching ratios $\mathcal{B}\left(\bar{B}_{s} \rightarrow \phi \eta\right)$ versus $M_{H^{+}}$and $1 / N_{c}^{\text {eff }}$ in the SM and models II and III by using the BSW form factors. For (a) and (b), we set $N_{c}^{\text {eff }}=3$ and $M_{H^{+}}=200 \mathrm{GeV}$, respectively. The four curves correspond to the theoretical predictions in the SM (dotted line), model II (dot-dashed curve), model III with $\theta=0^{\circ}$ (solid curve), and $\theta=30^{\circ}$ (short-dashed curve), respectively.
significant ( $\sim 70 \%$ ) in size and insensitive to the variations of $M_{H^{+}}$and $N_{c}^{\mathrm{eff}}$.

## V. CP-VIOLATING ASYMMETRIES OF $\boldsymbol{B}_{s}$ MESON DECAYS

In Ref. [25], Du et al. studied the branching ratios and $C P$-violating asymmetries for decay modes $B_{s} \rightarrow K^{-} \pi^{+}$, $K^{+} K^{-}, \bar{K}^{0} \pi^{0}, \phi \phi$, and $\bar{K}^{0} \phi$. Recently, Ali et al. [42] estimated the $C P$-violating asymmetries in 76 charmless hadronic decays of $B_{u}$ and $B_{d}$ mesons. The calculation of the $C P$-violating asymmetry $\mathcal{A}_{C P}$ for $B_{s}$ meson decays is theoretically very similar to that of the $B_{d}$ meson decays. For more details about the theoretical aspects of $C P$-violating asymmetries in $B_{u, d} \rightarrow h_{1} h_{2}$ decays, one should see Ref. [42] and reference therein. In this section, we calculate the $C P$-violating asymmetries of $B_{s} \rightarrow h_{1} h_{2}$ decays in the framework of the SM and the general two-Higgs-doublet models. We focus on evaluating the new physics effects on $\mathcal{A}_{C P}$ for $39 B_{s}$ decay channels induced by charged-Higgs-boson penguin diagrams appearing in the general two-Higgs-doublet models.


FIG. 2. Same as Fig. 1 but for the decay $\bar{B}_{s} \rightarrow K^{+} K^{-}$.

In models I and II, one does not expect sizable changes in $\mathcal{A}_{C P}$ of $B_{s}$ decays since there is no any new phase introduced when compared with the SM. In model III, although the introducing of a new phase $\theta$ played an important role in relaxing the constraint on the parameter space of model III due to the CLEO measurement of $B \rightarrow X_{s} \gamma$ decay as studied in Ref. [34], we still do not expect dramatic changes for the pattern of the $C P$-violating asymmetries of $B_{s}$ decays under consideration because this phase may alter the theoretical prediction of $\mathcal{A}_{C P}$ through loop diagrams only.

Analogous to the $B_{d}$ meson decays, the time-dependent $C P$ asymmetry for the decays of states that were tagged as pure $B_{s}^{0}$ or $\bar{B}_{s}^{0}$ at production is defined as

$$
\begin{equation*}
\mathcal{A}_{C P}(t)=\frac{\Gamma\left(B_{s}^{0}(t) \rightarrow f\right)-\Gamma\left(\bar{B}_{s}^{0}(t) \rightarrow \bar{f}\right)}{\Gamma\left(B_{s}^{0}(t) \rightarrow f\right)+\Gamma\left(\bar{B}_{s}^{0}(t) \rightarrow \bar{f}\right)} . \tag{52}
\end{equation*}
$$

Following Ref. [42], the neutral $B_{s}^{0}\left(\bar{B}_{s}^{0}\right)$ decays can be classified into three classes according to the properties of the final states $f$ and $\bar{f}$.
(i) Class- 1 decays: $B_{s}^{0} \rightarrow f, \bar{B}_{s}^{0} \rightarrow \bar{f}$, and the final states $f$ or $\bar{f}$ is not a common final state of $B_{s}^{0}$ and $\bar{B}_{s}^{0}$, for example,


FIG. 3. Same as Fig. 1 but for the decay $\bar{B}_{s} \rightarrow \eta \eta^{\prime}$.
$B_{s}^{0} \rightarrow K^{+} \pi^{-}$. The $C P$-violating asymmetry for class- 1 decays will be independent of time,

$$
\begin{equation*}
\mathcal{A}_{C P}=\frac{\Gamma\left(B_{s}^{0} \rightarrow f\right)-\Gamma\left(\bar{B}_{s}^{0} \rightarrow \bar{f}\right)}{\Gamma\left(B_{s}^{0} \rightarrow f\right)+\Gamma\left(\bar{B}_{s}^{0} \rightarrow \bar{f}\right)}, \tag{53}
\end{equation*}
$$

in terms of partial decay widths.
(ii) Class-2 and 3 decays: $\stackrel{(-)}{B}_{s}^{0} \rightarrow(f=\bar{f})$ with $f^{C P}= \pm f$ (class 2) or $f^{C P} \neq \pm f$ (class 3), the time-integrated $C P$ asymmetries are of the form

$$
\begin{equation*}
\mathcal{A}_{C P}=\frac{1}{1+x^{2}} \frac{1}{1+\left|\lambda_{C P}\right|^{2}}-2 \frac{x}{1+x^{2}} \frac{\operatorname{Im}\left(\lambda_{C P}\right)}{1+\left|\lambda_{C P}\right|^{2}}, \tag{54}
\end{equation*}
$$

with

$$
\begin{equation*}
\lambda_{C P}=\frac{V_{t b}^{*} V_{t s}}{V_{t b} V_{t s}^{*}} \frac{\langle f| H_{e f f}\left|\bar{B}_{s}^{0}\right\rangle}{\langle f| H_{e f f}\left|B_{s}^{0}\right\rangle}, \tag{55}
\end{equation*}
$$

where $x=\Delta M_{B_{s}^{0}} / \Gamma_{B_{s}^{0}} \approx 20$ is the preferred value in the SM


FIG. 4. Same as Fig. 1 but for the decay $\bar{B}_{s} \rightarrow K^{+} K^{-*}$.
[25] for the case of $B_{s}^{0}-\bar{B}_{s}^{0}$ mixing. ${ }^{3}$ Contrary to the $B_{d}$ meson decay where $x \approx 0.73$, it is easy to see that the parameter $x$ for $B_{s}^{0}$ decays is very large. The first and second terms in Eq. (54) are strongly suppressed by $1 / x^{2}$ and $1 / x$, respectively. We therefore do not expect large $C P$-violating asymmetries $\mathcal{A}_{C P}$ for the class-2 and class- $3 B_{s}^{0}$ decays. This expectation is confirmed by the numerical results given below.

In Tables VII and VIII, we present numerical results of $C P$-violating asymmetries $\mathcal{A}_{C P}$ for $39 B_{s} \rightarrow h_{1} h_{2}$ decay channels in the SM and 2 HDMs , using the input parameters as given in the Appendix and assuming that $k^{2}=m_{b}^{2} / 2, \rho$ $=0.16, \eta=0.34, M_{H^{+}}=200 \mathrm{GeV}, \theta=0^{\circ}, 30^{\circ}$, and $N_{c}^{\mathrm{eff}}=2$, $3, \infty$. We show the numerical results for the case of using BSW form factors only since the differences induced by using the BSW or LCSR form factors are small for almost all $B_{s}$ decay modes.

Among $39 B_{s}$ decay modes studied, we find that seven of them have $C P$-violating asymmetries larger than $20 \%$ in the SM and model III:

$$
\begin{equation*}
\bar{B}_{s} \rightarrow K^{0 *} \pi^{0}, K^{0} \rho^{0}, \bar{K}^{0 *} \eta^{\left({ }^{( }\right)}, K^{+} K^{-*}, K^{0 *} \rho^{0}, K^{0 *} \omega . \tag{56}
\end{equation*}
$$

[^3] C.L.


FIG. 5. Same as Fig. 1 but for the decay $\bar{B}_{s} \rightarrow K^{+*} K^{-*}$.
All these seven decay modes belong to the $C P$-class- 1 decay modes. On the other hand, all 24 class- 2 and -3 decay modes have small $C P$-violating asymmetries only, $\left|\mathcal{A}_{C P}\right| \lesssim 5 \%$, mainly due to the strong suppression of $1 / x^{2}$ as shown in Eq. (54).

In models I and II, the new physics corrections on $\mathcal{A}_{C P}$ for almost all $B_{s}$ decay modes studied here are negligibly small as can be seen from Table VIII and Figs. 6-8. In model III, the new physics correction is varying from channel to channel, as illustrated in Table VII and Figs. 6-8: (i) For $\bar{B}_{s} \rightarrow K^{+} K^{-}$decay, the new physics correction to its $\mathcal{A}_{C P}$ is very small in size and insensitive to the variations of $N_{c}^{\text {eff }}$ and $\theta$; (ii) for $\bar{B}_{s} \rightarrow K^{+} K^{-*}$ decay, the new physics correction to its $\mathcal{A}_{C P}$ is moderate in size, from $-20 \%$ to $-40 \%$ with $0^{\circ} \leqslant \theta \leqslant 30^{\circ}$, and insensitive to variations of $N_{c}^{\text {eff }}$; (iii) for $\bar{B}_{s} \rightarrow \bar{K}^{0 *} \eta^{\prime}$ and three remaining decays given in Eq. (56), the size and the sign of the new physics corrections strongly depend on both $N_{c}^{\text {eff }}$ and $\theta$; (iv) for $\bar{B}_{s} \rightarrow \eta \eta^{\prime}, \phi \phi$ and several other $C P$ class-2 and -3 decays, the new physics corrections can be as large as a factor of 30 , but have a very strong dependence on $N_{c}^{\text {eff }}$ and $\theta$. Despite the large new physics correction to these decay modes, their $\mathcal{A}_{C P}$ are still smaller than $5 \%$ because of strong suppression of $1 / x^{2}$.

For the QCD penguin-dominated $\bar{B}_{s} \rightarrow K^{+} K^{-*}$ decay, its decay amplitude is proportional to the combination of large
and $N_{c}^{\text {eff }}$ stable coefficients $a_{1}$ and $a_{4}[12]$ :

$$
\begin{equation*}
\mathcal{M}\left(\bar{B}_{s} \rightarrow K^{+} K^{-*}\right) \propto\left[V_{u b} V_{t s}^{*} a_{1}-V_{t b} V_{t s}^{*}\left(a_{4}+a_{10}\right)\right] . \tag{57}
\end{equation*}
$$

The imaginary parts of $\mathcal{M}$ for $b \rightarrow s$ and $\bar{b} \rightarrow \bar{s}$ transitions are very different, which in turn leads to a large $A_{C P}$. The numerical result indeed shows that this decay has a large and $N_{c}^{\text {eff }}$ stable $C P$-violating asymmetry,

$$
\begin{equation*}
\mathcal{A}_{C P}\left(\stackrel{(-)}{B_{s}} \rightarrow K^{ \pm} K^{\mp *}\right) \approx-30 \% \tag{58}
\end{equation*}
$$

for $2 \leqslant N_{c}^{\text {eff }} \leqslant \infty$. Another advantage of this decay mode is the large $(\sim 70 \%)$ new physics enhancement to its branching ratio $\mathcal{B}\left(\bar{B}_{s} \rightarrow K^{+} K^{-*}\right)$ in model III, as illustrated in Fig. 7. Taking into account the above facts, this decay mode $\bar{B}_{s}$ $\rightarrow K^{+} K^{-*}$ seems to be the 'best'" channel to find $C P$ violation of the $B_{s}$ system through studies of two-body charmless decays of $B_{s}$ meson.

Since the tree-dominated $\bar{B}_{s} \rightarrow K^{+} \pi^{-}$decay mode has a moderate $C P$-violating asymmetry $(\sim 10 \%)$, a large branching ratio $\left(\sim 7 \times 10^{-6}\right)$, negligible new physics correction, large detection efficiency, ${ }^{4}$ and a very weak $N_{c}^{\text {eff }}$ dependence, we therefore classify this decay mode as one of the promising decay channels for discovering the $C P$ violation in $B_{s}$ system.

For the decay $\bar{B}_{s} \rightarrow \bar{K}^{0 *} \eta^{\prime}$, although the SM prediction of its $\mathcal{A}_{C P}$ can be large, it is varying in the range of $-60 \%$ to $60 \%$ due to the strong dependence on $N_{c}^{\mathrm{eff}}$, as illustrated in Fig. 8. Another disadvantage of this decay is its small branching ratio $(0.02-0.16) \times 10^{-6}$, almost two orders smaller than that of $\bar{B}_{s} \rightarrow K^{+} \pi^{-}$and $K^{+} K^{-*}$ decays.

For the remaining five decay modes as given in Eq. (56), although the size of their $\mathcal{A}_{C P}$ can also be as large as $20 \%-30 \%$, but these decays can not be 'good'" channels for discovering the $C P$ violation in the $B_{s}$ system because of the strong $N_{c}^{\mathrm{eff}}$ dependence and very small branching ratios.

In Figs. 6 and 7, we show the mass and $N_{c}^{\text {eff }}$ dependence of $\mathcal{A}_{C P}$ for $\bar{B}_{s} \rightarrow K^{+} K^{-}$and $K^{+} K^{-*}$ decays. In these figures, the dotted and dot-dashed curves refer to the theoretical prediction in the SM and model II, while the solid and shortdashed curves correspond to the prediction in the model III for $\theta=0^{\circ}$ and $30^{\circ}$, respectively. As can be seen from Fig. 7, the $C P$-violating asymmetry of $\bar{B}_{s} \rightarrow K^{+} K^{-*}$ decay is large in size and has weak or moderate dependence on $M_{H^{+}}, N_{c}^{\mathrm{eff}}$, and $\theta$.

## VI. SUMMARY AND DISCUSSIONS

In this paper, we calculated the branching ratios and $C P$-violating asymmetries of two-body charmless hadronic decays of $B_{s}$ mesons in the standard model and the general

[^4]TABLE VII. $C P$-violating asymmetries $\mathcal{A}_{C P}\left(B_{s} \rightarrow h_{1} h_{2}\right)$ (in percent) in the SM and model III, with $k^{2}=m_{b}^{2} / 2, \rho=0.16, \eta=0.34$, $M_{H^{+}}=200 \mathrm{GeV}, \theta=0^{\circ}, 30^{\circ}$, and $N_{c}^{\text {eff }}=2,3, \infty$.

| Channel | $C P$ class | SM |  |  | Model III: $\theta=0^{\circ}$ |  |  | Model III: $\theta=30^{\circ}$ |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | 2 | 3 | $\infty$ | 2 | 3 | $\infty$ | 2 | 3 | $\infty$ |
| $\bar{B}_{s} \rightarrow K^{+} \pi^{-}$ | 1 | 10.2 | 10.2 | 10.3 | 9.95 | 9.96 | 9.97 | 10.2 | 10.2 | 10.3 |
| $\bar{B}_{s} \rightarrow K_{S}^{0} \pi^{0}$ | 2 | -1.99 | -3.98 | 4.50 | -3.18 | -3.87 | 4.48 | -2.60 | -4.51 | 4.36 |
| $\bar{B}_{s} \rightarrow K_{S}^{0} \eta$ | 2 | -4.73 | -3.62 | 2.93 | -4.94 | -3.59 | 2.17 | -4.76 | -4.19 | 1.67 |
| $\bar{B}_{s} \rightarrow K_{S}^{0} \eta^{\prime}$ | 2 | 0.31 | -2.86 | -4.74 | -0.50 | -2.99 | -4.96 | -1.44 | -3.84 | -4.77 |
| $\bar{B}_{s} \rightarrow K^{+} K^{-}$ | 2 | -1.71 | -1.74 | - 1.77 | -1.40 | -1.41 | - 1.43 | -2.38 | -2.40 | -2.45 |
| $\bar{B}_{s} \rightarrow \pi^{0} \eta$ | 2 | -2.72 | -0.24 | 2.93 | -2.69 | -0.24 | 2.92 | -2.69 | -0.24 | 2.92 |
| $\bar{B}_{s} \rightarrow \pi^{0} \eta^{\prime}$ | 2 | -2.72 | -0.24 | 2.93 | -2.69 | -0.24 | 2.92 | -2.69 | -0.24 | 2.92 |
| $\bar{B}_{s} \rightarrow \eta \eta^{\prime}$ | 2 | 0.07 | 0.05 | 0.01 | 0.05 | 0.03 | 0.01 | - 1.08 | - 1.09 | - 1.11 |
| $\bar{B}_{s} \rightarrow \eta^{\prime} \eta^{\prime}$ | 2 | -0.16 | 0.03 | 0.36 | -0.13 | 0.02 | 0.27 | -1.34 | -1.21 | -0.98 |
| $\bar{B}_{s} \rightarrow \eta \eta$ | 2 | 0.30 | 0.06 | -0.31 | 0.24 | 0.05 | -0.25 | -0.81 | -0.97 | - 1.24 |
| $\bar{B}_{s} \rightarrow K^{0} \bar{K}^{0}$ | 2 | 0.05 | 0.04 | 0.04 | 0.03 | 0.03 | 0.03 | -1.19 | - 1.18 | -1.17 |
| $\bar{B}_{s} \rightarrow K^{*+} \pi^{-}$ | 1 | 0.34 | 0.34 | 0.34 | 0.34 | 0.34 | 0.34 | 0.34 | 0.34 | 0.34 |
| $\bar{B}_{s} \rightarrow K^{+} \rho^{-}$ | 1 | 5.56 | 5.56 | 5.55 | 5.52 | 5.51 | 5.51 | 5.60 | 5.60 | 5.59 |
| $\bar{B}_{s} \rightarrow K^{0 *} \pi^{0}$ | 1 | -8.45 | -26.6 | 6.17 | -8.59 | -41.2 | 6.17 | -8.40 | -40.1 | 6.27 |
| $\bar{B}_{s} \rightarrow K_{S}^{0} \rho^{0}$ | 1 | -22.6 | -16.3 | 15.5 | -21.1 | -7.99 | 14.8 | -20.3 | - 16.6 | 15.4 |
| $\bar{B}_{s} \rightarrow K_{S}^{0} \omega$ | 2 | 3.88 | -1.60 | 4.53 | 3.15 | -2.01 | 4.65 | 2.78 | -2.90 | 4.64 |
| $\bar{B}_{s} \rightarrow \bar{K}^{0 *} \eta$ | 1 | -27.3 | -0.36 | 23.1 | - 19.5 | -0.01 | 18.6 | -21.6 | -4.60 | 18.7 |
| $\bar{B}_{s} \rightarrow \bar{K}^{0 *} \eta^{\prime}$ | 1 | 52.8 | 30.8 | -45.0 | 39.9 | 14.1 | -36.7 | 42.5 | -2.09 | -35.7 |
| $\bar{B}_{s} \rightarrow K^{+} K^{-*}$ | 1 | -30.5 | -30.9 | -31.7 | - 18.8 | - 19.0 | - 19.4 | -23.9 | -24.2 | -24.6 |
| $\bar{B}_{s} \rightarrow K^{+*} K^{-}$ | 1 | 1.40 | 1.45 | 1.53 | 1.37 | 1.42 | 1.49 | 1.37 | 1.41 | 1.49 |
| $\bar{B}_{s} \rightarrow \rho \eta$ | 2 | -3.03 | -0.26 | 3.02 | -3.02 | -0.26 | 3.01 | -3.02 | -0.26 | 3.01 |
| $\bar{B}_{s} \rightarrow \rho \eta^{\prime}$ | 2 | -3.03 | -0.26 | 3.02 | -3.02 | -0.26 | 3.01 | -3.02 | -0.26 | 3.01 |
| $\bar{B}_{s} \rightarrow \omega \eta$ | 2 | -0.87 | -1.01 | -0.84 | -0.71 | - 1.02 | -0.68 | -1.67 | -1.02 | -1.74 |
| $\bar{B}_{s} \rightarrow \omega \eta^{\prime}$ | 2 | -0.87 | -1.01 | -0.84 | -0.71 | - 1.02 | -0.68 | -1.67 | -1.02 | - 1.74 |
| $\bar{B}_{s} \rightarrow \pi^{0} \phi$ | 2 | -2.72 | -0.24 | 2.93 | -2.69 | -0.24 | 2.92 | -2.69 | -0.24 | 2.92 |
| $\bar{B}_{s} \rightarrow \phi \eta$ | 2 | 0.49 | 0.17 | 2.87 | -0.71 | - 1.02 | -0.68 | -1.67 | - 1.02 | - 1.74 |
| $\bar{B}_{s} \rightarrow \phi \eta^{\prime}$ | 2 | -0.31 | 0.25 | -0.74 | -0.71 | -1.02 | -0.68 | -1.67 | - 1.02 | - 1.74 |
| $\bar{B}_{s} \rightarrow K_{S}^{0} \bar{K}^{0 *}$ | 1 | -1.25 | -1.21 | -1.13 | - 1.02 | -0.98 | -0.93 | -6.98 | -6.46 | -5.72 |
| $\bar{B}_{s} \rightarrow K^{0 *} K_{S}^{0}$ | 1 | -0.02 | -0.03 | -0.04 | -0.02 | -0.03 | -0.04 | -0.02 | -0.03 | -0.04 |
| $\bar{B}_{s} \rightarrow K_{S}^{0} \phi$ | 2 | -3.38 | -3.45 | -3.27 | -2.88 | -3.45 | -3.30 | -4.42 | -3.47 | -3.84 |
| $\bar{B}_{s} \rightarrow K^{+*} \rho^{-}$ | 1 | 5.56 | 5.56 | 5.55 | 5.52 | 5.51 | 5.51 | 5.60 | 5.60 | 5.59 |
| $\bar{B}_{s} \rightarrow K^{0 *} \rho^{0}$ | 1 | -22.6 | -16.3 | 15.5 | -21.1 | -7.99 | 14.8 | -20.3 | -16.6 | 15.4 |
| $\bar{B}_{s} \rightarrow K^{0} * \omega$ | 1 | 25.5 | 13.1 | 5.43 | 20.6 | 8.57 | 5.37 | 20.9 | 4.39 | 5.46 |
| $\bar{B}_{s} \rightarrow K^{+*} K^{-} *$ | 3 | -3.86 | -3.87 | -3.90 | -3.25 | -3.27 | -3.29 | -4.05 | -4.07 | -4.10 |
| $\bar{B}_{s} \rightarrow \rho^{0} \phi$ | 3 | -3.03 | -0.26 | 3.02 | -3.02 | -0.26 | 3.01 | -3.02 | -0.26 | 3.01 |
| $\bar{B}_{s} \rightarrow \omega \phi$ | 3 | -0.87 | - 1.01 | -0.84 | -0.71 | - 1.02 | -0.68 | - 1.67 | - 1.02 | -1.74 |
| $\bar{B}_{s} \rightarrow K^{0 *} \bar{K}^{0 *}$ | 3 | 0.05 | 0.05 | 0.04 | 0.04 | 0.03 | 0.03 | - 1.18 | -1.13 | - 1.06 |
| $\bar{B}_{s} \rightarrow K^{0 *} \phi$ | 1 | 4.41 | 3.46 | 3.01 | 2.90 | 2.40 | 2.15 | -3.03 | -2.27 | -1.91 |
| $\bar{B}_{s} \rightarrow \phi \phi$ | 3 | 0.05 | 0.06 | 0.12 | 0.04 | 0.04 | 0.06 | -1.19 | -1.24 | -1.79 |

two-Higgs-doublet models (models I, II, and III) by employing the NLO effective Hamiltonian with generalized factorization. In Sec. III, we defined the effective Wilson coefficients $C_{i}^{\text {eff }}$ with the inclusion of new physics contributions,
and presented the formulas needed to calculate the branching ratios $\mathcal{B}\left(B_{s} \rightarrow h_{1} h_{2}\right)$.

In Sec. IV, we calculated the branching ratios for $39 B_{s}$ $\rightarrow h_{1} h_{2}$ decays in the SM and models I, II, and III, presented

TABLE VIII. $\mathcal{A}_{C P}\left(B_{s} \rightarrow h_{1} h_{2}\right)$ (in percent) in models I and II, with $k^{2}=m_{b}^{2} / 2, \rho=0.16, \eta=0.34, M_{H^{+}}=200 \mathrm{GeV}, \tan \beta=2$, and $N_{c}^{\text {eff }}$ $=2,3, \infty$.

| Channel | $C P$ class | SM |  |  | Model I |  |  | Model II |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | 2 | 3 | $\infty$ | 2 | 3 | $\infty$ | 2 | 3 | $\infty$ |
| $\bar{B}_{s} \rightarrow K^{+} \pi^{-}$ | 1 | 10.2 | 10.2 | 10.3 | 10.2 | 10.3 | 10.3 | 10.3 | 10.3 | 10.3 |
| $\bar{B}_{s} \rightarrow K_{S}^{0} \pi^{0}$ | 2 | -1.99 | -3.98 | 4.50 | -1.98 | -3.98 | 4.79 | -1.63 | -4.01 | 4.84 |
| $\bar{B}_{s} \rightarrow K_{S}^{0} \eta$ | 2 | -4.73 | -3.62 | 2.93 | -4.74 | -3.62 | 2.92 | -4.65 | -3.63 | 3.11 |
| $\bar{B}_{s} \rightarrow K_{S}^{0} \eta^{\prime}$ | 2 | 0.31 | -2.86 | -4.74 | 0.28 | -2.86 | -4.75 | 0.52 | -2.82 | -4.64 |
| $\bar{B}_{s} \rightarrow K^{+} K^{-}$ | 2 | - 1.71 | -1.74 | - 1.77 | - 1.70 | - 1.72 | - 1.76 | -1.80 | -1.82 | -1.86 |
| $\bar{B}_{s} \rightarrow \pi^{0} \eta$ | 2 | -2.72 | -0.24 | 2.93 | -2.59 | -0.23 | 2.84 | -2.59 | -0.23 | 2.84 |
| $\bar{B}_{s} \rightarrow \pi^{0} \eta^{\prime}$ | 2 | -2.72 | -0.24 | 2.93 | -2.59 | -0.23 | 2.84 | -2.59 | -0.23 | 2.84 |
| $\bar{B}_{s} \rightarrow \eta \eta^{\prime}$ | 2 | 0.07 | 0.05 | 0.01 | 0.07 | 0.05 | 0.01 | 0.07 | 0.05 | 0.01 |
| $\bar{B}_{s} \rightarrow \eta^{\prime} \eta^{\prime}$ | 2 | -0.16 | 0.03 | 0.36 | -0.16 | 0.03 | 0.36 | -0.16 | 0.04 | 0.39 |
| $\bar{B}_{s} \rightarrow \eta \eta$ | 2 | 0.30 | 0.06 | -0.31 | 0.30 | 0.06 | -0.31 | 0.31 | 0.06 | -0.32 |
| $\bar{B}_{s} \rightarrow K^{0} \bar{K}^{0}$ | 2 | 0.05 | 0.04 | 0.04 | 0.04 | 0.04 | 0.04 | 0.05 | 0.05 | 0.05 |
| $\bar{B}_{s} \rightarrow K^{*+} \pi^{-}$ | 1 | 0.34 | 0.34 | 0.34 | 0.34 | 0.34 | 0.34 | 0.34 | 0.34 | 0.34 |
| $\bar{B}_{s} \rightarrow K^{+} \rho^{-}$ | 1 | 5.56 | 5.56 | 5.55 | 5.57 | 5.57 | 5.56 | 5.58 | 5.57 | 5.57 |
| $\bar{B}_{s} \rightarrow K^{0 *} \pi^{0}$ | 1 | -8.45 | -26.6 | 6.17 | -8.44 | -25.3 | 6.18 | -8.40 | -22.9 | 6.18 |
| $\bar{B}_{s} \rightarrow K_{S}^{0} \rho^{0}$ | 1 | -22.6 | - 16.3 | 15.5 | -22.6 | -16.2 | 15.5 | -22.9 | - 19.8 | 15.7 |
| $\bar{B}_{s} \rightarrow K_{S}^{0} \omega$ | 2 | 3.88 | - 1.60 | 4.53 | 3.87 | -1.62 | 4.53 | 4.03 | - 1.49 | 4.50 |
| $\bar{B}_{s} \rightarrow \bar{K}^{0 *} \eta$ | 1 | -27.3 | -0.36 | 23.1 | -27.2 | -0.36 | 23.1 | -29.5 | -0.50 | 24.3 |
| $\bar{B}_{s} \rightarrow \bar{K}^{0 *} \eta^{\prime}$ | 1 | 52.8 | 30.8 | -45.0 | 52.6 | 30.0 | -44.8 | 55.4 | 37.1 | -46.5 |
| $\bar{B}_{s} \rightarrow K^{+} K^{-*}$ | 1 | -30.5 | -30.9 | -31.7 | -29.9 | -30.4 | -31.3 | -33.9 | -34.5 | -35.6 |
| $\bar{B}_{s} \rightarrow K^{+*} K^{-}$ | 1 | 1.40 | 1.45 | 1.53 | 1.43 | 1.47 | 1.53 | 1.44 | 1.47 | 1.54 |
| $\bar{B}_{s} \rightarrow \rho \eta$ | 2 | -3.03 | -0.26 | 3.02 | -2.96 | -0.25 | 2.96 | -2.96 | -0.25 | 2.96 |
| $\bar{B}_{s} \rightarrow \rho \eta^{\prime}$ | 2 | -3.03 | -0.26 | 3.02 | -2.96 | -0.25 | 2.96 | -2.96 | -0.25 | 2.96 |
| $\bar{B}_{s} \rightarrow \omega \eta$ | 2 | -0.87 | - 1.01 | -0.84 | -0.87 | - 1.05 | -0.84 | -0.91 | -1.05 | -0.89 |
| $\bar{B}_{s} \rightarrow \omega \eta^{\prime}$ | 2 | -0.87 | - 1.01 | -0.84 | -0.87 | -1.05 | -0.84 | -0.91 | - 1.05 | -0.89 |
| $\bar{B}_{s} \rightarrow \pi^{0} \phi$ | 2 | -2.72 | -0.24 | 2.93 | -2.59 | -0.23 | 2.84 | -2.59 | -0.23 | 2.84 |
| $\bar{B}_{s} \rightarrow \phi \eta$ | 2 | 0.49 | 0.17 | 2.87 | -0.87 | - 1.05 | -0.84 | -0.91 | - 1.05 | -0.89 |
| $\bar{B}_{s} \rightarrow \phi \eta^{\prime}$ | 2 | -0.31 | 0.25 | -0.74 | -0.87 | - 1.05 | -0.84 | -0.91 | - 1.05 | -0.89 |
| $\bar{B}_{s} \rightarrow K_{S}^{0} \bar{K}^{0 *}$ | 1 | -1.25 | -1.21 | -1.13 | -1.24 | -1.20 | -1.12 | -1.31 | -1.26 | - 1.18 |
| $\bar{B}_{s} \rightarrow K^{0 *} K_{S}^{0}$ | 1 | -0.02 | -0.03 | -0.04 | -0.02 | -0.03 | -0.04 | -0.02 | -0.03 | -0.04 |
| $\bar{B}_{s} \rightarrow K_{S}^{0} \phi$ | 2 | -3.38 | -3.45 | -3.27 | -3.39 | -3.45 | -3.27 | -3.67 | -3.45 | -3.26 |
| $\bar{B}_{s} \rightarrow K^{+*} \rho^{-}$ | 1 | 5.56 | 5.56 | 5.55 | 5.57 | 5.57 | 5.56 | 5.58 | 5.57 | 5.57 |
| $\bar{B}_{s} \rightarrow K^{0} * \rho^{0}$ | 1 | -22.6 | -16.3 | 15.5 | - 22.6 | -16.2 | 15.5 | -22.9 | -19.8 | 15.7 |
| $\bar{B}_{s} \rightarrow K^{0 *} \omega$ | 1 | 25.5 | 13.1 | 5.43 | 25.4 | 12.9 | 5.44 | 26.7 | 14.5 | 5.45 |
| $\bar{B}_{s} \rightarrow K^{+*} K^{-*}$ | 3 | -3.86 | -3.87 | -3.90 | -3.83 | -3.85 | -3.89 | -3.97 | -3.99 | -4.02 |
| $\bar{B}_{s} \rightarrow \rho^{0} \phi$ | 3 | -3.03 | -0.26 | 3.02 | -2.96 | -0.25 | 2.96 | -2.96 | -0.25 | 2.96 |
| $\bar{B}_{s} \rightarrow \omega \phi$ | 3 | -0.87 | - 1.01 | -0.84 | -0.87 | - 1.05 | -0.84 | -0.91 | - 1.05 | -0.89 |
| $\bar{B}_{s} \rightarrow K^{0 *} \bar{K}^{0 *}$ | 3 | 0.05 | 0.05 | 0.04 | 0.05 | 0.05 | 0.04 | 0.06 | 0.05 | 0.05 |
| $\bar{B}_{s} \rightarrow K^{0 *} \phi$ | 1 | 4.41 | 3.46 | 3.01 | 4.28 | 3.38 | 2.96 | 4.81 | 3.74 | 3.25 |
| $\bar{B}_{s} \rightarrow \phi \phi$ | 3 | 0.05 | 0.06 | 0.12 | 0.05 | 0.05 | 0.12 | 0.06 | 0.06 | 0.14 |

the numerical results in Tables III-VI, and displayed the $M_{H^{+}}$and $N_{c}^{\text {eff }}$ dependence of several interesting decay modes in Figs. 1-5. From the numerical results, one can see the following.
(i) In models I and II, the new physics corrections to the decay rates of all $B_{s} \rightarrow h_{1} h_{2}$ decay modes are small and will be masked by other larger known theoretical uncertainties.
(ii) In model III, the new physics corrections to QCD


FIG. 6. $C P$-violating asymmetries $A_{C P}$ of $\bar{B}_{s} \rightarrow K^{+} K^{-}$decay versus $M_{H^{+}}$and $1 / N_{c}^{\text {eff }}$ in the SM and models II and III. For (a) and (b), we set $N_{c}^{\text {eff }}=3$ and $M_{H^{+}}=200 \mathrm{GeV}$, respectively. The four curves correspond to the theoretical predictions in the SM (dotted line), model II (dot-dashed curve), model III with $\theta=0^{\circ}$ (solid curve), and $\theta=30^{\circ}$ (short-dashed curve), respectively.
penguin-dominated decays $B_{s} \rightarrow K^{0} \eta^{\left({ }^{\prime}\right)}, K^{+} K^{-*}, \phi \phi$, etc., are large in size, from $30 \%$ to $130 \%$ with respect to the SM predictions, and insensitive to the variations of the mass $M_{H^{+}}$and $N_{c}^{\text {eff }}$. For the tree- or electroweak penguindominated decay modes as listed in Eq. (46), however, the new physics corrections are very small in size: $\delta \mathcal{B} \leqslant 5 \%$.
(iii) For the decays $\bar{B}_{s} \rightarrow \eta \eta^{\prime}$ and $\bar{B}_{s} \rightarrow \eta^{\prime} \eta^{\prime}$, the analogue of $B_{d} \rightarrow K^{0} \eta^{\prime}$ decay, the branching ratios are large but in comparable size with the other six decay modes listed in Eq. (48). The new physics enhancements to $\mathcal{B}\left(\bar{B}_{s} \rightarrow \eta \eta^{\prime}\right)$ and $\mathcal{B}\left(\bar{B}_{s} \rightarrow \eta^{\prime} \eta^{\prime}\right)$ are significant in size, $\sim 70 \%$ in model III.
(iv) For decay modes $B_{s} \rightarrow \pi^{0} \phi, \phi \eta^{\prime}, \rho^{0} \phi, \omega \phi$ and $B_{s}$ $\rightarrow \phi \phi$, the variation of the branching ratios induced by using the BSW or LCSR form factors is about a factor of 2 , but small or moderate for all other decay modes. This feature remains basically unchanged after the inclusion of new physics contributions.
(v) For $B_{s} \rightarrow K^{+} K^{-}$and other decay modes as listed in Eq. (48), the branching ratios are at the level of $(1-3)$ $\times 10^{-5}$ in the SM and model III. These decay modes will be


FIG. 7. Same as Fig. 6 but for decay $\bar{B}_{s} \rightarrow K^{+} K^{-*}$.
measurable at the future hadron colliders with large $b$ production.

In Sec. V, we calculated the $C P$-violating asymmetries $\mathcal{A}_{C P}$ for $39 B_{s} \rightarrow h_{1} h_{2}$ decays in the SM and 2HDMs, presented the numerical results in Tables VII and VIII, and displayed the $M_{H^{+}}$and $N_{c}^{\text {eff }}$ dependence of $\mathcal{A}_{C P}$ for several typical decay modes in Figs. 6-8. From those tables and figures, the following conclusions can be drawn.
(i) For almost all $B_{s}$ decay modes, the new physics corrections on $\mathcal{A}_{C P}$ are negligibly small in models I and II. In model III, the new physics correction is varying from channel to channel, and has a strong dependence on the parameter $N_{c}^{\text {eff }}$ and the new phase $\theta$ for most decay modes.
(ii) For $24 C P$-class-2 and $-3 B_{s}$ meson decay modes, their $C P$-violating asymmetries are small, $\left|\mathcal{A}_{C P}\right| \leqslant 5 \%$, due to the strong $1 / x^{2}$ suppression.
(iii) Among the studied $39 B_{s}$ meson decay modes, seven of them can have a $C P$-violating asymmetry larger than $20 \%$ in magnitude.
(iv) The $\bar{B}_{s} \rightarrow K^{+} K^{-*}$ decay has a large and $N_{c}^{\text {eff }}$ - and $\theta$-stable $C P$-violating asymmetry, $\approx-30 \%$, and a large branching ratio. This mode seems to be the 'best'" channel to find $C P$ violation of $B_{s}$ system through studies of twobody charmless decays of $B_{s}$ meson. The tree-dominated $\bar{B}_{s} \rightarrow K^{+} \pi^{-}$decay is also a promising decay channel for discovering the $C P$ violation in $B_{s}$ system.


FIG. 8. Branching ratios and $C P$-violating asymmetries of $\bar{B}_{s}$ $\rightarrow K^{0 *} \eta^{\prime}$ decay versus $1 / N_{c}^{\text {eff }}$ in the SM and models II and III, assuming $M_{H^{+}}=200 \mathrm{GeV}$ and $\tan \beta=2$. The four curves correspond to the theoretical predictions in the SM (dotted curve), model II (short-dashed curve), model III with $\theta=0^{\circ}$ (long-dashed curve), and $\theta=30^{\circ}$ (solid curve), respectively.

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## APPENDIX: INPUT PARAMETERS AND FORM FACTORS

In this appendix we present relevant input parameters. The input parameters are similar to those used in Ref. [12].
(i) The coupling constants, $B$ meson masses, light meson masses, etc., are as follows (all masses in units of GeV ) [12,20]:

$$
\begin{aligned}
\alpha_{e m} & =1 / 128, \quad \alpha_{s}\left(M_{Z}\right)=0.118, \quad \sin ^{2} \theta_{W}=0.23, \\
G_{F} & =1.16639 \times 10^{-5}(\mathrm{GeV})^{-2},
\end{aligned}
$$

$$
\begin{align*}
M_{Z} & =91.188, \quad M_{W}=80.42, \quad m_{B_{s}^{0}}=5.369 \\
m_{\pi^{ \pm}} & =0.140, \\
m_{\pi^{0}} & =0.135, \quad m_{\eta}=0.547, \quad m_{\eta^{\prime}}=0.958 \\
m_{\rho} & =0.770, \quad m_{\omega}=0.782, \\
m_{\phi} & =1.019, \quad m_{K^{ \pm}}=0.494, \quad m_{K^{0}}=0.498 \\
m_{K^{*} \pm} & =0.892, \\
m_{K^{*}} & =0.896, \quad \tau\left(B_{s}^{0}\right)=1.493 \mathrm{ps} . \tag{A1}
\end{align*}
$$

(ii) For the elements of CKM matrix, we use Wolfenstein parametrization and fix the parameters $A, \lambda, \rho, \eta$ to their central values:

$$
\begin{equation*}
A=0.804, \quad \lambda=0.22, \quad \rho=0.16, \quad \eta=0.34 \tag{A2}
\end{equation*}
$$

(iii) Following Refs. [26,13], the current quark masses evaluated at the scale $\mu=m_{b}$ will be used in the numerical calculations:

$$
\begin{align*}
& m_{b}\left(m_{b}\right)=4.34 \mathrm{GeV}, \quad m_{c}\left(m_{b}\right)=0.95 \mathrm{GeV} \\
& m_{s}\left(m_{b}\right)=0.105 \mathrm{GeV} \\
& m_{d}\left(m_{b}\right)=6.4 \mathrm{MeV}, \quad m_{u}\left(m_{b}\right)=3.2 \mathrm{MeV} \tag{A3}
\end{align*}
$$

For the mass of the heavy top quark we also use $m_{t}$ $=\overline{m_{t}}\left(m_{t}\right)=168 \mathrm{GeV}$.
(iv) For the decay constants of light mesons, the following values are used in the numerical calculations (in units of MeV ):

$$
\begin{align*}
& f_{\pi}=133, \quad f_{K}=160, \quad f_{K^{*}}=221, \quad f_{\rho}=210 \\
& f_{\omega}=195, \quad f_{\phi}=237, \\
& f_{\eta}^{u}=f_{\eta}^{d}=78, \quad f_{\eta^{\prime}}^{u}=f_{\eta^{\prime}}^{d}=63, \quad f_{\eta}^{c}=-2.4 \\
& f_{\eta^{\prime}}^{c}=-6.3, \quad f_{\eta}^{s}=-112, \quad f_{\eta^{\prime}}^{s}=137 \tag{A4}
\end{align*}
$$

where $f_{\eta^{\left({ }^{\prime}\right)}}^{u}$ and $f_{\eta^{\left({ }^{\prime}\right)}}^{s}$ have been defined in the two-anglemixing formalism with $\theta_{0}=-9.2^{\circ}$ and $\theta_{8}=-21.2^{\circ}$ [43].
(v) In the calculation we use the following BSW form factors $F(0)$ (in the units of GeV ) $[25,16,12]$ :

$$
\begin{aligned}
& F_{0}^{B \rightarrow \pi}(0)=0.33, \quad F_{0}^{B \rightarrow K}(0)=0.274, \\
& F_{0}^{B \rightarrow \eta}(0)=-0.212, \quad F_{0}^{B \rightarrow \eta^{\prime}}(0)=0.218, \\
& A_{0,1,2}^{B \rightarrow \phi}(0)=0.273, \quad A_{0}^{B \rightarrow K^{*}}(0)=0.236, \\
& A_{1,2}^{B \rightarrow K^{*}}(0)=0.232,
\end{aligned}
$$

$$
\begin{equation*}
V^{B \rightarrow \phi}(0)=0.319, \quad V^{B \rightarrow K^{*}}(0)=0.2817 \tag{A5}
\end{equation*}
$$

We use the monopole $k^{2}$ dependence for form factors,

$$
\begin{equation*}
f_{i}\left(k^{2}\right)=\frac{f_{i}(0)}{1-k^{2} / m_{*}^{2}} \tag{A6}
\end{equation*}
$$

where $m_{*}$ is the pole mass given in [16]:

$$
\begin{array}{r}
\left\{m\left(0^{-}\right), m\left(1^{-}\right), m\left(1^{+}\right), m\left(0^{+}\right)\right\} \\
\quad=\{5.2789,5.3248,5.37,5.73\} \tag{A7}
\end{array}
$$

for $\bar{u} b$ and $\bar{d} b$ currents and
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$$
\begin{equation*}
\left\{m\left(0^{-}\right), m\left(1^{-}\right), m\left(1^{+}\right), m\left(0^{+}\right)\right\}=\{5.3693,5.41,5.82,5.89\} \tag{A8}
\end{equation*}
$$

for $\bar{s} b$ currents.
(vi) For the decays involving $B_{s} \rightarrow K^{*}$ and $B_{s} \rightarrow \phi$ transitions, we also consider the case of using LCSR form factors with the $k^{2}$ dependence as defined in Ref. [44]:

$$
\begin{equation*}
f\left(k^{2}\right)=\frac{f(0)}{1-a\left(k^{2} / M_{B_{s}}^{2}\right)+b\left(k^{2} / M_{B_{s}}^{2}\right)^{2}}, \tag{A9}
\end{equation*}
$$

where the values of $f(0)$ and coefficients $a$ and $b$ have been given in Ref. [44].
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[^1]:    ${ }^{1}$ For $b \rightarrow d \bar{q} q$ decays, one simply makes the replacement $s \rightarrow d$.

[^2]:    ${ }^{2}$ In the improved generalized factorization approach [18], these effective coefficients are renormalization scale and scheme independent, gauge invariant, and infrared safe.

[^3]:    ${ }^{3}$ From Ref. [20], the upper limit is $x=\Delta M_{B_{s}^{0}} / \Gamma_{B_{s}^{0}}>15.7$ at $95 \%$

[^4]:    ${ }^{4}$ In general, the detection efficiency for the two-body $B$ meson decays with charged final states is larger than that with neutral final states by a factor of 2 or 3 .

