S-wave $\Lambda\pi$ phase shift is not large

Ulf-G. Meißner and José A. Oller
Forschungszentrum Jülich, Institut für Kernphysik (Th), D-52425 Jülich, Germany
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We study the strong interaction *S*-wave $\Lambda\pi$ phase shift in the region of the Ξ mass in the framework of a relativistic chiral unitary approach based on coupled channels. All parameters have been previously determined in a fit to strangeness $S=-1$ *S*-wave kaon-nucleon data. We find $0^\circ \leqslant \delta_0 \leqslant 1.1^\circ$ in agreement with previous chiral perturbation theory calculations (or extensions thereof). We also discuss why a recent coupled channel *K*-matrix calculation gives a result for δ_0 that is negative and much bigger in magnitude. We argue why that value should not be trusted.

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Direct CP violation can be measured in the decay $\Xi \to \Lambda\pi \to p\pi\pi$ (for a recent experiment, see [1]). To extract the CP violating phase, one has to know the strong $\Lambda\pi$ *S*- and *P*-wave phase shifts at the mass of the cascade, denoted δ_0 and δ_1, respectively. While earlier calculations [2,3] were inconclusive on the value of δ_0, a leading order heavy baryon chiral perturbation theory (HBChPT) analysis led to a vanishing *S*-wave phase shift [4] and corrections including excited Σ intermediate states were shown to give a bound of $\delta_0 \sim 0.5^\circ$ [4,5]. Relativistic tree level calculations have also been performed, leading to a somewhat larger band of values for δ_0, but still $|\delta_0| \leqslant 2^\circ$ [6,7]. A more recent calculation using also dimension-2 operators [8] with the corresponding low-energy constants fixed from kaon-nucleon scattering [9] gave the range $-3.0^\circ \leqslant \delta_0 \leqslant +0.4^\circ$.[1] In that paper, the effect of channel coupling was also investigated, based on the observation that in SU(3), the $\Lambda\pi$ state is coupled to the $\Sigma\pi$, $N\overline{K}$, $\Sigma\eta$ and ΞK states with strangeness $S=-1$ and isospin $I=1$. A *K*-matrix approach was used to calculate the channel coupling effects and a surprisingly large $\delta_0 \simeq -7^\circ$ was found. The authors of Ref. [8] have been careful to point out that more refined coupled channel calculations based on chiral perturbation theory (ChPT) are necessary to further clarify this surprising result. We have recently presented a novel relativistic chiral unitary approach based on coupled channels [11]. Dispersion relations are used to perform the necessary resummation of the lowest order relativistic chiral Lagrangian. Within this framework, the *S*-wave kaon-nucleon interactions for strangeness $S=-1$ were studied and a good description of the data in the K^-p, $\pi\Sigma$ and $\pi\Lambda$ channels [cross sections, threshold ratios, mass distribution in the region of the $\Lambda(1405)$] was obtained. This method can be systematically extended to higher orders, emphasizing its applicability to any scenario of strong self-interactions where the perturbative series diverges even at low energies. It is straightforward to project out the $\Lambda\pi \to \Lambda\pi$ amplitude from our coupled channel solutions and extract in a parameter-free manner the corresponding *S*-wave phase shift. This is done here. To close the Introduction, we remark that our approach can also be used to calculate the *P* waves. Since there is no discrepancy in the corresponding predictions for δ_1, we focus here entirely on the *S* wave.

We briefly summarize our calculational scheme; for details see [11]. It is based on the fact that unitarity, above the pertinent thresholds, implies that the inverse of a partial wave amplitude satisfies

$$\mathrm{Im}\, T^{-1}(W)_{ij} = -\rho(W)_i \delta_{ij}\,, \tag{1}$$

where $\rho_i \equiv q_i/(8\pi W)$, $W=\sqrt{s}$ the center-of-mass (c.m.) energy, q_i is the modulus of the c.m. three-momentum and the subscripts i and j refer to the physical channels. The $\Lambda\pi$ states couple strongly to several channels. To be consistent with lowest order ChPT, where all the baryons belonging to the same SU(3) multiplet are degenerate, one should consider the whole set of states, K^-p (1), $\overline{K^0}n$ (2), $\pi^0\Sigma^0$ (3), $\pi^+\Sigma^-$ (4), $\pi^-\Sigma^+$ (5), $\pi^0\Lambda$ (6), $\eta\Lambda$ (7), $\eta\Sigma^0$ (8), $K^+\Xi$ (9), $K^0\Xi^0$ (10), where between brackets the channel number, to be used in a matrix notation, is given for each state. The unitarity relation in Eq. (1) gives rise to a cut in the T matrix of partial wave amplitudes which is usually called the unitarity or right-hand cut. Hence we can write down a dispersion relation for $T^{-1}(W)$, in a fairly symbolic language:

$$T^{-1}(W)_{ij} = -\delta_{ij}\left\{\widetilde{a}_i(s_0) + \frac{s-s_0}{\pi}\int_{s_i}^{\infty} ds' \frac{\rho(s')_i}{(s'-s)(s'-s_0)}\right\} + \mathcal{T}^{-1}(W)_{ij}\,, \tag{2}$$

where s_i is the value of the s variable at the threshold of channel i and $\mathcal{T}^{-1}(W)_{ij}$ indicates other contributions coming from local and pole terms as well as crossed channel dynamics but *without* a right-hand cut. These extra terms will be taken directly from ChPT after requiring the *matching* of our general result to the ChPT expressions. Notice also that the negative of the quantity in the curly brackets, denoted $g(s)_i$ from here on, is the familiar scalar loop integral

[1]Note that the parameters obtained in [9] need to be taken with some care since the important η channels were not considered, as stressed in [10].

$$g(s)_i = i\int \frac{d^4q}{(2\pi)^4}\frac{1}{(q^2-M_i^2+i\epsilon)((P-q)^2-m_i^2+i\epsilon)}$$
$$= \frac{1}{16\pi^2}\left\{ a_i(\mu) + \log\frac{m_i^2}{\mu^2} + \frac{M_i^2-m_i^2+s}{2s}\log\frac{M_i^2}{m_i^2} + \frac{q_i}{\sqrt{s}}\log\frac{m_i^2+M_i^2-s-2\sqrt{s}q_i}{m_i^2+M_i^2-s+2\sqrt{s}q_i}\right\}, \quad (3)$$

where M_i and m_i are, respectively, the meson and baryon masses in the state *i*. Notice that in order to calculate $g(s)_i$, we are using the physical masses both for mesons and baryons since the unitarity result in Eq. (1) is exact. In the usual chiral power counting, $g(s)_i$ is $\mathcal{O}(p)$ because the baryon propagator scales as $\mathcal{O}(p^{-1})$. Let us note that the important point here is to proceed systematically, guaranteeing that $\mathcal{T}$ is free of the right-hand cut and matching simultaneously with the ChPT expressions. We can further simplify the notation by employing a matrix formalism. We introduce the matrices $g(s)=\mathrm{diag}[g(s)_i]$, T and $\mathcal{T}$, the latter defined in terms of the matrix elements T_{ij} and $\mathcal{T}_{ij}$. In this way, from Eq. (2), the T matrix can be written as

$$T(W) = [I + \mathcal{T}(W)g(s)]^{-1}\mathcal{T}(W). \quad (4)$$

In this paper, we are considering the lowest order (tree level) ChPT amplitudes as input. Hence, expanding the previous equation, our final expression for the T matrix, taking as input the lowest order ChPT results, has the form

$$T(W) = [I + T_1(W)g(s)]^{-1}T_1(W). \quad (5)$$

For more details on this formalism, we refer the reader to Refs. [11,12]. We only want to remark that this approach is not just a unitarization scheme, like e.g. the *K*-matrix approach. The latter is, however, included as one particular approximation as discussed below.

Using the lowest order relativistic (tree level) ChPT amplitudes for $\phi_i B_a \to \phi_j B_b$ as input, where $\phi_i\,(B_a)$ denotes a member of the Goldstone boson (ground state baryon) octet, one obtains a very good description of the scattering data for $K^-p \to K^-p, K^0n, \pi^+\Sigma^-, \pi^-\Sigma^+, \Lambda\pi^0, \Sigma^0\pi^0$ (for kaon laboratory momenta below 250 MeV), the so-called threshold ratios γ, R_c and R_n, the K^-p scattering length and the $\pi^+\Sigma^-$ event distribution in the region of the $\Lambda(1405)$ in terms of three parameters (using fixed axial couplings, $D=0.80$ and $F=0.46$ [13]). These are the baryon octet mass in the chiral limit, m_0, the chiral limit value of the three-flavor meson decay constant,[2] F_0, and the subtraction constant $a(\mu)$; cf. Eq. (3). Note that it was shown in [11] that it suffices to take only one subtraction constant for *all* channels; thus the subscript ''*i*'' appearing in Eq. (3) for these constants will be dropped. In Ref. [11], we considered two sets of parameters, set I describing the best fit and set II using the so-called natural values (as discussed in that paper). The pertinent numbers are, for set I, $m_0=1.286$ GeV, $F_0=74.1$ MeV, $a(\mu)=-2.23$ and, for set II, $m_0=1.151$ GeV, $F_0=86.4$ MeV, $a(\mu)=-2$ at the scale $\mu=630$ MeV. Of course, physical observables are scale independent. It is now straightforward to extract the $\Lambda\pi$ phase shift as shown in Fig. 1 by the solid line (set I) and the dashed line (set II). The corresponding phases at the mass of the Ξ^0 and the Ξ^- are

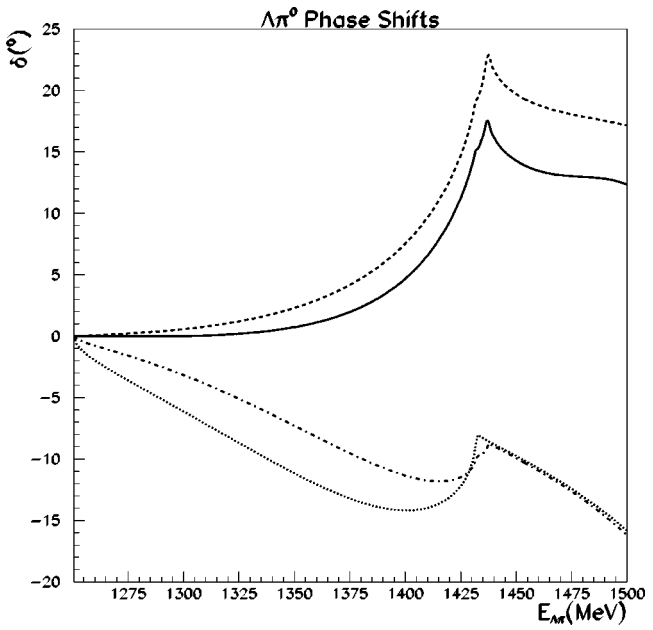

FIG. 1. The $\Lambda\pi$ phase shift in degrees versus the c.m. energy, $W=E_{\Lambda\pi}$. The various lines are explained in the text.

$$\text{set I:}\quad \delta_0(m_{\Xi^0})=0.10^\circ,\quad \delta_0(m_{\Xi^-})=0.16^\circ,$$
$$\text{set II:}\quad \delta_0(m_{\Xi^0})=0.92^\circ,\quad \delta_0(m_{\Xi^-})=1.11^\circ, \quad (6)$$

consistent with earlier ChPT findings [4–8]. We should stress that set I gives the better fit in the $\bar{K}N$ sector and should be preferred.

It is important to understand the large result obtained in the *K*-matrix formalism [8]. The *K*-matrix approach is one particular approximation to our scheme in that ones sets

$$g(s)_i = -\frac{i\,q_i}{8\pi W} \equiv -i\,\rho(s)_i\,. \quad (7)$$

Notice that $-\rho(s)_i$, above the threshold of channel *i*, is the imaginary part of $g(s)_i$; cf. Eq. (2). In order to see the importance of keeping the whole $g(s)_i$ function, compare the dashed and dot-dashed lines in Fig. 1. The latter is obtained for set II by making use of Eq. (5) but using the approximation given in Eq. (7) to the $g(s)_i$ function. The differences are huge and for the second case the results are similar to the findings of Ref. [8]. In fact, we can reproduce the results for their *K*-matrix calculation by means of Eq. (5) by consider-

[2]We remark that there are some indications that the order parameter of chiral symmetry breaking, F_0, decreases sizably when going from the two- to three-flavor sector; see e.g. [14].

ing only the dominant nonrelativistic seagull (Weinberg-Tomozawa) term to the tree level meson-baryon scattering and the *K*-matrix representation of the $g(s)_i$ function. This is given by the dotted line in Fig. 1. All these large differences nicely show that it is not sufficient to account only for the imaginary part of the scalar loop functions via unitarity but that a proper treatment of the real part by an appropriate dispersion relation is of equal importance. Consequently, the large and negative value for $\delta_0 \simeq -7°$ of Ref. [8] can be ruled out and is just a result of the simple representation of the function $g(s)_i$ used in that reference. This is, by far, not sufficiently accurate for this case and the full relativistic expression for $g(s)_i$ [cf. Eq. (3)] has to be used. Furthermore, the phases are sensitive to F_0 and m_0. We conclude from our approach that indeed δ_0 is narrowly bounded,

$$0° \leqslant \delta_0 \leqslant 1.1°, \tag{8}$$

and that the large value found in the *K*-matrix approach should not be used.

In summary, we have used a relativistic chiral unitary approach based on coupled channels to investigate the strong *S*-wave $\Lambda\pi$ phase shift in the region of the Ξ. All parameters have been previously determined from a good description of the kaon-nucleon data [11] and thus we arrive at a small band of values for δ_0; cf. Eq. (8). This number is consistent with earlier findings in ChPT (or extensions thereof) [4–8]. We have also shown why the *K*-matrix approach of Ref. [8] leads to a large value of δ_0 and why this number should not be trusted. The strong $\Lambda\pi$ *S*-wave phase in the region of the cascade mass is indeed small.

Reply to the note added in proof of Ref. [8]: We would like to emphasize here again a fundamental property not accounted for in the note added in proof by the authors of Ref. [8]: *The most simple version of a K-matrix parametrization, as the one used in Ref. [8], does not account for the real parts of the unitarity loops*. In our work, however, the latter have been included by using dispersion relations in terms of the known imaginary part. Thus an improved calculation of the coupled channel dynamics relevant for the *S*-wave $\pi\Lambda$ scattering to that of Ref. [8] is given. As a result, we have been able to show that the real parts of the unitarity loops turn out to be so important that they simply spoil all the consequences that could be derived by ignoring them as in Ref. [8]. In particular we have demonstrated that unitarity effects in coupled channels *do not enhance* the elastic $\pi\Lambda$ *S*-wave phase shift by themselves. In addition, we have given a very conservative estimate of the errors. In Ref. [11], we have performed a variety of fits not discussed in detail and never found any solution which would give a larger phase $\Lambda\pi$ than set II; thus we can confidentially repeat that the previously found small values for the phase should be correct. The seemingly large factor of 10 difference between sets I and II is simply an artifact caused by the almost zero of the phase at the mass of the Ξ. Further improvements to our work would come by considering crossed channel dynamics and higher order local chiral corrections as discussed in Ref. [11]. That would also allow a better discussion of the theoretical uncertainties.

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