Mining energy from a black hole by strings

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We discuss how cosmic strings can be used to mine energy from black holes. A string attached to the black hole gives rise to an additional channel for the energy release. It is demonstrated that when a string crosses the event horizon, its transverse degrees of freedom are thermally excited and thermal string perturbations propagate along the string to infinity. The internal metric induced on the 2D world sheet of the static string crossing the horizon describes a 2D black hole. For this reason thermal radiation of string excitations propagating along the string can be interpreted as Hawking radiation of the 2D black hole. It is shown that the rate of energy emission through the string channel is of the same order of magnitude as the bulk radiation of the black hole. Thus, for N_s strings attached to the black hole the efficiency of string channels is increased by factor N_s. We discuss restrictions on N_s which exist because of the finite thickness of strings, the gravitational back reaction and quantum fluctuations. Our conclusion is that the energy emission rate by strings can be increased as compared to the standard emission in the bulk by the factor 10^3 for GUT strings and up to the factor 10^{31} for electroweak strings.

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I. INTRODUCTION

A black hole behaves like a heated body. A black hole of mass M emits thermal radiation with temperature $(8\pi M)^{-1}$. But there exists a special property which singles out a black hole as a thermodynamical object. While the size of a usual radiating body can be arbitrary, the size of the black hole is determined by the same parameter M as its temperature. For a given temperature T the rate of radiation of the heated body, which is proportional to T^4R^2, can be made arbitrarily large by making its size R bigger and bigger. This is not true for a black hole. The rate of energy emission by the black hole is determined only by its mass and is proportional to $M^{-2}\sim T^2$.

Can one increase the rate of radiation of the black hole by an external influence? Unruh and Wald [1,2] demonstrated that in principle this is possible. They proposed a gedanken experiment when a small box with ideally conducting (mirrorlike) boundaries was taken close to black hole surface, opened there, closed again, and taken back to infinity. They demonstrated that in this process energy can be extracted from the black hole in the form of thermal radiation. By repeating this process, one can mine additional energy from a black hole and effectively increase its emission. Unruh and Wald conjectured that the highest possible rate of energy extraction from a black hole can reach $c^5/G\approx 3.6\times 10^{59}$ erg/s [3].

One can describe this process of energy mining in slightly different terms. An effective surface area of the black hole is determined by the position of the potential barrier. This potential barrier does not allow quanta of the thermal atmosphere of the black hole located close to the horizon to escape to the infinity. One can describe the effect of energy mining, at least in a quasistationary regime, by saying that a device which is used for this purpose simply increases the probability of the penetration through the potential barrier of modes in some frequency range. As the result, the effective radiative surface area increases.

In this paper we would like to propose a new mechanism of energy mining from black holes. Namely we propose to use for this purpose *cosmic strings*. We demonstrate that this mechanism is very efficient. In many aspects a ''device'' for energy mining by means of strings seems to be much more ''realistic'' than the Unruh-Wald gedanken device using idealized boxes.

II. MECHANISM OF ENERGY MINING

Different aspects of the black-hole energy mining by strings are discussed in other sections. Here we describe the main idea and give some order of magnitude estimations of the efficiency of this process.

Interaction of cosmological defects (cosmic strings, domain walls) with black holes was discussed in [4–6]. In particular it was shown that there exist stationary configurations of such defects which spread from infinity to a black hole and enter the black hole horizon.

To describe this situation in a more general setup let us first assume that the spacetime has D-dimensions and it contains a D-dimensional black hole. We call it a *bulk* black hole. Consider a d-dimensional world surface Σ representing a topological defect in this D-dimensional spacetime. We call it a *brane*. An internal geometry γ on the brane is generated by its embedding in the D-dimensional bulk spacetime. If the static surface Σ crosses the event horizon of the static bulk black hole, the induced geometry γ itself de-

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scribes a d-dimensional black hole. It follows from a simple fact that a Killing vector of the bulk space is tangent to Σ and thus it is the Killing vector of the induced metric γ. Hence the surface where Σ crosses the horizon of the bulk black hole is the Killing horizon of the d-dimensional black hole on the brane.

It can be shown that transverse perturbations of test branes can be described as a set of massless scalar fields propagating in the background metric γ. In the presence of the d-dimensional black hole these degrees of freedom are thermally excited and thermal radiation of brane excitations emitted by the d-dimensional black hole propagates along the brane to infinity. Thus such a brane serves as an additional channel and can be used for the energy mining from the bulk black hole.

Let us estimate the efficiency of this new additional ''channel.'' The required exact calculations are given later (Sec. III). Now we just make an order of magnitude estimation and explain why the mechanism of energy extraction from black holes by means of branes can be effective.

The energy flux from a black hole is

$$\dot{E}\equiv\frac{dE}{dt}=\frac{1}{2\pi}\int_0^\infty\frac{d\omega\omega\Gamma_\omega}{\exp(8\pi M\omega)-1}, \tag{2.1}$$

where Γ_ω is the transition probability for a mode of energy ω to penetrate the potential barrier and to reach the infinity. This formula is valid for any number of spacetime dimensions. Let us apply it to the radiation of the d-dimensional black hole on the brane. Notice that the internal d-dimensional black-hole geometry on the static brane crossing the bulk black hole is characterized by only one parameter M, related to the mass of the bulk black hole, or what is equivalent, by its gravitational radius.[1] Thus, the equation for a massless field propagating in this background can be easily rewritten in the dimensionless form where the dependence on M enters only through dimensionless quantity $\varpi=\omega/T_{BH}=8\pi M\omega$.[2] Because the factor Γ_ω is dimensionless and depends on ϖ one has

$$\dot{E}=\frac{J_d}{128\pi^3M^2},\quad J_d=\int_0^\infty\frac{d\varpi\varpi\Gamma(\varpi)}{\exp(\varpi)-1}. \tag{2.2}$$

This observation shows that the energy density emitted by a d-dimensional black hole is always proportional to M^{-2}, and the dimensionality d appears only in a dimensionless coefficient J_d. This important simple property was recently discussed by Emparan, Horowitz and Myers [8] who examined Hawking radiation for black holes on a brane in a world with large extra dimensions.

Thus, a static d-dimensional topological defect attached to the bulk black hole gives rise to additional radiation with the flux J_dM^{-2}. The rate of energy emission through the new channel has the same order of magnitude as the emission rate of the bulk black hole. The more branes are attached to the bulk black hole the higher efficiency of the energy mining. One can expect that the number of such branes which can be attached to the bulk black hole without their mutual intersections is bigger for small d.

Hence, returning to the discussion of the physically most interesting case of 4-dimensional bulk spacetime one can conclude that the highest efficiency of the black hole energy mining would be obtained if one uses cosmic strings ($d=2$). In the present paper we discuss this idea and some of its consequences.

III. THERMAL STRING EXCITATIONS

To discuss quantum radiation of string excitations let us consider a static string of tension μ which enters radially a Schwarzschild black hole of mass M through its north pole. The other part of the string (string segment) which enters radially the black hole through the south pole is not of interest for the moment. Denote by n_R^μ, $R=1,2$, two mutually orthogonal normal vectors to the string world sheet Σ. Small perturbation of the string δx^μ can be decomposed in terms of

$$\Phi_R=\delta x^\mu n_R^\mu \tag{3.1}$$

and a component along the world sheet, which is a pure gauge. By linearizing the Nambu-Goto action[3]

$$I=-\mu\int d^2\zeta\sqrt{\gamma} \tag{3.2}$$

(γ_{AB} is the induced metric on Σ), one obtains the following quadratic action for string perturbations:

$$I[\Phi]=-\mu\int d^2\zeta\sqrt{\gamma}\left[\gamma^{AB}\Phi_{P,A}\Phi_{R,B}+\frac{1}{2}R\Phi_P\Phi_R\right]\delta^{PR}, \tag{3.3}$$

where R is a scalar curvature of the two-dimensional induced metric γ. We call these string excitations *stringons*. The induced metric for the unperturbed string is

$$d\gamma^2=\gamma_{AB}d\zeta^Ad\zeta^B=-Fdt^2+F^{-1}dr^2,\quad F=1-\frac{2M}{r}, \tag{3.4}$$

$\zeta^A=(\zeta^0,\zeta^1)=(t,r)$, and $R=4M/r^3$. By redefining the fields Φ_R

$$\varphi_R=\sqrt{2\mu}\Phi_R, \tag{3.5}$$

[1]The parameter M is proportional to the gravitational radius. M coincides with the black hole mass M_{BH} in four dimensions. In higher dimensions D the relation is more complicated $M_{BH}\sim M^{D-3}$, see [7].

[2]We use units in which $G=c=\hbar=1$.

[3]Here the constant μ is the tension of the string measured in units $G=c=1$. We keep in mind that it corresponds to the dimensionless combination $G\mu/c^2$.

we can write Eq. (3.3) as a sum of two identical actions each being of the form

$$I[\varphi]=-\frac{1}{2}\int d^2\zeta\sqrt{\gamma}\left[(\nabla\varphi)^2+\frac{1}{2}R\varphi^2\right]. \tag{3.6}$$

This is nothing but a massless nonminimally coupled ($\xi=-1/2$) scalar field in the spacetime of a two-dimensional Schwarzschild black hole of mass M.

The field equation

$$\left[\Box-\frac{1}{2}R\right]\varphi=0, \tag{3.7}$$

in the tortoise coordinates $dr_*=dr/F$ takes the form

$$\left[-\frac{\partial^2}{\partial t^2}+\frac{\partial^2}{\partial r_*^2}+U\right]\varphi=0, \quad U=-\frac{2M}{r^3}\left(1-\frac{2M}{r}\right), \tag{3.8}$$

which is identical to the equation for s-mode of a four-dimensional massless scalar field in the Schwarzschild geometry. Since stringons have two independent states of polarization, any string segment attached to the bulk black hole brings with it 2 additional channels of emission. The rate of emission through each of these channels is identical to the s-mode contribution to the rate of emission for a scalar massless field in the bulk space.

To estimate this contribution one can use the following approximation for Γ at low frequency obtained by Starobinsky [9] and Page [10]:[4]

$$\Gamma_\omega=16\omega^2M^2. \tag{3.9}$$

The corresponding rate of emission is

$$\dot{E}^{\rm SP}\equiv\frac{16M^2}{2\pi}\int_0^\infty\frac{d\omega\,\omega^3}{\exp(8\pi M\omega)-1}=\frac{1}{7680\pi M^2}. \tag{3.10}$$

Numerical calculations give for the exact value of the s-mode contribution the following result:

$$\dot{E}^{\text{s-mode}}=1.62\dot{E}^{\rm SP}. \tag{3.11}$$

To compare this result with the rate of bulk emission of the black hole we recall that the approximate expression can be obtained by using DeWitt's ansatz [12,13]

$$\Gamma_\omega\approx 27M^2\omega^2. \tag{3.12}$$

The emission rate in this approximation is

$$\dot{E}^{\rm DeWitt}\equiv\frac{27M^2}{2\pi}\int_0^\infty\frac{d\omega\,\omega^2}{\exp(8\pi M\omega)-1}=\frac{9}{40960\pi M^2}. \tag{3.13}$$

Numerical calculations [14,15] show that this result is very accurate. In fact the exact value $\dot{E}^{\rm scalar}_{\rm bulk}$ of the bulk rate of emmision is

$$\dot{E}^{\rm scalar}_{\rm bulk}=1.06\dot{E}^{\rm DeWitt}. \tag{3.14}$$

By using these results we can write

$$\dot{E}^{\text{s-mode}}=0.91\dot{E}^{\rm scalar}_{\rm bulk}. \tag{3.15}$$

In other words, the additional rate of quantum radiation $\dot{E}^{\text{string segment}}$ through each segment of the string attached to the bulk black hole is 1.8 times bigger than the contribution of the scalar massless field to the emission rate of the bulk black hole.[5] We took into account that there are two states of polarization of the string excitations (stringons).

Above we assumed that the bulk black hole radiated a massless scalar field. In fact a realistic black hole emits all possible species of particles which can be emitted at the given Hawking temperature. Black holes of mass greater than 10^{17} g effectively emit only massless particles (neutrinos, photons, and gravitons). Using the results obtained by Page [10,17] one has

$$\dot{E}^{\text{bulk BH}}=A_{m=0}\dot{E}^{\text{scalar field}}_{\rm bulk}, \tag{3.16}$$

where

$$A_{m=0}=0.54h(1/2)+0.22h(1)+0.03h(2), \tag{3.17}$$

and $h(s)$ is the number of distinct polarizations of spin-s particles. For photon, graviton and 3 sorts of massless neutrino $A_{m=0}=3.74$. Thus for a black hole of the mass $M>10^{17}$ g which emits only massless particles one has

$$\dot{E}^{\text{string segment}}\approx\frac{1}{2}\dot{E}^{\text{bulk BH}}. \tag{3.18}$$

For smaller black holes the right-hand side of this relation contains an additional factor $A_{m=0}/A_{\rm total}$, where $A_{\rm total}$ is the factor A, Eq. (3.17), calculated for all particles which can be effectively emitted at the temperature $(8\pi M)^{-1}$.

IV. EFFICIENCY OF BLACK HOLE MINING BY STRINGS

Let us discuss now how many strings can be attached to the black hole. There are several kinds of restrictions on the number of strings. The most important of them are effects connected with the string's thickness, and effects connected with the back reaction.

[4]Notice that this result is 10 times smaller than the rate emission calculated in the absence of the potential barrier which can be obtained by using the Polyakov effective action [11].

[5]The fact that the energy flux from a black hole in string excitations is approximately that of field modes in the bulk was pointed out earlier by Lawrence and Martinec [16].

Before discussing these effects let us make the following remark. A string heated to the temperature T higher than the energy of the corresponding phase transition $\sim\mu^{1/2}$ becomes unstable and can decay. One can expect this kind of instability if the string is attached to the black hole with Hawking temperature $(8\pi M)^{-1}$ higher than $\mu^{1/2}$. The thickness of the string with tension μ is proportional to $\mu^{-1/2}$. Hence one can expect that this effect of thermal instability is not dangerous if the thickness of the string is much smaller than the gravitational radius of the black hole. We shall always assume that this condition is satisfied.

Let us consider now the effects connected with the thickness of strings.[6] Denote by N_s the number of string segments attached to the black hole of mass M. Then the total surface area of string cross section is proportional to $\pi N_s \mu^{-1}$. If the strings do not touch each other this quantity must be smaller than the surface area of the black hole, $16\pi M^2$. This condition gives the following restriction on N_s:

$$N_s < 16 M^2 \mu. \tag{4.1}$$

The second kind of restrictions on N_s are connected with the back reaction effect. The gravitational field of a string creates an angle deficit $\alpha = 4\mu$, that is the length of a unit circle around the string becomes $\beta = 2\pi(1-\alpha)$. It can be shown by using the Gauss-Bonnet theorem that N_s string segments of the equal tension μ which enter the black hole horizon change the solid angle around the black hole from 4π to $2\pi(2 - N_s\alpha)$. This means that we have to consider only situations when

$$a \equiv 2\mu N_s \ll 1. \tag{4.2}$$

For a certain mass $M = M^* = (a/32)^{1/2}/\mu$ the both restrictions give the same number N_s. For $M < M^*$ ($M > M^*$) restriction on N_s follows from relation (4.1) [from relation (4.2)]. To give estimations we consider two values of the string tensions, μ_{GUT} and μ_{EW}, which correspond to the energy scale of the grand unified theory (GUT) (10^{16} GeV) and electroweak (100 GeV) phase transitions, respectively. One has

$$\mu_{GUT} = 10^{-6}, \quad \mu_{EW} = 10^{-34}. \tag{4.3}$$

For estimations we also choose $a = 1/100$. Then we have

$$M^*_{GUT} = 1.77\times 10^4 m_{Pl} \approx 0.2 \ \text{g},$$
$$M^*_{EW} = 1.77\times 10^{32} m_{Pl} \approx 2\times 10^{27} \ \text{g}. \tag{4.4}$$

It is interesting that the mass M^*_{EW} is close to the Earth's mass 10^{28} g.

For GUT strings M^*_{GUT} is so small that for all black holes which are not at the stage of their explosion one has $M > M^*_{GUT}$ and hence the upper limit on N_s is

$$N_s^{GUT} \sim 5\times 10^3. \tag{4.5}$$

For N_s^{EW} the restrictions are

$$N_s^{EW} \sim 5\times 10^{31} \quad \text{for} \ \ M > M^*_{EW}, \tag{4.6}$$

and

$$N_s^{EW} \sim 5\times 10^{31}\left(\frac{M}{M^*_{EW}}\right)^2 \quad \text{for} \ \ M < M^*_{EW}. \tag{4.7}$$

One can conclude that even with GUT strings mining energy from black holes can be quite efficient. For electroweak or lighter strings the effect of amplification of the rate of quantum radiation is very high.

Let us estimate now the total energy rate of emission through the attached strings. For $M > M^*$ the number of attached string segments is $N_s^* = a/(2\mu)$ and

$$\dot{E}^{\text{string}}_{\text{total}} = \frac{B N_s^*}{M^2}, \tag{4.8}$$

where $B \approx 1/(2400\pi)$. $\dot{E}^{\text{string}}_{\text{total}}$ grows when M becomes smaller until it reaches M^*. After that its value remains constant. Denote this maximum value of the energy rate by $\dot{E}^*$, then one has

$$\dot{E}^* \approx 2.5\times 10^{-3} \mu \dot{E}^{\max}, \tag{4.9}$$

where $\dot{E}^{\max} = c^5/G$ is the highest rate of energy extraction discussed in the Introduction.

V. THERMAL FLUCTUATIONS OF STRINGS IN BLACK HOLE GEOMETRY

Thermal quantum fluctuations of the string near the black hole horizon can change effectively the string cross section. If this effect would be large we had to reconsider estimations of the previous section. Let us analyze this effect in more detail. The effective width Δx generated by quantum fluctuations of the string can be defined as

$$(\Delta x)^2 = \langle \delta x^\mu \delta x_\mu \rangle, \tag{5.1}$$

where δx^μ are transverse perturbations of the string. The quantum average can be defined according to Eqs. (3.1) and (3.5) as

$$\langle \delta x^\mu \delta x_\mu \rangle = \sum_R \langle \hat{\Phi}_R^2 \rangle = \frac{1}{\mu} \langle \hat{\varphi}^2 \rangle, \tag{5.2}$$

where φ is a two-dimensional quantum scalar field described by Eq. (3.7). To estimate the effective width of the string fluctuations we have to find the correlator $\langle \hat{\varphi}^2 \rangle$ on the hori-

[6] A detailed picture of how a Nielsen-Olesen string of a finite width interacts with a black hole is discussed in [18,19]. The effects connected with the string thickness are also important for high-frequency waves propagating along the string when the wavelength becomes comparable with $\mu^{-1/2}$.

zon of the 2D black hole. We will do it by calculating explicitly the Green function $G(p,p')$ of the field φ in the Euclidean theory.

The 2-dimensional Euclidean black hole metric can be written as

$$d\gamma_E^2=4M^2d\tilde{\gamma}^2, \quad d\tilde{\gamma}^2=(1-x^{-1})d\tau^2+(1-x^{-1})^{-1}dx^2, \tag{5.3}$$

where $x=r/2M$ and τ is periodic with the period $2\pi/\kappa=4\pi$. Here $\kappa=1/2$ is the surface gravity of the horizon in the metric $d\tilde{\gamma}^2$. Near the horizon the metric $d\tilde{\gamma}^2$ takes the Rindler form

$$d\tilde{\gamma}^2=\frac{l^2}{4}d\tau^2+dl^2, \tag{5.4}$$

where l is the dimensionless proper distance.

To calculate thermal fluctuations at the horizon of a string attached to a black hole it is sufficient to know the Green function $G(p,p_0)$ with one point at the horizon $x=x_0=1$. Such a Green function does not depend on τ and can be written as $G(x,x_0)$. The Green function equation

$$\Box G(p,p')=-\delta(p,p'), \tag{5.5}$$

takes the form

$$\left[\frac{d}{dx}\left(\left(1-\frac{1}{x}\right)\frac{d}{dx}\right)-\frac{1}{x^3}\right]G(x,x_0)=-\frac{\delta(x-x_0)}{2\pi}. \tag{5.6}$$

A general solution of this equation is

$$G(x,x_0)=Cx+\frac{1}{4\pi}x\ln\frac{x}{x-1}. \tag{5.7}$$

To single out a solution we impose a boundary condition $G(L,x_0)=0$, that is we suppose that a position of the string is fixed at the point $x=L$. Then we have

$$G(x,x_0)=\frac{1}{4\pi}x\left[\ln\frac{x}{x-1}-\ln\frac{L}{L-1}\right]. \tag{5.8}$$

It is not difficult to see that if the point p is approaching p_0 then $(x-1)=(\Delta l)^2/(4M)^2$ where Δl is the proper distance between the two points. The distance Δl can be considered as a regularization parameter, a cutoff which enables one to avoid short-distance or ultraviolet divergences in the average of the operator $\hat{\varphi}^2$. Thus, the regularized correlator can be written as

$$\langle\hat{\varphi}^2\rangle=\frac{1}{4\pi}\left[\ln\left(\frac{4M}{\Delta l}\right)^2-\ln\frac{L}{L-1}\right]. \tag{5.9}$$

In the considered situation the value of ultraviolet cutoff is related to the thickness of the string. The high-frequency waves with the wavelength comparable to or smaller than the string width cannot propagate along the string, or at least their propagation should be different from the propagation of the low-frequency waves. Hence, it is natural to relate Δl with the string width $1/\sqrt{\mu}$.

The quantity $b\equiv 4M\sqrt{\mu}$ is large for the macroscopical black holes. For instance, for a solar mass black hole $b=4\times10^{35}$ for GUT strings and $b=4\times10^{21}$ for electroweak strings. However, b appears in the right-hand side of Eq. (5.9) under the logarithm. Because of this additional logarithm the effective width of the string Δx related to quantum fluctuations can be only order or two greater than the string width $1/\sqrt{\mu}$. This factor can be easily taken into account in the estimations of the previous section of the upper number of strings which can be attached to the black hole but this does not change essentially the main results.

VI. POLYHEDRAL STRING CONFIGURATIONS

Our device for energy extraction consists of a black hole and N_s segments[7] of strings attached to the black hole which are located along radial directions.

Until now we did not discuss gravitational back reaction and possible mutual interaction of strings attached to the black hole. String interactions can arise for different reasons. Consider two parallel Nielsen-Olesen strings separated by distance l greater than μ^{-1}. Strictly speaking the fields do not vanish outside the radius μ^{-1}, but become exponentially small. Because of the nonlinearity of the field equations one can expect a force $\sim\exp(-l\mu)$ between two strings. Quantum effects also results in the force between 2 parallel strings [20]. In a general case in the presence of these and mutual gravitational interaction forces (proportional to μ^2) a system of strings is not static. One can expect that when the distances between strings are large in comparison with their width these forces are negligible.

Quite remarkably, there is a special case when one can guarantee that the forces acting on the strings identically vanish because of the symmetry [21]. It happens when there exists a symmetry transformation (rotation) which transforms the system (the black hole with attached strings) into itself. The force acting on the string (which is always orthogonal to the string) must vanish since it must remain invariant under this rotation. The string configurations which respect this symmetry are discussed in detail in [21]. We just mention here that for these special (polyhedral) configurations strings are directed along radii and coincide with the symmetry axes of regular polyhedra. Therefore, there exist only a few types of string configurations. The number of strings corresponding to a tetrahedron, octahedron and icosahedron is 14, 26, and 62, respectively.[8] There is also a so called ''double pyramid configuration'' when an even num-

[7]If a black hole captures an infinite cosmic string, 2 new string segments would be attached to the black hole. For this reason N_s must be an even number.

[8]The number of strings differs from the order of the symmetry group (12, 24, 60) of the corresponding solids. This difference is discussed in [21].

ber of strings goes inside the black hole in the equatorial plane at the equal angles between two neighbor strings and the two strings are attached at the south and the north poles. The number of strings in this configuration can be arbitrary. All strings have equal tension.[9] For an arbitrary string tension the symmetry guarantees that such configurations are exact solutions of the Einstein equations and that outside the strings the metric is locally isometric to the Schwarzschild metric.[10]

Returning to the problem of the energy mining from black holes we see that the icosahedron configuration gives about 10^2 independent channels for the energy mining. For the tetrahedron and octahedron configurations this number is less. The maximal number of strings is possible for the double pyramid configuration. As we discussed above there is a restriction on the total number N_s of strings in this configuration determined by value of the solid angle around the black hole, $a=2\mu N_s \ll 1$. The other restriction is the upper bound on the number of ''equatorial'' strings of the width $\mu^{-1/2}$ which can be attached to the black hole of the horizon radius $2M$

$$N_s \simeq 4\pi M\sqrt{\mu}. \tag{6.1}$$

Both restrictions result to the same upper bound N_s at $M = M^* = a/(8\pi)\mu^{-3/2}$. For $M<M^*(M>M^*)$ restriction on N_s follows from relation (6.1) [from relation (4.2)]. We can again give estimations for two values μ_{GUT} and μ_{EW}, which correspond to grand unified and electroweak scales, respectively. By choosing as before $a=1/100$ we have

$$M^*_{GUT} \sim 4 \text{ g}, \quad M^*_{EW} \sim 4\times 10^{42} \text{ g}. \tag{6.2}$$

For a black hole with the mass $M>10^{17}$ g we get the same upper bound (4.5) for GUT strings ($N_s \sim 10^3$). For electroweak strings the restrictions are

$$N_s^{EW} \sim 5\times 10^{31} \quad \text{for } M>M^*_{EW}, \tag{6.3}$$

and

$$N_s^{EW} \sim 5\times 10^{31}\frac{M}{M^*_{EW}} \quad \text{for } M<M^*_{EW}. \tag{6.4}$$

Bound (6.3) coincides with Eq. (4.6). For a stellar mass black hole $M \sim 10^{34}$ g one has to use Eq. (6.4) which differs from Eq. (4.7). The number of strings in this case is $N_s \sim 10^{23}$.

We have thus demonstrated that there are black hole solutions with many strings where the back reaction effect is taken into account explicitly and where the upper bound on the number of strings either coincides with our general estimations or somewhat different but still very high.

It should be emphasized that one can expect the existence of wider variety of static solutions of Einstein equations describing a static black hole with attached radial cosmic strings than polyhedral configurations. We discuss this problem elsewhere [21].

VII. DISCUSSION

We demonstrated that mining energy from black holes by cosmic string can be very effective. In principle, a black hole with attached cosmic strings can be used as an energy generator. To make it ''technically'' simpler, one may assume that strings in the black hole exterior have end points with monopoles which are attached to a spherical construction with the size of a few gravitational radius. Thermal energy released in this construction can be transformed into useful work. If we use the relation (4.9) to estimate the energy generated by this device we get for the electroweak strings the result $\dot{E}^* \sim 9\times 10^{22}$ erg/sec. This is one order of magnitude smaller than the total solar radiation through the Earth surface which is approximately 1.7×10^{24} erg/sec. Such a device is able to produce energy $\dot{E}^*$ during period of time $T \sim Mc^2/\dot{E}^*$. For example for a black hole of the same mass as the mass of Earth, $\sim 6\times 10^{27}$ g, the time T is approximately 10^{26} sec, that is much greater than the lifetime of the Universe.

In our discussion we assumed that black holes are nonrotating. One can expect that for rotating black holes the estimations obtained above are also valid. Stationary configurations of test strings near a rotating black hole were discussed in [4]. According to the uniqueness theorem proved there, the only stationary strings which enter a stationary rotating black hole have world sheets spread by a timelike at infinity Killing vector and a principle null ray of the Kerr geometry. Such a string belongs to a cone $\theta=$const. Different nonintersecting strings which belong to the same cone can be obtained by a rigid rotation at some angle ϕ. Strings which belong to different cones do not intersect one another. Thus such strings form a two-parameter family which is parametrized by angles (θ,ϕ), the coordinates on the infinite redshift surface of the points where a string crosses this surface. As it was demonstrated in [4] the internal geometry on such stationary string surfaces is again 2D black-hole geometry so that there is thermal flux of stringons along the strings.

A black hole and a cosmic string are relativistic objects with well known (at least theoretically) properties. The assumptions we have made to study the strings attached to the black hole can be easily controlled. This makes this system much simpler for analysis than the Unruh-Wald gedanken experiment with mirror boxes [1–3]. It is interesting that quite general arguments allows one to obtain the relation (4.9) for the maximal rate of radiation in the string–black-hole system, which besides the universal ''maximal'' rate c^5/G contains only one additional factor μ, the string tension.

[9]In a more general case strings corresponding to three different types of the symmetry axes can have different tensions.

[10]If the force between two strings in the Schwarzschild geometry is repulsive polyhedral string configurations are stable. The problem of stability of polyhedral strings requires further investigation.

Finally, we want to mention another problem which is not directly related to strings in four dimensions but where the above results may be of some interest as well. There is an analog of stringlike defects in higher dimensional gravity theories. As was shown in [22], it is possible to obtain a four-dimensional gravity from a six-dimensional gravity localized on a stringlike defect. This scenario may be an alternative to localizing gravity on branes. Our arguments then can be used to show that the Hawking emission rate of a six-dimensional black hole along a stringlike defect (i.e., into a four-dimensional world) would be comparable to its radiation in the bulk. This is similar to what one can say about radiation of a black hole along the brane [8].

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