Mass generation for non-Abelian antisymmetric tensor fields in a three-dimensional space-time

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(Received 13 December 2000; published 17 May 2001)

Starting from a recently proposed Abelian topological model in 2+1 dimensions, which involve the Kalb-Ramond two form field, we study a non-Abelian generalization of the model. An obstruction for the generalization is detected. However, we show that the goal is achieved if we introduce a vectorial auxiliary field. Consequently, a model is proposed, exhibiting a non-Abelian topological mass generation mechanism in $D=3$, that provides mass for the Kalb-Ramond field. The covariant quantization of this model requires ghosts for ghosts. Therefore, in order to quantize the theory, we construct a complete set of Becchi-Rouet-Stora-Tyutin (BRST) and anti-BRST equations using the horizontality condition.

DOI: 10.1103/PhysRevD.63.127702 PACS number(s): 11.10.Kk, 12.90.+b

Antisymmetric tensor gauge fields provide a natural extension of the usual vector gauge fields, appearing as a mediator of string interaction and having an important key role in supergravity. Also, they are fundamental to the well-known topological mass generation mechanism [1] for Abelian vector bosons in four dimensions, through a BF term [2]. This term is characterized by the presence of an antisymmetric gauge field $B_{\mu\nu}$ (Kalb-Ramond field) and the field strength $F_{\mu\nu}$. Non-Abelian extensions of models involving antisymmetric gauge fields in four-dimensional space-time were introduced first by Lahiri [3,4] and later by Hwang and Lee [5], in the context of topological mass generation models. Both procedures require the introduction of an auxiliary vector field, justified by the need to untie the constraint between two and three form curvatures F and H, and to the best of our knowledge, that is the first approach in the literature to considering invariant non-Abelian field strength for an antisymmetric tensor gauge field. A non-Abelian theory involving an antisymmetric tensor field coupled to a gauge field, appears as an alternative mechanism for generating vector bosons masses, similar to the theory of a heavy Higgs particle. It is worth mentioning a generalization to a compact non-Abelian gauge group of an Abelian mechanism in the context of non-Abelian quantum hair on black holes [6].

Kalb-Ramond fields arise naturally in string coupled to the area element of the two-dimensional worldsheet [7] and a string Higgs mechanism was introduced by Rey in Ref. [8].

Recently, we have shown a topological mass generation in an Abelian three-dimensional model involving a two form gauge field $B_{\mu\nu}$ and a scalar field φ, rather than the usual Maxwell-Chern-Simons model [9,10]. The action for the model just mentioned reads as

$$S^A_{inv}=\int d^3x\left(\frac{1}{12}H_{\mu\nu\alpha}H^{\mu\nu\alpha}+\frac{1}{2}\partial_\mu\varphi\partial^\mu\varphi+\frac{m}{2}\epsilon^{\mu\nu\alpha}B_{\mu\nu}\partial_\alpha\varphi\right), \tag{1}$$

where $H_{\mu\nu\alpha}$ is the totally antisymmetric tensor $H_{\mu\nu\alpha}=\partial_\mu B_{\nu\alpha}+\partial_\alpha B_{\mu\nu}+\partial_\nu B_{\alpha\mu}$.

The action (1) is invariant under the transformation

$$\delta\varphi=0,\quad \delta B_{\mu\nu}=\partial_{[\mu}\omega_{\nu]}, \tag{2}$$

and its equations of motion give the massive equations

$$(\Box+m^2)\partial_\mu\varphi=0,\quad (\Box+m^2)H_{\mu\nu\alpha}=0. \tag{3}$$

The model described by action (1) can be consistently obtained by dimensional reduction of a four-dimensional $B\wedge F$ model if we discard the Chern-Simons-like terms [9].

The purpose of this paper is to construct a non-Abelian version of the action (1). The only possibility is via an introduction of an auxiliary vector field, as we have proved in Ref. [11], using the method of consistent deformations. We obtain here the Becchi-Rouet-Stora-Tyutin (BRST) and anti-BRST equations by applying the horizontality condition, including an auxiliary vectorial field, which allows the sought non-Abelian generalization. In this way, the need to add an auxiliary field is put in more rigorous grounds. In addition, we show a non-Abelian topological mass generation mechanism for the Kalb-Ramond field in three dimensions.

It is interesting to remark that the introduction of a one form gauge connection A is required to go further in the non-Abelian generalization of our model (1), although our original Abelian action (1) does not contain this field. Note that, as pointed out by Thierry-Mieg and Ne'eman [12] for the non-Abelian case, the field strength for B is[1]

$$H=dB+[A,B]\equiv DB, \tag{4}$$

where $d=dx^\mu(\partial/\partial x^\mu)$ is the exterior derivative.

Resorting to Ref. [12], we can define a new $\mathcal{H}$ given by

$$\mathcal{H}=dB+[A,B]+[F,C], \tag{5}$$

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[1]Here and in the rest of the paper, in order to handle BRST transformations, we use differential forms formalism for convenience.

where C is the one form auxiliary field required and $F = dA + A \wedge A$.

The obstruction to the non-Abelian generalization lies only on the kinetic term for the antisymmetric field, but the topological term must be conveniently redefined. So the non-Abelian version of the action (1) can be written as

$$\int_{M_3} \mathrm{Tr}\left\{\frac{1}{2}\mathcal{H}\wedge{}^*\mathcal{H} + m\mathcal{H}\wedge\varphi + \frac{1}{2}D\varphi\wedge{}^*D\varphi\right\}, \tag{6}$$

where * is the Hodge star operator.

The action above is invariant under the following transformations:

$$\delta A = -D\theta, \quad \delta\varphi = [\theta,\varphi], \quad \delta B = D\Lambda + [\theta,B],$$
$$\delta C = \Lambda + [\theta, C], \tag{7}$$

where θ and Λ are zero and one form transformation parameters, respectively.

Here we shall use a formalism developed by Thierry-Mieg *et al.* [12,13] in order to obtain the BRST and anti-BRST transformation rules. In general lines, we closely follow the treatment of Ref. [12] or [5], since the new object introduced here, namely, the scalar field, does not modify the approach.

The presence of a scalar field in topological invariants is not so uncommon. A three-dimensional Yang-Mills topological action was proposed by Baulieu and Grossman [14] for magnetic monopoles by gauge fixing the following topological invariant:

$$S_{top} = \int_{M_3} \mathrm{Tr}\{F\wedge D\varphi\}. \tag{8}$$

In the work of Thierry-Mieg and Ne'eman [12], a geometrical BRST quantization scheme was developed where the base space is extended to a second fiber-bundle space so that it contains unphysical (fiber-gauge orbit) directions and physical (space-time) directions. Using a double fiber-bundle structure, Quiros *et al.* [15] extended the principal fiber-bundle formalism in order to include anti-BRST symmetry. Basically the procedure consists in extending the space-time to take into account a pair of scalar anticommuting coordinates denoted by y and $\bar{y}$, which correspond to coordinates in the directions of the gauge group of the principal fiber bundle. Then the so-called ''horizontality condition'' is imposed. This condition enforces the curvature components containing vertical (fiber) directions to vanish. Only the horizontal components of physical curvature in the extended space survive.

Let us define the following form fields in the extended space and valued in the Lie algebra $\mathcal{G}$ of the gauge group:

$$\tilde{\varphi} = \varphi, \tag{9}$$

$$\tilde{A} \equiv A_\mu dx^\mu + A_N dy^N + A_{\bar{N}} d\bar{y}^{\bar{N}} \equiv A + \alpha + \bar{\alpha}, \tag{10}$$

$$\tilde{B} \equiv \tfrac{1}{2}B_{\mu\nu}dx^\mu\wedge dx^\nu + B_{\mu N}dx^\mu\wedge dy^N + B_{\mu\bar{N}}dx^\mu\wedge d\bar{y}^{\bar{N}} + \tfrac{1}{2}B_{MN}dy^M\wedge dy^N + B_{M\bar{N}}dy^M\wedge d\bar{y}^{\bar{N}} + \tfrac{1}{2}B_{\bar{M}\bar{N}}d\bar{y}^{\bar{M}}\wedge d\bar{y}^{\bar{N}}$$
$$\equiv B - \beta - \bar{\beta} + \gamma + h + \bar{\gamma}, \tag{11}$$

and

$$\tilde{C} \equiv C_\mu dx^\mu + C_N dy^N + C_{\bar{N}} d\bar{y}^{\bar{N}} \equiv C + c + \bar{c}. \tag{12}$$

Note that we identify the components in unphysical directions with new fields, namely, α, β, and c ($\bar{\alpha}$, $\bar{\beta}$, and $\bar{c}$) as anticommuting ghosts (antighosts) and the commuting ghosts (antighost) γ and h ($\bar{\gamma}$). Since B has three degrees of freedom in three dimensions, the number of DOF described by the set of fields B, β, $\bar{\beta}$, γ, $\bar{\gamma}$, and h is 3-3-3+1+1+1=0. Obviously, the scalar field φ has only one DOF.

The curvatures 2 form $\tilde{F}$ and 3 form $\tilde{\mathcal{H}}$ in the fiber-bundle space are

$$\tilde{F} \equiv \tilde{d}\tilde{A} + \tilde{A}\wedge\tilde{A}, \tag{13}$$

and

$$\tilde{\mathcal{H}} \equiv \tilde{d}\tilde{B} + [\tilde{A},\tilde{B}] + [\tilde{F},\tilde{C}], \tag{14}$$

where $\tilde{d} = d + s + \bar{s}$. The exterior derivatives in the gauge group directions are denoted by $s = dy^N(\partial/\partial y^N)$ and $\bar{s} = d\bar{y}^{\bar{N}}(\partial/\partial\bar{y}^{\bar{N}})$.

It is important to remark here that since we are focusing a mass generation mechanism or, in other words, the action (6), the extra symmetries that appear in the pure topological model have no room in the present discussion.

The horizontality condition, or equivalently, the Maurer-Cartan equation for the field strength F, can be written as

$$\tilde{F} \equiv \tilde{d}\tilde{A} + \tilde{A}\wedge\tilde{A} = F, \tag{15}$$

and for the three form $\mathcal{H}$ is

$$\tilde{\mathcal{H}} \equiv \tilde{d}\tilde{B} + [\tilde{A},\tilde{B}] + [\tilde{F},\tilde{C}] = \mathcal{H}. \tag{16}$$

Also we can impose the horizontality condition for the one form $D\varphi$, which may be written as

$$\tilde{D}\tilde{\varphi} = \tilde{d}\varphi + [\tilde{A},\varphi] = D\varphi. \tag{17}$$

By expanding both sides of Eq. (15) over the pairs of two forms, one can obtain the following transformation rules:

$$sA_\mu = D_\mu\alpha, \quad \bar{s}A_\mu = D_\mu\bar{\alpha},$$
$$s\alpha = -\alpha\wedge\alpha, \quad \overline{s\alpha} = -\bar{\alpha}\wedge\bar{\alpha}, \tag{18}$$
$$s\bar{\alpha} + \bar{s}\alpha = -\alpha\wedge\bar{\alpha}.$$

In order to close the algebra, we introduce an extra scalar commuting field, b valued in the Lie algebra $\mathcal{G}$, such that

$$s\bar{\alpha}=b, \tag{19}$$

and consequently

$$\bar{s}\alpha=-b-\bar{\alpha}\wedge\alpha,\quad \bar{s}b=-\bar{\alpha}\wedge b,\quad sb=0. \tag{20}$$

On the other hand, expanding Eq. (16) over the basis of three forms yields

$$
\begin{aligned}
&sB_{\mu\nu}=-[\alpha,B_{\mu\nu}]-D_{[\mu}\beta_{\nu]}+[F_{\mu\nu},c],\\
&\bar{s}B_{\mu\nu}=-[\bar{\alpha},B_{\mu\nu}]-D_{[\mu}\bar{\beta}_{\nu]}-[F_{\mu\nu},\bar{c}],\\
&s\beta_\mu=-[\alpha,\beta_\mu]+D_\mu\gamma,\quad \overline{s\beta_\mu}=-[\bar{\alpha},\bar{\beta}_\mu]\\
&\quad +D_\mu\bar{\gamma},\\
&s\bar{\beta}_\mu+\bar{s}\beta_\mu=-[\alpha,\bar{\beta}_\mu]-[\bar{\alpha},\beta_\mu]+D_\mu h,\\
&s\gamma=-[\alpha,\gamma],\quad \overline{s\gamma}=-[\bar{\alpha},\bar{\gamma}],\\
&\bar{s}\gamma+sh=-[\alpha,h]-[\bar{\alpha},\gamma],\quad s\bar{\gamma}+\bar{s}h=-[\bar{\alpha},h]\\
&\quad -[\alpha,\bar{\gamma}].
\end{aligned}
\tag{21}
$$

Note that when we treat two odd forms, the [,] must be reading as an anticommutator.

The action of s and $\bar{s}$ upon c, $\bar{c}$, and C is not defined in Eqs. (21). However, condition (16) leads us to

$$\tilde{B}+\tilde{D}\tilde{C}=B+DC. \tag{22}$$

Condition (22) yields the BRST and anti-BRST transformations for the auxiliary field C and its ghosts c and $\bar{c}$:

$$
\begin{aligned}
&sC_\mu=-[\alpha,C_\mu]+D_\mu c+\beta_\mu,\\
&\bar{s}C_\mu=-[\bar{\alpha},C_\mu]+D_\mu\bar{c}+\bar{\beta}_\mu,\\
&sc=-[\alpha,c]-\gamma,\quad \overline{sc}=-[\bar{\alpha},\bar{c}]-\bar{\gamma},\\
&s\bar{c}+\bar{s}c=-[\bar{\alpha},c]-[\alpha,\bar{c}]-h.
\end{aligned}
\tag{23}
$$

However, as usual, the action of s and $\bar{s}$ on the ghosts and antighosts is not completely specified by Eqs. (21) and (23). Therefore, a set of additional fields is required, namely, a commuting vector field t_μ, two anticommuting scalar fields ω and $\bar{\omega}$, and a commuting scalar field n. These fields are used to solve Eqs. (21). Then, we get

$$
\begin{aligned}
&s\bar{\beta}_\mu=t_\mu,\quad \bar{s}\beta_\mu=-t_\mu-[\alpha,\bar{\beta}_\mu]-[\bar{\alpha},\beta_\mu]+D_\mu h,\\
&sh=\omega,\quad \bar{s}\gamma=-\omega-[\alpha,h]-[\bar{\alpha},\gamma],\\
&s\bar{\gamma}=\bar{\omega},\quad \bar{s}h=-\bar{\omega}-[\alpha,\bar{\gamma}]-[\bar{\alpha},h],\\
&s\bar{c}=n,\quad \bar{s}c=-n-[\alpha,\bar{c}]-[\bar{\alpha},c]-h,\\
&st_\mu=s\omega=s\bar{\omega}=sn=0,\\
&\bar{s}t_\mu=-[\bar{\alpha},t_\mu]-[D_\mu\alpha,\bar{\gamma}]-D_\mu\bar{\omega}-[\bar{\beta}_\mu,t],\\
&\bar{s}n=-[\bar{\alpha},n]-[\bar{c},b]+\bar{\omega},\\
&\bar{s}\omega=-[\bar{\alpha},\omega]-[\alpha\alpha,\bar{\gamma}]-[\alpha,\bar{\omega}]-[h,b],\\
&\overline{s\omega}=-[\bar{\alpha},\bar{\omega}]-[\bar{\gamma},b].
\end{aligned}
\tag{24}
$$

The nilpotency of the s and $\bar{s}$ operators was used to obtain the last eight relations.

Finally, by expanding Eq. (17), we obtain

$$s\varphi=[\alpha,\varphi],\bar{s}\varphi=[\bar{\alpha},\varphi]. \tag{25}$$

Therefore, a complete set of BRST and anti-BRST equations, namely, Eqs. (18)–(20), (23)–(25), and (21), associated with the classical symmetry defined by Eq. (7), was obtained.

It is important to point out the difference between the fields that do not belong to the principal fiber-bundle expansion of the ''physical'' fields (b,t_μ,n,ω, and $\bar{\omega}$) (introduced in order to complete the BRST and anti-BRST algebra) and the auxiliary one form field C introduced in order to overcome the obstruction to the non-Abelian generalization. Note that here the *a priori* introduction of the auxiliary field C was necessary in order to fix the BRST and anti-BRST transformation rules. Furthermore, the obstruction to non-Abelian generalization of the four-dimensional BF model, namely, the existence of the constraint $[F,{}^*H]=0$, appears in the context of our model as $[F,{}^*H-m\varphi]=0$, as can be seen from the equations of motion of the action (6), considered in the absence of the auxiliary field.

The simplest scenario to study mass generation is to consider the equations of motion of the action (6). For convenience, we define a new one form field as $K\equiv D\varphi$. Therefore, the equations of motion can be written as

$$D^*\mathcal{H}=mK,\quad D^*K=-m\mathcal{H}. \tag{26}$$

Equations (26) can be combined into the following second-order equations:

$$(D^*D^*+m^2)\mathcal{H}=0,\quad (D^*D^*+m^2)K=0. \tag{27}$$

Considering only linear terms for the fields, we get

$$(d^*d^*+m^2)H=0,\quad (d^*d^*+m^2)d\varphi=0, \tag{28}$$

which are similar to Eqs. (3), and exhibit mass generation for H and φ.

On the other hand, by looking to the pole structures of the propagators of the model, mass generation can also be established. In order to obtain them, we use the action (6) added with convenient gauge fixing terms, namely,

$$S_T=\int_{M_3}\mathrm{Tr}\Big\{\frac{1}{2}\mathcal{H}\wedge{}^*\mathcal{H}+m\mathcal{H}\wedge\varphi+\frac{1}{2}D\varphi\wedge{}^*D\varphi+\mathcal{J}\wedge{}^*B+j\wedge{}^*\varphi+J\wedge{}^*M+J_p\wedge{}^*p+p\wedge{}^*dM+M\wedge{}^*dB\Big\}, \tag{29}$$

where $\mathcal{J}$, J, J_p, and j are currents related to the fields B, M, p, and φ, respectively, which generate propagators in the path-integral formulation. The Lagrange multiplier fields M and p are introduced in order to implement the Landau gauge fixing.

Therefore, the tree-level effective propagators for the Kalb-Ramond and scalar fields are

$$\langle \varphi\varphi\rangle_{a,b} = -\frac{\delta_{ab}}{p^2-m^2} \tag{30}$$

and

$$\langle BB\rangle_{a\mu\nu,b\rho\sigma} = \frac{\delta_{ab}}{p^2-m^2}\left[g_{\mu[\rho}g_{\sigma]\nu} - \frac{g_{\mu[\rho}p_{\sigma]}p_\nu}{p^2} + \frac{g_{\nu[\rho}p_{\sigma]}p_\mu}{p^2}\right], \tag{31}$$

where a and b are group indices, and μ,ν,ρ, and σ are space-time indices. It is interesting to note that, here, the gauge field B absorbs the scalar field (not a Higgs field, however) and acquires a longitudinal degree of freedom and a mass. The inverse process is possible too.

In this paper we have succeeded in extending a tridimensional Abelian topological model to the non-Abelian case. The model considered here couples a second-rank antisymmetric tensor field and a scalar field in a topological way. We introduce two new fields in the model in order to obtain the pursued non-Abelian version. One field is a one form gauge connection (A) which allows us to define a Yang-Mills covariant derivative. The other auxiliary field (C) is a vectorial one, which is required in order to resolve the constraint that prevents the correct non-Abelianization.

A formal framework to consider the introduction of these fields and the consequent new symmetries is furnished by BRST and anti-BRST transformation rules, which are obtained using the horizontality condition. Although quite similar to other topological models, it is worth mentioning that, in this case, we have constructed transformation rules for the Kalb-Ramond field, for two one form fields and for a scalar field.

Here it is worthwhile to mention that a similar mass generation model was presented by Jackiw and Pi in Ref. [16], where a non-Abelian version of the mixed Chern-Simons term was considered. However, a one form field was declared to carry odd parity, so preserving the parity of the model. Besides, due to the parity constraint, the Jackiw-Pi model has less gauge symmetries than ours, and in these two features reside the essential difference between the models considered. Furthermore, once Jackiw and Pi change the two form field by a one form field, they do not have the obstruction to non-Abelianization of the kinetic term detected by Lahiri (four-dimensional case) and us (three-dimensional case).

Finally, the topological mass generation mechanism for an Abelian model found in a previous paper was extended for the non-Abelian case, and we end up with an effective theory describing massive Kalb-Ramond gauge fields in $D=3$ space-time.

We conclude by mentioning the possible relevance of the present discussion to string theory. Indeed, the Kalb-Ramond field couples directly to the worldsheet of strings, and bosonic string condensation into the vacuum realize the Higgs mechanism to the Kalb-Ramond gauge field [8]. Therefore, an alternative scenario to give mass to the Kalb-Ramond field in the context of strings may be an interesting continuation of our present results.

This work was supported in part by Conselho Nacional de Desenvolvimento Científico e Tecnológico-CNPq and Fundação Cearense de Amparo à Pesquisa-FUNCAP.

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