Lifetime of the B_c meson and some relevant problems

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The lifetime of the B_c meson is estimated by means of consistent considerations of all of the heavy mesons ($B^0, B^\pm, B_s, D^0, D^\pm, D_s$) and the double heavy meson B_c itself as well. The framework, where the nonspectator effects for inclusive decays in terms of the optical theorem are taken into account properly, is adopted, and the parameters appearing in the estimate are treated phenomenologically by global fitting all the existing data for all the heavy mesons. In this estimation, special attention is paid to the masses of the heavy flavors and the bound-state effects as well. The present approach to the problem is meaningful because the numerical results show that the lifetime of the meson B_c can reach to the center value of the experimental observation. Because of the interference between the terms of the QCD correction from penguin diagrams and the main ones $c_1 O_1$, $c_2 O_2$, the total width of the meson B_c is enhanced by a factor about 3–4% that is different from the other heavy mesons and can be tested experimentally in the future.

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I. INTRODUCTION

Recently the meson B_c has been observed by the Collider Detector at Fermilab (CDF) Collaboration at the Tevatron [1], so careful and further studies of the B_c meson are motivated very much with this fresh reason. It is known that B_c is the meson of the ground state of a double heavy flavored system with a c quark and b antiquark (there is no light flavored quark involved), and its decay should be either through the decays of each component (a heavy flavor), or through the annihilation of the two components (two heavy flavors), that is very different from the heavy mesons B, B_s, D, and D_s, etc. Of the decays, the contributions from the two components ($\bar{b}$ quark and c quark) individually to the total width happen to be comparable each other in amount. So it is accessible in future experiments that with the meson B_c one may investigate the two different heavy flavors simultaneously. Especially, certain decay mechanisms in B_c decays play a similar role in D decays and B decays; so some parameters appearing in B_c decays should be the same as those in B decays or in D decays. Therefore when estimating the B_c decay, we may rely on the consistent considerations and phenomenologically use the experimental available data of $D^0, D^\pm, D_s$ decays and $B^0, B^\pm, B_s$ decays as input to determine them. Obviously in this way the estimate for the B_c lifetime should be comparatively reasonable. The meson B_c certainly can be used to test the approach for the estimates of the lifetime and the inclusive leptonic decays, and it will be an independent complement to the heavy mesons: B mesons and D mesons for study of the two heavy flavor b and c decays. Furthermore, the meson B_c has unique advantages, if one carries on a comparative study of the two heavy flavors b and c, since one may do it simultaneously in a meson.

There have been quite a lot of studies of the lifetimes of the mesons D, B, and B_c as well [2–13]. The reason in part is that a lifetime estimate is to calculate an inclusive process, and one may do it in terms of the optical theorem so it is comparatively ''easy,'' due to the ''duality'' for quark and hadron states:

$$\sum_{i,j} |q_i, g_j\rangle\langle q_i, g_j| = \sum_k |h_k\rangle\langle h_k|,$$

where h_k, q_i, and g_j denote hadrons, quarks, and gluons, respectively, and, for an inclusive process the optical theorem, being applied to the level of hadrons, can be turned onto the level of quark-gluons, so that the estimate can be made on the quark-gluon level. As a result, one can avoid certain difficulties from the hadronization in the final state. In general, the hadronization from quarks into hadrons, being of nonperturbative nature, is attributed to evaluate a relevant hadron matrix element, which cannot be handled reliably so far.[1] Now when the optical theorem and the ''duality'' are applied, the problem will be ''solved'' in part: the nonperturbative part in the final state of the decay may be summed by the theorem, i.e., the calculation of the part of the final state may be completed totally at the quark-gluon level, so only those nonperturbative (hadronization) parts in the initial state are left to be dealt with. Thus in the studies of the lifetime and the other inclusive processes with this approach, one may pay more attention to the decay mechanisms and the ''hadronization'' in the initial state.

To estimate the lifetimes and inclusive decays for the heavy flavor, first the effective Lagrangian with QCD corrections should be known; here we adopted that as in Refs. [2–4]. With the effective Lagrangian for c and b decays,

[1]In principle the lattice gauge simulation might deal with the nonperturbative effects as well as one wishes if the power of computers were infinitely strong, but in practice the computer ability now still is at quite sizable ''distance'' to obtain sufficiently accurate results for calculating such hadron matrix elements.

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phenomenological analyses of D meson lifetimes and B-meson lifetimes have been made in Ref. [8]. Generally for all the heavy meson decays, the contributions can be decomposed into three categories: the dominant one, i.e., the direct decay of the heavy quark with the light quark inside the heavy meson being as a spectator (this contribution is very sensitive to the heavy quark mass, i.e., proportional to m_Q^5); the nonspectator one from W annihilation (WA) [or exchange (WE)], and the Pauli interference (PI) one [3]. The parameters appearing in the formulas for the lifetimes are (a) the quark masses, (b) the matrix element $\langle 0|J_{\mu 5}|M_{(B_c,B,D)}\rangle$ relating to the decay constant, (c) the relevant nonfactorizability parameters [3], etc. All the parameters are not well known, or say their precise values are not calculable, except some of them may be fixed phenomenologically by fitting data.

In the present paper, in order to have an estimate of the lifetime of B_c which may be better than before, we will take a "consistent" view of the parameters appearing in the estimates of the lifetimes for all of the heavy mesons D, D_s, B, B_s and those for the meson B_c, thus we fix the parameters for B_c meson by fitting the available data for the lifetimes and the inclusive semileptonic decays of the heavy mesons D and B. In particular, we would like to see the consistency of the approach for the lifetime estimates and the study of the inclusive leptonic decays, namely, to estimate the lifetime of the meson B_c with the parameters which are fixed phenomenologically by fitting the available existing data for the other heavy mesons. Here we also try to discuss some effects and uncertainties of the estimate as possible.

In the literature, to obtain a comparable value for lifetime and inclusive leptonic decays with the experimental one, the charm quark mass m_c appearing in the estimate for D and B decays takes different values [8,15]. We think it is reasonable if considering the renormalon and bound-state effects, since the c-quark mass appears in different situations: in the initial state for D decays but in the final state for B decays. In general, for the quark (antiquark) in the parent meson of a concerned decay mode, the mass should be close to its "pole" value if some bound-state effects are ignored, whereas, for the masses of the product quarks (antiquarks) in the final state of an inclusive process, it is reasonable to take the relevant running masses to avoid the renormalon and bound-state effects with the duality for hadrons and quark-gluons mentioned at the beginning. Furthermore, the running energy scale of the running masses should be taken at the mass of the decaying quark (or mesons for WA and PI). Indeed, this problem is somewhat subtle and causes substantial theoretical uncertainties. In the earlier estimates for inclusive processes, the quark decays are considered only as if the quark is "free." In fact, the bound-state effects on the effective mass of the heavy quark should be taken into account. Namely, the heavy quark effective mass, appearing in the formulation, should deviate from the pole value by an amount corresponding to the binding energy [16–18]. In our work, we take the pole masses and the running masses properly as mentioned here and use one-loop formular to connect them each other. We also pay attention to the bound state effects and use a parametrization which is a bit different from that of Ref. [17] to account for the effects on the mass of the decaying quark (see the text below for details).

The relation between the pole mass and the running $\overline{\mathrm{MS}}$ to one loop level reads

$$m=\overline{m}(\overline{m})\left(1+\frac{4}{3}\,\frac{\alpha_s(\overline{m})}{\pi}\right), \tag{1}$$

where the running coupling constant (at leading logarithm level) is

$$\alpha_s=\frac{12\pi}{(33-2n_f)\ln\dfrac{Q^2}{\Lambda^2}}, \tag{2}$$

with $\alpha_s(m_z^2)=0.118$ [14]. The running mass runs as

$$\overline{m}(Q^2)=\frac{m}{\left(\dfrac{1}{2}\ln\dfrac{Q^2}{\Lambda^2}\right)^{d_m}}, \tag{3}$$

where $d_m=12/(33-2n_f)$.

For the lifetimes of B and D mesons, the contributions from penguin terms of the effective Lagrangian generally are not important [4] because of the smallness of their coefficients $c_3\cdots c_6$. But as pointed out in Ref. [19], the penguin contributions to the charmless decays of B mesons are not negligible. The reason is that for those modes the main contributions (since they are not zero even at tree level, thus we will call them "tree parts" as in most literature later on) will suffer a cancellation (c_1+c_2/N_c) or (c_2+c_1/N_c), and the "tree part" $c_1O_1+c_2O_2$ does not contribute, thus the penguin contributions become important. As for the B_c meson, the problem has not been investigated very carefully yet. In the earlier paper [9], for instance, the lifetime of B_c was estimated, though there the bound state effect was carefully handled in terms of the Bethe-Salpeter equation; the penguin contributions from the effective Lagrangian were ignored. Recently, Beneke and Buchalla [11] presented an evaluation of the B_c lifetime, where they also ignored the "penguin" terms. In fact, for the spectator mechanism, the contributions from the penguin terms in B decays have been estimated by Bagan *et al.* [7], and their results show that only a few thousandths of changes are made, so in general we can neglect them altogether. However, for the WA and PI terms of B_c decays, the operators induced by the penguin diagrams are $\Sigma_{i=3}^{6}c_iO_i$ which contain terms $(\overline{s}_ib_i)(\overline{c}_jc_j)$ and $(\overline{s}_ib_j)\times(\overline{c}_jc_i)$, where i,j are color indices, so the interference of the penguin diagram with the tree part

$$L_{\mathrm{eff}}^{(\mathrm{tree})}=V_{cb}V_{cs}^{*}[c_1\overline{c}_L\gamma_\mu b_L\overline{s}_L\gamma^\mu c_L+c_2\overline{s}_L\gamma_\mu b_L\overline{c}_L\gamma^\mu c_L]$$

makes substantial contributions. Namely, in B and D decays, the penguin contributions, being proportional to $|c_i|^2$ or $|c_i^* c_j|(i,j=3\sim 6)$, are small [4], whereas, in some B_c decays there may exist the interference terms $|c_i^* c_1|$ and $|c_i^* c_2|$, which are not so small, so that the interference may bring up a few percents of corrections in lifetime. Indeed our numerical results show that the interference can make a change in lifetime of B_c so large as 3–4 % of the total.

In the paper with the approach described here, we reestimate the lifetime, the branching ratios of the semileptonic and pure leptonic decays for the meson B_c, and expect to gain more knowledge about decay mechanisms and QCD corrections for B_c meson. The paper is organized as follows: after the introduction, we present a useful formulation in Sec. II, and give the numerical results and the concerned phenomenological parameters in Sec. III, then we draw conclusions and put discussions into the last section. For convenience, we collect some useful formulas in the Appendix.

II. FORMULATION

In this section let us describe the mechanisms for the lifetimes of the mesons D, B, and B_c, etc., and present useful formulas for later numerical calculations.

A. The spectator components and the contributions from *b* or *c* decays

In terms of the quark-hadron duality and the optical theorem, the ''full'' inclusive decay width (the lifetime) of a heavy hadron H_Q (containing a heavy quark $Q=b,c$) is related to the absorptive part of the matrix element for ''forward scattering'' operator $\hat{T}$.

$$\mathbf{\Gamma}(H_Q\to X)=\frac{1}{m_{H_Q}}\mathrm{Im}\int d^4x\langle H_Q|\hat{T}|H_Q\rangle$$
$$=\frac{1}{2m_{H_Q}}\langle H_Q|\hat{\Gamma}|H_Q\rangle, \tag{4}$$

where

$$\hat{T}=T\{i\mathcal{L}_{\rm eff}(x),\mathcal{L}_{\rm eff}(0)\},$$

and $\mathcal{L}_{\rm eff}$ is the relevant effective Lagrangian which is responsible for the decay. For the concerned final state X with designated quark-antiquark combination and up to order $1/m_Q^3$, we have

$$\Gamma(H_Q\to X)=\frac{G_F^2 m_Q^2}{192\pi^3}|V(CKM)|^2\times\Bigg\{c_3^X\langle H_Q|\bar{Q}Q|H_Q\rangle+c_5^X\frac{\langle H_Q|\bar{Q}i\sigma GQ|H_Q\rangle}{m_Q^2}+\sum_i c_{6,i}^X\frac{\langle H_Q|(\bar{Q}\Gamma_i q)(\bar{q}\Gamma_i Q)|H_Q\rangle}{m_Q^3}+\mathcal{O}(1/m_Q^4)\Bigg\}. \tag{5}$$

Here only the heavy quark (b,c quark) decays are concerned. For the spectator components of the heavy meson decays, the other flavor in the meson remains as a spectator. Of the B_c-meson decays there are two possibilities: $\bar{b}$ decays with the c quark as a spectator, and c decays with the $\bar{b}$-quark as a spectator. Furthermore, in principle, in each spectator component there are two components: the semileptonic one and the nonleptonic one as follows:

$$\Gamma(b\to c)=\sum_{l=e,\mu,\tau}\Gamma_{b\to cl\nu}+\sum_{q=u,d,s,c}\Gamma_{b\to cq\bar{q}} \tag{6}$$

for b decay and

$$\Gamma(c\to s)=\sum_{l=e,\mu}\Gamma_{c\to s\bar{l}\nu}+\sum_{q=u,d,s}\Gamma_{c\to sq\bar{q}} \tag{7}$$

for c decay. As for the concerned B_c meson, as a double heavy meson, its two components $\bar{b}$ quark and c quark, each plays the decay role and the spectator role once in turn, so both of Eqs. (6),(7) as the spectator components make contributions to B_c decay.

The semileptonic and nonleptonic decay rates of the b quark up to order $1/m_b^2$ have been evaluated by many authors [2,10,22]. Since in our numerical computations later on, we will use their formulas, we quote the relevant useful formulas in Appendix A. For $c\to s$, the formulation is similar (even simpler), we include the relevant useful formulas in the Appendix also.

B. The nonspectator components in *D* and *B* meson decays

The nonspectator contributions are crucially important to the D inclusive decays. For instance, the PI contribution may explain the data why $\tau_{D^\pm}\sim 2\tau_{D^0}$, but $\tau_{B^\pm}\sim\tau_{B^0}$. The penguin contributions in $D(D_s)$ and $B(B_s)$ decays may be negligible as aforementioned, but the bound state effects emerge. To determine the nonfactorization effects well still is an open problem.

With straightforward calculations, the precise operators for the nonspectator contributions may be obtained. (a) For the $D(D_s)$ decays,

$$\Gamma^{\mathrm{WA}}(D^0)=-\Gamma_0\,\eta_{\mathrm{nspec}}\frac{m_D^2}{m_c^2}(|V_{cs}|^2|V_{ud}|^2+|V_{cd}|^2|V_{us}|^2)(1-x_+)^2\left\{\left(\frac{c_1^2}{N}+2c_1c_2+Nc_2^2\right)\left[\left(1+\frac{x_+}{2}\right)B_1-(1+2x_+)B_2\right]\right.$$
$$\left.+2c_1^2\left[\left(1+\frac{x_+}{2}\right)\epsilon_1-(1+2x_+)\epsilon_2\right]\right\}-\Gamma_0\,\eta_{\mathrm{nspec}}\frac{m_D^2}{m_c^2}|V_{cs}|^2|V_{us}|^2\sqrt{1-4x_+}\left\{\left(\frac{c_1^2}{N}+2c_1c_2+Nc_2^2\right)\right.$$
$$\left.\times[(1-x_+)B_1-(1+2x_+)B_2]+2c_1^2[(1-x_+)\epsilon_1-(1+2x_+)\epsilon_2]\right\},$$

$$\Gamma^{\mathrm{PI}}(D^+)=\Gamma_0\,\eta_{\mathrm{nspec}}\frac{p_-^2}{m_c^2}|V_{ud}|^2(|V_{cs}|^2(1-x_-)^2+|V_{cd}|^2)\cdot[(c_1^2+c_2^2)(B_1+6\epsilon_1)+6c_1c_2B_1],$$

$$\Gamma^{\mathrm{WA}}(D_s^+)=-\Gamma_0\,\eta_{\mathrm{nspec}}\frac{m_{D_s}^2}{m_c^2}|V_{cs}|^2|V_{ud}|^2\left\{\left(\frac{c_2^2}{N}+2c_1c_2+Nc_1^2\right)(B1-B2)+2c_2^2(\epsilon_1-\epsilon_2)\right\}$$
$$-\Gamma_0\,\eta_{\mathrm{nspec}}\frac{m_{D_s}^2}{m_c^2}|V_{cs}|^2|V_{us}|^2(1-x_+)^2\left\{\left[\frac{c_2^2}{N}+2c_1c_2+Nc_2^2\right]\left[\left(1+\frac{x_+}{2}\right)B_1-(1+2x_+)B_2\right]\right.$$
$$\left.+2c_2^2\left[\left(1+\frac{x_+}{2}\right)\epsilon_1-(1+2x_+)\epsilon_2\right]\right\},$$

$$\Gamma^{\mathrm{PI}}(D_s^+)=\Gamma_0\,\eta_{\mathrm{nspec}}\frac{p_-^2}{m_c^2}|V_{us}|^2(|V_{cs}|^2(1-x_-)^2+|V_{cd}|^2)[(c_1^2+c_2^2)(B_1+6\epsilon_1)+6c_1c_2B_1],$$

$$\Gamma(D_s^+\to\tau\nu_\tau)=\frac{G_F^2m_\tau^2f_{D_s}^2m_{D_s}}{8\pi}|V_{cs}|^2\left(1-\frac{m_\tau^2}{m_{D_s}^2}\right)^2, \tag{8}$$

where

$$\Gamma_0=\frac{G_F^2m_c^5}{192\pi^3},\qquad \eta_{\mathrm{nspec}}=16\pi^2\frac{f_{D_q}^2m_{D_q}}{m_c^3},$$
$$x_+=\frac{\overline{m}_s^2}{p_+^2},\quad p_+=p_c+p_{\bar q},$$
$$x_-=\frac{\overline{m}_s^2}{p_-^2},\quad p_-=p_c-p_{\bar q}. \tag{9}$$

In the equations, the hadronic parameters are defined as follows:

$$\frac{g^{\mu\nu}}{2m_{D_q}}\langle D_q|O^q_{\mu\nu}|D_q\rangle\equiv\frac{f_{D_q}^2m_{D_q}}{8}B_1,$$
$$\frac{g^{\mu\nu}}{2m_{D_q}}\langle D_q|T^q_{\mu\nu}|D_q\rangle\equiv\frac{f_{D_q}^2m_{D_q}}{8}\epsilon_1,$$
$$\frac{p^\mu p^\nu}{2m_{D_q}^3}\langle D_q|O^q_{\mu\nu}|D_q\rangle\equiv\frac{f_{D_q}^2m_{D_q}}{8}B_2,$$
$$\frac{p^\mu p^\nu}{2m_{D_q}^3}\langle D_q|T^q_{\mu\nu}|D_q\rangle\equiv\frac{f_{D_q}^2m_{D_q}}{8}\epsilon_2, \tag{10}$$

where

$$O^q_{\mu\nu}=\bar{c}\gamma_\mu Lq\bar{q}\gamma_\nu Lc,$$
$$T^q_{\mu\nu}=\bar{c}\gamma_\mu T^aLq\bar{q}\gamma_\nu T^aLc, \tag{11}$$

with $T^a=\lambda^a/2$ and λ^a being the Gell-Mann matrices. (b) For the B and B_s decays,

$$\Gamma^{\mathrm{WA}}(B_d^0)=-\Gamma_0\,\eta_{\mathrm{nspec}}|V_{ud}|^2(1-z_+)^2\left\{\left(\frac{c_1^2}{N}+2c_1c_2+Nc_2^2\right)\left[\left(1+\frac{z_+}{2}\right)B_1-(1+2z_+)B_2\right]\right.$$
$$+2c_1^2\left[\left(1+\frac{z_+}{2}\right)\epsilon_1-(1+2z_+)\epsilon_2\right]\Bigg\}-\Gamma_0\,\eta_{\mathrm{nspec}}|V_{cd}|^2\sqrt{1-4z_+}\left\{\left(\frac{c_1^2}{N}+2c_1c_2+Nc_2^2\right)\right.$$
$$\left.\times[(1-z_+)B_1-(1+2z_+)B_2]+2c_1^2[(1-z_+)\epsilon_1-(1+2z_+)\epsilon_2]\right\},$$

$$\Gamma^{\mathrm{PI}}(B^-)=\Gamma_0\,\eta_{\mathrm{nspec}}\frac{p_-^2}{m_B^2}(1-z_-)^2[(c_1^2+c_2^2)(B_1+6\epsilon_1)+6c_1c_2B_1],$$

$$\Gamma^{\mathrm{WA}}(B_s^0)=-\Gamma_0\,\eta_{\mathrm{nspec}}|V_{us}|^2(1-z_+)^2\left\{\left(\frac{c_1^2}{N}+2c_1c_2+Nc_2^2\right)\left[\left(1+\frac{z_+}{2}\right)B_1-(1+2z_+)B_2\right]\right.$$
$$+2c_1^2\left[\left(1+\frac{z_+}{2}\right)\epsilon_1-(1+2z_+)\epsilon_2\right]\Bigg\}-\Gamma_0\,\eta_{\mathrm{nspec}}|V_{cs}|^2\sqrt{1-4z_+}\left\{\left(\frac{c_1^2}{N}+2c_1c_2+Nc_2^2\right)\right.$$
$$\left.\times[(1-z_+)B_1-(1+2z_+)B_2]+2c_1^2[(1-z_+)\epsilon_1-(1+2z_+)\epsilon_2]\right\}. \tag{12}$$

where

$$\Gamma_0=\frac{G_F^2m_b^5}{192\pi^3}|V_{cb}|^2,\quad \eta_{\mathrm{nspec}}=16\pi^2\frac{f_{B_q}^2m_{B_q}^3}{m_b^5},$$

$$z_+=\frac{\overline{m}_c}{m_{B_q}^2},\quad z_-=\frac{\overline{m}_c^2}{p_-^2}=\frac{\overline{m}_c^2}{(p_b-p_{\overline{u}})^2}. \tag{13}$$

Similar to the D meson, the hadronic parameters B_1, B_2, ϵ_1, and ϵ_2 are defined

$$\frac{g^{\mu\nu}}{2m_{B_q}}\langle B_q|O^q_{\mu\nu}|B_q\rangle\equiv\frac{f_{B_q}^2m_{B_q}}{8}B_1,$$

$$\frac{g^{\mu\nu}}{2m_{B_q}}\langle B_q|T^q_{\mu\nu}|B_q\rangle\equiv\frac{f_{B_q}^2m_{B_q}}{8}\epsilon_1,$$

$$\frac{p^\mu p^\nu}{2m_{B_q}^3}\langle B_q|O^q_{\mu\nu}|B_q\rangle\equiv\frac{f_{B_q}^2m_{B_q}}{8}B_2,$$

$$\frac{p^\mu p^\nu}{2m_{B_q}^3}\langle B_q|T^q_{\mu\nu}|B_q\rangle\equiv\frac{f_{B_q}^2m_{B_q}}{8}\epsilon_2, \tag{14}$$

where

$$O^q_{\mu\nu}=\overline{b}\gamma_\mu Lq\overline{q}\gamma_\nu Lb,$$

$$T^q_{\mu\nu}=\overline{b}\gamma_\mu T^aLq\overline{q}\gamma_\nu T^aLb. \tag{15}$$

C. The nonspectator components in B_c decays

As pointed out above, the spectator contributions to the B_c lifetime should be a sum of those from $\overline{b}$ and c individual decays

$$\Gamma^{\mathrm{spectator}}=\Gamma_b^{\mathrm{spectator}}+\Gamma_{\overline{c}}^{\mathrm{spectator}}, \tag{16}$$

where $\Gamma_b^{\mathrm{spectator}}$ and $\Gamma_{\overline{c}}^{\mathrm{spectator}}$ are the same as they are in B and D decays, respectively, and given in Eqs. (6),(7). Now let us deal with the nonspectator contributions which are different from those in B and D decays.

To estimate the nonspectator components in the B_c decays, let us write the relevant effective Lagrangian precisely here:

$$L_{\mathrm{eff}}^{\Delta C=1}(\mu=m_c)=-\frac{4G_F}{\sqrt{2}}V_{cs}V_{ud}^*\{c_1(\mu)(\overline{s}\gamma_\mu Lc)(\overline{u}\gamma^\mu Ld)$$
$$+c_2(\mu)(\overline{u}\gamma_\mu Lc)(\overline{s}\gamma^\mu Ld)\}+\mathrm{H.c.}, \tag{17}$$

and

$$L_{\mathrm{eff}}^{\Delta B=1}(\mu=m_b)=-\frac{4G_F}{\sqrt{2}}\left\{V_{cb}\left[V_{ud}^*(c_1(\mu)O_1^u+c_2(\mu)O_2^u)\right.\right.$$
$$+V_{cs}^*(c_1(\mu)O_1^c+c_2(\mu)O_2^c)$$
$$+\sum_{l=e,\tau,\mu}\overline{l}\gamma_\mu L\nu\overline{c}\gamma^\nu Lb$$
$$\left.\left.+V_{cs}^*\sum_{i=3}^{6}c_iO_i\right]\right\}+\mathrm{H.c.}, \tag{18}$$

where the operators are

$$O_1^c=\overline{s}\gamma_\mu Lc\overline{c}\gamma^\mu Lb,$$

$$O_1^u=\overline{d}\gamma_\mu Lu\overline{c}\gamma^\mu Lb,$$

$$O_2^c=\overline{s_i}\gamma_\mu Lc_j\overline{c_j}\gamma^\mu Lb_i,$$

$$O_2^u=\overline{d_i}\gamma_\mu Lu_j\overline{c_j}\gamma^\mu Lb_i,$$

$$O_3=\overline{s}\gamma_\mu Lb\overline{c}\gamma^\mu Lc,$$

$$O_4=\overline{s_i}\gamma_\mu Lb_j\overline{c_j}\gamma^\mu Lc_i,$$

$$O_5=\overline{s}\gamma_\mu Lb\overline{c}\gamma^\mu Rc,$$

$$O_6=\overline{s_i}\gamma_\mu Lb_j\overline{c_j}\gamma^\mu Rc_i, \tag{19}$$

and $c_i(i=1,2,\dots)$, denoting the Wilson coefficients, due to QCD corrections, will take the same values as those in Ref. [4]. Here we consider the nonspectator components in B_c decays by two steps. The first step is to compute the relevant operators up to the order $O(1/m_Q^4)$ and then to evaluate the contributions precisely.

1. Pauli interference (PI) operators

The Pauli interference (PI) operators $\hat{\Gamma}^{\rm PI}_{\rm tree}$ and $\hat{\Gamma}^{\rm PI}_{\rm penguin}$ which correspond to the nonleptonic decay induced by the tree part and penguin, respectively, are given by

$$\hat{\Gamma}^{\rm PI}_{\rm tree}=\frac{2G_F^2}{\pi}|V_{cb}|^2|V_{cs}|^2(1-z_-)^2p_-^2\cdot\{2c_1c_2\cdot\overline{b^i}\gamma_\mu Lc^i\overline{c^j}\gamma^\mu Lb^j+(c_1^2+c_2^2)\times\overline{b^i}\gamma_\mu Lc^j\overline{c^j}\gamma^\mu Lb^i\},$$

$$\begin{aligned}\hat{\Gamma}^{\rm PI}_{\rm penguin}=&\frac{2G_F^2}{\pi}|V_{cb}|^2|V_{cs}|^2(1-z_-)^2p_-^2\cdot\{(2c_1c_3+2c_2c_4+2c_3c_4)\cdot\overline{b^i}\gamma_\mu Lc^i\overline{c^j}\gamma^\mu Lb^j\\&+(c_3^2+c_4^2+2c_1c_4+2c_2c_3)\cdot\overline{b^i}\gamma_\mu Lc^j\overline{c^j}\gamma^\mu Lb^i\}+\frac{G_F^2}{3\pi}|V_{cb}|^2|V_{cs}|^2(1-z_-)^2\cdot\{(1-z_-)p_-^2g^{\mu\nu}+2(1+2z_-)p_-^\mu p_-^\nu\}\\&\times\{2c_5c_6\cdot\overline{b^i}\gamma_\mu Lb^j\overline{c^j}\gamma_\nu Rc^i+(c_5^2+c_6^2)\cdot\overline{b^i}\gamma_\mu Lb^i\overline{c^j}\gamma_\nu Rc^j\}-\frac{G_F^2}{\pi}|V_{cb}|^2|V_{cs}|^2(1-z_-)^2\overline{m}_cp^\alpha\\&\times\{[c_2c_6+c_3c_6+c_1c_5+c_4c_5][\overline{b^i}\gamma^\mu Lc^i\overline{c^j}\gamma_\alpha\gamma_\mu Lb^j+\overline{b^i}\gamma_\mu\gamma_\alpha Rc^i\overline{c^j}\gamma^\mu Lb^j]+[c_2c_5+c_3c_5+c_1c_6+c_4c_6]\\&\times[\overline{b^i}\gamma^\mu Lc^j\overline{c^j}\gamma_\alpha\gamma_\mu Lb^i+\overline{b^i}\gamma_\mu\gamma_\alpha Rc^j\overline{c^j}\gamma^\mu Lb^i]\},\end{aligned}\tag{20}$$

where

$$z_-=\frac{\overline{m}_c^2}{p_-^2},\quad p_-=p_b-p_{\overline{c}}. \tag{21}$$

2. Weak annihilation (WA) operators

The weak annihilation operators are $\hat{\Gamma}^{\rm WA}_{\rm tree}$, $\hat{\Gamma}^{\rm WA}_{\rm penguin}$, and $\hat{\Gamma}^{\rm WA}(B_c\to\tau\nu_\tau)$ which correspond to the nonleptonic decay induced by the tree part, penguin, and the pure leptonic (PL) decay, respectively.[2]

$$\begin{aligned}\hat{\Gamma}^{\rm WA}_{\rm tree}=&-\frac{2G_F^2}{3\pi}|V_{cb}|^2|V_{cs}|^2(1-z_+)^2\cdot\left\{\left(1+\frac{z_+}{2}\right)p_+^2g^{\mu\nu}-(1+2z_+)p_+^\mu p_+^\nu\right\}\\&\times\{(Nc_1^2+2c_1c_2)\overline{\cdot b^i}\gamma_\mu Lc^i\overline{c^j}\gamma_\nu Lb^j+c_2^2\cdot\overline{b^i}\gamma_\mu Lc^j\overline{c^j}\gamma_\nu Lb^i\},\end{aligned}\tag{22}$$

[2]Because of helicity suppression, the decays $B_c\to l(e,\mu)+\nu$ are neglectable for the lowest order estimate of the lifetime; thus we do so here.

$$\hat{\Gamma}^{\rm WA}_{\rm penguin}=-\frac{2G_F^2}{3\pi}|V_{cb}|^2|V_{cs}|^2(1-z_+)^2\left\{\left(1+\frac{z_+}{2}\right)p_+^2g^{\mu\nu}-(1+2z_+)p_+^\mu p_+^\nu\right\}$$
$$\times\{(Nc_4^2+2Nc_1c_4+2c_3c_4+2c_1c_3+2c_2c_4)\cdot\overline{b^i}\gamma_\mu Lc^i\overline{c^j}\gamma_\nu Lb^j+(c_3^2+2c_2c_3)\cdot\overline{b^i}\gamma_\mu Lc^j\overline{c^j}\gamma_\nu Lb^i\}$$
$$+\frac{4G_F^2}{\pi}|V_{cb}|^2|V_{cs}|^2(1-z_+)^2p_+^2[(Nc_6^2+2c_5c_6)\cdot\overline{b^i}Rc^i\overline{c^j}Lb^j+c_5^2\cdot\overline{b^i}Rc^j\overline{c^j}Lb^i]$$
$$+\frac{2G_F^2}{\pi}|V_{cb}|^2|V_{cs}|^2(1-z_+)^2\overline{m}_cp_+^\mu\{[Nc_6(c_1+c_4)+c_5(c_1+c_4)+c_6(c_2+c_3)]\cdot[\overline{b^i}Rc^i\overline{c^j}\gamma_\mu Lb^j$$
$$+\overline{b^i}\gamma_\mu Lc^i\overline{c^j}Lb^j]+(c_2c_5+c_3c_5)\cdot[\overline{b^i}Rc^j\overline{c^j}\gamma_\mu Lb^i+\overline{b^i}\gamma_\mu Lc^j\overline{c^j}Lb^i]\}, \tag{23}$$

$$\hat{\Gamma}^{\rm WA}(B_c\to\tau\nu_\tau)=-\frac{2G_F^2}{3\pi}|V_{cb}|^2(1-z_\tau)^2\left\{\left(1+\frac{z_\tau}{2}\right)p_+^2g^{\mu\nu}-(1+2z_\tau)p_+^\mu p_+^\nu\right\} \tag{24}$$
$$\times\overline{b^i}\gamma_\mu Lc^i\overline{c^j}\gamma_\nu Lb^j,$$

where the parameters p_+, z_+, and z_τ are defined by

$$p_+=p_b+p_c,$$
$$z_+=\frac{\overline{m}_c^2}{p_+^2}=\frac{\overline{m}_c^2}{M_{B_c}^2},$$
$$z_\tau=\frac{m_\tau^2}{p_+^2}=\frac{m_\tau^2}{M_{B_c}^2}. \tag{25}$$

3. The contributions from the nonspectator WA and PI to the lifetime for B_c meson

Substituting all the above operators $\hat{\Gamma}^{\rm WA}$, $\hat{\Gamma}^{\rm PI}$ into the relevant matrix element and taking imaginary part, we may estimate the nonspectator contributions to the lifetime of B_c meson:

$$\Gamma=\frac{1}{2M_{B_c}}\langle B_c|\hat{\Gamma}|B_c\rangle, \tag{26}$$

where $\hat{\Gamma}$ denotes the relevant operators for PI and WA given in the above subsections.

According to Eq. (26), when evaluating the lifetime some hadronic matrix elements appear and their values need to be determined, whereas, having nonpertubative nature, they cannot be determined by well-established theories as yet. Let us discuss phenomenological determination of them here.

First of all, the parameters, such as $B_1,B_2,\tilde{B}_1,\tilde{B}_2,\epsilon_1$, $\epsilon_2,\tilde{\epsilon}_1$, and $\tilde{\epsilon}_2$, appear in the corresponding estimates for B and D decays too. Precisely for B_c decays,[3] they are

$$\frac{1}{2M_{B_c}}\langle B_c|O^c_{V-A}|B_c\rangle\equiv\frac{f^2_{B_c}M_{B_c}}{8}B_1,$$

$$\frac{1}{2M_{B_c}}\langle B_c|O^c_{S-P}|B_c\rangle\equiv\frac{f^2_{B_c}M_{B_c}}{8}B_2,$$

$$\frac{1}{2M_{B_c}}\langle B_c|T^c_{V-A}|B_c\rangle\equiv\frac{f^2_{B_c}M_{B_c}}{8}\epsilon_1,$$

$$\frac{1}{2M_{B_c}}\langle B_c|T^c_{S-P}|B_c\rangle\equiv\frac{f^2_{B_c}M_{B_c}}{8}\epsilon_2,$$

$$\frac{1}{2M_{B_c}}\langle B_c|\tilde{O}^c_{V-A}|B_c\rangle\equiv\frac{f^2_{B_c}M_{B_c}}{8}\tilde{B}_1,$$

$$\frac{1}{2M_{B_c}}\langle B_c|\tilde{O}^c_{S-P}|B_c\rangle\equiv\frac{f^2_{B_c}M_{B_c}}{8}\tilde{B}_2,$$

$$\frac{1}{2M_{B_c}}\langle B_c|\tilde{T}^c_{V-A}|B_c\rangle\equiv\frac{f^2_{B_c}M_{B_c}}{8}\tilde{\epsilon}_1,$$

$$\frac{1}{2M_{B_c}}\langle B_c|\tilde{T}^c_{S-P}|B_c\rangle\equiv\frac{f^2_{B_c}M_{B_c}}{8}\tilde{\epsilon}_2,$$

where the relevant four-quark operators are

[3]In fact, throughout the paper for convenience we use $B_1,B_2,\tilde{B}_1\ldots$, only for definitions in cases of D, B, and B_c mesons. Thus in principle, for different mesons they take different values.

$$O^c_{V-A}=\overline{b}\gamma_\mu Lc\overline{c}\gamma^\mu Lb,$$

$$O^c_{S-P}=\overline{b}Lc\overline{c}Rb,$$

$$T^c_{V-A}=\overline{b}\gamma_\mu LT^a c\overline{c}\gamma^\mu LT^a b,$$

$$T^c_{S-P}=\overline{b}LT^a c\overline{c}RT^a b,$$

$$\tilde{O}^c_{V-A}=\overline{b}\gamma_\mu Rc\overline{c}\gamma^\mu Rb,$$

$$\tilde{O}^c_{S-P}=\overline{b}Rc\overline{c}Lb,$$

$$\tilde{T}^c_{V-A}=\overline{b}\gamma_\mu RT^a c\overline{c}\gamma^\mu RT^a b,$$

$$\tilde{T}^c_{S-P}=\overline{b}RT^a c\overline{c}LT^a b. \tag{27}$$

There are eight extra matrix elements corresponding to the new operators in the B_c case. The ‘‘new’’ matrix elements relate to the above parameters or new ones $(\epsilon_3,\epsilon_4,\epsilon_5,\epsilon_6)$ as follows:

$$\frac{1}{2M_{B_c}}\langle B_c|\overline{b}Lc\overline{c}Lb|B_c\rangle\equiv\frac{-f^2_{B_c}M_{B_c}}{8}B_2,$$

$$\frac{1}{2M_{B_c}}\langle B_c|\overline{b}Rc\overline{c}Rb|B_c\rangle\equiv\frac{-f^2_{B_c}M_{B_c}}{8}B_2,$$

$$\frac{1}{2M_{B_c}}\langle B_c|\overline{b}\gamma_\mu Lc\overline{c}\gamma^\mu Rb|B_c\rangle\equiv\frac{-f^2_{B_c}M_{B_c}}{8}B_1,$$

$$\frac{1}{2M_{B_c}}\langle B_c|\overline{b}\gamma_\mu Rc\overline{c}\gamma^\mu Lb|B_c\rangle\equiv\frac{-f^2_{B_c}M_{B_c}}{8}B_1,$$

$$\frac{1}{2M_{B_c}}\langle B_c|\overline{b}LT^a c\overline{c}LT^a b|B_c\rangle\equiv\frac{-f^2_{B_c}M_{B_c}}{8}\epsilon_3,$$

$$\frac{1}{2M_{B_c}}\langle B_c|\overline{b}RT^a c\overline{c}RT^a b|B_c\rangle\equiv\frac{-f^2_{B_c}M_{B_c}}{8}\epsilon_4,$$

$$\frac{1}{2M_{B_c}}\langle B_c|\overline{b}\gamma_\mu LT^a c\overline{c}\gamma^\mu RT^a b|B_c\rangle\equiv\frac{-f^2_{B_c}M_{B_c}}{8}\epsilon_5,$$

$$\frac{1}{2M_{B_c}}\langle B_c|\overline{b}\gamma_\mu RT^a c\overline{c}\gamma^\mu LT^a b|B_c\rangle\equiv\frac{-f^2_{B_c}M_{B_c}}{8}\epsilon_6. \tag{28}$$

For the nonspectator component PI, we have

$$\Gamma^{\rm PI}_{\rm tree}=\frac{G_F^2}{4\pi}f^2_{B_c}M_{B_c}|V_{cb}|^2|V_{cs}|^2(1-z_-)^2p_-^2\cdot\left\{\left[2c_1c_2+\frac{1}{N}(c_1^2+c_2^2)\right]B_1+2(c_1^2+c_2^2)\epsilon_1\right\}, \tag{29}$$

$$\begin{aligned}\Gamma^{\rm PI}_{\rm penguin}=&\frac{G_F^2}{4\pi}f^2_{B_c}M_{B_c}|V_{cb}|^2|V_{cs}|^2(1-z_-)^2p_-^2\left\{\left[2c_2c_4+2c_1c_3+2c_3c_4+\frac{1}{N}(c_3^2+c_4^2+2c_2c_3+2c_1c_4)\right]B_1\right.\\&\left.+2(c_3^2+c_4^2+2c_2c_3+2c_1c_4)\epsilon_1\right\}-\frac{G_F^2}{4\pi}f^2_{B_c}M_{B_c}|V_{cb}|^2|V_{cs}|^2(1-z_-)^2\left\{\left[2c_5c_6+\frac{1}{N}(c_5^2+c_6^2)\right]\right.\\&\times\left[\frac{2+z_-}{3}p_-^2\tilde{B}_2-\frac{1+2z_-}{6}(m_b^2\tilde{B}_1+m_c^2B_1-4m_bm_cB_2+2m_bm_cB_1)\right]\\&\left.+2(c_5^2+c_6^2)\left[\frac{2+z_-}{3}p_-^2\tilde{\epsilon}_2-\frac{1+2z_-}{6}(m_b^2\tilde{\epsilon}_1+m_c^2\epsilon_1-2m_bm_c(\epsilon_3+\epsilon_4)+m_bm_c(\epsilon_5+\epsilon_6))\right]\right\}\\&-\frac{G_F^2}{8\pi}f^2_{B_c}M_{B_c}|V_{cb}|^2|V_{cs}|^2(1-z_-)^2\overline{m}_c\left\{\left[c_1c_5+c_2c_6+c_3c_6+c_4c_5+\frac{1}{N}(c_1c_6+c_2c_5+c_4c_6\right.\right.\\&\left.+c_3c_5)\right][2m_cB_1+m_b(-4B_2+2B_1)]\\&\left.+2(c_1c_6+c_2c_5+c_4c_6+c_3c_5)2m_c\epsilon_1-2m_b(\epsilon_3+\epsilon_4)+m_b(\epsilon_5+\epsilon_6)]\right\},\end{aligned} \tag{30}$$

and for WA, we have

$$\Gamma^{\rm WA}_{\rm tree}=-\frac{G_F^2}{12\pi}|V_{cb}|^2|V_{cs}|^2 f_{B_c}^2 M_{B_c}(1-z_+)^2\left\{\left[Nc_1^2+2c_1c_2+\frac{c_2^2}{N}\right]\right.$$
$$\times\left[\left(1+\frac{z_+}{2}\right)M_{B_c}^2 B_1-(1+2z_+)(m_b^2B_2+m_c^2\tilde{B}_2+2m_bm_cB_2)\right]$$
$$\left.+2c_2^2\left[\left(1+\frac{z_+}{2}\right)M_{B_c}^2\epsilon_1-(1+2z_+)(m_b^2\epsilon_2+m_c^2\tilde{\epsilon}_2+m_bm_c(\epsilon_3+\epsilon_4))\right]\right\}, \tag{31}$$

$$\Gamma^{\rm WA}(B_c\to\tau\nu)=-\frac{G_F^2}{12\pi}|V_{cb}|^2|V_{cs}|^2 f_{B_c}^2 M_{B_c}(1-z_+)^2\left\{\left(1+\frac{z_\tau}{2}\right)M_{B_c}^2B_1-(1+2z_\tau)(m_b^2B_2+m_c^2\tilde{B}_2+2m_bm_cB_2)\right\}, \tag{32}$$

$$\Gamma^{\rm WA}_{\rm penguin}=-\frac{G_F^2}{12\pi}|V_{cb}|^2|V_{cs}|^2 f_{B_c}^2 M_{B_c}(1-z_+)^2\left\{\left[\left(\frac{2c_2+c_3}{N}+2c_1+c_4\right)(c_3+Nc_4)\right]\right.$$
$$\times\left[\left(1+\frac{z_+}{2}\right)M_{B_c}^2 B_1-(1+2z_+)(m_b^2B_2+m_c^2\tilde{B}_2+2m_bm_cB_2)\right]$$
$$\left.+2(2c_2+c_3)c_3\left[\left(1+\frac{z_+}{2}\right)p_+^2\epsilon_1-(1+2z_+)(m_b^2\epsilon_2+m_c^2\tilde{\epsilon}_2+m_bm_c(\epsilon_3+\epsilon_4))\right]\right\}$$
$$+\frac{G_F^2}{2\pi}|V_{cb}|^2|V_{cs}|^2 f_{B_c}^2 M_{B_c}^3(1-z_+)^2\left\{\left[\frac{c_5^2}{N}+2c_5c_6+Nc_6^2\right]\tilde{B}_2+2c_5^2\tilde{\epsilon}_2\right\}$$
$$-\frac{G_F^2}{4\pi}|V_{cb}|^2|V_{cs}|^2 f_{B_c}^2 M_{B_c}\overline{m}_c(1-z_+)^2\left\{\left[\left(\frac{c_2+c_3}{N}+c_1+c_4\right)(c_5+Nc_6)\right]\right.$$
$$\left.\times[2m_bB_2+2m_c\tilde{B}_2]+2(c_2+c_3)c_5[m_b(\epsilon_3+\epsilon_4)+2m_b\tilde{\epsilon}_2]\right\}. \tag{33}$$

D. The effective mass of the decaying heavy quark

The masses of the acting heavy quarks in the decays must be treated carefully although the bound-state effects make the problem complicated and obscure. It is commonly accepted that if the charm quark appears as a decay product, the mass should be its running one at the energy scale of the decaying quark or the meson, whereas, if it appears as the ''parent(s)'' of the decay, the quark (antiquark) is not ''free,'' but in a bound state; thus the pole mass should be taken and the bound-state effects on the mass must be taken into account too. Especially in the spectator mechanism the decay possibility of the heavy quark is very sensitive to the value of its ''adopted'' mass; hence what value of the quark mass adopted in the estimate must be paid special attention. Narison [16] used the QCD sum rules to estimate the mass difference $M^{\rm NR}_{b(c)}-M^{\rm PT2}_{b(c)}$ where $M^{\rm PT2}$ is the short-distance perturbative pole mass and $M^{\rm NR}$ is the long-distance QCD-related effective mass up to two-loops. The authors of Ref. [17] attributed such effects into a factor which is multiplied to the decay width of the ''free'' quark.

Here instead of deriving the modification factor with a relatively large uncertainty, we treat the problem phenomenologically, i.e., by introducing a parametrization

$$M_Q^{\rm eff}=M_Q^{\rm pole}-\Delta, \tag{34}$$

where Δ manifests the bound-state effects, and it will be fixed phenomenologically. Note here that for each heavy meson there are three quantities: lifetime (total width), inclusive semileptonic branching ratio, and pure leptonic branching ratio which may be used for phenomenological analysis, so the estimates here are still well determined even when we introduce the parameter Δ here.

In the next section, we will discuss Δ and other related parameters more precisely. With all the formulas derived above and the hadronic matrix elements, we can make numerical evaluation of the lifetime of B_c straightforwardly.

III. NUMERICAL RESULTS

Since we carry out the estimate of the lifetime of B_c with a ''global'' comparison to all of the heavy and double heavy mesons, so the determination of all of the parameters by fitting the existence experimental data is ''over-determined'' for our goal and has certain level tests. Therefore we evaluate the lifetimes, the semileptonic branching ratios and the pure leptonic branching ratios for all the mesons $D^{\pm},D^0,D_s,B^{\pm},B^0,B_s$, and B_c in this section in turn and present the numerical results in this section.

Generally speaking, we need to make some assumption and conjecture to decrease the number of the parameters which need to be determined and we may assume that

$$\tilde{B}_{1(2)}=B_{1(2)}, \quad \tilde{\epsilon}_{1(2)}=\epsilon_{1(2)}, \tag{35}$$

with symmetry consideration. As for the parameters $\epsilon_{3,4}$ and $\epsilon_{5,6}$, we try to make the conjecture that $\epsilon_{3,4}\simeq\epsilon_2$ and $\epsilon_{5,6}\simeq\epsilon_1$ instead of precise computation.[4]

In the earlier literature, usually $B_1\approx B_2\sim 1$, $\epsilon_1\sim -0.15$ (from the lattice calculations) and $\epsilon_2=0$ are taken. In our numerical computations and trials to fit the data of the lifetimes of the heavy mesons $D^{\pm},D^0,D_s,B^{\pm},B^0$ and B_s and their semileptonic decay branching ratios as well, we find that to adjust the values of the parameters and the pole masses of b and c quarks, indeed when the parameter $\epsilon_2\neq 0$, etc., is taken, a better fit is obtained. Now let us present the fitting and determination of the parameters for the heavy mesons in detail.

A. For the heavy mesons *D* and *B*

To evaluate the lifetimes of $D^0,D^{\pm},D_s,B^0,B^{\pm},B_s$ mesons and their branching ratios of the semileptonic decays, we use the formulas given in Secs. II A, II B, and the Appendix. The values of the parameter set with the assumption and conjecture as indicated above are taken as follows: $|V_{cs}|=0.974$, $|V_{ud}|=0.975$, $\alpha_s(m_c)=0.29$, $c_1(m_c)=1.30$, $c_2(m_c)=-0.57$ [8], $B1=B2=1$, $\epsilon_1\simeq -0.05$, $\epsilon_2=0$ [8], the decay constants of D mesons $f_D=160$ MeV, $f_{D_s}=190$ MeV. In the evaluation of the Pauli interference contribution to D decay width, we take the $p_-^2=(p_c-p_{\bar{q}})^2$ value as 0.5 M_D^2 as done in Ref. [24]. We take $m_b^{\rm pole}=5.02$ GeV, $m_c^{\rm pole}=1.88$ GeV [26,27]. By Eq. (1), we have the running mass of the charm quark at various energy scales as

$$\bar{m}_c(m_c)=1.67 \text{ GeV}, \quad \bar{m}_c(m_b)=1.41 \text{ GeV},$$

$$\bar{m}_c(m_{B_c})=1.37 \text{ GeV}.$$

To fit the data as done in Ref. [8], the quark masses as $m_s=125$ MeV, $m_c^{\rm eff}=1.65$ GeV are desired, respectively. Then the lifetimes for the D mesons are obtained: $\tau(D^0)=0.419$ ps, $\tau(D^-)=1.06$ ps, $\tau(D_s^-)=0.446$ ps, and the branching ratio for the semileptonic decay of D^0 meson $B_{SL}(D^0)=6.9\%$. Comparing to the experimental data: $\tau(D^0)=0.415\pm0.04$ ps; $\tau(D^{\pm})=1.057\pm0.015$ ps; $\tau(D_s)=0.467\pm0.017$ ps and $B_{SL}(D^0)=6.75\pm0.29\%$, one can see the fit is quite good.

In the estimate of B-meson lifetimes, the values of the parameters are adopted as follows: $|V_{cb}|=0.04$, $\alpha_s(m_b)=0.20$, $c_1(m_b)=1.150$, $c_2(m_b)=-0.313$ [4], $\alpha=1.06$, $\beta=1.32$ [28], $B1=B2=1$, $\epsilon_1=-0.14$, $\epsilon_2=-0.08$ [29], the b-quark pole mass $m_b^{\rm pole}=5.02$ GeV and $m_b^{\rm eff}=4.89\sim4.91$ GeV. The decay constants $f_B=200$ MeV and $f_{B_s}=220$ MeV. We think that the mass of the charm quark, being different from the D-meson decays in the initial state but now in the final state, should take the value of its running one instead, namely, the running value at the energy scale m_b, i.e., $\bar{m}_c(m_b)$ should be taken here. To correspond to the pole mass value as taken above in estimating D-meson decays, the precise value of $\bar{m}_c(m_b)=1.41$ GeV is obtained. Furthermore, when calculating the PI contribution to B^- width, we take the value of $p_-^2=(p_b-p_{\bar{u}})^2$ approximately to be 0.8 M_B^2 as done in Ref. [24].

With the chosen parameters the results as follows are obtained:

$$\tau(B^0)=1.54 \text{ ps}, \quad \tau(B^-)=1.74 \text{ ps}, \quad \tau(B_s^-)=1.56 \text{ ps},$$

$$B_{sl}(B^0)=11.2\%, \quad \text{if} \quad m_b^{\rm eff}=4.89 \text{ GeV},$$

$$\tau(B^0)=1.52 \text{ ps}, \quad \tau(B^-)=1.71 \text{ ps}, \quad \tau(B_s^-)=1.54 \text{ ps},$$

$$B_{sl}(B^0)=11.2\%, \quad \text{if} \quad m_b^{\rm eff}=4.90 \text{ GeV},$$

$$\tau(B^0)=1.50 \text{ ps}, \quad \tau(B^-)=1.68 \text{ ps}, \quad \tau(B_s^-)=1.51 \text{ ps},$$

$$B_{sl}(B^0)=11.2\%, \quad \text{if} \quad m_b^{\rm eff}=4.91 \text{ GeV},$$

where $B_{\rm sl}$ indicates the branching ratio of the semileptonic decay. Comparing the results with the experimental data $\tau(B^0)=1.56\pm0.04$ ps, $\tau(B^{\pm})=1.65\pm0.04$ ps, $\tau(B_s)=1.54\pm0.07$ ps and $B_{SL}(B^0)=10.5\pm0.008\%$ we can see the fit is quite good. Note that with the definition of Δ, we have taken $\Delta_c\equiv m_c^{\rm pole}-m_c^{\rm eff}=0.23$ GeV and $\Delta_b\equiv m_b^{\rm pole}-m_b^{\rm eff}=0.11\sim0.13$ GeV, which is understandable.

Let us start the next step that with all the parameters as obtained above, i.e., in the sense obtained by fitting the lifetimes of $B^0,B^{\pm},B_s,D^0,D^{\pm},D_s$ and the branching ratios of the semileptonic decays of B and D mesons, we proceed to evaluate the lifetime of B_c meson and its semileptonic decay rate.

B. For the double heavy meson B_c

The spectator component contribution to the B_c width is a sum of the decays of the $\bar{b}$ and c quark while the other one is left as a spectator in the meson B_c. When evaluating this contribution, m_b and m_c take their effective values. The energy scale for $m_b^{\rm pole}(B_c)$ is $M_B\sim m_b(m_b)$ whereas that for $m_c^{\rm pole}(B_c)$ should be $M_D\sim m_c(m_c)$. We adopt the effective masses for b and $\bar{c}$ as the values gained in B and D decays.

[4]We make the assumption Eq. (35) and some conjectures here only to decrease the number of the parameters which need to be determined, so as to carry on the phenomenological study further. Certainly, they should be tested. Our approach here is to make certain assumption and conjectures first and then to explore their consequences, to make comparison with experimental data, and to see if the assumption and conjectures are reasonable or not so as to guide us in further study. Moreover, we know that in general the vacuum saturation for the matrix elements of a product of two current picks up substantial contribution, so we think our conjectures and assumption should be not too wild. Certainly they are substantial sources of theoretical uncertainties for the present estimate. Since the uncertainties are of nonperturbative nature and to estimate them is not easy, we leave them for elsewhere [23].

TABLE I. The results for the B_c meson.

f_{B_c}	τ_{B_c}	$\Gamma^{\rm pen}$	$\Gamma^{b\to c}$	$\Gamma^{c\to s}$	$\Gamma^{\rm WA}$	$\Gamma^{\rm PI}$	$\Gamma(\tau\nu)$	B_{SL}
440 MeV	0.362 (ps)	3.4%	22.8%	70.9%	13.4%	−7.1%	0.078 ps^{-1}	8.7%
500 MeV	0.357 (ps)	4.3%	22.4%	69.7%	16.9%	−9.0%	0.100 ps^{-1}	8.4%

For the nonspectator contributions, i.e., the WA and PI pieces, the charm-quark mass in the final state should be taken its running one at the corresponding energy scale M_{B_c}. Now let us take the relevant parameters for B_c as follows: $M_{B_c}=6.25$ GeV, $M^*_{B_c}=6.33$ GeV,[5] $B_1\sim B_2\sim 1$ are assumed and as conjecture we take $\epsilon_1=-0.14$ and $\epsilon_2=-0.08$. For the decay constant, we adopt Eichten and Quigg's $f_{B_c}=500$ MeV [20] based on potential model and the lattice $f_{B_c}=440$ MeV [21], respectively. Furthermore in the calculation of the PI contribution, the quantity $p_-^2=(p_b-p_{\bar c})^2\simeq 2m_b^2+2m_c^2-M_{B_c}^2$ is taken approximately. With these parameters, we obtain the numerical results and tabulate them in Table I.

In the table τ_{B_c} denotes the total lifetime of B_c, $\Gamma^{\rm pen}$ denotes the contribution from the interference between the penguin and ''tree'' terms, $\Gamma(\tau\nu)$ is the width of the pure leptonic decay (τ channel only but almost equal to the total), and $B_{\rm SL}$ is the branching ratio of the semileptonic decay of the meson B_c. From the table one may see that the estimated lifetime of B_c is around 0.36 ps, which is shorter than the center value measured by CDF [1].

Since both the ''parent'' $\bar b$ and c quarks reside in the bound state, B_c meson, as in the cases of the heavy mesons D and B, etc., if the bound-state effects are involved, the problem how to determine the value of the masses m_b and m_c emerges. We may fix the parameters by fitting data of the heavy mesons D and B, i.e., those presented in Table I, so now the problem becomes whether the values obtained from D and B mesons can be applied to the case of B_c directly. More precisely, let us discuss the bound-state effects on the effective masses of $\bar b$ and c in the B_c meson further.

Because B_c includes two heavy quarks, i.e., it is a double heavy meson; the bound-state effects might be greater than those in the heavy mesons B,D. We suspect that the values $m_c^{\rm eff}$ and $m_b^{\rm eff}$ might be a little smaller than $m_c^{\rm eff}=1.65$ GeV and $m_b^{\rm eff}=4.9$ GeV those obtained from B and D decays. Phenomenologically, if in B_c meson, $m_c^{\rm eff}(B_c)=1.55$ GeV, $m_b^{\rm eff}(B_c)=4.85$ GeV are taken, we will obtain $\tau(B_c)\approx 0.47$ ps, which occasionally is closer to the center value of the lifetime measured recently [1]. In this case, $\Delta_c=0.33$ GeV and $\Delta_b=0.17$ GeV. This result may be understood: since the rates of direct $\bar b$ and c decays dominate the lifetime of B_c meson, and are proportional to $(M_Q^{\rm eff})^5$, the results are so sensitive to the effective masses.

[5]Here considering the consistency among the parameters, we do not take the experimental central value $M_{B_c}=6.4$ GeV, but the ones obtained by potential model.

IV. CONCLUSION AND DISCUSSIONS

In this work we try to estimate the lifetime of B_c in a more consistent way for all of the heavy and double heavy mesons. Our approach is that we adopt the ''unique'' theoretical framework under which the nonspectator effects are taken into account properly for all of the heavy flavor inclusive decays. The parameters appearing in the estimate of B_c meson are determined phenomenologically, i.e., by fitting the existing data of the heavy mesons $B^0,B^\pm,B_s$, D^0, $D^\pm,D_s$ correspondingly and by assumption and conjectures. Furthermore, the effective mass for a heavy quark inside a heavy meson or the meson B_c is treated carefully in two ways: pole and running masses are taken according to the different roles in the decays; in various binding systems the available effective decay masses may vary. The uncertainties in the estimate are discussed roughly.

In the estimation of the lifetime and inclusive semileptonic decay for the meson B_c, not all of the parameters, especially those relating to the matrix elements for two currents, can be fixed by fitting the available data of the heavy mesons $B^0,B^\pm,B_s,D^0,D^\pm,D_s$; nevertheless in order to carry out the estimate we also make some reasonable assumptions or conjectures. Certainly these assumptions and conjectures should be computed by suitable approaches and tested experimentally. We leave the task and discuss them more carefully elsewhere [23], but here only argue them as below instead.

In fact, in terms of the lattice gauge simulation, the QCD sum rule method and other approaches, these parameters can be computed in practice or in principle. As for the parameters B_1 and B_2, i.e., the factors in the hadronic matrix elements, for instance, their values manifest the deviation from the vacuum saturation, so they should not be very great. Some other parameters, such as ϵ_1 and ϵ_2, seen to relate to the nonfactorization effects [33], may be calculated for D mesons and B mesons, respectively, in terms of the QCD sum rules [8,28,29]. It is known that the numerical values obtained by the QCD sum rules may have errors about 10–15 %, but they still can be used in phenomenological calculations and do not cause a very large uncertainty. Thus in this work we take the values from D mesons and B mesons as references for the B_c meson that in fact is a conjecture. The consistency of our numerical results for B and D mesons give us certain confidence of the conjecture for the parameters, i.e., the validity of the parameter regions. Furthermore, having carefully considered the quark masses and taken into account the bound-state effects, one may learn how crucial is the value of the effective mass for the acting quark in the decays.

The earlier estimates on the lifetimes of B and D mesons and the semileptonic decay rates obviously deviate from the data. Luke, Savage, and Wise [5] pointed out that in the decay $c \to X \overline{e} \nu_e$, the contribution of α_s^2 order is of the same magnitude as that of $\mathcal{O}(\alpha_s)$ and this higher order correction suppresses the semileptonic decay rate of the D meson. Bearing this fact in mind, we compare our numerical results for the lifetimes of D mesons and their semileptonic decay rates, and find that they may be satisfactorily consistent with data. Whereas, for B-meson decays, the α_s^2 order correction, as well as the $\mathcal{O}(\alpha_s)$ correction, are smaller. Whereas with these corrections concerned, the results for B mesons are also consistent with data within the experiment tolerance region. All these imply that the parameters taken as the above are reasonable.

When evaluating the B_c lifetime and its inclusive semileptonic decay rates, some new aspects must be taken into account. First there are several new operators in the effective Lagrangian playing roles. Their appearance is due to nonnegligible charm mass m_c, whereas in B and D cases, the light quark mass m_q may be ignored with quite high accuracy. Correspondingly, several new hadronic matrix elements are induced by these operators. Some of them are also proportional to B_1 and B_2, which appear in the expressions for B and D meson decays, as long as the factorization theorem and the vacuum saturation approximately work well enough. In order to consider the nonfactorization contributions, new parameters appear and $\epsilon_3 \sim \epsilon_6$ is assumed, whereas in this work, we have taken a naive symmetry consideration and let $\epsilon_{3,4} \simeq \epsilon_2$ and $\epsilon_{5,6} \simeq \epsilon_1$, although it needs to be tested further.

As pointed out in the Introduction, in the case of the meson B_c, the interference between the penguin and tree terms is not negligible. Namely, the penguin contribution to B_c lifetime is much more important than that to B and D decays. Our results confirm this allegation and we have found the contribution from the interference can be as large as 3–4 % of the total width. Since direct measurements of penguin diagram contributions through specific decays are interesting, this sizable value certainly encourages future experimental studies.

The lifetime of the B_c meson is estimated to be around 0.36 ps if $f_{B_c} \sim 440$–500 MeV and the values of the parameters taken as discussed above, which is smaller than the central value of the measurement: $\tau_{B_c} = 0.46^{+0.18}_{-0.16}(\text{stat}) \pm 0.063(\text{syst})$ ps [1]. In our present estimation, we use the values of M_{B_c} and $M^*_{B_c}$ as 6.25 and 6.33 GeV [2], whereas the measurement is $M_{B_c} \sim 6.40 \pm 0.39(\text{stat}) \pm 0.13(\text{syst})$ GeV. When the bound-state effects on the masses of $\overline{b}$ and c quarks are reasonably taken into account, we can have $\tau_{B_c} \sim 0.47$ ps, which is very close to the present experimental center value for the B_c-meson lifetime. As noted, the change of f_{B_c} itself does not influence the result much, e.g., as f_{B_c} changes from 440 MeV to 500 MeV, τ_{B_c} varies 1% only. Considering the fact that our earlier estimation was $\tau_{B_c} = 0.4$ ps [9] and the estimation in terms of the light-front constituent quark model was $\tau_{B_c} = 0.59 \pm 0.06$ ps [34], which is larger than the measured value, and the uncertainty in masses of the decay quarks alone would result in an estimated change about 4%, the deviation between the values of theoretical estimates and the center value of the measurement in the B_c lifetime is not a serious problem, but leaves room for further studies.

The more accurate experimental measurements may shed new light on the approach, assumption and conjectures adopted here, especially, the consistent consideration of the effective heavy flavor theory, the parameters and the duality between quark states and hadronic states:

$$\sum_{i,j} |q_i, g_j\rangle\langle q_i, g_j| = \sum_k |h_k\rangle\langle h_k|.$$

Therefore more precise theoretical studies are needed.

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APPENDIX

The semileptonic and nonleptonic decay rates of b quark through order $1/m_Q^2$ are given as [2,25]

$$\Gamma_{SL}(H_b) = \Gamma_0^{(b)} \eta(x_c, x_l, 0) \left[I_0(x_c,0,0) \langle H_b|\overline{b}b|H_b\rangle - \frac{2\langle \mu_G^2\rangle_{H_b}}{m_b^2} I_1(x_c,0,0) \right],$$

$$\Gamma_{NL}(H_b) = \Gamma_0^{(b)} N \left\{ \left(c_1^2 + c_2^2 + \frac{2c_1c_2}{N} \right) \left[(\alpha I_0(x_c,0,0) + \beta I_0(x_c, x_c, 0)) \langle H_b|\overline{b}b|H_b\rangle - \frac{2\langle \mu_G^2\rangle_{H_b}}{m_b^2} (I_1(x_c,0,0) + I_1(x_c, x_c, 0)) \right] - 8 \frac{\langle \mu_G^2\rangle_{H_b}}{m_b^2} \frac{2c_1c_2}{N} [I_2(x_c,0,0) + I_2(x_c, x_c, 0)] \right\}, \tag{A1}$$

where

$$\Gamma_0^{(b)} \equiv \frac{G_F^2 m_b^5}{192\pi^3} |V_{cb}|^2; \tag{A2}$$

and the following notation has been used: I_0, I_1, and I_2 are phase-space factors, namely,

$$I_0(x,0,0) = (1-x^2)(1-8x+x^2) - 12x^2 \log x,$$

$$I_1(x,0,0)=\frac{1}{2}\left(2-x\frac{d}{dx}\right)I_0(x,0,0),$$

$$I_2(x,0,0)=(1-x)^3,$$

$$I_0(x,x,0)=v(1-14x-2x^2-12x^3)+24x^2(1-x^2)\log\frac{1+v}{1-v},$$

$$I_1(x,x,0)=\frac{1}{2}\left(2-x\frac{d}{dx}\right)I_0(x,x,0),$$

$$I_2(x,x,0)=v\left(1+\frac{x}{2}+3x^2\right)-3x(1-2x^2)\log\frac{1+v}{1-v},$$

$$x_c=(\overline{m}_c/m_b)^2,\quad v=\sqrt{1-4x}, \tag{A3}$$

with $I_{0,1,2}(x,x,0)$ describing the $b\to c\bar{c}s$ transitions.

For $\eta(x_c,x_l,0)$, which is the QCD radiative correction to the semileptonic decay rate, the general analytic expression is given in Ref. [30]. The special case $\eta(x,0,0)$ is given in Ref. [31] and it can be approximated numerically by [25,32]

$$\eta(x,0,0)\cong 1-\frac{2\alpha_s}{3\pi}\left[\left(\pi^2-\frac{31}{4}\right)(1-\sqrt{x})^2+\frac{3}{2}\right]. \tag{A4}$$

For the decay $b\to c\tau\nu$, according to [7] we roughly have

$$\Gamma(b\to c\tau\nu)\sim 0.25\Gamma(b\to ce\nu). \tag{A5}$$

The expressions are simpler for $c\to s$:

$$\Gamma_{\rm SL}(H_c)=\Gamma_0^{(c)}\cdot\eta(x_s,x_l,0)\left[I_0(x_s,0,0)\langle H_c|\bar{c}c|H_c\rangle-\frac{2\langle\mu_G^2\rangle_{H_c}}{m_c^2}I_1(x_s,0,0)\right], \tag{A6}$$

$$\Gamma_{\rm NL}(H_c)=\Gamma_0^{(c)}\cdot N\left\{\left(c_1^2+c_2^2+\frac{2c_1c_2}{N}\right)\left[\alpha I_0(x_s,0,0)\times\langle H_c|\bar{c}c|H_c\rangle-\frac{2\langle\mu_G^2\rangle_{H_c}}{m_c^2}I_1(x_s,0,0)\right]-8\frac{\langle\mu_G^2\rangle_{H_c}}{m_c^2}\frac{2c_1c_2}{N}\cdot I_2(x_s,0,0)\right\}. \tag{A7}$$

where

$$\Gamma_0^{(c)}\equiv\frac{G_F^2m_c^5}{192\pi^3}|V_{cs}|^2,\quad x_s=\frac{\overline{m}_s^2}{m_c^2}, \tag{A8}$$

and for the correction $\eta(x_s,x_l,0)$ in the c-decay case, we adopt a numerical expression from [5]. It reads

$$\eta_{\rm SL}=1-2.08\left(\frac{\alpha_s(m_c)}{\pi}\right)-22.7\left(\frac{\alpha_s(m_c)}{\pi}\right)^2. \tag{A9}$$

For the dimension-3 operator $\bar{Q}Q$, the expectation value can be expressed as follows:

$$\langle H_Q|\bar{Q}Q|H_Q\rangle=1-\frac{\langle(\mathbf{p}_Q)^2\rangle_{H_Q}}{2m_Q^2}+\frac{\langle\mu_G^2\rangle_{H_Q}}{2m_Q^2}+\mathcal{O}(1/m_Q^3); \tag{A10}$$

where $\langle(\mathbf{p}_Q)^2\rangle\equiv\langle H_Q|\bar{Q}(iD)^2Q|H_Q\rangle$ denotes the average kinetic energy of the quark Q moving inside the hadron and $\langle\mu_G^2\rangle_{H_Q}\equiv\langle H_Q|\bar{Q}(i/2)\sigma\cdot GQ|H_Q\rangle$.

Based on Refs. [2,10] the kinetic terms take the values respectively as follows:

$$\frac{\langle(\mathbf{p}_b)^2\rangle_B}{m_b^2}\simeq 0.016,\quad\frac{\langle(\mathbf{p}_c)^2\rangle_D}{m_c^2}\simeq 0.21;$$

$$\frac{\langle(\mathbf{p}_b)^2\rangle_{B_c}}{m_b^2}\simeq 0.04,\quad\frac{\langle(\mathbf{p}_c)^2\rangle_{B_c}}{m_c^2}\simeq 0.4. \tag{A11}$$

For the chromomagnetic operator one finds $\langle\mu_G^2\rangle_{P_Q}\simeq\frac{3}{2}m_Q(M_{V_Q}-M_{P_Q})$, where P_Q and V_Q denote the pseudoscalar and vector mesons, respectively.

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