Remarks on flavor-neutrino propagators and oscillation formulas

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(Received 12 October 2000; published 4 June 2001)

We examine the general structure of the formulas of neutrino oscillations proposed by Blasone and Vitiello (BV). Reconstructing their formulas with the retarded propagators of the flavor-neutrino fields for the case of many flavors, we can get easily the formulas which satisfy the suitable boundary conditions and are independent of arbitrary mass parameters $\{\mu_\rho\}$, as obtained by BV for the case of two flavors. In this two-flavor case, our formulas reduce to those obtained by BV under a T-invariance condition. Furthermore, the reconstructed probabilities are shown to coincide with those derived with recourse to the mass Hilbert space $\mathcal{H}_m$ which is unitarily inequivalent to the flavor Hilbert space $\mathcal{H}_f$. Such a situation is not found in the corresponding construction in the manner of BV. Then the new factors in BV's formulas, which modify the usual oscillation formulas, are not the trace of the flavor Hilbert space construction, but come from Bogolyubov transformation among the operators of spin-$\frac{1}{2}$ neutrinos with different masses.

DOI: 10.1103/PhysRevD.64.013011 PACS number(s): 14.60.Pq

I. PURPOSE AND FUNDAMENTAL ASSUMPTION

The field theoretical descriptions of neutrino oscillations have been examined from various viewpoints [1–6]. When we want to reformulate straightforwardly, in the framework of field theory, the familiar quantum-mechanical derivation of the neutrino oscillation formula [7], we encounter the problem of how to define field theoretically one (anti)neutrino state with a definite flavor. Giunti *et al.* [2] gave a negative answer to this problem on the basis of the observation that the Hilbert space of the weak eigenstates with definite flavors can be constructed approximately only in extremely relativistic case.[1] In Ref. [3], the authors asserted that the flavor (or weak) as well as the mass Hilbert spaces $\mathcal{H}_f$ and $\mathcal{H}_m$ can be really constructed by employing the Bogolyubov transformation among creation and annihilation operators of the flavor and mass eigenstates of neutrinos. The unitary inequivalence between those two Hilbert spaces leads to a certain effect in the neutrino oscillation formulas, which is to be observed in the low-energy experiment.

In order to determine the coefficients appearing in the Bogolyubov transformation mentioned above, the masses of the electron- and muon-neutrinos (in the two-flavor case) were taken in Ref. [3] to be the mass eigenvalues m_1 and m_2, respectively. To this prescription, the present authors gave a criticism [4]. Its essence lies in the point that the masses of flavor neutrinos are inherently arbitrary and such arbitrariness should not remain in any observed quantities; thus, it is unphysical that the oscillation formulas of neutrinos depend on the arbitrarily chosen mass parameters.

In connection with this criticism [4], Blasone and Vitiello (BV) [5] have remarked that there exist some quantities leading to the neutrino oscillation formulas, which satisfy the necessary boundary conditions and also are independent of the mass parameters of flavor neutrinos even when we start with the theory including such arbitrary parameters.

The purpose of the present paper is to present clearly, on a general basis of the field theory, the logical feature of the remark given by BV in Ref. [5]. The considerations developed in the following are based on the ⟨Setup⟩: The relation of the flavor-neutrino field operator $\nu_\rho(x)$ to the neutrino field operator $\nu_j(x)$ is expressed as

$$\nu_\rho(x)=\sum_{j=1}^{N_f} z_{\rho j}^{1/2}\nu_j(x), \quad \rho=e,\mu,\tau,\ \ldots; \tag{1.1}$$

here, $\nu_j(x)$ satisfies the free Dirac equation with a definite mass m_j,

$$(\partial\!\!\!/+m_j)\nu_j(x)=0, \tag{1.2}$$

and the matrix $Z^{1/2}=(z_{\rho j}^{1/2})$ satisfies $\Sigma_{j=1}^{N_f} z_{\rho j}^{1/2} z_{\sigma j}^{1/2*}=\delta_{\rho\sigma}$; N_f = the number of flavors.

The linear combinations (1.1) are determined so as to diagonalize the mass term in the Lagrangian. [In other words, this is so as to diagonalize the pole part of the propagator matrix constructed from the flavor neutrino field operators [8]; when only the repetitions of the bilinear-mass-type interaction are taken into account, the unitarity of $Z^{1/2}$ is obtained [4]. If CP (or T) invariance is required, we obtain the reality of $Z^{1/2}$ [8].]

Because of the above setup, we obtain the c-number property of the anticommutators

$$\{\nu_\rho(x),\nu_\sigma^\dagger(y)\}=c\ \text{ number} \tag{1.3}$$

and

$$\{\nu_\rho(x),\nu_\sigma(y)\}=0. \tag{1.4}$$

[1]This assertion seems to be not so convincing, the reason for which will be explained in Appendix C.

Further, the canonical commutation relations among the flavor-neutrino field operators at an equal time are consistently obtained

$$\{\nu_\rho(\vec{x},t),\nu_\sigma^\dagger(\vec{y},t)\}=\delta_{\rho\sigma}\delta(\vec{x}-\vec{y}),\quad \{\nu_\rho(\vec{x},t),\nu_\sigma(\vec{y},t)\}=0, \tag{1.5}$$

due to the unitarity of $Z^{1/2}$ and due to Eq. (1.4), respectively.

Therefore, from Eq. (1.1) we see that

$$\langle \mathrm{vac}|\{\nu_\rho(x),\overline{\nu_\sigma}(y)\}|\mathrm{vac}\rangle \tag{1.6}$$

does not depend on the choice of the vacuum state when this state is equally normalized as $\langle \mathrm{vac}|\mathrm{vac}\rangle=1$. In other words, the expectation value

$$_f\langle 0(T)|\{\nu_\rho(x),\overline{\nu_\sigma}(y)\}|0(T)\rangle_f, \tag{1.7}$$

where $|0(T)\rangle_f$ is the vacuum state (at an arbitrary time T) belonging to the flavor Hilbert space $\mathcal{H}_f$ specified by a set of the mass parameters $\{\mu_e,\mu_\mu,\ldots\}=\{\mu_\lambda\}$, is equal to

$$_m\langle 0|\{\nu_\rho(x),\overline{\nu_\sigma}(y)\}|0\rangle_m,\quad |0\rangle_m\in\mathcal{H}_m; \tag{1.8}$$

this equality holds irrespectively of both $\{\mu_\lambda\}$ and the time T, since the c number in Eq. (1.3) depends on $\{m_j\}$ but not on $\{\mu_\lambda\}$ due to the ⟨Setup⟩. (As to the definitions of the vacuum states, see the next section.)

In the following sections, we will explain that these facts described above provide a general field theoretical basis for understanding the implications included in the remark given in Ref. [5].

In Sec. II, we summarize the essence of Ref. [5] after giving the necessary definitions of the notation and relations. In Sec. III we make clear, on the basis of the ⟨Setup⟩, the general implications of BV's remark [5]. Section IV is devoted to a summarizing discussion. In the Appendixes, relevant mathematical details are given.

II. REFORMULATION OF BV's WORK AND RELATED REMARK

A. Notation and definitions

We summarize the notation and definitions of the related quantities in accordance with Ref. [4].

The relation (1.1) between the flavor eigenfields ν_F and the mass eigenfields ν_M is expressed by the transformation as

$$\nu_F(x)\equiv\begin{pmatrix}\nu_e(x)\\ \nu_\mu(x)\\ \vdots\end{pmatrix}=G^{-1}(x^0)\begin{pmatrix}\nu_1(x)\\ \nu_2(x)\\ \vdots\end{pmatrix}G(x^0)$$
$$=\begin{pmatrix}z_{e1}^{1/2} & z_{e2}^{1/2} & \\ z_{\mu1}^{1/2} & z_{\mu2}^{1/2} & \\ & & \ddots\end{pmatrix}\begin{pmatrix}\nu_1(x)\\ \nu_2(x)\\ \vdots\end{pmatrix}\equiv Z^{1/2}\nu_M(x), \tag{2.1}$$

where

$$Z^{1/2}=[z_{\rho j}^{1/2}],\quad Z^{1/2}Z^{1/2\dagger}=I; \tag{2.2}$$

the concrete form of $G(x^0)$ in the two-flavor case is given by BV [3]. Let us expand the neutrino field in terms of helicity-momentum eigenfunctions as

$$\nu_a(x)=\frac{1}{\sqrt{V}}\sum_{\vec{k}r}\{u_a(kr)\alpha_a(kr;t)e^{i\vec{k}\cdot\vec{x}}+v_a(kr)\beta_a^\dagger(kr;t)e^{-i\vec{k}\cdot\vec{x}}\}$$
$$=\frac{1}{\sqrt{V}}\sum_{\vec{k}r}e^{i\vec{k}\cdot\vec{x}}\{u_a(kr)\alpha_a(kr;t)+v_a(-kr)\beta_a^\dagger(-kr;t)\}, \tag{2.3}$$

where in the Kramers representation

$$(i\not{k}+\mu_a)u_a(kr)=0,\quad (i\not{k}-\mu_a)v_a(kr)=0,$$
$$k_0=\sqrt{\vec{k}^2+\mu_a^2}\equiv\omega_a(k),$$
$$u_a^*(kr)u_b(ks)=v_a^*(-kr)v_b(-ks)=\rho_{ab}(k)\delta_{rs},$$
$$u_a^*(kr)v_b(-ks)=v_a^*(-kr)u_b(ks)=i\lambda_{ab}(k)\delta_{rs}, \tag{2.4}$$
$$\rho_{ab}(k)\equiv\cos\left(\frac{\chi_a-\chi_b}{2}\right),$$
$$\lambda_{ab}(k)\equiv\sin\left(\frac{\chi_a-\chi_b}{2}\right),\quad \cot\chi_a=\frac{|\vec{k}|}{\mu_a}.$$

[For the case that $\nu_a(x)$ represents the mass eigenfield $\nu_j(x)$, we write μ_j as m_j. Note that, for $a=\lambda(=e,\mu,\ldots)$, μ_λ is an arbitrarily fixed parameter [4]. As to the concrete forms of $u_a(kr)$ and $v_a(kr)$, see Appendix C.] Here we use the notation

$$\alpha_F(kr;t)\equiv\begin{pmatrix}\alpha_e(kr;t)\\ \alpha_\mu(kr;t)\\ \vdots\end{pmatrix},\quad \beta_F(-kr;t)\equiv\begin{pmatrix}\beta_e(-kr;t)\\ \beta_\mu(-kr;t)\\ \vdots\end{pmatrix},$$
$$\alpha_M(kr;t)\equiv\begin{pmatrix}\alpha_1(kr;t)\\ \alpha_2(kr;t)\\ \vdots\end{pmatrix},\quad \beta_M(-kr;t)\equiv\begin{pmatrix}\beta_1(-kr;t)\\ \beta_2(-kr;t)\\ \vdots\end{pmatrix}. \tag{2.5}$$

We have

$$\begin{pmatrix}\alpha_F(kr;t)\\ \beta_F^\dagger(-kr;t)\end{pmatrix}=K^{-1}(t)\begin{pmatrix}\alpha_M(kr;t)\\ \beta_M^\dagger(-kr;t)\end{pmatrix}K(t)$$
$$=\mathcal{K}(k)\begin{pmatrix}\alpha_M(kr;t)\\ \beta_M^\dagger(-kr;t)\end{pmatrix}, \tag{2.6}$$

with

$$\mathcal{K}(k)=\begin{pmatrix} P(k) & i\Lambda(k) \\ i\Lambda(k) & P(k) \end{pmatrix}, \quad \mathcal{K}(k)\mathcal{K}^{\dagger}(k)=I,$$

$$P(k)=[P(k)_{\rho j}]=[z_{\rho j}^{1/2}\rho_{\rho j}(k)],$$

$$\Lambda(k)=[z_{\rho j}^{1/2}\lambda_{\rho j}(k)]. \tag{2.7}$$

$\mathcal{K}$ is independent of the time t, but depends, of course, on $\{\mu_\lambda\}$ and $[z^{1/2}]_{\rho j}$ other than $|\vec{k}|$, and we dropped such a dependence for simplicity. The relation between the vacuum states $|0(t)\rangle_f \in \mathcal{H}_f$ and $|0\rangle_m \in \mathcal{H}_m$ is expressed with this $K(t)$ as

$$|0(t)\rangle_f = K(t)^{-1}|0\rangle_m. \tag{2.8}$$

Here, these vacua are defined for $^\forall\vec{k}$ and r as

$$\alpha_F(kr;t)|0(t)\rangle_f=\beta_F(kr;t)|0(t)\rangle_f=0,$$

$$\alpha_M(kr;t)|0\rangle_m=\beta_M(kr;t)|0\rangle_m=0, \tag{2.9}$$

with the normalization ${}_f\langle 0(t)|0(t)\rangle_f={}_m\langle 0|0\rangle_m=1$.

From Eq. (2.6), we obtain the relations connecting the creation and annihilation operators with different times, expressed as

$$\begin{pmatrix} \alpha_F(kr;0) \\ \beta_F^{\dagger}(-kr;0) \end{pmatrix} = \mathcal{K}(k)\begin{pmatrix} \alpha_M(kr;0) \\ \beta_M^{\dagger}(-kr;0) \end{pmatrix} = W(k;t)\begin{pmatrix} \alpha_F(kr;t) \\ \beta_F^{\dagger}(-kr;t) \end{pmatrix}, \tag{2.10}$$

with

$$W(k;t)\equiv \mathcal{K}(k)\Phi(k;t)\mathcal{K}^{\dagger}(k) = \begin{pmatrix} P\phi P^{\dagger}+\Lambda\phi^{*}\Lambda^{\dagger} & i(-P\phi\Lambda^{\dagger}+\Lambda\phi^{*}P^{\dagger}) \\ i(\Lambda\phi P^{\dagger}-P\phi^{*}\Lambda^{\dagger}) & \Lambda\phi\Lambda^{\dagger}+P\phi^{*}P^{\dagger} \end{pmatrix},$$

$$\Phi(k;t)\equiv\begin{pmatrix} \phi(t) & 0 \\ 0 & \phi^{\dagger}(t) \end{pmatrix}, \quad \phi(t)=\begin{pmatrix} e^{i\omega_1 t} & & 0 \\ & e^{i\omega_2 t} & \\ 0 & & \ddots \end{pmatrix}. \tag{2.11}$$

Therefore we obtain

$$K(t)\begin{pmatrix} \alpha_F(kr;0) \\ \beta_F^{\dagger}(-kr;0) \end{pmatrix}K^{-1}(t)=W(k;t)\begin{pmatrix} \alpha_M(kr;t) \\ \beta_M^{\dagger}(-kr;t) \end{pmatrix} \tag{2.12}$$

or

$$K(0)\begin{pmatrix} \alpha_F(kr;t) \\ \beta_F^{\dagger}(-kr;t) \end{pmatrix}K^{-1}(0)=W^{\dagger}(k;t)\begin{pmatrix} \alpha_M(kr;0) \\ \beta_M^{\dagger}(-kr;0) \end{pmatrix}. \tag{2.13}$$

We write the matrix elements of $W(k;t)$ as

$$W(k;t)=\begin{pmatrix} W_{\rho\sigma}(k;t) & W_{\rho\bar{\sigma}}(k;t) \\ W_{\bar{\rho}\sigma}(k;t) & W_{\bar{\rho}\bar{\sigma}}(k;t) \end{pmatrix}. \tag{2.14}$$

From Eqs. (2.11), we have

$$W_{\rho\sigma}(k;t)^{*}=W_{\bar{\sigma}\bar{\rho}}(k;t),$$

$$W_{\rho\bar{\sigma}}(k;t)^{*}=W_{\sigma\bar{\rho}}(k;t),$$

$$W_{\bar{\rho}\sigma}(k;t)^{*}=W_{\bar{\sigma}\rho}(k;t). \tag{2.15}$$

Due to the unitarity of $Z^{1/2}$, we have

$$W(k;T-t)W^{\dagger}(k;T)=W(k;-t)=W^{\dagger}(k;t), \tag{2.16}$$

in addition to the unitarity of $W(k;t)$,

$$W(k;t)W^{\dagger}(k;t)=I. \tag{2.17}$$

B. BV's results

Let us review briefly the main contents of BV's paper [5]. For the two-flavor case, we consider that an initial electron neutrino evolves (oscillates) in time with the two relevant propagators (for $t\geqslant 0$)

$$iG_{ee}^{>}(\vec{x},t;\vec{y},0)={}_f\langle 0(0)|\nu_e(\vec{x},t)\overline{\nu_e}(\vec{y},0)|0(0)\rangle_f, \tag{2.18}$$

$$iG_{\mu e}^{>}(\vec{x},t;\vec{y},0)={}_f\langle 0(0)|\nu_\mu(\vec{x},t)\overline{\nu_e}(\vec{y},0)|0(0)\rangle_f. \tag{2.19}$$

By employing their Fourier components

$$iG_{\rho\sigma}^{>}(k;t)\equiv\frac{1}{V}\int d\vec{x}\int d\vec{y}\, iG_{\rho\sigma}^{>}(\vec{x},t;\vec{y},0)e^{-i\vec{k}(\vec{x}-\vec{y})},$$

$$\rho \text{ (and } \sigma)=e,\mu, \tag{2.20}$$

we define

$$\widetilde{\mathcal{P}}_{ee}^{r}(k;t)\equiv i\,u_e^{\dagger}(kr)G_{ee}^{>}(k;t)\gamma^0 u_e(kr) = \{\alpha_e(kr;t),\alpha_e^{\dagger}(kr;0)\}, \tag{2.21}$$

$$\mathcal{P}_{\bar{e}e}^{r}(k;t)\equiv i\,v_e^{\dagger}(-kr)G_{ee}^{>}(k;t)\gamma^0 u_e(kr) = \{\beta_e^{\dagger}(-kr;t),\alpha_e^{\dagger}(kr;0)\}, \tag{2.22}$$

$$\widetilde{\mathcal{P}}_{\mu e}^{r}(k;t)\equiv i\,u_\mu^{\dagger}(kr)G_{\mu e}^{>}(k;t)\gamma^0 u_e(kr) = \{\alpha_\mu(kr;t),\alpha_e^{\dagger}(kr;0)\}, \tag{2.23}$$

$$\widetilde{\mathcal{P}}_{\bar{\mu}e}^{r}(k;t)\equiv i\,v_\mu^{\dagger}(-kr)G_{\mu e}^{>}(k;t)\gamma^0 u_e(kr) = \{\beta_\mu^{\dagger}(-kr;t),\alpha_e^{\dagger}(kr;0)\}. \tag{2.24}$$

Then the quantities defined by

$$P_{\nu_e\to\nu_e}(k;t)\equiv|\widetilde{\mathcal{P}}_{ee}^{r}(k;t)|^2+|\widetilde{\mathcal{P}}_{\bar{e}e}^{r}(k;t)|^2,$$

$$P_{\nu_e\to\nu_\mu}(k;t)\equiv|\tilde{\mathcal{P}}^r_{\mu e}(k;t)|^2+|\tilde{\mathcal{P}}^r_{\bar{\mu}e}(k;t)|^2 \tag{2.25}$$

are seen to be interpreted as the observable oscillation probabilities in the sense that these quantities satisfy the necessary boundary conditions as

$$P_{\nu_e\to\nu_\rho}(k;t=0)=\delta_{e\rho}, \quad \sum_\rho P_{\nu_e\to\nu_\rho}(k;t)=1, \tag{2.26}$$

and also are shown to be ''μ_λ independent.'' Therefore, the special choice of the mass parameters in Ref. [5], $\mu_e=m_1$ and $\mu_\mu=m_2$, is justified and free from the criticism of Ref. [4].

The resultant formulas of the probabilities are

$$P_{\nu_e\to\nu_e}(k;t)=1-\sin^2(2\theta)\left[\rho_{12}^2(k)\sin^2\left(\frac{\omega_2(k)-\omega_1(k)}{2}t\right)+\lambda_{12}^2(k)\sin^2\left(\frac{\omega_2(k)+\omega_1(k)}{2}t\right)\right],$$

$$P_{\nu_e\to\nu_\mu}(k;t)=1-P_{\nu_e\to\nu_e}(k;t). \tag{2.27}$$

[θ is the mixing angle in the two flavor case; see Eq. (2.40).] In the framework of Ref. [5], the new factors $\rho_{12}^2=1-\lambda_{12}^2$ appearing in the above oscillation formulas are thought to be a result of the unitary inequivalence between $\mathcal{H}_f$ and $\mathcal{H}_m$.

It is pointed out further by BV [5] that the quantities in Eq. (2.27) coincide with the expectation values of the charge operators

$$Q_\sigma(t=0)\equiv\sum_{\vec{k},r}[\alpha_\sigma^\dagger(kr;0)\alpha_\sigma(kr;0)-\beta_\sigma^\dagger(-kr;0)\beta_\sigma(-kr;0)],$$

$$\sigma=e,\mu, \tag{2.28}$$

on the electron-neutrino state at a time t:

$$|\nu_e(kr;t)\rangle\equiv\alpha_e^\dagger(kr;t)|0(t)\rangle_f; \tag{2.29}$$

we have

$$\langle\nu_e(kr;t)|Q_\sigma(0)|\nu_e(kr;t)\rangle=|\{\alpha_\sigma(kr;0),\alpha_e^\dagger(kr;t)\}|^2+|\{\beta_\sigma^\dagger(-kr;0),\alpha_e^\dagger(kr;t)\}|^2=P_{\nu_e\to\nu_\sigma}(k;t), \tag{2.30}$$

$${}_f\langle0(t)|Q_\sigma(0)|0(t)\rangle_f=0,$$

$$\langle\nu_e(kr;t)|[Q_e(0)+Q_\mu(0)]|\nu_e(kr;t)\rangle=1. \tag{2.31}$$

Those consequences of Ref. [5] summarized above are confirmed by straightforward calculations with the use of the concrete form of $W(k;t)$ in the two-flavor case. It seems, however, necessary for us to make clear a simple (or general) reason why the above consequences are obtained. With this aim, we first rewrite the above consequences in the many-flavor case, which is given in the next subsection, and will consider in Sec. III the structure of the retarded propagators of flavor-neutrino fields.

Before entering the next subsection, it may be worthwhile to make a remark on the quantities appearing in Eqs. (2.21)–(2.24). The definitions of $\tilde{\mathcal{P}}^r_{\rho e}(k;t)$ and $\tilde{\mathcal{P}}^r_{\bar{\rho}e}(k;t)$ employed by BV [5] seem to be somewhat misleading. These quantities are introduced only for convenience, and should not be understood as representing transitions such as neutrino-antineutrino transitions to occur in the neutrino oscillation process. It is helpful for us to note that each of $\{\alpha_\rho(kr;t),\alpha_e^\dagger(kr;0)\}$ and $\{\beta_\rho^\dagger(-kr;t),\alpha_e^\dagger(kr;0)\}$ has only a nonvanishing term proportional to $\{\alpha_\rho(kr;0),\alpha_\sigma^\dagger(kr;0)\}$. [Concretely, see Eqs. (2.35).] Further we note that we obtain

$$i\overline{G}^>_{\rho\sigma}(\vec{x},t;\vec{y},0)\equiv{}_f\langle0(0)|\nu_\rho(\vec{x},t)\nu_\sigma(\vec{y},0)|0(0)\rangle_f=0 \tag{2.32}$$

due to $\{\alpha_\rho(qs;t),\beta_\sigma^\dagger(-kr;0)\}=\{\beta_\rho^\dagger(-qs;t),\beta_\sigma^\dagger(-kr;0)\}=0$ obtained from Eq. (1.4) or (2.10).

C. Rewriting BV's formulas in the many-flavor case

In order to study the general structures of BV's results, let us define the quantity, in the many-flavor case,

$$i\mathcal{G}^>_{\rho\sigma}(\vec{x},t;\vec{y},0)\equiv\theta(t)\,{}_f\langle0(0)|\nu_\rho(\vec{x},t)\overline{\nu_\sigma}(\vec{y},0)|0(0)\rangle_f,$$

$$\rho \text{ (and } \sigma)=e,\mu,\tau,\ldots . \tag{2.33}$$

[$i\mathcal{G}^>_{\rho\sigma}(\vec{x},0;\vec{y},0)\equiv\lim_{t\to+0}i\mathcal{G}^>_{\rho\sigma}(\vec{x},t;\vec{y},0)$.] One can extract the component with the momentum $\vec{k}$ from this quantity as

$$i\mathcal{G}^>_{\rho\sigma}(k;t)\equiv\frac{1}{V}\int d\vec{x}\int d\vec{y}\,i\mathcal{G}^>_{\rho\sigma}(\vec{x},t;\vec{y},0)e^{-i\vec{k}(\vec{x}-\vec{y})}$$
$$=\theta(t)_f\langle0(0)|\sum_r[\{\alpha_\rho(kr;t),\alpha_\sigma^\dagger(kr;0)\}\times u_\rho(kr)\overline{u_\sigma}(kr)+\{\beta_\rho^\dagger(-kr;t),\alpha_\sigma^\dagger(kr;0)\}\times v_\rho(-kr)\overline{u_\sigma}(kr)]|0(0)\rangle_f. \tag{2.34}$$

From Eq. (2.10), we obtain

$$\{\alpha_\rho(kr;t),\alpha_\sigma^\dagger(kr;0)\}=\sum_\kappa\{[W^\dagger_{\rho\kappa}(k;t)\alpha_\kappa(kr;0)+W^\dagger_{\rho\bar{\kappa}}(k;t)\beta_\kappa^\dagger(-kr;0)],\alpha_\sigma^\dagger(kr;0)\}=W^\dagger_{\rho\sigma}(k;t)=W_{\sigma\rho}(k;t)^*,$$

$$\{\beta_\rho^\dagger(-kr;t),\alpha_\sigma^\dagger(kr;0)\}=\sum_\kappa\{[W^\dagger_{\bar{\rho}\kappa}(k;t)\alpha_\kappa(kr;0)+W^\dagger_{\bar{\rho}\bar{\kappa}}(k;t)\beta_\kappa^\dagger(-kr;0)],\alpha_\sigma^\dagger(kr;0)\}=W^\dagger_{\bar{\rho}\sigma}(k;t)=W_{\sigma\bar{\rho}}(k;t)^*. \tag{2.35}$$

Thus $\mathcal{G}^>_{\rho\sigma}(k;t)$ is given by

$$i\mathcal{G}^{>}_{\rho\sigma}(k;t)=\theta(t)\sum_r\,[W_{\sigma\rho}(k;t)^*u_\rho(kr)\overline{u}_\sigma(kr)+W_{\sigma\bar{\rho}}(k;t)^*v_\rho(-kr)\overline{u}_\sigma(kr)]. \quad (2.36)$$

With this quantity, we can define, for $t\geqslant 0$,

$$P^{>}_{\nu_\sigma\to\nu_\rho}(k;t)\equiv\frac{1}{2}\mathrm{Tr}[\mathcal{G}^{>}_{\rho\sigma}(k;t)\mathcal{G}^{>\dagger}_{\rho\sigma}(k;t)]=|\{\alpha_\rho(kr;t),\alpha^\dagger_\sigma(kr;0)\}|^2+|\{\beta^\dagger_\rho(-kr;t),\alpha^\dagger_\sigma(kr;0)\}|^2=|W_{\sigma\rho}(k;t)^*|^2+|W_{\sigma\bar{\rho}}(k;t)^*|^2, \quad (2.37)$$

which is equal to Eq. (2.27) for the two-flavor case. (Here, ''Tr'' means to take the trace with respect to the indices of Dirac spinors.) Along the same line as BV's which has been described in the previous subsection, let us call this quantity the probability, since it satisfies automatically the normalization as

$$\sum_\rho P^{>}_{\nu_\sigma\to\nu_\rho}(k;t)=1 \quad (2.38)$$

due to the unitarity of $W(k;t)$ (or $Z^{1/2}$) and the boundary conditions as

$$P^{>}_{\nu_\sigma\to\nu_\rho}(k;t=0)=\delta_{\rho\sigma} \quad (2.39)$$

due to the canonical communication relation at an equal time or the property of $W(k;t=0)=I$.

We cannot see straightforwardly that the right-hand sides (RHS's) of Eq. (2.37) are independent of $\{\mu_\lambda\}$. Such independence in the many-flavor case is shown under the reality condition on $Z^{1/2}=[z^{1/2}_{\rho j}]$. This condition is implicitly used in Ref. [4], where the two-flavor case is examined; in this case,

$$Z^{1/2}=\begin{pmatrix}\cos\theta & \sin\theta\\ -\sin\theta & \cos\theta\end{pmatrix}. \quad (2.40)$$

Under the condition of real $Z^{1/2}$, $P^{>}_{\nu_\sigma\to\nu_\rho}(k;t)$ is also equal to the expectation value of the number operator:

$$\langle N_\sigma;kr;t\rangle_{\rho-f}\equiv{}_f\langle 0(t)|\alpha_\rho(kr;t)N_\sigma(t=0)\alpha^\dagger_\rho(kr;t)|0(t)\rangle_f. \quad (2.41)$$

Hereafter we use the notation $N_\sigma(t)$ instead of $Q_\sigma(t)$.

Detailed proofs of the $\{\mu_\lambda\}$ independence of Eq. (2.37) and the equality of the expectation value of the number operator to $P^{>}_{\nu_\sigma\to\nu_\rho}(k;t)$ are given in Appendixes A and B.

D. Corresponding propagator on $\mathcal{H}_m$

It may be useful to note that the corresponding propagator defined on $\mathcal{H}_m$ does not have the same properties as $i\mathcal{G}^{>}_{\rho\sigma}(\vec{x},t;\vec{y},0)$.

The propagator which is constructed on $\mathcal{H}_m$ corresponding to Eq. (2.33) may be

$$iS^{>}_{\rho\sigma}(\vec{x},x^0;\vec{y},y^0)\equiv\theta(x^0-y^0)_m\langle 0|\nu_\rho(x)\overline{\nu}_\sigma(y)|0\rangle_m=\sum_j z^{1/2}_{\rho j}z^{1/2*}_{\sigma j}\theta(x^0-y^0)\times{}_m\langle 0|(\nu_j(x)\overline{\nu}_j(y))|0\rangle_m, \quad (2.42)$$

which is a part of the Feynman propagator, $iS_{F\rho\sigma}(x-y)={}_m\langle 0|T(\nu_\rho(x)\overline{\nu}_\sigma(y))|0\rangle_m$. By employing the quantity, in the same way as the case of the previous section,

$$iS^{>}_{\rho\sigma}(k;t)\equiv\frac{1}{V}\int d\vec{x}\int d\vec{y}\, iS^{>}_{\rho\sigma}(\vec{x},t;\vec{y},0)e^{-i\vec{k}(\vec{x}-\vec{y})}, \quad (2.43)$$

with $iS^{>}_{\rho\sigma}(k;t=0)\equiv\lim_{t\to+0}iS^{>}_{\rho\sigma}(k;t)$, we obtain, for $t\geqslant 0$,

$$\Pi^{>}_{\nu_\sigma\to\nu_\rho}(k;t)\equiv\frac{1}{2}\mathrm{Tr}|S^{>}_{\rho\sigma}(k;t)S^{>\dagger}_{\rho\sigma}(k;t)|=\sum_{i,j}z^{1/2}_{\rho j}z^{1/2*}_{\sigma j}z^{1/2*}_{\rho i}z^{1/2}_{\sigma i}\rho^2_{ji}e^{i(\omega_i-\omega_j)t}. \quad (2.44)$$

Note that this formula does not include the term proportional to $e^{\pm i(\omega_i+\omega_j)t}$. Although $\Sigma_\rho\Pi^{>}_{\nu_e\to\nu_\rho}(k;t)=1$ is satisfied, $\Pi^{>}_{\nu_e\to\nu_\rho}(k;t)$ does not satisfy the initial conditions as is easily seen in the two-flavor case,[2]

$$\Pi^{>}_{\nu_e\to\nu_e}(k;t)\overset{t\to+0}{\to}1-\frac{1}{2}\sin^2(2\theta)\lambda^2_{12}\neq 1,$$

$$\Pi^{>}_{\nu_e\to\nu_\mu}(k;t)\overset{t\to+0}{\to}\frac{1}{2}\sin^2(2\theta)\lambda^2_{12}\neq 0, \quad (2.45)$$

and then is different from $P^{>}_{\nu_e\to\nu_\rho}(k;t)$ given in Sec. II C.

One may say that this difference means the necessity of the flavor Hilbert space, $\mathcal{H}_f$. We will examine in the next section whether it is true or not.

III. RETARDED KERNEL AND AMPLITUDE

Let us consider two types of the retarded propagators defined on $\mathcal{H}_f$ and $\mathcal{H}_m$, respectively, as

$$i\mathcal{G}^{(ret)}_{\rho\sigma}(\vec{x},t;\vec{y},0)\equiv\theta(t)_f\langle 0(0)|\{\nu_\rho(\vec{x},t),\overline{\nu}_\sigma(\vec{y},0)\}|0(0)\rangle_f, \quad (3.1)$$

[2] Similar discussions can be found in Ref. [6].

$$i\mathcal{S}^{(ret)}_{\rho\sigma}(\vec{x},t;\vec{y},0) \equiv \theta(t)_m\langle 0|\{\nu_\rho(\vec{x},t),\overline{\nu_\sigma}(\vec{y},0)\}|0\rangle_m, \tag{3.2}$$

where these quantities at the time $t=0$ are defined by

$$\lim_{t\to+0} i\mathcal{G}^{(ret)}_{\rho\sigma}(\vec{x},t;\vec{y},0), \quad \lim_{t\to+0} i\mathcal{S}^{(ret)}_{\rho\sigma}(\vec{x},t;\vec{y},0). \tag{3.3}$$

As a result of the c-number property of the anticommutator, the $\vec{k}$ components of these quantities are equal to each other[3] as

$$ig^{(ret)}_{\rho\sigma}(\vec{k},t) = is^{(ret)}_{\rho\sigma}(\vec{k},t); \tag{3.4}$$

therefore, one obtains the oscillation formula, which is independent of $\{\mu_\lambda\}$, as will be seen concretely in the following.

A. Case of $i\mathcal{G}^{(ret)}_{\rho\sigma}(\vec{x},t)$

The $\vec{k}$ components of $i\mathcal{G}^{(ret)}_{\rho\sigma}(\vec{x}-\vec{y},t)$ defined as

$$ig^{(ret)}_{\rho\sigma}(k;t) \equiv \frac{1}{V}\int d\vec{x}\int d\vec{y}\, i\mathcal{G}^{(ret)}_{\rho\sigma}(\vec{x},t;\vec{y},0)e^{-i\vec{k}\vec{x}}e^{i\vec{k}\vec{y}} \tag{3.5}$$

become

$$\begin{aligned} ig^{(ret)}_{\rho\sigma}(k;t) = \theta(t)\sum_r \, [&\{\alpha_\rho(kr;t),\alpha^\dagger_\sigma(kr;0)\}u_\rho(kr)\overline{u}_\sigma(kr) \\ &+\{\beta^\dagger_\rho(-kr;t),\alpha^\dagger_\sigma(kr;0)\}v_\rho(-kr)\overline{u}_\sigma(kr) \\ &+\{\alpha_\rho(kr;t),\beta_\sigma(-kr;0)\}u_\rho(kr)\overline{v}_\sigma(-kr) \\ &+\{\beta^\dagger_\rho(-kr;t),\beta_\sigma(-kr;0)\} \\ &\times v_\rho(-kr)\overline{v}_\sigma(-kr)]. \end{aligned} \tag{3.6}$$

With this quantity, we can define

$$P^{(ret)}_{\nu_\sigma\to\nu_\rho}(k;t) \equiv \frac{1}{4}\mathrm{Tr}[g^{(ret)}_{\rho\sigma}(k;t)g^{(ret)\dagger}_{\rho\sigma}(k;t)], \tag{3.7}$$

and call it the probability on the basis of its properties, similar to Eqs. (2.38) and (2.39), as explained below. For $t\geqslant 0$, we obtain

$$\begin{aligned} P^{(ret)}_{\nu_\sigma\to\nu_\rho}(k;t) = \frac{1}{4}\sum_r \, [&|\{\alpha_\rho(kr;t),\alpha^\dagger_\sigma(kr;0)\}|^2 \\ &+|\{\beta^\dagger_\rho(-kr;t),\alpha^\dagger_\sigma(kr;0)\}|^2 \\ &+|\{\alpha_\rho(kr;t),\beta_\sigma(-kr;0)\}|^2 \\ &+|\{\beta^\dagger_\rho(-kr;t),\beta_\sigma(-kr;0)\}|^2] \\ = \frac{1}{2}[&|W_{\sigma\rho}(k;t)|^2+|W_{\sigma\bar{\rho}}(k;t)|^2+|W_{\bar{\sigma}\rho}(k;t)|^2 \\ &+|W_{\overline{\sigma\rho}}(k;t)|^2], \end{aligned} \tag{3.8}$$

due to Eq. (2.10) [or due to Eqs. (A5) and (A6)]. Clearly we can confirm the following properties of this quantity:

(1) Fundamentally due to the canonical commutation relation (1.5),

$$P^{(ret)}_{\nu_\sigma\to\nu_\rho}(k;t=0) = \delta_{\rho\sigma}; \tag{3.9}$$

(2) due to the unitarity of $Z^{1/2}$ [or $W(k;t)$],

$$\sum_\rho P^{(ret)}_{\nu_\sigma\to\nu_\rho}(k;t) = 1; \tag{3.10}$$

(3) from Eq. (2.15),

$$P^{(ret)}_{\nu_\sigma\to\nu_\rho}(k;t) = P^{(ret)}_{\nu_\rho\to\nu_\sigma}(k;t); \tag{3.11}$$

(4) under the condition of real $Z^{1/2}$ [i.e., due to Eqs. (A8)]

$$P^{(ret)}_{\nu_\sigma\to\nu_\rho}(k;t) = [|W_{\sigma\rho}(k;t)|^2+|W_{\sigma\bar{\rho}}(k;t)|^2] = P^{>}_{\nu_\sigma\to\nu_\rho}(k;t). \tag{3.12}$$

B. Case of $i\mathcal{S}^{(ret)}_{\rho\sigma}(\vec{x},t)$

The $\vec{k}$ components of $i\mathcal{S}^{(ret)}_{\rho\sigma}(\vec{x}-\vec{y},t)$ defined as

$$is^{(ret)}_{\rho\sigma}(k;t) \equiv \frac{1}{V}\int d\vec{x}\int d\vec{y}\, i\mathcal{S}^{(ret)}_{\rho\sigma}(\vec{x},t;\vec{y},0)e^{-i\vec{k}\vec{x}}e^{i\vec{k}\vec{y}} \tag{3.13}$$

become

$$\begin{aligned} is^{(ret)}_{\rho\sigma}(k;t) = \theta(t)\sum_{j,r} \, z^{1/2}_{\rho j}z^{1/2*}_{\sigma j}[&\{\alpha_j(kr;t),\alpha^\dagger_j(kr;0)\} \\ &\times u_j(kr)\overline{u_j}(kr)+\{\beta^\dagger_j(-kr;t),\beta_j(-kr;0)\} \\ &\times v_j(-kr)\overline{v_j}(-kr)]. \end{aligned} \tag{3.14}$$

With this quantity we can define

$$\Pi^{(ret)}_{\nu_\sigma\to\nu_\rho}(k;t) \equiv \frac{1}{4}\mathrm{Tr}[s^{(ret)}_{\rho\sigma}(k;t)s^{(ret)\dagger}_{\rho\sigma}(k;t)], \tag{3.15}$$

and call it the probability due to the same reasoning as before. For $t\geqslant 0$, we obtain

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[3]We obtain not only for $T=0$ but also for an arbitrary time T:

$${}_f\langle 0(T)|\{\nu_\rho(\vec{x},t),\overline{\nu_\sigma}(\vec{y},0)\}|0(T)\rangle_f = {}_m\langle 0|\{\nu_\rho(\vec{x},t),\overline{\nu_\sigma}(\vec{y},0)\}|0\rangle_m,$$

where the vacuum states $|0(T)\rangle_f$ and $|0\rangle_m$ are equally normalized. Thus, the quantity

$$\mathcal{G}^{(ret)}_{\sigma\rho}(\vec{x},t;\vec{y},0;T) \equiv {}_f\langle 0(T)|\{\nu_\rho(\vec{x},t),\overline{\nu_\sigma}(\vec{y},0)\}|0(T)\rangle_f$$

does not depend on T.

$$\Pi^{(ret)}_{\nu_\sigma\to\nu_\rho}(k;t)\equiv\frac{1}{4}\sum_{j,i,r} z^{1/2}_{\rho j}z^{1/2*}_{\sigma j}z^{1/2*}_{\rho i}z^{1/2}_{\sigma i}\,\mathrm{Tr}\{[u_j(kr)u_j^\dagger(kr)e^{-i\omega_j t}+v_j(-kr)v_j^\dagger(-kr)e^{i\omega_j t}]$$
$$\times[u_i(kr)u_i^\dagger(kr)e^{i\omega_i t}+v_i(-kr)v_i^\dagger(-kr)e^{-i\omega_i t}]\}$$
$$=\frac{1}{2}\sum_{j,i} z^{1/2}_{\rho j}z^{1/2*}_{\sigma j}z^{1/2*}_{\rho i}z^{1/2}_{\sigma i}[\rho^2_{ij}(k)(e^{-i(\omega_j-\omega_i)t}+\mathrm{c.c.})+\lambda^2_{ij}(k)(e^{-i(\omega_j+\omega_i)t}+\mathrm{c.c.})]$$
$$=\delta_{\rho\sigma}-2\sum_{j,i} z^{1/2}_{\rho j}z^{1/2*}_{\sigma j}z^{1/2*}_{\rho i}z^{1/2}_{\sigma i}\left[\rho^2_{ij}(k)\sin^2\left(\frac{\omega_j-\omega_i}{2}t\right)+\lambda^2_{ij}(k)\sin^2\left(\frac{\omega_j+\omega_i}{2}t\right)\right]=P^{(ret)}_{\nu_\sigma\to\nu_\rho}(k;t), \quad (3.16)$$

and this is equal to Eq. (2.27) in the two-flavor case. Furthermore, we have the properties of this quantity,

$$(1)\quad \Pi^{(ret)}_{\nu_\sigma\to\nu_\rho}(k;t=0)=\delta_{\rho\sigma}, \quad (3.17)$$

$$(2)\quad \sum_\rho \Pi^{(ret)}_{\nu_\sigma\to\nu_\rho}(k;t)=1, \quad (3.18)$$

$$(3)\quad \Pi^{(ret)}_{\nu_\sigma\to\nu_\rho}(k;t)=\Pi^{(ret)}_{\nu_\rho\to\nu_\sigma}(k;t), \quad (3.19)$$

due to essentially the same reasons mentioned for deriving Eqs. (3.9), (3.10), and (3.11). As expected,

$$(4)\quad \Pi^{(ret)}_{\nu_\sigma\to\nu_\rho}(k;t)\to P^{>}_{\nu_\sigma\to\nu_\rho}(k;t)\ \ \text{for the real}\ \ Z^{1/2},$$

given explicitly by Eq. (A12).

IV. DISCUSSION AND FINAL REMARKS

We have generalized BV's formulas (2.27), $P_{\nu_e\to\nu_\rho}(k;t)$, to $P^{>}_{\nu_\sigma\to\nu_\rho}(k;t)$ of the many-flavor case; as a result of the unitarity of $Z^{1/2}$, these $P^{>}_{\nu_\sigma\to\nu_\rho}(k;t)$ satisfy the boundary conditions which are required for the probability interpretation, but are $\{\mu_\lambda\}$ dependent generally. With those formulas, it has been shown that they are $\{\mu_\lambda\}$ independent and equal to the expectation values of the number operators for the real $Z^{1/2}$, which is of course the case of BV, i.e., the two-flavor case. At the same time, we have shown that the corresponding quantities $\Pi^{>}_{\nu_\sigma\to\nu_\rho}(k;t)$ constructed on $\mathcal{H}_m$ cannot satisfy the boundary conditions.

On the other hand, we could construct the other quantities $P^{(ret)}_{\nu_\sigma\to\nu_\rho}(k;t)$ by employing the anticommutators $\{\nu_\rho(\vec{x},x^0),\nu^\dagger_\sigma(\vec{y},y^0)\}$; these $P^{(ret)}_{\nu_\sigma\to\nu_\rho}(k;t)$ satisfy the boundary conditions stated above, due to the unitarity of $Z^{1/2}$. They are automatically $\{\mu_\lambda\}$ independent because of the $\{\mu_\lambda\}$-independent c-number property of the anticommutators and are equal to the corresponding quantities $\Pi^{(ret)}_{\nu_\sigma\to\nu_\rho}(k;t)$ constructed on $\mathcal{H}_m$.

Those quantities $P^{(ret)}_{\nu_\sigma\to\nu_\rho}(k;t)$ reduce to $P^{>}_{\nu_\sigma\to\nu_\rho}(k;t)$ for the real $Z^{1/2}$ and to BV's formulas $P_{\nu_e\to\nu_\rho}(k;t)$ for the two-flavor case. Then we conclude that the $\{\mu_\lambda\}$ independence of the "oscillation formulas" asserted by BV [5] is essentially based on the ⟨Setup⟩, and that the new factors in the BV's formulas, which are different from the usual oscillation formulas, are not the trace of the flavor Hilbert space construction, but come from the field theoretical treatment of mixing fields using Bogolyubov transformation. In the present case, the coefficients of the new factors are fixed by the spin-$\frac{1}{2}$ property of neutrino.

The interrelationship among the relevant quantities is summarized in Fig. 1.

Let us make some remarks as follows. Concerning the construction of $\mathcal{H}_f$, we remark in Appendix C that we cannot necessarily eliminate the possibility to construct $\mathcal{H}_f$ within the extent of the paper by Giunti *et al.* [2]. According to the context of the present paper, the construction of $\mathcal{H}_f$ is not always excluded, since there are some quantities, such as $\langle 0(0)|\{\nu_\rho(x),\overline{\nu_\sigma}(y)\}|0(0)\rangle$, which are obtained on the basis of $\mathcal{H}_m$ and equal to ones obtained on the basis of $\mathcal{H}_f$. For the quantities constructed on $\mathcal{H}_f$, $\{\mu_\lambda\}$ independence seems to suggest that there is no difference of those quantities from the ones constructed on $\mathcal{H}_m$. It is a future task to make clearer the field theoretical basis of such an anticipation.

In the present paper, we examined only the contributions from the propagator by extracting a part of neutrino propagation from the full transition amplitudes corresponding to various neutrino experiments. The relation between the oscillation probability discussed above and the full transition probability is not clear yet, and we cannot decide which propagator one should use to calculate the oscillation probability. It may be a meaningful fact that $\Pi^{(ret)}_{\nu_\sigma\to\nu_\rho}(k;t)$ satisfies the boundary conditions while $\Pi^{>}_{\nu_\sigma\to\nu_\rho}(k;t)$ does not.

We give a comment here on the relationship between our Eq. (3.16) and the usual oscillation probability [9]. We get Eq. (2.44) [Eq. (2.37)] by dropping the contribution of $\overline{\nu_\sigma}\nu_\rho$ from Eq. (3.2) [Eq. (3.1)]. By setting $\rho_{ij}=1$ obtained in the extremely relativistic limit and replacing the time t with the distance traveled by the neutrino, Eq. (2.44) [Eq. (2.37) or (A11)] goes to the usual oscillation probability. Further, there is an interesting difference between our framework and the usual oscillation probability concerning CP violation. The equalities (3.11) and (3.19) are obtained irrespective of T invariance. This means that the difference between $\mathrm{Prob}(\nu_\rho\to\nu_\sigma)$ and $\mathrm{Prob}(\nu_\sigma\to\nu_\rho)$ as a CP-violation effect cannot be observed in the neutrino oscillation in the present framework. Thus, if this difference is confirmed in the ex-

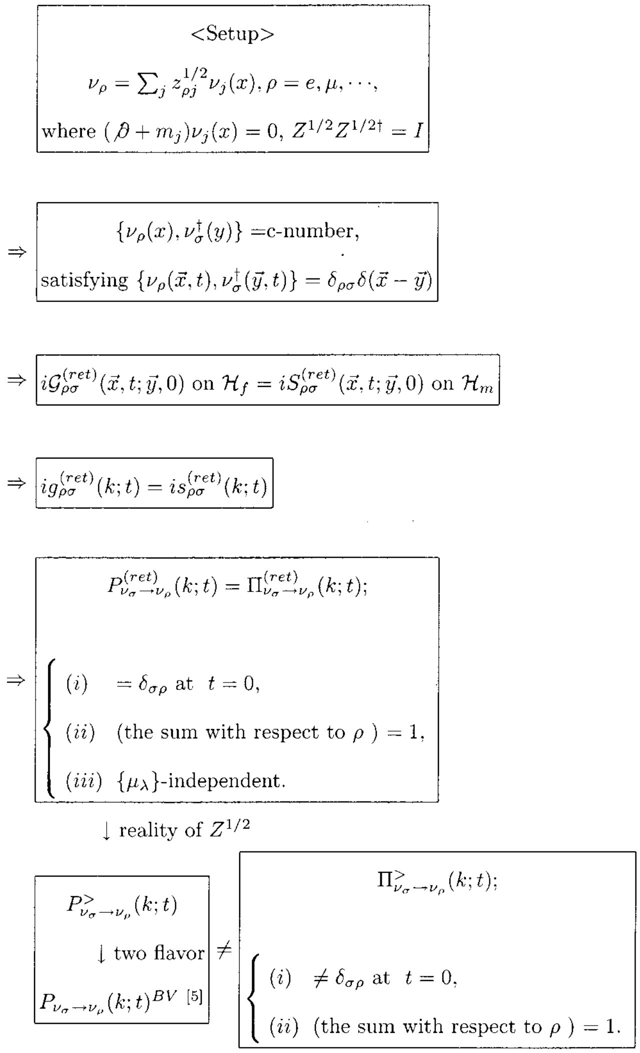

FIG. 1. Without the reality of $Z^{1/2}$, $P^{>}_{\nu_\sigma\to\nu_\rho}(k;t) \neq \langle N_\sigma;kr;t\rangle_{\rho-f}$, and both $P^{>}_{\nu_\sigma\to\nu_\rho}(k;t)$ and $\langle N_\sigma;kr;t\rangle_{\rho-f}$ are $\{\mu_\lambda\}$ dependent for $N_f \geqslant 3$.

periments to be nonzero, the present approach, in which only the neutrino propagation part is taken into consideration, becomes excluded.

Here it seems worthy to remark on the treatment of low-energy weak processes with accompanying neutrinos, such as $\pi^+ \to l_\rho^+ \nu_\rho$, $\mu^+ \to e^+ \nu_e \overline{\nu}_\mu$, and $n \to p e \nu_e$. In the lowest-order calculation with respect to the weak interaction, each participating flavor neutrino has been treated as an asymptotic field with a definite mass nearly equal to zero, and we have obtained important information on the structure of the weak interactions. As an illustration, we examine the decay probabilities of $\pi^+ \to l_\rho^+ \nu_\rho$, $\rho = \mu, e$. By employing the usual $V-A$ weak interaction, we have the decay probabilities for $\pi^+ \to l_\rho^+ \nu_\rho$:

$$P(\pi^+ \to l_\rho^+ \nu_\rho) = \frac{f_\pi^2 G_\beta^2}{8\pi} m_\rho^2 \left(1 + \frac{\mu_\rho^2}{m_\rho^2} - \frac{(m_\rho^2 - \mu_\rho^2)^2}{m_\pi^2 m_\rho^2}\right) m_\pi \times \left[\left(1 - \frac{(m_\rho + \mu_\rho)^2}{m_\pi^2}\right) \times \left(1 - \frac{(m_\rho - \mu_\rho)^2}{m_\pi^2}\right)\right]^{1/2}; \tag{4.1}$$

here, f_π is the pion decay constant defined by $\langle 0|A_\alpha(0)|\pi^+(p)\rangle = i f_\pi p_\alpha$, $A_\alpha(x) =$ the weak axial vector current such as $i\overline{d}(x)\gamma_\alpha\gamma_5 u(x)$, $(f_\pi)_{exp} \simeq 131$ MeV; $G_\beta \simeq (10^{-5}/M_p^2)$ is the weak Fermi coupling constant (including the Cabbibo-Kobayashi-Maskawa angle); m_ρ and μ_ρ are the masses of l_ρ and ν_ρ, respectively. For $\mu_\rho = 0$, Eq. (4.1) reduces to

$$P(\pi^+ \to l_\rho^+ \nu_\rho) = \frac{f_\pi^2 G_\beta^2}{8\pi} m_\rho^2 m_\pi \left(1 - \frac{m_\rho^2}{m_\pi^2}\right)^2; \tag{4.2}$$

then, we obtain numerical values which are in good agreement with the experimental ones.

The calculation above is based on the existence of asymptotic flavor neutrino fields with definite masses. But we cannot regard the flavor neutrino fields $\nu_\rho(x)$'s to define the asymptotic fields, leading to time-independent creation and annihilation operators with definite four-momenta, since in accordance with $\langle$Setup$\rangle$ with mass differences among m_j's, each $\alpha_\rho(kr;t)$ and $\beta_\rho^\dagger(-kr;t)$ in the expansion of $\nu_\rho(x)$ does not have a simple time dependence, since $\alpha_\rho(kr;t) = \Sigma_j\{\mathcal{K}(k)_{\rho j}\alpha_j(kr;t) + \mathcal{K}(k)_{\rho\bar{j}}\beta_j^\dagger(-kr;t)\}$. Contrarily, each of $\alpha_j(kr;t)$ and $\beta_j^\dagger(-kr;t)$ in the expansion of $\nu_j(x)$ has a simple time dependence as $\exp(\mp i\omega_j(k)t)$ with $\omega_j(k) = (\vec{k}^2 + m_j^2)^{1/2}$; thus we can construct the Hilbert space $\mathcal{H}_m$, independently of the time, by employing the operators $\alpha_j^\dagger(kr) \equiv \alpha_j^\dagger(kr;t)e^{-i\omega_j(k)t}$ and $\beta_j^\dagger(-kr) \equiv \beta_j^\dagger(-kr;t)e^{-i\omega_j(k)t}$. The probabilities $P(\pi^+ \to l_\rho^+ \nu_\rho)$ are given by

$$P(\pi^+ \to l_\rho^+ \nu_\rho) = \frac{f_\pi^2 G_\beta^2}{8\pi} m_\rho^2 m_\pi {\sum_j}' |z_{\rho j}^{1/2}|^2 \left(1 + \frac{m_j^2}{m_\rho^2} - \frac{(m_\rho^2 - m_j^2)^2}{m_\pi^2 m_\rho^2}\right) \times \left\{\left(1 - \frac{(m_\rho + m_j)^2}{m_\pi^2}\right)\left(1 - \frac{(m_\rho - m_j)^2}{m_\pi^2}\right)\right\}^{1/2}, \tag{4.3}$$

where the summation Σ_j' is performed over j's which are allowed under four-momentum conservation. We see that Eq. (4.3) reduces to Eq. (4.2), when (1) the masses of the relevant neutrinos are negligibly small, and even if a neutrino has a non-negligible mass, the corresponding mixing angle $|z_{\rho j}^{1/2}|$ is very small, and (2) $\Sigma_j' |z_{\rho j}^{1/2}|^2 = 1$ holds almost precisely.

Though Eqs. (4.1) and (4.3) give the same formula (4.2) in the limit of massless neutrinos, one should adopt the well-defined formula (4.3). It is necessary for us to examine whether all experimental data are consistent or not with those conditions as noted above.

The oscillation formulas usually employed are possibly modified due to various reasons, e.g., due to certain effects coming from a more detailed quantum-mechanical or field theoretical description [1] reflecting real experimental situations,[4] or due to the mixing of the known left-handed neutrinos with some right-handed neutrinos which may propagate in the bulk space including some extra dimensions [11]. Anyhow, it may be necessary to investigate the neutrino oscillation by applying the field theory in accordance with respective experimental situations.

ACKNOWLEDGMENTS

The authors would like to express their thanks to Professor K. Ishikawa and the other members of the particle theory group in Hokkaido University for their helpful discussions. Thanks are also due to Professor G. Vitiello and Dr. M. Blasone, Salerno University, for their communications and remarks.

APPENDIX A: μ_λ INDEPENDENCE OF $P^{>}_{\nu_\sigma\to\nu_\rho}$ UNDER THE *CP*-INVARIANCE CONDITION

As an example to see the μ_λ independence of Eq. (2.37), we can derive the equality

$$\begin{aligned}&|\{\alpha_\rho(kr;t),\alpha_\sigma^\dagger(kr;0)\}|^2+|\{\beta_\rho^\dagger(kr;t),\alpha_\sigma^\dagger(kr;0)\}|^2\\&\quad=|\{\alpha_\rho^{BV}(kr;t),\alpha_\sigma^{BV\dagger}(kr;0)\}|^2\\&\qquad+|\{\beta_\rho^{BV}(kr;t),\alpha_\sigma^{BV\dagger}(kr;0)\}|^2\end{aligned}\tag{A1}$$

under the reality condition of Z. Here α_ρ^{BV} and β_ρ^{BV} are the operators employed in Ref. [4],

$$\begin{pmatrix}\alpha_F(kr;t)\\ \beta_F^\dagger(-kr;t)\end{pmatrix}=\begin{pmatrix}\rho_F(k) & i\lambda_F(k)\\ i\lambda_F(k) & \rho_F(k)\end{pmatrix}\begin{pmatrix}\alpha_F^{BV}(kr;t)\\ \beta_F^{BV\dagger}(-kr;t)\end{pmatrix},\tag{A2}$$

where $\rho_F(k)$ and $\lambda_F(k)$ are $N_f\times N_f$ diagonal matrices:

$$\rho_F(k)=\begin{pmatrix}\rho_{e1}(k) & & 0\\ & \rho_{\mu 2}(k) & \\ 0 & & \ddots\end{pmatrix},$$

$$\lambda_F(k)=\begin{pmatrix}\lambda_{e1}(k) & & 0\\ & \lambda_{\mu 2}(k) & \\ 0 & & \ddots\end{pmatrix}.\tag{A3}$$

For simplicity, we use the shortcut notations as

$$\begin{aligned}&\alpha_\rho(kr;t)\to\alpha_\rho(t),\\&\beta_\rho(kr;t)\to\beta_\rho(t).\end{aligned}\tag{A4}$$

Then,

$$\begin{aligned}\{\alpha_\rho(t),\alpha_\sigma^\dagger(0)\}&=\Big\{\sum_j[z_{\rho j}^{1/2}\rho_{\rho j}\alpha_j(t)\\&\quad+iz_{\rho j}^{1/2}\lambda_{\rho j}\beta_j^\dagger(t)],\sum_l[z_{\sigma l}^{1/2*}\rho_{\sigma l}\alpha_l^\dagger(0)\\&\quad-iz_{\sigma l}^{1/2*}\lambda_{\sigma l}\beta_l(0)]\Big\}\\&=\sum_j z_{\rho j}^{1/2}z_{\sigma j}^{1/2*}(\rho_{\rho j}\rho_{\sigma j}e^{-i\omega_j t}+\lambda_{\rho j}\lambda_{\sigma j}e^{i\omega_j t})\\&=W_{\sigma\rho}(k;t)^*\end{aligned}\tag{A5}$$

and

$$\begin{aligned}\{\beta_\rho^\dagger(t),\beta_\sigma(0)\}&=\sum_j z_{\rho j}^{1/2}z_{\sigma j}^{1/2*}(\lambda_{\rho j}\lambda_{\sigma j}e^{-i\omega_j t}+\rho_{\rho j}\rho_{\sigma j}e^{i\omega_j t})\\&=W_{\overline{\sigma\rho}}(k;t)^*,\\ \{\beta_\rho^\dagger(t),\alpha_\sigma^\dagger(0)\}&=\sum_j z_{\rho j}^{1/2}z_{\sigma j}^{1/2*}i(\lambda_{\rho j}\rho_{\sigma j}e^{-i\omega_j t}-\rho_{\rho j}\lambda_{\sigma j}e^{i\omega_j t})\\&=W_{\sigma\bar\rho}(k;t)^*,\end{aligned}\tag{A6}$$

$$\begin{aligned}\{\alpha_\rho(t),\beta_\sigma(0)\}&=\sum_j z_{\rho j}^{1/2}z_{\sigma j}^{1/2*}i(-\rho_{\rho j}\lambda_{\sigma j}e^{-i\omega_j t}\\&\quad+\lambda_{\rho j}\rho_{\sigma j}e^{i\omega_j t})=W_{\bar\sigma\rho}(k;t)^*.\end{aligned}$$

For the real $Z^{1/2}$, there are some relations among these anti-commutators as

$$\begin{aligned}&\{\beta_\rho^\dagger(t),\beta_\sigma(0)\}=\{\alpha_\rho^\dagger(t),\alpha_\sigma(0)\},\\&\{\alpha_\rho(t),\beta_\sigma(0)\}=-\{\beta_\rho(t),\alpha_\sigma(0)\},\end{aligned}\tag{A7}$$

i.e.,

$$W_{\overline{\sigma\rho}}(k;t)^*=W_{\sigma\rho}(k;t),\qquad W_{\bar\sigma\rho}(k;t)^*=-W_{\sigma\bar\rho}(k;t).\tag{A8}$$

[These relations (A8) are also confirmed directly from Eqs. (2.11) by noting $P^\dagger=(z_{\rho j}^{1/2}\rho_{\rho j})^\dagger=P^T$ and $\Lambda^\dagger=(z_{\rho j}^{1/2}\lambda_{\rho j})^\dagger=\Lambda^T$ for real $Z^{1/2}$.] By taking $(e,1)$, $(\mu,2),(\tau,3),\dots$ for (ρ,j) and (σ,l),

[4]Recently, along this line, a possible approach has been done by one of the present authors (T.Y.) and Ishikawa [10].

$$\text{LHS of Eq. (A1)} = |\{\rho_{\rho j}\alpha_\rho^{BV}(t)+i\lambda_{\rho j}\beta_\rho^{BV\dagger}(t),\rho_{\sigma l}\alpha_\sigma^{BV\dagger}(0)-i\lambda_{\sigma l}\beta_\sigma^{BV}(0)\}|^2$$
$$+|\{i\lambda_{\rho j}\alpha_\rho^{BV}(t)+\rho_{\rho j}\beta_\rho^{BV\dagger}(t),\rho_{\sigma l}\alpha_\sigma^{BV\dagger}(0)-i\lambda_{\sigma l}\beta_\sigma^{BV}(0)\}|^2$$
$$=\rho_{\sigma l}^2|\{\alpha_\rho^{BV}(t),\alpha_\sigma^{BV\dagger}(0)\}|^2+\lambda_{\sigma l}^2|\{\beta_\rho^{BV\dagger}(t),\beta_\sigma^{BV}(0)\}|^2+\lambda_{\sigma l}^2|\{\alpha_\rho^{BV}(t),\beta_\sigma^{BV}(0)\}|^2$$
$$+\rho_{\sigma l}^2|\{\beta_\rho^{BV\dagger}(t),\alpha_\sigma^{BV\dagger}(0)\}|^2+i\rho_{\sigma l}\lambda_{\sigma l}\{\alpha_\rho^{BV}(t),\alpha_\sigma^{BV\dagger}(0)\}\{\alpha_\rho^{BV\dagger}(t),\beta_\sigma^{BV\dagger}(0)\}+\text{H.c.}$$
$$+i\rho_{\sigma l}\lambda_{\sigma l}\{\beta_\rho^{BV\dagger}(t),\alpha_\sigma^{BV\dagger}(0)\}\{\beta_\rho^{BV}(t),\beta_\sigma^{BV\dagger}(0)\}+\text{H.c.} \tag{A9}$$

Employing Eqs. (A7),

$$\text{(A9)}=|\{\alpha_\rho^{BV}(kr;t),\alpha_\sigma^{BV\dagger}(kr;0)\}|^2+|\{\beta_\rho^{BV\dagger}(kr;t),\alpha_\sigma^{BV\dagger}(kr;0)\}|^2, \tag{A10}$$

leading to Eq. (A1). It is needless to note that $P^{(ret)}_{\nu_\sigma\to\nu_\rho}(k;t)$ given by Eq. (3.8) is shown to be equal to that calculated for the special choice of μ_λ's as above without requiring the reality of $Z^{1/2}$.

On the other hand, we obtain

$$\text{the last side of Eq. (2.37)}=\sum_{i,j}[\mathcal{K}_{\sigma j}\phi_j\mathcal{K}^*_{\rho j}+\mathcal{K}_{\sigma\bar{j}}\phi_j^*\mathcal{K}^*_{\rho\bar{j}}][\mathcal{K}_{\rho i}\phi_i^*\mathcal{K}^*_{\sigma i}+\mathcal{K}_{\rho\bar{i}}\phi_i\mathcal{K}^*_{\sigma\bar{i}}]+\sum_{i,j}[\mathcal{K}_{\sigma j}\phi_j\mathcal{K}^*_{\rho\bar{j}}+\mathcal{K}_{\sigma\bar{j}}\phi_j^*\mathcal{K}^*_{\rho\bar{j}}][\mathcal{K}_{\rho\bar{i}}\phi_i^*\mathcal{K}^*_{\sigma i}$$
$$+\mathcal{K}_{\rho\bar{i}}\phi_i\mathcal{K}^*_{\sigma\bar{i}}]$$
$$=\sum_{i,j}z_{\sigma j}^{1/2}z_{\rho j}^{1/2*}z_{\rho i}^{1/2}z_{\sigma i}^{1/2*}[\rho_{\sigma j}\rho_{\sigma i}\rho_{ji}\phi_j\phi_i^*+\lambda_{\sigma j}\lambda_{\sigma i}\rho_{ji}\phi_j^*\phi_i+\rho_{\sigma j}\lambda_{\sigma i}\lambda_{ji}\phi_j\phi_i-\lambda_{\sigma j}\rho_{\sigma i}\lambda_{ji}\phi_j^*\phi_i^*]. \tag{A11}$$

For the case of real $Z^{1/2}$,

$$\text{Eq. (A11)}=\sum_{i,j}z_{\sigma j}^{1/2}z_{\rho j}^{1/2}z_{\rho i}^{1/2}z_{\sigma i}^{1/2}[\rho_{ji}^2(\phi_j\phi_i^*+\phi_j^*\phi_i)+\lambda_{ji}^2(\phi_j\phi_i+\phi_j^*\phi_i^*)]/2$$
$$=\delta_{\sigma\rho}-2\sum_{i,j}z_{\sigma j}^{1/2}z_{\rho j}^{1/2}z_{\rho i}^{1/2}z_{\sigma i}^{1/2}\left[\rho_{ji}^2\sin^2\left(\frac{\Delta\omega_{ji}}{2}t\right)+\lambda_{ji}^2\sin^2\left(\frac{\omega_j+\omega_i}{2}t\right)\right], \tag{A12}$$

and then we get a μ_λ-independent quantity explicitly.

APPENDIX B: NUMBER OPERATOR

Let us define the expectation values of the number operator of neutrinos,

$$\langle N_\sigma;kr;t\rangle_{\rho-f}\equiv{}_f\langle 0(t)|\alpha_\rho(kr;t)N_\sigma(t=0)\alpha_\rho^\dagger(kr;t)|0(t)\rangle_f, \tag{B1}$$

where

$$N_\sigma(t)\equiv\sum_{\vec{q},s}[n_\sigma(qs;t)-\bar{n}_\sigma(-qs;t)],$$
$$n_\sigma(qs;t)\equiv\alpha_\sigma^\dagger(qs;t)\alpha_\sigma(qs;t),$$
$$\bar{n}_\sigma(-qs;t)\equiv\beta_\sigma^\dagger(-qs;t)\beta_\sigma(-qs;t), \tag{B2}$$
$$\langle n_\sigma(qs;0);kr;t\rangle_{\rho-f}\equiv{}_f\langle 0(t)|\alpha_\rho(kr;t)n_\sigma(qs;0)\alpha_\rho^\dagger(kr;t)|0(t)\rangle_f,$$
$$\langle\bar{n}_\sigma(-qs;0);kr;t\rangle_{\rho-f}\equiv{}_f\langle 0(t)|\alpha_\rho(kr;t)\bar{n}_\sigma(-qs;0)\alpha_\rho^\dagger(kr;t)|0(t)\rangle_f.$$

From Eq. (2.10),

$$n_\sigma(qs;0)=\sum_{\lambda,\kappa}\{W_{\sigma\lambda}(q;t)^*\alpha_\lambda^\dagger(qs;t)+W_{\sigma\bar{\lambda}}(q;t)^*\beta_\lambda(-qs;t)\}\{W_{\sigma\kappa}(q;t)\alpha_\kappa(qs;t)+W_{\sigma\bar{\kappa}}(q;t)\beta_\kappa^\dagger(-qs;t)\},$$

$$\langle n_\sigma(qs;0);kr;t\rangle_{\rho-f}=|W_{\sigma\rho}(k;t)|^2\delta_{rs}\delta(\vec{k},\vec{q})+\sum_{\lambda}|W_{\sigma\bar{\lambda}}(q;t)|^2, \tag{B3}$$

$$\overline{n}_\sigma(-qs;0)=\sum_{\lambda,\kappa}\{W_{\bar{\sigma}\lambda}(q;t)\alpha_\lambda(qs;t)+W_{\overline{\sigma\lambda}}(q;t)\beta_\lambda^\dagger(-qs;t)\}\{W_{\bar{\sigma}\kappa}(q;t)^*\alpha_\kappa^\dagger(qs;t)+W_{\overline{\sigma\kappa}}(q;t)^*\beta_\kappa(-qs;t)\},$$

$$\langle \overline{n}_\sigma(-qs;0);kr;t\rangle_{\rho-f}=\sum_{\lambda}|W_{\bar{\sigma}\lambda}(q;t)|^2-|W_{\bar{\sigma}\rho}(k;t)|^2\delta_{rs}\delta(\vec{k},\vec{q}). \tag{B4}$$

We have

$$\langle N_\sigma;kr;t\rangle_{\rho-f}=|W_{\sigma\rho}(k;t)|^2+|W_{\bar{\sigma}\rho}(k;t)|^2+2\sum_{\vec{q},\lambda}[|W_{\sigma\bar{\lambda}}(q;t)|^2-|W_{\bar{\sigma}\lambda}(q;t)|^2]; \tag{B5}$$

then, for $t\geqslant 0$,

$$\langle N_\sigma;kr;t\rangle_{\rho-f}=P^{>}_{\nu_\sigma\to\nu_\rho}(k;t)+|W_{\bar{\sigma}\rho}(k;t)|^2-|W_{\sigma\bar{\rho}}(k;t)|^2+2\sum_{\vec{q},\lambda}[|W_{\sigma\bar{\lambda}}(q;t)|^2-|W_{\bar{\sigma}\lambda}(q;t)|^2], \tag{B6}$$

from which we obtain, under the reality condition of $Z^{1/2}$ [or Eq. (A8)],

$$\langle N_\sigma;kr;t\rangle_{\rho-f}=P^{>}_{\nu_\sigma\to\nu_\rho}(k;t), \tag{B7}$$

$$\sum_\sigma \langle N_\sigma;kr;t\rangle_{\rho-f}=1. \tag{B8}$$

Thus we see that, in order to derive the oscillation formulas for N_f flavors as the generalization of those given by BV [5], we have to assume the reality of $Z^{1/2}$, supported by T (or CP) invariance [8].

APPENDIX C: $\mathcal{H}_f$ CONSTRUCTION

We make a remark on the construction of the flavor Hilbert space $\mathcal{H}_f$ in connection with the work of Giunti *et al.* [2]. In Ref. [2], the Majorana neutrino field is considered for simplicity, but we consider the Dirac one in accordance with the representations as explained in Sec. II.

First of all, we consider the expansion of the flavor-neutrino field $\nu_\rho(x)$ as Eq. (1.1):

$$\nu_\rho(x)=\sum_{j=1}^{N_f} z_{\rho j}^{1/2}\nu_j(x)$$
$$=\frac{1}{\sqrt{V}}\sum_{j=1}^{N_f} z_{\rho j}^{1/2}\sum_{\vec{k}r} e^{i\vec{k}\cdot\vec{x}}\{u_j(kr)\alpha_j(kr;t)+v_j(-kr)\beta_j^\dagger(-kr;t)\}, \tag{C1}$$

where $\rho=e,\mu,\tau,\ldots$. With the two-component spinors of spin eigenstates $w(k\uparrow)$, the momentum-helicity eigensolutions are given as

$$u_j(k\uparrow)=\begin{pmatrix}c_j\\ s_j\end{pmatrix}\otimes w(k\uparrow),\quad u_j(k\downarrow)=\begin{pmatrix}s_j\\ c_j\end{pmatrix}\otimes w(k\downarrow),$$
$$v_j(k\uparrow)=\begin{pmatrix}-s_j\\ c_j\end{pmatrix}\otimes w(k\downarrow),\quad v_j(k\downarrow)=\begin{pmatrix}c_j\\ -s_j\end{pmatrix}\otimes w(k\uparrow), \tag{C2}$$

where

$$w(k\uparrow)\equiv\begin{pmatrix}a(\vec{k})\\ b(\vec{k})\end{pmatrix},\quad w(k\downarrow)\equiv\begin{pmatrix}-b^*(\vec{k})\\ a^*(\vec{k})\end{pmatrix}, \tag{C3}$$

$$\frac{\vec{\sigma}\vec{k}}{|\vec{k}|}w(k,r)=\begin{Bmatrix}+1\\ -1\end{Bmatrix}w(k,r)\quad \text{for}=\begin{Bmatrix}\uparrow\\ \downarrow\end{Bmatrix}; \tag{C4}$$

$a(\vec{k})=\cos(\vartheta/2)e^{-i\varphi/2}$, $b(\vec{k})=\sin(\vartheta/2)e^{i\varphi/2}$ for $k_z=k\cos(\vartheta)$, $k_x+ik_y=k\sin(\vartheta)e^{i\varphi}$; $c_j=\cos(\chi_j/2)$, $s_j=\sin(\chi_j/2)$ with $\cot(\chi_j)=|\vec{k}|/m_j$. By noting $a(-\vec{k})=-ib^*(\vec{k})$ and $b(-\vec{k})=ia^*(\vec{k})$, we have

$$v_j(-k\uparrow)=\begin{pmatrix}-s_j\\ c_j\end{pmatrix}\otimes iw(k\uparrow),$$
$$v_j(-k\downarrow)=\begin{pmatrix}c_j\\ -s_j\end{pmatrix}\otimes iw(k\downarrow). \tag{C5}$$

Following the notation employed in Ref. [2], c_j and s_j are written as κ_{j+} and κ_{j-}, respectively;

$$\kappa_{j\pm}=\sqrt{\frac{\omega_j(k)\pm|\vec{k}|}{2\omega_j(k)}}. \tag{C6}$$

Thus Eq. (C1) is expressed as

$$\nu_\rho(x)=\frac{1}{\sqrt{V}}\sum_j z_{\rho j}^{1/2}\sum_{\vec{k}} e^{i\vec{k}\cdot\vec{x}}\Bigg[\alpha_j(k\uparrow;t)\begin{pmatrix}\kappa_{j+}\\ \kappa_{j-}\end{pmatrix}\otimes w(k\uparrow)+\alpha_j(k\downarrow;t)\begin{pmatrix}\kappa_{j-}\\ \kappa_{j+}\end{pmatrix}\otimes w(k\downarrow)+i\beta_j^\dagger(-k\uparrow;t)\begin{pmatrix}-\kappa_{j-}\\ \kappa_{j+}\end{pmatrix}\otimes w(k\uparrow)+i\beta_j^\dagger(-k\downarrow;t)\begin{pmatrix}\kappa_{j+}\\ -\kappa_{j-}\end{pmatrix}\otimes w(k\downarrow)\Bigg]. \tag{C7}$$

By defining further

$$A_{\rho\pm}(kr;t)\equiv\sum_j z_{\rho j}^{1/2}\alpha_j(kr;t)\kappa_{j\pm},$$

$$B_{\rho\pm}^\dagger(-kr;t)\equiv\pm i\sum_j z_{\rho j}^{1/2}\beta_j^\dagger(-kr;t)\kappa_{j\pm}, \tag{C8}$$

Eq. (C7) is rewritten as

$$\nu_\rho(x)=\frac{1}{\sqrt{V}}\sum_{\vec{k}} e^{i\vec{k}\cdot\vec{x}}\Bigg[\begin{pmatrix}A_{\rho+}(k\uparrow;t)\\ A_{\rho-}(k\uparrow;t)\end{pmatrix}\otimes w(k\uparrow)+\begin{pmatrix}A_{\rho-}(k\downarrow;t)\\ A_{\rho+}(k\downarrow;t)\end{pmatrix}\otimes w(k\downarrow)+\begin{pmatrix}B_{\rho-}^\dagger(-k\uparrow;t)\\ B_{\rho+}^\dagger(-k\uparrow;t)\end{pmatrix}\otimes w(k\uparrow)+\begin{pmatrix}B_{\rho+}^\dagger(-k\downarrow;t)\\ B_{\rho-}^\dagger(-k\downarrow;t)\end{pmatrix}\otimes w(k\downarrow)\Bigg]. \tag{C9}$$

From the canonical commutation relations at equal time among α_j, $\alpha_j^\dagger$, β_j, and $\beta_j^\dagger$, we obtain

$$\{A_{\sigma\pm}(kr;t),A_{\rho\pm}^\dagger(qs;t)\}=\{B_{\rho\pm}(kr;t),B_{\sigma\pm}^\dagger(qs;t)\}=\sum_j z_{\sigma j}^{1/2}z_{\rho j}^{1/2*}\frac{\omega_j(k)\pm|\vec{k}|}{2\omega_j(k)}\delta_{rs}\delta(\vec{k},\vec{q}),$$

$$\{A_{\sigma\pm}(kr;t),A_{\rho\mp}^\dagger(qs;t)\}=\{B_{\rho\pm}(kr;t),B_{\sigma\mp}^\dagger(qs;t)\}=\sum_j z_{\sigma j}^{1/2}z_{\rho j}^{1/2*}\frac{|m_j|}{2\omega_j(k)}\delta_{rs}\delta(\vec{k},\vec{q}), \tag{C10}$$

$$\text{others}=0,$$

but these are not the canonical commutation relations at equal time among $A_{\sigma\pm}$, $A_{\sigma\pm}^\dagger$, $B_{\sigma\pm}$, and $B_{\sigma\pm}^\dagger$, and we cannot construct the flavor Hilbert space $\mathcal{H}_f$ with these operators.

In the extremely relativistic limit, Eq. (C10) reduces to

$$\{A_{\sigma+}(kr;t),A_{\rho+}^\dagger(qs;t)\}\to\delta_{\sigma\rho}\delta_{rs}\delta(\vec{k},\vec{q}),$$

$$\{B_{\sigma+}(kr;t),B_{\rho+}^\dagger(qs;t)\}\to\delta_{\sigma\rho}\delta_{rs}\delta(\vec{k},\vec{q}), \tag{C11}$$

$$\text{others}=0.$$

Then the high-momentum part of the RHS of Eq. (C9) becomes

$$\nu_\rho(x)\to\frac{1}{\sqrt{V}}\sum_{\vec{k}} e^{i\vec{k}\cdot\vec{x}}\Bigg\{\begin{pmatrix}A_{\rho+}(k\uparrow;t)\\ B_{\rho+}^\dagger(-k\uparrow;t)\end{pmatrix}\otimes w(k\uparrow)+\begin{pmatrix}B_{\rho+}^\dagger(-k\downarrow;t)\\ A_{\rho+}(k\downarrow;t)\end{pmatrix}\otimes w(k\downarrow)\Bigg\}, \tag{C12}$$

with

$$A_{\rho+}(kr;t)=\sum_j z_{\rho j}^{1/2}\alpha_j(kr;t),$$

$$B_{\rho+}^\dagger(-kr;t)=\sum_j z_{\rho j}^{1/2}\beta_j^\dagger(-kr;t). \tag{C13}$$

If we follow the assertion in Ref. [2], we can construct the Hilbert space $\mathcal{H}_f$ only in the extremely relativistic limit. This assertion, however, is self-evident, since in this limit the mass differences among neutrinos do not play any physical role. Further, the transformation (C8) is not canonical, and the number of operators $(A_{\rho\pm}(kr;t),B_{\rho\pm}^\dagger(-kr;t))$ is twice larger than that of $(\alpha_j(kr;t),\beta_j^\dagger(-kr;t))$. Thus, the assertion against the $\mathcal{H}_f$ construction seems not appropriate.

Here we show an example which has the canonical commutation relations and the possibility of constructing the flavor Hilbert space $\mathcal{H}_f$.

We define a kind of Bogolyubov transformation by

$$\tilde{A}_\rho(kr;t)\equiv\sum_j z_{\rho j}^{1/2}[\alpha_j(kr;t)\kappa_{j+}-i\beta_j^\dagger(-kr;t)\kappa_{j-}],$$

$$\tilde{B}_\rho^\dagger(-kr;t)\equiv\sum_j z_{\rho j}^{1/2}[-i\alpha_j(kr;t)\kappa_{j-}+\beta_j^\dagger(-kr;t)\kappa_{j+}]. \tag{C14}$$

We have canonical communication relations as

$$\{\tilde{A}_\rho(kr;t),\tilde{A}_\rho^\dagger(qs;t)\}=\delta_{rs}\delta(\vec{k},\vec{q})=\{\tilde{B}_\rho(-kr;t),\tilde{B}_\rho^\dagger(-qs;t)\},$$

$$\text{others}=0, \tag{C15}$$

and in this case the neutrino field is expanded as

$$\nu_\rho(x)=\frac{1}{\sqrt{V}}\sum_{\vec{k}} e^{i\vec{k}\cdot\vec{x}}\Bigg\{\Bigg[\tilde{A}_\rho(k\uparrow;t)\begin{pmatrix}1\\0\end{pmatrix}+i\tilde{B}_\rho^\dagger(-k\uparrow;t)\begin{pmatrix}0\\1\end{pmatrix}\Bigg]\otimes w(k\uparrow)+\Bigg[\tilde{A}_\rho(k\downarrow;t)\begin{pmatrix}0\\1\end{pmatrix}+i\tilde{B}_\rho^\dagger(-k\downarrow;t)\begin{pmatrix}1\\0\end{pmatrix}\Bigg]\otimes w(k\downarrow)\Bigg\}, \tag{C16}$$

which corresponds to the plane-wave expansion with $\mu_\lambda = 0$. Then physical quantities, which should be $\{\mu_\lambda\}$ independent, are allowed to be calculated by employing the expansion (C16) with Eq. (C14); the quantities such as $\rho_{\sigma j}(k) = \cos[(\chi_\sigma - \chi_j)/2]$ and $\lambda_{\sigma j}(k) = \sin[(\chi_\sigma - \chi_j)/2]$ are now replaced by $\cos(\chi_j/2) = c_j$ and $\sin(-\chi_j/2) = -s_j$; thus the matrices $[P(k)_{\rho j}]$ and $[\Lambda(k)_{\rho j}]$ appearing in $\mathcal{K}(k)$ are replaced by $[z_{\rho j}^{1/2} \rho_j(k)]$ and $[-z_{\rho j}^{1/2} \lambda_j(k)]$.

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