Pseudo Dirac scenario for neutrino oscillations

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We argue how the pseudo Dirac scenario for neutrinos leads to rich neutrino oscillation phenomena, including oscillation inside each generation. The pseudo Dirac scenario is generalized by incorporating generation mixings and formulas for the various neutrino oscillations are derived. As an application we compare the formulas with the corresponding data. We find that the observed pattern of mixings, such as almost maximal mixing in the atmospheric neutrino oscillation, is naturally explained in the generalized pseudo Dirac scenario with small generation mixings. We, however, also point out that there remain some problems to be settled for this scenario to be viable. The possible theoretical framework to realize the pseudo Dirac scenario is also briefly commented on.

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I. INTRODUCTION

The presence of neutrino oscillations, strongly suggested by the recent SuperKamiokande results on atmospheric neutrinos [1], is almost a unique clue to physics beyond the well-established standard model. More precisely, recent data on neutrino oscillations seem to have put forward the following challenging theoretical problems, which may lead to physics beyond the standard model.

(a) The data from (Super-)Kamiokande on atmospheric neutrinos necessitate a large or almost maximal mixing angle [1]. The solar neutrino deficit may also be explained by the ‘‘large-angle solution’’ [2]. How can such a large or maximal mixing be naturally derived theoretically?

(b) If we further accept the Liquid Scintillation Neutrino Detector (LSND) result [3], in addition to the solar and atmospheric neutrino data, the scheme with only three light neutrino states clearly gets into trouble. What kind of theoretical framework or model is needed to accommodate all of these neutrino oscillations? It has been argued that we should introduce at least one ‘‘sterile state.’’

(c) There seems to be a large disparity among the magnitudes of mixing angles implied by these experiments; the data on atmospheric neutrino and possibly that on solar neutrino indicate the necessity of large or almost maximal mixing angles [1,2], while the neutrino oscillation at LSND experiment is well described by a small mixing angle [3]. How can such disparity be naturally explained theoretically?

One clear thing is that these problems altogether suggest that flavor mixing or mass matrices in the leptonic sector are quite different from those in the quark sector. In particular the presence of large mixings and the necessity of extending the scheme with only three light neutrinos are specific new features in the leptonic sector, not shared by the quark sector, and may lead to a drastic modification of the standard model.

The main purpose of the present paper is to generalize the pseudo Dirac scenario for neutrino masses, which we argue to imply rich neutrino oscillation phenomena, by incorporating generation mixings and to derive general formulas for neutrino oscillations in the generalized pseudo Dirac scenario. We will show that the above problems, (a), (b), and (c), are naturally (without any fine tuning) solved simultaneously in the generalized scheme without enlarging the number of generations. It is worth noticing that small generation mixings are shown to be exactly what we need to solve the problems. We, however, will also point out some serious problems encountered by the atmospheric and solar neutrino oscillations into sterile states [4,2] and try to discuss some possible ways to cure these problems. We also investigate briefly what kind of model or theoretical framework is possible for the pseudo Dirac scenario to be realized, for the purpose of searching for some direction to the physics beyond the standard model. The key ingredient for our scenario is the ‘‘pseudo Dirac’’ property of neutrinos, whose precise meaning is now discussed in some detail.

It will be natural to suspect that the specific features of flavor mixing or mass matrices in the leptonic sector stated above should be related with the peculiarity of the leptonic masses, i.e., the fact that only neutrinos may have Majorana masses. Once neutrinos are allowed to have Majorana masses, we may think of three typical cases for neutrino masses, which we will discuss successively below. In the base of weak eigenstates, ψ_{wL}, where active states are put in the upstairs and ‘‘sterile’’ states are put in the downstairs,

$$\psi_{wL}=\begin{pmatrix}\nu_{\alpha L}\\ \overline{\nu}_{\alpha L}\end{pmatrix}\quad[\alpha=e,\mu,\tau;\quad \overline{\nu}_{\alpha L}=(\nu_{\alpha R})^C],\tag{1}$$

the neutrino mass term is generally written as

$$L_{\mathrm{mass}}=\frac{1}{2}\psi_{wL}^{t}CM\psi_{wL},\tag{2}$$

where C is the charge-conjugation matrix and the 6×6 mass matrix M takes a form of

$$M=\begin{pmatrix} M_L & M_D^t \\ M_D & M_R^* \end{pmatrix}, \tag{3}$$

with the 3×3 matrices M_D, M_L, and M_R being those for Dirac masses, and left- and right-handed Majorana masses, respectively. Depending on the extent of lepton number violation, or relative magnitudes of Majorana masses to those of Dirac masses, we can think of three typical cases.

(1) Pure Dirac—Imposing lepton number conservation, or ignoring all Majorana masses ($M_L=M_R=0$), we get pure Dirac neutrinos. We all know that there are only three mass eigenvalues for three generations, although the mass matrix M should have six eigenvalues, in general. What's really happening is that there are three degenerate pairs of mass eigenstates. The situation may be easily seen for the simplified one generation case

$$M=\begin{pmatrix} 0 & m_D \\ m_D & 0 \end{pmatrix}. \tag{4}$$

It is easy to see that M has eigenvalues $m_D, -m_D$ and the angle θ, in the orthogonal matrix to diagonalize M is just $\pi/4$. If processes we are interested in have no chirality flip, as in the case of neutrino oscillations (in the absence of magnetic field), the sign of mass is irrelevant and we have degenerate mass squared. It should also be noted that in this case a maximal mixing, $\theta=\pi/4$, between an active state and a sterile state, ν_L and $\overline{\nu_L}$, has been realized. Unfortunately, this maximal mixing does not lead to any neutrino oscillation, just because the mass squared are degenerated.

(2) ''Pseudo Dirac''—What happens if we allow small lepton number violation, i.e., if we switch on very small Majorana masses, $M_L, M_R \ll M_D$ (with the magnitudes of matrices being compared by taking typical orders of magnitudes of the matrix elements). Neutrinos are still almost Dirac particles and are called ''pseudo Dirac'' neutrinos [5]. The small Majorana masses, however, slightly lift the degeneracy of mass eigenvalues, and we get almost degenerate pairs of eigenstates with tiny mass differences. As far as the Majorana masses are small, the mixing angles should remain almost maximal, $\theta \simeq \pi/4$. To understand the situation, we consider the one generation case again. The mass matrix now reads as

$$M=\begin{pmatrix} m_L & m_D \\ m_D & m_R \end{pmatrix}, \tag{5}$$

with $m_L, m_R \ll m_D$. We now get $|\tan 2\theta| = |2m_D/(m_R-m_L)| \gg 1$, leading to almost maximal mixing $\theta \simeq \pi/4$. We have two mass eigenstates, which are almost symmetric and antisymmetric combinations of active and sterile states, i.e., $\nu_S=\sin\theta\nu_L+\cos\theta\overline{\nu_L}\simeq(1/\sqrt{2})(\nu_L+\overline{\nu_L})$, and $\nu_A=(-i)(\cos\theta\nu_L-\sin\theta\overline{\nu_L})\simeq(1/\sqrt{2}i)(\nu_L-\overline{\nu_L})$. Their masses are almost degenerate but are slightly different; $m_S\sim m_A\sim m_D$, $\Delta m\equiv m_S-m_A\sim m_L+m_R \ll m_D$. Now the tiny mass difference and the almost maximal mixing will lead to a neutrino oscillation between an active state and a sterile state, even if we have only one generation! Actually even for the one generation case two kinds of neutrino oscillations, without and with chirality flip, are possible (though the latter oscillation necessitates the presence of magnetic field and we ignore the magnetic field unless otherwise stated in this paper), i.e., (i) $\nu_L\to\overline{\nu_L}$ and (ii) $\nu_L\to\nu_R$ [6]. If matter effects are included, the oscillations of (i) and (ii) become resonant oscillations, which are quite similar to those in Mikheyev-Smirnov-Wolfenstein (MSW) [7] and resonant spin flavor precession (RSFP) [8] scenarios. For instance, the Hamiltonian in the base of $(\nu_L, \overline{\nu_L})$ for the former resonant oscillation, is given by [6]

$$H=\begin{pmatrix} a & \dfrac{\Delta m^2}{4E}\sin 2\theta \\ \dfrac{\Delta m^2}{4E}\sin 2\theta & \dfrac{\Delta m^2}{2E}\cos 2\theta \end{pmatrix}, \tag{6}$$

where the matter effect a is given for, e.g., ν_e as $a_e = (G_F/\sqrt{2})(2N_e-N_n)$, with N_e and N_n denoting the number densities of electron and neutron, respectively. As the matter of fact, in the limit of maximal mixing ($\theta=\pi/4$), the matter effect in the sun may be regarded as irrelevant, while the matter effect in the Earth results in a significant day-night effect.

(3) See-saw—The last possibility is famous see-saw scenario [9] in which $SU(2)$ invariant Majorana masses, M_R, are supposed to be much larger than the Dirac masses: $M_R \gg M_D, M_L\simeq 0$. The sterile states $\overline{\nu_L}$ approximately become mass eigenstates and are decoupled from low-energy processes such as neutrino oscillation. Thus only lighter three mass eigenstates ($\simeq\nu_{\alpha L}$) participate in neutrino oscillation phenomena. In the see-saw scenario, therefore, the mixings relevant for the neutrino oscillations are generation mixings and there seems to be no immediate reason to expect large mixing angles. It is also worth noting that as far as chirality preserving oscillations are concerned, there is no observable distinction between the cases of (1) and (3); in both cases only three light neutrino states participate in the oscillations.

From the above discussion we learn that only in the pseudo Dirac scenario six neutrino states fully participate in low-energy processes, and rich neutrino oscillation phenomena, both intergenerational and active↔sterile, are expected. In fact, a trial to explain existing data on neutrino oscillations based on the pseudo Dirac scenario was made some time ago [6,10]. (For the recent revived interest in this scenario, refer to Refs. [11,12]. See also Ref. [13] for the recent discussions in the framework of four neutrinos with only one sterile state.) In our previous attempt [6] generation mixings were switched off, for brevity, and three generations shared their roles to account for neutrino oscillations; solar neutrino oscillation was mainly due to $\nu_{eL}\to\overline{\nu_{eL}}$ in the first generation (in Ref. [6] the effect of magnetic field was also taken into account). The atmospheric neutrino oscillation could be naturally explained by $\nu_{\mu L}\to\overline{\nu_{\mu L}}$ with almost maximal mixing angle. At that time there was a datum to suggest the existence of a 17 keV neutrino [14], which enforced us to rely on the pseudo Dirac scenario, as otherwise a three generation scheme could not explain all of these data simultaneously. The 17 keV neutrino has been ruled out, and instead there has appeared the LSND data [3], which again necessitates three independent mass differences and modifying the ordinary three generation scheme with either see-saw or pure Dirac neutrinos.

In the present paper we generalize our previous argument [6] including generation mixings. We will find that small generation mixings, just as in the Cabibbo-Kobayashi-Maskawa (CKM) matrix in the quark sector, is exactly what we need; the nice features of pseudo Dirac scenario, such as maximal mixings in atmospheric and solar neutrino oscillations, are known to remain basically intact, while LSND data is naturally explained by the small generation mixings. In such a sense, we may say that *''The recent data on neutrino oscillations may be natural consequences of the property that lepton number is only slightly violated and generation mixings are small.''*

II. MASS EIGENSTATES AND MIXINGS

In the present paper we allow arbitrary generation mixings or arbitrary off-diagonal mass matrices, except for the assumption of pseudo Dirac property $M_L, M_R \ll M_D$. Thus we may naively expect that the diagonalization of the 6×6 mass matrix is quite complicated and various formulas for the probabilities of neutrino oscillations are expressed by use of 6 mass eigenvalues and 15 mixing angles (together with possible multiple CP violating phases). It, however, turns out that under the assumption of pseudo Dirac, all probabilities of neutrino oscillations are describable in terms of 6 mass eigenvalues and just one 3×3 unitary matrix U, ''Maki-Nakagawa-Sakata (MNS) matrix'' [15], which has three mixing angles and one CP violating phase and just corresponds to the CKM matrix in the quark sector.

To see this let us now discuss the diagonalization of the mass matrix M. Ignoring magnetic field, only chirality preserving transitions are important. So what we should diagonalize is $M^\dagger M$, rather than M itself. Keeping terms up to the first order in Majorana masses, which may be justified in the pseudo Dirac hypothesis, $M^\dagger M$ reads as

$$M^\dagger M \simeq \begin{pmatrix} M_D^\dagger M_D & M_L^* M_D^t + M_D^\dagger M_R^* \\ M_D^* M_L + M_R M_D & M_D^* M_D^t \end{pmatrix}. \qquad (7)$$

Because of the pseudo Dirac property, the dominant matrix is M_D. Thus we first diagonalize it by biunitary transformation;

$$U_R^\dagger M_D U_L = \mathrm{diag}(m_1, m_2, m_3) \equiv \hat{M},$$

$$\nu_{\alpha L} = (U_L)_{\alpha i}\nu_{iL}, \quad \overline{\nu}_{\alpha L} = (U_R^*)_{\alpha i}\overline{\nu}_{iL}. \qquad (8)$$

Accordingly, $M^\dagger M$ is cast into the following form by a unitary transformation due to a 6×6 unitary matrix V

$$V^\dagger (M^\dagger M) V = \begin{pmatrix} \hat{M}^2 & U_L^\dagger M_L^\dagger U_L^* \hat{M} + \hat{M} U_R^\dagger M_R^* U_R^* \\ \hat{M} U_L^t M_L U_L + U_R^t M_R U_R \hat{M} & \hat{M}^2 \end{pmatrix},$$

$$V = \begin{pmatrix} U & 0 \\ 0 & U_R^* \end{pmatrix}, \quad U \equiv U_L, \qquad (9)$$

where U_L has been rewritten simply as U, as it is the only matrix that appears in the formulas of neutrino oscillations. $\hat{M}^2$ is a diagonal matrix, while the matrices in the off-diagonal position, e.g., $\hat{M} U_L^t M_L U_L + U_R^t M_R U_R \hat{M}$, have not been diagonalized yet. Such off-diagonal matrices, however, are much smaller than $\hat{M}^2$ due to the pseudo Dirac property and seem to be negligible, anyway. It is not quite right, since once we ignore these off-diagonal matrices there appear degenerate pairs in the eigenvalues of $M^\dagger M$, i.e., each of m_i^2 $(i=1,2,3)$ appear twice. It is a general wisdom in perturbation theory that when there is a degeneracy in eigenvalues, eigenvalues and eigenstates can be fixed only after we include first-order perturbation, that connects the members of the pair, while other perturbations connecting different pairs may be safely ignored. This means that $V^\dagger(M^\dagger M)V$ can be effectively decomposed into three independent block-diagonal matrices. Each block-diagonal matrix takes a form of

$$\begin{pmatrix} m_i^2 & m_i\epsilon_i^* \\ m_i\epsilon_i & m_i^2 \end{pmatrix} \quad (i=1,2,3), \qquad (10)$$

where $\epsilon_i \equiv (U_L^t M_L U_L + U_R^t M_R U_R)_{ii}$, and $|\epsilon_i| \ll m_i$, because of the pseudo Dirac property.

It is now easy to see that we obtain, in total, six mass eigenstates

$$\nu_{iS} \equiv \frac{1}{\sqrt{2}}(\nu_{iL} + e^{i\phi_i}\overline{\nu}_{iL}), \quad \nu_{iA} \equiv \frac{1}{\sqrt{2}i}(\nu_{iL} - e^{i\phi_i}\overline{\nu}_{iL}),$$

$$(i=1,2,3), \qquad (11)$$

where $e^{i\phi_i} = \epsilon_i/|\epsilon_i|$. Their mass eigenvalues are given as

$$m_{iS}^2 = m_i^2 + m_i|\epsilon_i|, \quad m_{iA}^2 = m_i^2 - m_i|\epsilon_i|, \quad (i=1,2,3). \qquad (12)$$

To summarize, ψ_{wL} is related to mass eigenstates as

$$\psi_{wL}=\begin{pmatrix}\nu_{\alpha L}\\ \bar{\nu}_{\alpha L}\end{pmatrix}=\hat{V}\begin{pmatrix}\nu_{iS}\\ \nu_{iA}\end{pmatrix}, \tag{13}$$

where

$$\hat{V}\equiv\begin{pmatrix}U & 0\\ 0 & U_R\end{pmatrix}\cdot\begin{pmatrix}\frac{1}{\sqrt{2}} & 0 & 0 & \frac{i}{\sqrt{2}} & 0 & 0\\ 0 & \frac{1}{\sqrt{2}} & 0 & 0 & \frac{i}{\sqrt{2}} & 0\\ 0 & 0 & \frac{1}{\sqrt{2}} & 0 & 0 & \frac{i}{\sqrt{2}}\\ \frac{1}{\sqrt{2}}e^{-i\phi_1} & 0 & 0 & -\frac{i}{\sqrt{2}}e^{-i\phi_1} & 0 & 0\\ 0 & \frac{1}{\sqrt{2}}e^{-i\phi_2} & 0 & 0 & -\frac{i}{\sqrt{2}}e^{-i\phi_2} & 0\\ 0 & 0 & \frac{1}{\sqrt{2}}e^{-i\phi_3} & 0 & 0 & -\frac{i}{\sqrt{2}}e^{-i\phi_3}\end{pmatrix}, \tag{14}$$

and correspondingly the mass-squared matrix is diagonalized as

$$\hat{V}^{\dagger}(M^{\dagger}M)\hat{V}=M^2_{\rm diag},$$

$$M_{\rm diag}=\mathrm{diag}(m_{1S},m_{2S},m_{3S},m_{1A},m_{2A},m_{3A}). \tag{15}$$

Now the neutrinos emitted by weak interactions (weak eigenstates) are expressed in terms of mass eigenstates ν_{jS},ν_{jA} $(j=1-3)$ and a unitary matrix U, as follows:

$$\nu_{\alpha L}=U_{\alpha j}\frac{\nu_{jS}+i\nu_{jA}}{\sqrt{2}}. \tag{16}$$

The fact that there appears only single 3×3 unitary matrix U, even though we started from an arbitrary 6×6 mass matrix M, is one of our main results based on the pseudo Dirac property.

III. FORMULAS FOR NEUTRINO OSCILLATIONS

We will now derive the formulas for neutrino oscillations in terms of the differences of six mass-squared and single unitary matrix U. Though there is no reason to expect *a priori*, some specific pattern of neutrino masses, we can still get some useful information on the pattern from the reported data on neutrino oscillations [1–3]. Namely, once we regard the mixing angles in U as small, as suggested by the CKM matrix in the quark sector, the mass-squared difference, responsible for each observed neutrino oscillation, is given as

$$\text{solar neutrino:}\quad m_1|\epsilon_1|\sim10^{-5}-10^{-4}\ \ (\mathrm{eV}^2),$$

$$\text{atmospheric neutrino:}\quad m_2|\epsilon_2|\sim10^{-3}-10^{-2}\ \ (\mathrm{eV}^2),$$

$$\text{LSND:}\quad \Delta m^2_{12}\sim10^{-1}-1\ \ (\mathrm{eV}^2). \tag{17}$$

This knowledge suggests (with a little prejudice) a hierarchical structure of mass differences

$$m_1|\epsilon_1|\ll m_2|\epsilon_2|\ll m_3|\epsilon_3|\ll\Delta m^2_{12}\ll\Delta m^2_{13}, \tag{18}$$

where $\Delta m^2_{ij}\equiv m^2_j-m^2_i$. The hierarchical structure makes the formulas for neutrino oscillations simple and easy to be compared with the data. The 6 mass-eigenstates and 3 mass dif-

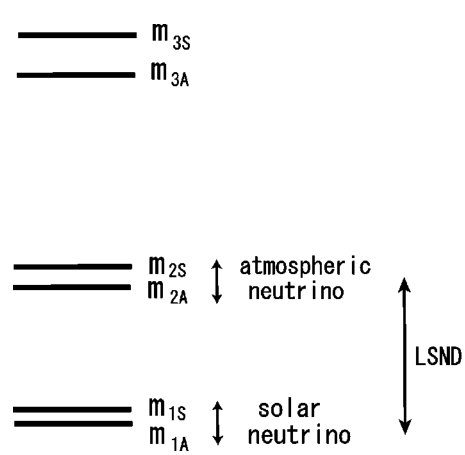

FIG. 1. The mass differences relevant for each neutrino oscillation.

ferences, relevant for each neutrino oscillation, are shown in Fig. 1.

A. A general formula for vacuum oscillation

We first note that except for the case of solar neutrino oscillation, both atmospheric neutrino oscillation and the oscillation in the LSND experiment are well described by vacuum oscillations. This is definitely true for the LSND case, but may need some care in the case of atmospheric neutrino, as will be commented on below.

In general, the probability of finding a state, born as an active state $\nu_{\alpha L}$ at time 0, in an active state $\nu_{\beta L}$ (α and β may be the same) at time t, is given by

$$P(\nu_\alpha\to\nu_\beta)=\left|\left(\hat{V}\exp\left\{i\frac{M^2_{\rm diag}}{2E}t\right\}\hat{V}^\dagger\right)_{\beta\alpha}\right|^2=\frac{1}{4}\left|\sum_{j=1}^{3}U_{\beta j}\left\{\exp\left(i\frac{m^2_{jS}}{2E}t\right)+\exp\left(i\frac{m^2_{jA}}{2E}t\right)\right\}U^*_{\alpha j}\right|^2. \quad (19)$$

B. Formulas for atmospheric neutrino oscillation

As the oscillation of atmospheric neutrino is sensitive to the mass difference $m_2|\epsilon_2|\sim10^{-3}$ (eV^2) [1], under the mass hierarchy Eq. (18), $m_1|\epsilon_1|$ may be ignored and ν_1 can be regarded as pure Dirac particle, i.e., $\nu_1=1/\sqrt{2}\{(\nu_{1S}+\text{c.c.})+i(\nu_{1A}+\text{c.c.})\}$ with a unique mass $m_{1S}=m_{1A}=m_1$. The oscillation of atmospheric neutrino is due to the interference between ν_{2S} and ν_{2A}, and the matter waves of other states, ν_1, ν_{3S}, and ν_{3A}, do not interfere with ν_{2S},ν_{2A} or with each other, when time average is taken for the high-frequency modes in the oscillation probability. Thus the formula for the probability of atmospheric ν_μ to survive till time t, relevant for the zenith angle distribution, simply reads as

$$P(\nu_{\mu L}\to\nu_{\mu L})_{\rm atm}=|U_{\mu1}|^4+|U_{\mu2}|^4\times\cos^2\left(\frac{m_2|\epsilon_2|}{2E}t\right)+\frac{1}{2}|U_{\mu3}|^4. \quad (20)$$

Let us note that there are constant terms $|U_{\mu1}|^4,|U_{\mu3}|^4$ coming from the time average of the high-frequency modes, in sharp contrast to the conventional formula in a simplified two states system. The following formulas are also relevant for the analysis of atmospheric neutrino oscillation:

$$P(\nu_{eL}\to\nu_{eL})_{\rm atm}=|U_{e1}|^4+|U_{e2}|^4\times\cos^2\left(\frac{m_2|\epsilon_2|}{2E}t\right)+\frac{1}{2}|U_{e3}|^4, \quad (21)$$

$$P(\nu_{\mu L}\to\nu_{eL})_{\rm atm}=|U_{\mu1}|^2|U_{e1}|^2+|U_{\mu2}|^2|U_{e2}|^2\cos^2\left(\frac{m_2|\epsilon_2|}{2E}t\right)+\frac{1}{2}|U_{\mu3}|^2|U_{e3}|^2. \quad (22)$$

C. Formula for the liquid scintillation neutrino detector neutrino oscillation

As the neutrino oscillation observed by the LSND is sensitive to the mass difference $\Delta m^2_{12}\sim10^{-1}-1$ (eV^2) [3], under the hierarchy (18) all neutrino states can be regarded as pure Dirac particles, i.e., $m_i|\epsilon_i|=0$ and $m^2_{iS}=m^2_{iA}=m^2_i$ ($i=1,2,3$). The LSND neutrino oscillation is, therefore, due to the interference between ν_1 and ν_2, and the matter waves of another state ν_3, does not interfere with ν_1,ν_2, when time average is taken. Thus the formula for the transition probability of the LSND neutrino simply reads as

$$\begin{aligned}P(\nu_{\mu L}\to\nu_{eL})_{\rm LSND}&=|U_{\mu1}|^2|U_{e1}|^2+|U_{\mu2}|^2|U_{e2}|^2+|U_{\mu3}|^2|U_{e3}|^2\\&\quad+2\,{\rm Re}(U_{\mu1}U^*_{e1}U^*_{\mu2}U_{e2})\cos\left(\frac{\Delta m^2_{21}}{2E}t\right)\\&\quad-2\,{\rm Im}(U_{\mu1}U^*_{e1}U^*_{\mu2}U_{e2})\sin\left(\frac{\Delta m^2_{21}}{2E}t\right)\\&=4\{|U_{\mu2}U_{e2}|^2+{\rm Re}(U^*_{\mu2}U_{e2}U_{\mu3}U^*_{e3})\}\sin^2\left(\frac{\Delta m^2_{21}}{4E}t\right)\\&\quad+2\,{\rm Im}(U^*_{\mu2}U_{e2}U_{\mu3}U^*_{e3})\sin\left(\frac{\Delta m^2_{21}}{2E}t\right)\\&\quad+2|U_{\mu3}U_{e3}|^2.\end{aligned} \quad (23)$$

D. The matter oscillation of solar neutrino

The time evolution of the system in the presence of matter effects are governed by

$$i\frac{d}{dt}\begin{pmatrix}\nu_{\alpha L}\\ \overline{\nu_{\alpha L}}\end{pmatrix}=H\begin{pmatrix}\nu_{\alpha L}\\ \overline{\nu_{\alpha L}}\end{pmatrix}, \quad (24)$$

where

$$\begin{aligned}H&=\frac{1}{2E}(\hat{V}M^2_{\rm diag}\hat{V}^\dagger)+A,\\&=\hat{V}\left\{\frac{1}{2E}M^2_{\rm diag}+\hat{V}^\dagger A\hat{V}\right\}\hat{V}^\dagger,\\A&\equiv{\rm diag}(a_e,a_\mu,a_\tau,0,0,0),\end{aligned} \quad (25)$$

and the elements of the matrix A denoting the matter effects of ν_e, ν_μ, and ν_τ are given as $a_e=(G_F/\sqrt{2})(2N_e-N_n)$, $a_\mu=a_\tau=(G_F/\sqrt{2})(-N_n)$. It is easy to see that for the energy range of solar neutrinos and the mass hierarchy of Eq. (18), the matter effects inside the sun satisfy

$$Ea_\alpha \sim m_1|\epsilon_1| \ll m_2|\epsilon_2| \ll \cdots \ll \Delta m_{13}^2 \quad (\alpha=e,\mu,\tau). \tag{26}$$

Under the hierarchical structure of mass differences and the matter effects, in the Hamiltonian in the base of the mass eigenstates, $(1/2E)M_{\rm diag}^2+\hat{V}^\dagger A\hat{V}$, all the off-diagonal matrix elements due to the matter effects can be safely ignored except the ones in 2×2 subsystem of ν_{1S} and ν_{1A}. Thus the heavier states, ν_{iS},ν_{iA} $(i=2,3)$ are decoupled from the two states subsystem of (ν_{1S},ν_{1A}), and the time evolution of the subsystem is governed by a 2×2 Hamiltonian

$$\frac{1}{2E}\begin{pmatrix} m_{1S}^2 & 0 \\ 0 & m_{1A}^2 \end{pmatrix}+\frac{1}{2}\left(\sum_{\alpha=e}^{\tau}|U_{\alpha 1}|^2 a_\alpha\right)\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}. \tag{27}$$

This Hamiltonian can be rewritten in the base of $(\nu_{1L},\overline{\nu_{1L}})$ (ignoring a piece proportional to a unit matrix) as

$$\begin{pmatrix} \sum_{\alpha=e}^{\tau}|U_{\alpha 1}|^2 a_\alpha & \frac{\Delta m^2}{4E}\sin 2\theta \\ \frac{\Delta m^2}{4E}\sin 2\theta & \frac{\Delta m^2}{2E}\cos 2\theta \end{pmatrix}, \tag{28}$$

with $\Delta m^2=2m_1|\epsilon_1|$ and $\theta=\pi/4$. This Hamiltonian just corresponds to the one in the MSW mechanism [7], though in our case the mixing angle is maximal and the matter effect has been modified into $\Sigma_{\alpha=e}^{\tau}|U_{\alpha 1}|^2 a_\alpha$.

Now the survival probability of solar neutrino (time averaged) can be written in a simple form

$$\overline{P}(\nu_{eL}\to\nu_{eL})_{\rm solar}=|U_{e1}|^4\overline{P}(\nu_{1L}\to\nu_{1L})_{\rm eff}+\frac{1}{2}|U_{e2}|^4+\frac{1}{2}|U_{e3}|^4, \tag{29}$$

where the survival probability $\overline{P}(\nu_{1L}\to\nu_{1L})$ in the effective two states system of $(\nu_{1L},\overline{\nu_{1L}})$ is calculable by use of the Hamiltonian (28). A similar reduction formula based on a hierarchical mass structure was obtained in Ref. [16] in order to reduce solar neutrino oscillation in a three generation model into that of an effective two generation model. Though the above formula itself can be applicable for arbitrary θ, in the limit of maximal mixing $(\theta=\pi/4)$ of our present interest, the matter effect in the sun may be regarded as irrelevant (both vacuum and matter oscillations give survival probabilities of the solar neutrino of $\simeq 1/2$), while the matter effect in the Earth is known to result in a significant day-night effect.

IV. PSEUDO DIRAC SCENARIO CONFRONTED BY THE DATA ON NEUTRINO OSCILLATIONS

A. Comparing the formulas with the data

As the application of the formulas we have derived for the neutrino oscillations, we are now going to compare them with the corresponding experimental data. As we have already advertised, we will see that small generation mixing angles in the unitary matrix U is just what we need to explain the pattern of mixing angles observed in neutrino oscillations of our interest.

Thus we first consider the case where generation mixings are small, though the formulas we have derived above are applicable for arbitrary generation mixings. Retaining only the leading contributions for small generation mixing angles (say $\theta_1,\theta_2,\theta_3$ just as the angles in CKM matrix), we get the following formulas, relevant for each neutrino oscillation:

solar neutrino:
$$\overline{P}(\nu_{eL}\to\nu_{eL})_{\rm solar}\simeq\overline{P}(\nu_{1L}\to\nu_{1L})_{\rm eff}, \tag{30}$$

atmospheric neutrino:

$$1-P(\nu_{\mu L}\to\nu_{\mu L})_{\rm atm}\simeq\sin^2\left(\frac{m_2|\epsilon_2|}{2E}t\right),$$
$$P(\nu_{eL}\to\nu_{eL})_{\rm atm}\simeq 1,$$
$$P(\nu_{\mu L}\to\nu_{eL})_{\rm atm}\simeq 0, \tag{31}$$

LSND:
$$P(\nu_{\mu L}\to\nu_{eL})_{\rm LSND}\simeq 4|U_{e2}|^2\sin^2\left(\frac{\Delta m_{21}^2}{4E}t\right). \tag{32}$$

In the formula for LSND $|U_{e3}U_{\mu 3}|$ has been neglected compared with $|U_{e2}U_{\mu 2}|$, as is suggested by the hierarchical mixing angles, $\theta_1\gg\theta_2\gg\theta_3$, in the quark sector. In the case of solar neutrino oscillation, $\overline{P}(\nu_{1L}\to\nu_{1L})_{\rm eff}$ is obtainable from the time evolution governed by the effective Hamiltonian Eq. (28), which is very similar to that in the MSW mechanism with maximal mixing angle $\pi/4$. To be precise, in the Hamiltonian (28) the matter effect is not $\sqrt{2}G_F N_e$ as in the case of the MSW mechanism. The difference, however, is not large, as long as the generation mixings are small and also because the contribution of neutral current, the term proportional to N_n, is relatively suppressed compared with that of charged current by a factor of $\sim 1/12$. On the other hand, very recently we have heard of the news [2] that the data of SuperKamiokande on solar neutrinos favors the MSW-type solution with large mixing angle. Thus our scenario of pseudo Dirac provides a natural framework to derive the large-angle solution suggested by the data. In the case of atmospheric neutrino oscillation, the factor in front of $\sin^2[(m_2|\epsilon_2|/2E)t]$ is $1=\sin^2(2\times\pi/4)$, which just corresponds to a vacuum oscillation with maximal mixing, strongly suggested by the data on atmospheric neutrinos [1].

On the contrary, in the case of the LSND, our formula gives that of an ordinary 2 generation scheme with small generation mixing, if we identify $4|U_{e2}|^2$ with $\sin^2 2\theta_1$. This is just consistent with the experimental data [3], which says $\sin^2 2\theta_1 \leqslant 0.04$, when combined with the data from the BUGEY experiment. Another meaningful constraint on the generation mixing may come from the data of the CHOOZ experiment [17]. As the mass-squared difference, which is sensitive to the CHOOZ experiment, is comparable to that in atmospheric neutrino oscillation, we may write down a similar formula to Eq. (20) for the disappearance of $\overline{\nu}_e$,

$$1-P(\nu_{eL}\to\nu_{eL})_{\rm CHOOZ}=1-|U_{e1}|^4-|U_{e2}|^4\cos^2\left(\frac{m_2|\epsilon_2|}{2E}t\right)-\frac{1}{2}|U_{e3}|^4 \simeq 2|U_{e2}|^2, \tag{33}$$

where the unitarity of the U matrix was used and an approximation of $|U_{e3}|\ll|U_{e2}|$ was useful to simplify the result. The upper bound on the probability of disappearance from CHOOZ puts a bound

$$|U_{e2}|^2\sim\sin^2\theta_1\leqslant 0.05, \tag{34}$$

which is a little weaker than the upper bound stated above.

B. Problems to be settled

We have seen that the pseudo Dirac scenario just provides the favored solutions to the solar and the atmospheric neutrino problems with (almost) maximal mixings, suggested by the recent data, invoking the oscillations mainly into sterile states, while LSND data is naturally explained by ordinary generation mixing between active states with a small mixing. We naively expect that neutrino oscillations of solar or atmospheric neutrino into an active state and a sterile state cannot be clearly discriminated, since basically these experiments are disappearance experiments. Roughly speaking this expectation is certainly true, but the data from SuperKamiokande experiment have reached the precision, which is enough to distinguish these two cases. The most recent data seem to regard the maximal mixing solutions of neutrino oscillations into sterile states with disfavor [2,4]. We would like to discuss some possible ways to avoid the difficulty for each case.

1. Atmospheric neutrino

Though we have neglected the matter effect in the atmospheric neutrino oscillation, the matter effect of the Earth becomes non-negligible for higher neutrino energies. It has been pointed out that ν_μ oscillations into ν_τ and a sterile state have different zenith-angle dependence, as only in the case of the oscillation into the sterile state does the matter effect affect the time evolution of the neutrino states. Compared with the data, combining the analysis of neutral current enriched events, the SuperKamiokande collaboration claims that the oscillation into the sterile state with maximal mixing is regarded with disfavor [4]. Possible ways to evade this problem, that we can think of, are the following.

(a) When the oscillation to the sterile state is analyzed, simplified two states system of (ν_μ,ν_S) (ν_S denoting a sterile state) is assumed. In the scenario of the pseudo Dirac, however, we have 6 neutrino states to participate in the oscillation, and the formula for the oscillation, as seen in Eq. (20), is different from that in the simple 2 states system, typically having additional constant terms (for nonvanishing generation mixings). Including the matter effect of the Earth, our formula is modified into

$$\overline{P}(\nu_{\mu L}\to\nu_{\mu L})_{\rm atm}=|U_{\mu 2}|^4\overline{P}(\nu_{2L}\to\nu_{2L})_{\rm eff}+|U_{\mu 1}|^4+\frac{1}{2}|U_{\mu 3}|^4, \tag{35}$$

where the survival probability $\overline{P}(\nu_{2L}\to\nu_{2L})_{\rm eff}$ in the effective 2 states system $(\nu_{2L},\overline{\nu_{2L}})$ is that for the mass-squared difference $2m_2\epsilon_2$ and the matter effect $\Sigma_{\alpha=e}^{\tau}|U_{\alpha 2}|^2 a_\alpha$. Thus both the depletion rate of atmospheric ν_μ and the zenith angle dependence should be reanalyzed by use of this formula before some definite conclusion is derived.

(b) We just would like to point out that there is a claim that $\nu_\mu\to\nu_S$ oscillation with almost maximal mixing may not be ruled out, even in the simple 2 states system of (ν_μ,ν_S). It has been pointed out that a χ^2 analysis of the recent data does not exclude the maximal $\nu_\mu\to\nu_S$ oscillation solution with any significant confidence level, once various theoretical uncertainties and experimental systematic errors are included [18].

2. Solar neutrino

The SuperKamiokande Collaboration claims that the solar neutrino oscillation into the sterile state with maximal mixing is not favored [2]. The point is that $\nu_e\to\nu_\mu,\nu_\tau$ and $\nu_e\to\nu_S$ oscillations give slightly different contributions in the SuperKamiokande detector, as the final active states contribute to the event rate, while the sterile state does not. Accordingly, the survival probability of solar neutrino should be relatively higher in the case of oscillation into the sterile state. Thus the ''Be neutrino problem'' in the chlorine experiment becomes more severe in the sterile case. The possible ways out of this problem, we can think of, are the following.

(a) Again the claim that the solar neutrino oscillation to the sterile state is not favored is based on the analysis assuming a simplified two states system of (ν_e,ν_S). In the scenario of pseudo Dirac, however, we have 6 neutrino states to participate in the oscillation, and the formula Eq. (29) should be utilized to see whether the oscillation into the sterile state can accommodate all data of solar neutrino experiments or not.

(b) In the case of solar neutrino oscillation, the presence of solar magnetic field is potentially important, though we have ignored it in the above discussions. It has been pointed out that in the presence of the magnetic field, MSW type oscillation $\nu_{eL}\to\overline{\nu}_{eL}$ may be followed by a RSFP type oscillation $\overline{\nu}_{eL}\to\overline{\nu}_{eR}$, ignoring generation mixing [6]. The final state, now being an active state, contributes to the event rate and may remedy the ''Be neutrino problem.''

(c) In the above argument, Majorana masses have been treated as small perturbation, but it may not be unnatural to expect that in the first generation the Dirac mass is so small that the effect of Majorana masses is relatively enhanced, leading to a relatively small mixing between the active and sterile states.

3. Cosmological issues

There is another type of problem, i.e., cosmological problem. The well-known limit of number of effective neutrino species during nucleosynthesis puts a stringent bound on the mass-squared differences and mixings of neutrino oscillations into sterile states [19]. It, however, has been pointed out that once relatively large relic neutrino asymmetry L_ν, say $L_\nu \geqslant 10^{-4}$, is realized, such a problem can be evaded. For the details of the argument refer to Ref. [20].

Another issue of interest concerns dark matter in the universe. The mass hierarchy shown in Eqs. (17) and (18) suggests that there are pairs of Majorana neutrinos with almost degenerate masses, m_2 and m_3 [$0.1\ (\mathrm{eV}) \leqslant m_2 \ll m_3$], which may significantly contribute to the hot dark matter. The fraction of these neutrinos' contribution to the energy density of the universe is given by $\Omega_\nu = 2(m_2+m_3)/45$ eV [taking $H_0 = 70(\mathrm{km\ s^{-1}\ Mpc^{-1}})$] [21]. Thus too ''heavy'' ν_3 with $m_3 \gg 10$ eV will be excluded, as long as it remains as a stable particle.

We finally briefly comment on the related issue, i.e., neutrinoless double β decay. In our pseudo Dirac scenario, the relevant lepton number violating Majorana mass is given as

$$\frac{1}{2}\sum_i (U_{ei})^2(m_{iS}-m_{iA}) \simeq \frac{1}{2}\sum_i (U_{ei})^2|\epsilon_i|, \qquad (36)$$

which is well below the experimental upper bound under the mass hierarchy (17) and (18) with $\epsilon_i \ll m_i$ and for small generation mixings. This is basically because the pseudo Dirac neutrinos are almost Dirac particles and the lepton number is only slightly violated by their masses.

C. Predicted neutrino oscillations

Pseudo Dirac neutrinos have a rich phenomena of neutrino oscillation. As is seen in Fig. 1, in addition to the mass differences, relevant for solar, atmospheric, and LSND neutrino oscillations, there are two independent mass differences, i.e., the pseudo Dirac type mass splitting $m_3|\epsilon_3|$ and a mass difference between third and first or second generations Δm^2_{13}. These mass differences should predict neutrino oscillations that are characteristic to the pseudo Dirac scenario.

The characteristic prediction of the pseudo Dirac scenario should be $\nu_{\tau L} \to \overline{\nu}_{\tau L}$ due to the mass-squared difference $m_3|\epsilon_3|$, i.e., the pseudo Dirac property of the third generation neutrino. Keeping only the leading term for small generation mixing angles, we get the probabilities

$$1-P(\nu_{\tau L}\to\nu_{\tau L}) \simeq P(\nu_{\tau L}\to\overline{\nu}_{\tau L}) \simeq \sin^2\left(\frac{m_3|\epsilon_3|}{2E}t\right). \qquad (37)$$

If we take a reference value $m_3|\epsilon_3| \sim 10^{-2}\ (\mathrm{eV}^2)$, suggested by the hierarchy (17) and (18), $(L/\mathrm{km})/(E/\mathrm{GeV}) \geqslant 40$ (L is the length from the beam to the detector) will be needed, however, for the oscillation to be visible.

The presence of the largest mass-squared difference Δm^2_{13} predicts oscillations $\nu_{\tau L} \leftrightarrow \nu_{\mu L}$ or $\nu_{\tau L} \leftrightarrow \nu_{eL}$. In contrast to the case of an ordinary scenario with three light neutrino states, where $\nu_{\mu L} \to \nu_{\tau L}$ with maximal mixing should be responsible for the atmospheric neutrino oscillation, the probabilities of these oscillations between $\nu_{\tau L}$ and active states $\nu_{eL}, \nu_{\mu L}$ are suppressed by small generation mixing angles in our scenario. Neglecting smaller mass-squared differences the probabilities for these oscillations read as

$$P(\nu_{\tau L}\leftrightarrow\nu_{\mu L}) \simeq 4|U_{\mu 3}|^2|U_{\tau 3}|^2 \sin^2\left(\frac{\Delta m^2_{13}}{4E}t\right), \qquad (38)$$

$$P(\nu_{\tau L}\leftrightarrow\nu_{eL}) \simeq 4|U_{e3}|^2|U_{\tau 3}|^2 \sin^2\left(\frac{\Delta m^2_{13}}{4E}t\right), \qquad (39)$$

which will be suppressed by $|U_{\mu 3}|^2$ or $|U_{e3}|^2$, i.e., by small mixing angles between third and first or second generations.

V. THEORETICAL FRAMEWORK FOR THE PSEUDO DIRAC NEUTRINOS

Though there have already appeared a few attempts to construct models for the pseudo Dirac neutrinos [12], in this paper we instead list the problems to be resolved before a realistic model is constructed, and argue about a possible theoretical framework to provide natural mechanisms to solve the problems.

First of all it is worth noticing that the smallness of Majorana masses, needed to realize the pseudo Dirac neutrinos, satisfies the naturalness condition of 't Hooft [22], since if Majorana masses are absent, $M_L = M_R = 0$, the symmetry of the theory, i.e., the lepton number symmetry is enhanced. So the smallness will be stable under the radiative correction.

We, however, still have the following problems to be settled at the classical level: (1) How to explain the relation $M_R \ll M_D$? (2) How to explain the relation $M_L \ll M_D$? and (3) How to explain the smallness of M_D itself? We will discuss the possible theoretical frameworks to resolve these problems successively below.

A. Problem 1

1. Four-dimensional framework

In considering the possible theoretical framework to resolve this problem, it may be helpful to reconsider the conventional see-saw mechanism in the language of gauge-invariant operators. Suppose that the gauge symmetry of our world is $U(1)_{em}$ (QED), then the Majorana mass term $m_L \nu_L^2$ is gauge invariant. Therefore, there is no reason to expect that m_L should be small. As a matter of fact, the gauge symmetry of the standard model does not allow the mass operator, as it is gauge variant. Thus the Majorana mass is provided by an irrelevant operator $(1/M)L_L^2H^2$, where H

denotes the Higgs doublet and $L_L=(\nu_L,l_L)^t$ is a lepton doublet. As M is a gauge-invariant mass, it can be arbitrarily large. Thus the essence of the see-saw mechanism may be understood as the decoupling of some gauge singlet heavy particle with mass M (which need not to be ν_R). It, therefore, will be not unnatural to expect that a similar thing happens for the right-handed Majorana masses, as well. The Majorana mass term $m_R\nu_R^2$ is gauge invariant in the standard model and m_R is regarded to be quite large. This may not be true in some physics beyond the standard model. For instance in the left-right symmetric model $SU(2)_L\times SU(2)_R\times U(1)$ [23] with a $SU(2)_R$ doublet Higgs field H_R [in addition to the ordinary $SU(2)_L$ doublet Higgs field], the $m_R\nu_R^2$ operator is no longer gauge invariant, and will be replaced by an irrelevant operator $(1/M')L_R^2H_R^2$, with $L_R=(\nu_R,l_R)^t$. Thus the decoupling of a gauge singlet heavy particle whose mass M' is much larger than the scale of $SU(2)_R$ breaking, $M'\gg\langle H_R\rangle$, may imply the smallness of m_R. The heavy particle can be identified with a gauge singlet fermion S, having a Yukawa coupling L_RSH_R and a large Majorana mass M'. A diagram with the exchange of S yields the irrelevant operator $L_R^2H_R^2$.

2. Framework with extra dimensions

Recently there has appeared revived interest in higher-dimensional theories with extra dimensions as a possible solution to hierarchy problem [24–26]. In the scenario of large extra dimension [24], the higher-dimensional Planck scale is regarded as comparable to the weak scale, while in the scenario of small extra dimension [26] all masses in the visible brane are claimed to be strongly suppressed by the ''warp factor.'' Therefore the conventional see-saw mechanism [9], which needs large right-handed Majorana masses ($m_R\gg M_W$), may not work. In the case of the pseudo Dirac scenario, on the contrary, what we need is very small or even vanishing m_R. It is interesting to note that the presence of extra dimensions may provide a natural mechanism to realize this. For instance, let us consider a theory in five-dimensional space time, where the particles of the standard model, including ν_L, are assumed to reside on a three brane, while gauge singlet fields, such as ν_R, may reside in the bulk [27,28]. We note that in five-dimensional space-time, a Majorana spinor is known not to exist. [Majorana spinors exist only in the space time of $D=2,3,4\ (\mathrm{mod}\,8)$.] Thus the mass term $m_R\nu_R^2+\mathrm{H.c.}$ is not Lorentz invariant in this space time and we have vanishing m_R. Actually in such odd dimensional space-time chiral fermions do not exist, and ν_R should be accompanied by a gauge singlet $\tilde{\nu}_L$ to form a full spinor $\psi=(\nu_R,\tilde{\nu}_L)^t$. The full spinor may have a large Dirac mass and ν_R may be decoupled from low-energy phenomena. This difficulty may be evaded when the extra space is an orbifold S^1/Z_2, the extra space suggested by the recent works [24,26,29]. This is because the discrete symmetry Z_2 just corresponds to a symmetry under the transformation $\psi\to\gamma_5\psi$, which in turn behaves as a chiral transformation in the four-dimensional sense, thus making the Dirac mass term prohibited. Strictly speaking, ψ may also have masses due to nonzero K-K modes. For relatively small sizes of the extra space, such nonzero modes tend to be decoupled from the system and do not significantly affect the above argument.

B. Problem 2

In some sense, this problem may not be a real challenge; in any viable model the relation $M_L\ll M_D$ must be automatically built in, since otherwise custodial symmetry is significantly violated by the vacuum expectation value (VEV) of a $SU(2)$ triplet representation and $\Delta\rho=\rho-1$ gets sizable contribution, in contradiction with the data. Furthermore, if we have already got some mechanism to realize small but nonvanishing m_R in the mechanism discussed above, we are satisfied with vanishing m_L just as in the standard model. In a model with vanishing m_R, however, it becomes crucial for the pseudo Dirac scenario to slightly violate the lepton number by small but nonvanishing m_L. It is worth noticing that even in a higher-dimensional model with brane picture the ν_L is allowed to live only on the brane, and the issue concerning m_L is essentially four-dimensional.

If we wish to get the small m_L at the classical level as the form of a renormalizable operator we should introduce a $SU(2)_L$ triplet Higgs field H_T, whose VEV should be small so it does not contradict with the custodial symmetry. Then a marginal operator $L_L^2H_T$ gives small m_L. Or we may invoke an irrelevant operator $(1/M)L_L^2H^2$, as we discussed above, with a large mass scale $M\gg\langle H\rangle$. The operator may be the result of the exchange of a hypothetical gauge singlet S', which has a Yukawa coupling $L_LS'H$ and a large Majorana mass M. The S' should not be identified with ν_R. If we work in the framework of extra dimension with a brane, however, there may not be a good reason to assume that the gauge singlet S' resides only on the brane. Even if m_L is forbidden at the classical level, the small m_L may still be produced at the loop level, as long as there is some seed to violate the lepton number. The prototype model of this kind may be the model where the presence of a charged $SU(2)$ singlet scalar violates the lepton number explicitly [30].

C. Problem 3

In pseudo Dirac scenario, Dirac masses provide mean masses of neutrinos. Thus the crucial problem is how to explain the smallness of the neutrino Dirac masses compared with those of charged leptons or quarks.

1. Four-dimensional framework

In the four-dimensional framework the ''Dirac see-saw'' mechanism has been put forward [12], in which the marginal operator to give Dirac mass term $H\overline{\nu_L}\nu_R$ is forbidden, by a discrete symmetry, and the Dirac masses are provided by an irrelevant operator with $d>4$, just as happens in the conventional see-saw mechanism. Thus if the coefficient is sufficiently suppressed by the inverse of some large mass scale, we get small Dirac masses.

2. Framework with extra dimensions

Another intriguing possibility to realize the small neutrino Dirac masses is to invoke the presence of extra space [27,28]. For instance in the $(4+n)$-dimensional theory with a

3 brane, the original higher-dimensional Yukawa coupling $f_0 H \overline{\nu_L} \nu_R$ has a Yukawa coupling constant f_0, which behaves as $1/\sqrt{M_f^n}$, with M_f being the fundamental mass scale of the theory. The four-dimensional Yukawa coupling f_4 is thus given by $f_4 \sim 1/\sqrt{M_f^n V}$ (V is the volume of the extra dimension). The factor $1/\sqrt{V}$ comes from the overlap of the three fields on the brane, and f_4 may be suppressed by the largeness of the extra dimension. In fact in the Arkani-Hamed–Dimopoulos–Dvali model [24], by use of the relation $M_{pl}^2 = M_f^{2+n} V$, we get an n-independent result

$$f_4 \sim \frac{M_f}{M_{pl}}, \tag{40}$$

which is $\sim 10^{-16}$ for, e.g., $M_f \sim 1$ TeV. In this way, a small Dirac mass, $\sim f_4 M_W \sim f_4 M_f$, is achievable.

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[1] T. Toshito, in *Proceedings of the 30th International Conference on High Energy Physics*, Osaka, Japan, 2000, edited by C. S. Lim and T. Yamanaka (World Scientific, Singapore, 2001); Y. Fukuda *et al.*, Phys. Rev. Lett. **82**, 2644 (1999); Phys. Lett. B **467**, 185 (1999).

[2] Y. Takeuchi, in *Proceedings of the 30th International Conference on High Energy Physics*, Osaka, Japan, 2000 [1]; Y. Fukuda *et al.*, Phys. Rev. Lett. **82**, 1810 (1999); **82**, 2430 (1999).

[3] R. L. Imlay, in *Proceedings of the 30th International Conference on High Energy Physics*, Osaka, Japan, 2000 [1]; C. Athanassopoulos *et al.*, Phys. Rev. C **54**, 2685 (1996); **58**, 2489 (1998).

[4] S. Fukuda *et al.*, Phys. Rev. Lett. **85**, 3999 (2000).

[5] L. Wolfenstein, Nucl. Phys. **B186**, 147 (1981).

[6] M. Kobayashi, C. S. Lim, and M. M. Nojiri, Phys. Rev. Lett. **67**, 1685 (1991).

[7] S. P. Mikheyev and A. Yu. Smirnov, Yad. Fiz. **42**, 1441 (1985) [Sov. J. Nucl. Phys. **42**, 913 (1985)]; L. Wolfenstein, Phys. Rev. D **17**, 2369 (1978).

[8] C. S. Lim and W. J. Marciano, Phys. Rev. D **37**, 1368 (1988); E. Kh. Akhmedov, Phys. Lett. B **213**, 64 (1988).

[9] T. Yanagida, in *Proceedings of Workshop on Unified Theory and Baryon Number in the Universe*, edited by O. Sawada and A. Sugamoto (KEK, Japan, 1979); M. Gell-Mann, P. Ramond, and R. Slansky, in *Supergravity*, edited by P. van Nieuwenhuizen and D. Z. Freedman (North Holland, New York, 1979).

[10] H. Minakata and H. Nunokawa, Phys. Rev. D **45**, 3316 (1992); C. Giunti, C. W. Kim, and U. W. Kim, *ibid.* **46**, 3034 (1992). For an earlier related discussion on the pseudo-Dirac neutrinos, see also S. T. Petcov, Phys. Lett. **110B**, 245 (1982).

[11] R. Foot and R. R. Volkas, Phys. Rev. Lett. **75**, 4350 (1995); A. Geiser, Phys. Lett. B **444**, 358 (1998); W. Krolikowski, Acta Phys. Pol. B **31**, 663 (2000).

[12] D. Chang and O. C. W. Kong, Phys. Lett. B **477**, 416 (2000).

[13] C. Giunti, M. C. Gonzalez-Garcia, and C. Pena-Garay, Phys. Rev. D **62**, 013005 (2000); O. Yasuda, hep-ph/0006319.

[14] A. Hime and N. A. Jelley, Phys. Lett. B **257**, 441 (1991); B. Sur *et al.*, Phys. Rev. Lett. **66**, 2444 (1991).

[15] Z. Maki, M. Nakagawa, and S. Sakata, Prog. Theor. Phys. **28**, 870 (1962).

[16] C. S. Lim, in Proceedings of *BNL Neutrino Workshop 1987*, BNL report 52079, 1987.

[17] CHOOZ Collaboration, M. Apollonio *et al.*, Phys. Lett. B **420**, 397 (1998).

[18] R. Foot, Phys. Lett. B **496**, 169 (2000).

[19] R. Barbieri and A. Dolgov, Phys. Lett. B **237**, 440 (1990); K. Enqvist, K. Kainulainen, and J. Maalampi, *ibid.* **249**, 531 (1990).

[20] R. Foot and R. R. Volkas, Phys. Rev. Lett. **75**, 4350 (1995).

[21] Particle Data Group, D. E. Groom *et al.*, Eur. Phys. J. C **15**, 1 (2000).

[22] G. 't Hooft, *The Proceedings of Cargese Summer Institute* (Cargese Summer Inst., Cargese, 1979), p. 135.

[23] J. C. Pati and A. Salam, Phys. Rev. D **10**, 275 (1974); R. N. Mohapatra and J. C. Pati, *ibid.* **11**, 566 (1975); **11**, 2558 (1975); G. Senjanovic and R. N. Mohapatra, *ibid.* **12**, 1502 (1975).

[24] N. Arkani-Hamed, S. Dimopoulos, and G. Dvali, Phys. Lett. B **429**, 263 (1998).

[25] H. Hatanaka, T. Inami, and C. S. Lim, Mod. Phys. Lett. A **13**, 2601 (1998).

[26] L. Randall and R. Sundrum, Phys. Rev. Lett. **83**, 3370 (1999).

[27] N. Arkani-Hamed, S. Dimopoulos, G. Dvali, and J. March-Russell, ''Neutrino Masses from Large Extra Dimensions,'' hep-ph/9811448.

[28] G. Dvali and A. Yu. Smirnov, Nucl. Phys. **B563**, 63 (1999).

[29] P. Horava and E. Witten, Nucl. Phys. **B460**, 506 (1996).

[30] A. Zee, Phys. Lett. **93B**, 389 (1980).