Supersymmetric Randall-Sundrum scenario

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(Received 23 May 2000; published 24 May 2001)

We present the supersymmetric version of the minimal Randall-Sundrum model with two opposite tension branes.

DOI: 10.1103/PhysRevD.63.125025 PACS number(s): 12.60.Jv, 04.65.+e

I. INTRODUCTION

The two-brane Randall-Sundrum scenario [1] provides an appealing way to generate the electroweak gauge hierarchy as a consequence of spacetime geometry. The basic idea is to start with five dimensional anti–de Sitter (AdS) space, take the region between two slices parallel to the AdS horizon, and add a 3-brane along each slice. By tuning the brane tensions, the resulting configuration can be made stable against gravitational collapse.

In this model, the ratio of the weak to the Planck scale is determined by the distance between the two branes. The distance is fixed by the expectation value of a modulus field, called the radion. The usual hierarchy problem now appears in a new guise: What fixes the radion vacuum expectation valve (VEV), and what protects the VEV against large radiative corrections?

In a recent paper, Goldberger and Wise [2] proposed a way to stabilize the radion using five dimensional bulk matter. Supersymmetry provides another possibility. In this paper we will take some first steps in that direction, and show how to supersymmetrize the minimal Randall-Sundrum scenario.

In what follows we will use coordinates in which the five dimensional background metric takes the following form:

$$ds^2=e^{-2\sigma(\phi)}\eta_{mn}dx^m dx^n+r^2 d\phi^2. \tag{1}$$

The coordinate $x^5=r\phi$ parametrizes an orbifold S^1/Z_2, where the circle S^1 has radius r and the orbifold identification is $\phi\leftrightarrow-\phi$. For fixed ϕ, the coordinates x^m $(m=0,1,2,3)$ span flat Minkowski space, with metric $\eta_{mn}=\mathrm{diag}(-1,1,1,1)$. We choose to work on the orbifold covering space, so we take $-\pi<\phi\leqslant\pi$.

For the gravitational part of our action, we follow Randall and Sundrum and take the action to be the sum of bulk plus brane pieces,

$$S=S_{\text{bulk}}+S_{\text{brane}}. \tag{2}$$

The bulk action is that of pure five dimensional AdS gravity, while S_{brane} arises from the presence of two opposite tension branes.

The gravitational bulk action is given by

$$S_{\text{bulk}}=\frac{\Lambda}{\kappa^2}\int d^5x\,e\left[-\frac{1}{2}R+6\Lambda^2\right], \tag{3}$$

where κ is related to the four dimensional Planck constant, $e=\det e_M{}^A$, and $e_M{}^A$ is the five dimensional fünfbein.[1] In this expression, Λ is the bulk cosmological constant and R is the five dimensional Ricci scalar,

$$R=e_A{}^M e_B{}^N R_{MN}{}^{AB}. \tag{4}$$

The Riemann curvature $R_{MN}{}^{AB}$ is built from the spin connection according to the following conventions:

$$R_{MN}{}^{AB}=\partial_M\omega_N{}^{AB}-\partial_N\omega_M{}^{AB}-\omega_M{}^{AC}\omega_{NC}{}^B+\omega_N{}^{AC}\omega_{MC}{}^B. \tag{5}$$

The brane action serves as a source for the bulk gravitational fields. It arises from the 3-branes located at the orbifold points $\phi=0,\pi$. For the case at hand, the brane action is simply

$$S_{\text{brane}}=-6\frac{\Lambda^2}{r\kappa^2}\int d^5x\,\hat{e}[\delta(\phi)-\delta(\phi-\pi)], \tag{6}$$

where $\hat{e}=\det e_m{}^a$, and the $e_m{}^a$ are the components of the five dimensional fünfbein, restricted to the appropriate brane.

From this action it is not hard to show that the metric (1), with

$$\sigma(\phi)=r\Lambda|\phi|, \tag{7}$$

is a solution to the five dimensional Einstein equations,

$$R_{MN}-\frac{1}{2}g_{MN}R=-6g_{MN}\Lambda^2+6g_{mn}\delta_M^m\delta_N^n\left(\frac{\Lambda}{r}\right)\left(\frac{\hat{e}}{e}\right)\times[\delta(\phi)-\delta(\phi-\pi)]. \tag{8}$$

[1] We adopt the convention that capital letters run over the set {0,1,2,3,5} and lower-case letters run from 0 to 3. Tangent space indices are taken from the beginning of the alphabet; coordinate indices are from the middle. We follow the conventions of [3].

Away from the branes, the bulk metric is just that of five dimensional AdS space, with cosmological constant Λ. On the branes, the four dimensional metric is flat. As shown in [1], the effective theory of the gravitational zero modes is just ordinary four dimensional Einstein gravity, with a vanishing cosmological constant. The effective four dimensional squared Planck mass is $\kappa_{\rm eff}^{-2}=\kappa^{-2}(1-e^{-2\pi r\Lambda})$.

II. SUPERSYMMETRIC BULK

In what follows we will supersymmetrize the action (2). We start with the bulk action (3). Its supersymmetric extension can be found from the five dimensional supersymmetric AdS action [4]

$$\begin{aligned}S_{\rm bulk}=\Lambda\int d^5xe\Bigg[&-\frac{1}{2\kappa^2}R+{\rm i}\epsilon^{MNOPQ}\overline{\Psi}_M\Sigma_{NO}D_P\Psi_Q\\&-\frac{1}{4}F_{MN}F^{MN}-3\Lambda\overline{\Psi}_M\Sigma^{MN}\Psi_N+6\frac{\Lambda^2}{\kappa^2}\\&-{\rm i}\kappa\sqrt{\frac{3}{2}}\frac{1}{2}F_{MN}\overline{\Psi}^M\Psi^N-\kappa\frac{1}{6\sqrt{6}}\epsilon^{MNOPQ}F_{MN}F_{OP}B_Q\\&+{\rm i}\kappa\sqrt{\frac{3}{2}}\frac{1}{4}\epsilon^{MNOPQ}F_{MN}\overline{\Psi}_O\Gamma_P\Psi_Q\\&-\kappa\Lambda\sqrt{\frac{3}{2}}\epsilon^{MNOPQ}\overline{\Psi}_M\Sigma_{NO}\Psi_PB_Q\\&+\text{four-Fermi terms}\Bigg],\end{aligned}\tag{9}$$

where the ϵ tensor is defined to have tangent-space indices, and $\epsilon^{01235}=1$. This action contains the physical fields associated with the supergravity multiplet in five dimensions: the fünfbein $e_M{}^A$, the gravitino Ψ_M, and a vector field B_M. The covariant derivative $D_M\Psi_N=\partial_M\Psi_N+\frac{1}{2}\Sigma^{AB}\omega_{MAB}\Psi_N$ and the matrix $\Sigma^{AB}=\frac{1}{4}(\Gamma^A\Gamma^B-\Gamma^B\Gamma^A)$.

This action is invariant under the following supersymmetry transformations:

$$\begin{aligned}\delta e_M{}^A&={\rm i}\kappa(\overline{\eta}\Gamma^A\Psi_M-\overline{\psi}_M\Gamma^A\eta)\\ \delta B_M&=-{\rm i}\sqrt{\frac{3}{2}}(\overline{\eta}\psi_M-\overline{\psi}_M\eta)\\ \delta\psi_M&=\frac{2}{\kappa}D_M\eta+{\rm i}\frac{\Lambda}{\kappa}\Gamma_M\eta-{\rm i}\sqrt{6}\Lambda B_M\eta\\&\quad-\sqrt{\frac{2}{3}}\Bigg(\Gamma^NF_{NM}\\&\quad-\frac{1}{4}\epsilon_{MNOPQ}F^{NO}\Sigma^{PQ}\Bigg)\eta\\&\quad+\text{three-Fermi terms.}\end{aligned}\tag{10}$$

Since we work in the orbifold covering space, the spacetime manifold has no boundary, and we can freely integrate by parts. We use the 1.5 order formalism, so the spin connection obeys its own equation of motion and does not need to be varied.

For the case at hand, we must define the action of the orbifold symmetry on the AdS fields. We start by writing the five dimensional spinors in a four dimensional language, where

$$\Psi_M\to\begin{pmatrix}\psi^1_{M\alpha}\\ \overline{\psi}^{\dot\alpha}_{2M}\end{pmatrix}\tag{11}$$

and

$$\Gamma^a\to\begin{pmatrix}0&\sigma^a_{\alpha\dot\alpha}\\ \overline{\sigma}^{a\dot\alpha\alpha}&0\end{pmatrix}\qquad\Gamma^5\to\begin{pmatrix}-{\rm i}&0\\0&{\rm i}\end{pmatrix}.\tag{12}$$

The fields ψ^i_M (for $i=1,2$) are two-component Weyl spinors, in the notation of [3]. We then define $\psi^{\pm}_M=1/\sqrt{2}(\psi^1_M\pm\psi^2_M)$, and likewise for $\eta^{\pm}$.

In terms of these fields, the bulk supersymmetry transformations can be written in the following form:

$$\begin{aligned}\delta e_M{}^a&={\rm i}\kappa(\eta^+\sigma^a\overline{\psi}^+_M+\eta^-\sigma^a\overline{\psi}^-_M)+\text{H.c.}\\ \delta e_M{}^{\hat 5}&=\kappa(\eta^+\psi^-_M-\eta^-\psi^+_M)+\text{H.c.}\\ \delta B_M&=-{\rm i}\sqrt{\frac{3}{2}}(\eta^+\psi^-_M-\eta^-\psi^+_M)+\text{H.c.}\\ \delta\psi^{\pm}_m&=\frac{2}{\kappa}D_m\eta^{\pm}\mp\frac{{\rm i}}{\kappa}\omega_{ma\hat 5}\sigma^a\overline{\eta}^{\mp}\pm{\rm i}\frac{\Lambda}{\kappa}e_m{}^a\sigma_a\overline{\eta}^{\pm}\\&\quad+e_{m\hat 5}\frac{\Lambda}{\kappa}\eta^{\mp}-{\rm i}\sqrt{6}\Lambda B_m\eta^{\mp}\\&\quad-\sqrt{\frac{2}{3}}\Bigg(\mp e_a{}^NF_{Nm}\sigma^a\overline{\eta}^{\mp}-{\rm i}e_{\hat 5}{}^NF_{Nm}\eta^{\pm}\\&\quad-\frac{1}{4}\epsilon_{ABCde}e_m{}^Ae^{BN}e^{CO}F_{NO}\sigma^{de}\eta^{\pm}\\&\quad\pm\frac{{\rm i}}{4}\epsilon_{abcd}e_m{}^ae^{bN}e^{cO}F_{NO}\sigma^d\overline{\eta}^{\mp}\Bigg)\\ \delta\psi^{\pm}_5&=\frac{2}{\kappa}D_5\eta^{\pm}\mp\frac{{\rm i}}{\kappa}\omega_{5a\hat 5}\sigma^a\overline{\eta}^{\mp}+e_{5\hat 5}\frac{\Lambda}{\kappa}\eta^{\mp}\\&\quad\pm{\rm i}e_5{}^a\frac{\Lambda}{\kappa}\sigma_a\overline{\eta}^{\pm}-{\rm i}\sqrt{6}\Lambda B_5\eta^{\mp}\\&\quad+\sqrt{\frac{2}{3}}\Bigg(\mp e_a{}^nF_{5n}\sigma^a\overline{\eta}^{\mp}-{\rm i}e_{\hat 5}{}^nF_{5n}\eta^{\pm}\\&\quad+\frac{1}{4}\epsilon_{ABCde}e_5{}^Ae^{BN}e^{CO}F_{NO}\sigma^{de}\eta^{\pm}\\&\quad\pm\frac{{\rm i}}{4}\epsilon_{abcd}e_5{}^ae^{bN}e^{cO}F_{NO}\sigma^d\overline{\eta}^{\mp}\Bigg).\end{aligned}\tag{13}$$

In these expressions, all fields depend on the five dimensional coordinates. The symbol $\hat{5}$ denotes the fifth tangent space index, and all covariant derivatives contain the spin connection ω_{Mab}. Here and hereafter, we ignore all three- and four-Fermi terms.

From these transformations it is not hard to find a consistent set of Z_2 parity assignments under the orbifold transformation $\phi \to -\phi$. The assignments must leave the action and transformation laws invariant under the Z_2 symmetry. We assign even parity to

$$e_m{}^a, \quad e_{5\hat{5}}, \quad B_5, \quad \psi_m^+, \quad \psi_5^-, \quad \eta^+$$

and odd parity to

$$e_5{}^a, \quad e_m{}^{\hat{5}}, \quad B_m, \quad \psi_m^-, \quad \psi_5^+, \quad \eta^-.$$

The bulk supergravity action is invariant under $N=1$ supersymmetry in five dimensions. The branes break all but one four dimensional supersymmetry. To find its form, we shall study the supersymmetry transformations in the orbifold background, where $e_{5\hat{5}}=1$, $e_m{}^a = e^{-\sigma(\phi)}\delta_m^a$, and all other fields equal zero. This configuration satisfies the gravitational equations of motion when $\sigma(\phi)=r\Lambda|\phi|$. Note that this background is consistent with the orbifold symmetry.

In the orbifold background, the supersymmetry variations of the bosonic fields are obviously zero. The variations of the fermions are a little trickier. In this background, the spin connection evaluates to

$$\omega_{mAM}\Sigma^{AM} = \mathrm{sgn}(\phi)\Lambda\Gamma_m\Gamma^{\hat{5}}, \tag{14}$$

with all other components zero. The supersymmetry variations of the fermions reduce to the following form:

$$\delta\psi_m^{\pm} = \frac{2}{\kappa}\partial_m\eta^{\pm} \mp \mathrm{i}\,\mathrm{sgn}(\phi)\frac{\Lambda}{\kappa}\sigma_m\overline{\eta^{\mp}} \pm \mathrm{i}\frac{\Lambda}{\kappa}\sigma_m\overline{\eta^{\pm}}$$

$$\delta\psi_5^{\pm} = \frac{2}{\kappa}\partial_5\eta^{\pm} + \frac{\Lambda}{\kappa}\eta^{\mp}. \tag{15}$$

The unbroken supersymmetries are found by setting these variations to zero. The resulting Killing equations can then be solved for the Killing spinors $\eta^{\pm}$. The solution that reduces to a flat-space supersymmetry in four dimensions is simply

$$\eta^+ = \frac{1}{\sqrt{2}}e^{-\sigma(\phi)/2}\eta(x), \qquad \eta^- = \frac{1}{\sqrt{2}}e^{-\sigma(\phi)/2}\mathrm{sgn}(\phi)\,\eta(x), \tag{16}$$

where $\mathrm{sgn}(\phi)$ is the step function,[2] which evaluates to $(-1,0,1)$, depending on the sign of ϕ. In this expression, the spinor η contains four Grassmann components and is a function of $x^0,\ldots,x^3$, but not x^5. We shall see that it describes the one unbroken supersymmetry of the Randall-Sundrum scenario.

It is not hard to check that the spinors (16) are a solution to the Killing equations, for constant η, except for delta-function singularities at the orbifold points $\phi=0,\pi$. These singularities are very important. They motivate us to change the ψ_5^- supersymmetry transformation so that the spinors (16) are Killing spinors everywhere. We take

$$\begin{aligned}
\delta\psi_5^- ={}& \frac{2}{\kappa}D_5\eta^- + \frac{\mathrm{i}}{\kappa}\omega_{5a\hat{5}}\sigma^a\overline{\eta^+} + e_{5\hat{5}}\frac{\Lambda}{\kappa}\eta^+ - \mathrm{i}e_5{}^a\frac{\Lambda}{\kappa}\sigma_a\overline{\eta^-}\\
&- \mathrm{i}\sqrt{6}\Lambda B_5\eta^+ + \sqrt{\frac{2}{3}}\bigg(e_a{}^nF_{5n}\sigma^a\overline{\eta^+} - \mathrm{i}e_{\hat{5}}{}^nF_{5n}\eta^-\\
&+ \frac{1}{4}\epsilon_{ABCde}e_5{}^Ae^{BN}e^{CO}F_{NO}\sigma^{de}\eta^-\\
&- \frac{\mathrm{i}}{4}\epsilon_{abcd}e_5{}^ae^{bN}e^{cO}F_{NO}\sigma^d\overline{\eta^+}\bigg)\\
&- \frac{4}{r\kappa}[\delta(\phi)-\delta(\phi-\pi)]\eta^+.
\end{aligned} \tag{17}$$

In the orbifold background, this reduces to

$$\delta\psi_5^- = \frac{2}{\kappa}\partial_5\eta^- + \frac{\Lambda}{\kappa}\eta^+ - \frac{4}{r\kappa}[\delta(\phi)-\delta(\phi-\pi)]\eta^+. \tag{18}$$

The spinors (16) satisfy the modified Killing equations, for constant η, even at the orbifold points $\phi=0,\pi$. Furthermore, the supersymmetry transformations still close into the $N=1$ supersymmetry algebra.

III. SUPERSYMMETRIC BRANE

In the previous section, we changed the gravitino supersymmetry transformations so that the Killing spinors satisfy the Killing equations at every point in ϕ. Because of this, the bulk action is no longer invariant under the supersymmetry transformations (17). In this section we will find a brane

[2]The distribution $\mathrm{sgn}(\phi)$ obeys the following properties:

$$\int_{-\epsilon}^{\epsilon} d\phi\,\mathrm{sgn}(\phi)=0, \qquad \int_{-\epsilon}^{\epsilon} d\phi\,\mathrm{sgn}^2(\phi)=2\epsilon,$$

when integrated against smooth functions, and

$$\int_{-\epsilon}^{\epsilon} d\phi\,\mathrm{sgn}(\phi)\,\delta(\phi)=0, \qquad \int_{-\epsilon}^{\epsilon} d\phi\,\mathrm{sgn}^2(\phi)\,\delta(\phi)=\frac{1}{3},$$

when integrated against $\delta(\phi)$. The last relation ensures that

$$\int_{-\epsilon}^{\epsilon} d\phi\,\frac{d}{d\phi}\,\mathrm{sgn}^3(\phi)=2.$$

We thank Jan Conrad for a discussion on this point.

action whose variation precisely cancels that of the bulk.

We first compute the variation of the bulk action. Comparing Eq. (13) with Eq. (17), we see that the bulk variation vanishes except on the branes. Therefore, to compute the variation, we need to project the bulk fields onto the branes. For even fields, this is easy: The brane fields are just the bulk fields evaluated at the appropriate value of ϕ. For odd fields, the situation is more subtle: The brane fields must obey jump conditions across the delta function singularities and these conditions are determined by the brane action.

In what follows we will present the brane action and verify that it restores the supersymmetry of the bulk-plus-brane system. We assert that the brane action is simply

$$S_{\text{brane}}=\frac{\Lambda}{r\kappa^2}\int d^5x\hat{e}(-3\Lambda+2\kappa^2\psi_m^+\sigma^{mn}\psi_n^+)\times[\delta(\phi)-\delta(\phi-\pi)]+\text{H.c.} \tag{19}$$

where the fields $e_m{}^a$ and ψ_m^+ are projections of the corresponding five dimensional fields.

Given this brane action, it is easy to compute the jump conditions. From the equations of motion for $e_m{}^a$ and ψ_m^+, we find

$$[\omega_{ma5}]=\pm 2\Lambda e_{ma}, \quad [\psi_m^-]=\pm 2\psi_m^+, \tag{20}$$

where the square brackets denote the discontinuity across the singularity, and the $\pm$ applies to the brane at $\phi=0$ and π, respectively. A consistent solution is given by

$$\omega_{ma5}=\text{sgn}(\phi)\Lambda e_{ma}, \quad \psi_m^-=\text{sgn}(\phi)\psi_m^+, \tag{21}$$

in the neighborhood of the branes. All other odd fields vanish on the branes.

Now that we have the solutions to the jump conditions, we are free to compute the variation of the bulk action. A small calculation gives

$$\delta S_{\text{bulk}}=\frac{\Lambda}{r\kappa}\int d^5x e e^{\hat{5}5}[(8\eta^+\sigma^{mn}D_m\psi_n^+-i\kappa\sqrt{6}F^{\hat{5}m}\eta^+\psi_m^+ +6i\Lambda(1-\text{sgn}^2(\phi))\eta^+\sigma^m\overline{\psi}_m^+)[\delta(\phi)-\delta(\phi-\pi)]] +\text{H.c.} \tag{22}$$

where η^+ is the spinor (16). In what follows we will show that the variation of the brane action precisely cancels this term.

The supersymmetry variation of the brane action is not hard to find. The supersymmetry transformations are those of the bulk fields, as projected on the branes, subject to the jump conditions (20). From Eqs. (13) and (21), we compute

$$\delta e_m{}^a=i\kappa(1+\text{sgn}^2(\phi))\eta^+\sigma^a\overline{\psi}_m^+ +\text{H.c.}$$

$$\delta\psi_m^+=\frac{2}{\kappa}D_m\eta^+ +i\frac{\Lambda}{\kappa}(1-\text{sgn}^2(\phi))e_m{}^a\sigma_a\overline{\eta}^+ +i\sqrt{\frac{2}{3}}F_{\hat{5}m}\eta^+ +i\sqrt{\frac{2}{3}}F^{\hat{5}n}\sigma_{mn}\eta^+. \tag{23}$$

As above, η^+ is given by Eq. (16). In all fields, the coordinate ϕ is evaluated at $\phi=0$ or π, depending on the location of the brane. Substituting Eq. (23) into Eq. (19), we find

$$\delta S_{\text{brane}}=-\frac{\Lambda}{r\kappa}\int d^5x\hat{e}[(8\eta^+\sigma^{mn}D_m\psi_n^+-i\kappa\sqrt{6}F^{\hat{5}m}\eta^+\psi_m^+ +12i\Lambda\,\text{sgn}^2(\phi)\eta^+\sigma^m\overline{\psi}_m^+)[\delta(\phi)-\delta(\phi-\pi)]] +\text{H.c.} \tag{24}$$

The variation of the brane action, Eq. (24), cancels the variation of the bulk action, Eq. (22), because $e=e_{5\hat{5}}\hat{e}$ and $\text{sgn}^2(\phi)=1/3$ when integrated against a delta function. This proves that the full bulk-plus-brane Randall-Sundrum action is invariant under the four dimensional supersymmetry parametrized by the Killing spinor η in Eq. (16).

IV. MINIMAL EFFECTIVE ACTION

We will now derive the effective four dimensional action for the supergravity zero modes. We will see that it is nothing but the usual on-shell four dimensional flat-space supergravity action.

The zero modes of the four dimensional theory must satisfy the massless equations of motion in four dimensions. For the vierbein, the zero mode was given by Randall and Sundrum [1]:

$$e_M{}^A=\begin{pmatrix} 1 & 0 \\ 0 & e^{-\sigma(\phi)}\overline{e}_m{}^a(x)\end{pmatrix}, \tag{25}$$

where $\sigma(\phi)=r\Lambda|\phi|$ and the vierbein $\overline{e}_m{}^a$ is a function of $x^0,\dots,x^3$, but not x^5. The five dimensional Einstein equations, with brane sources, reduce to the usual four dimensional source-free Einstein equations for the vierbein $\overline{e}_m{}^a$.

The gravitino zero modes can be found in a similar way. One starts with the five dimensional gravitino equations of motion,

$$\partial_5\psi_m^+ +\frac{3}{2}\Lambda\psi_m^- -\text{sgn}(\phi)\Lambda\psi_m^+=0$$

$$\partial_5\psi_m^- +\frac{3}{2}\Lambda\psi_m^+ -\text{sgn}(\phi)\Lambda\psi_m^-=\frac{2}{r}[\delta(\phi)-\delta(\phi-\pi)]\psi_m^+, \tag{26}$$

and assumes the following ansatz:

$$\psi_m^+=\frac{1}{\sqrt{2}}\left(\frac{\kappa_{\text{eff}}}{\kappa}\right)e^{-\sigma(\phi)/2}\psi_m(x),$$

$$\psi_m^- = \frac{1}{\sqrt{2}}\left(\frac{\kappa_{\text{eff}}}{\kappa}\right)e^{-\sigma(\phi)/2}\text{sgn}(\phi)\psi_m(x). \tag{27}$$

Substituting Eq. (27) into Eq. (26), one recovers the usual four dimensional equations of motion for the gravitino field ψ_m.

In what follows, we will derive the effective four dimensional action for the supergravity zero mode fields. We start by setting all other fields to zero. This truncation is consistent with the supersymmetry transformations (10). We then substitute the zero-mode expressions into the supersymmetric bulk-plus-brane action and integrate over the coordinate x^5. We use the fact that

$$R = e^{2\sigma}\overline{R} + 20\Lambda^2 - 16\frac{\Lambda}{r}[\delta(\phi) - \delta(\phi - \pi)] \tag{28}$$

and

$$\omega_{mAB}\Sigma^{AB} = \text{sgn}(\phi)\Lambda\Gamma_m\Gamma^{\hat{5}} + \overline{\omega}_{mab}\sigma^{ab} \tag{29}$$

to find

$$S_{\text{eff}} = \int d^4x\overline{e}\left[-\frac{1}{2\kappa_{\text{eff}}^2}\overline{R} + \epsilon^{mnpq}\overline{\psi}_m\overline{\sigma}_n D_p\psi_q\right], \tag{30}$$

up to four-Fermi terms. This is nothing but the on-shell action for flat-space $N=1$ supergravity in four dimensions.

The supersymmetry transformation laws can be found in a similar way. We start with the supersymmetry transformation parameters η^+ and η^- as above, in Eq. (16). We then substitute the zero mode expressions into the supersymmetry transformations (10). All x^5 dependent terms cancel, leaving

$$\delta e_m{}^a = i\kappa_{\text{eff}}\eta\sigma^a\overline{\psi}_m + \text{H.c.},$$

$$\delta\psi_m = \frac{2}{\kappa_{\text{eff}}}D_m\eta. \tag{31}$$

These are nothing but the transformations of $N=1$ supergravity in four dimensions (up to three-Fermi terms), with an effective four dimensional squared Planck mass, $\kappa_{\text{eff}}^{-2} = \kappa^{-2}(1 - e^{-2\pi r\Lambda})$.

V. SUMMARY AND OUTLOOK

In this paper we supersymmetrized the minimal Randall-Sundrum scenario. We found the supersymmetric bulk-plus-brane action in five dimensions, as well as the corresponding supersymmetry transformations. We solved for the Killing spinor that describes the unbroken $N=1$ supersymmetry of the four dimensional effective theory. We derived the supergravitational zero modes, and showed that the low energy effective theory reduces to ordinary $N=1$ supergravity in four dimensions.

This work represents a first step towards a deeper understanding of supersymmetry in the context of warped compactifications. To study stability, one would like, of course, to include the radion multiplet, which reduces to $N=1$ matter in four dimensions. For phenomenology, one would also like to add supersymmetric matter on the branes and in the bulk. Work along all these lines is in progress.

Note added. On the same day this paper was submitted to the archive, a similar paper was posted by Gherghetta and Pomarol [5]. This paper used an x^5-dependent bulk gravitino mass to supersymmetrize the two-brane Randall-Sundrum scenario. The resulting construction can be interpreted as a truncation of a more fundamental theory with matter in the bulk. We did not take this approach because our goal was to supersymmetrize the purely gravitational case. For more on the difficulties of constructing brane-like solutions in matter-coupled five dimensional supergravity, see [6] and [7].

ACKNOWLEDGMENTS

We are pleased to acknowledge helpful conversations with Jan Conrad, Dan Freedman, Erich Poppitz, Raman Sundrum, and Max Zucker. This work was supported by the National Science Foundation, grant NSF-PHY-9404057, and the U.S. Department of Energy, contract DE-FG03-84ER40168.

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