Late time acceleration in Brans-Dicke cosmology

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In this work we investigate the possibility of having a late time accelerated phase of the universe, suggested by recent supernova observation, in the context of Brans-Dicke theory with a potential having a time dependent mass squared term which has recently become negative and a matter field. We find that while a perfect fluid (pressureless and with pressure) cannot support this acceleration, a fluid with dissipative pressure can drive this late time acceleration for a simple power-law expansion of the universe. We have also calculated some cosmological parameters in our model to match with observations.

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I. INTRODUCTION

A lot of activity has been triggered by two recent observations [1,2] on the explosion of type Ia supernovae. These data favor the existence of a new kind of matter with a positive energy density dominant in the present Universe and are also responsible for the present acceleration of the Universe accounted for by its negative pressure. This, along with the observed location of the first acoustic peak of cosmic microwave background (CMB) temperature fluctuation corroborated by the latest BOOMERANG and MAXIMA data [3,4], favors a spatially flat universe whose energy density is dominated by a cosmological constantlike term. Obviously the first natural choice to represent such special matter is the cosmological constant Λ [5,6]. For a flat matter dominated universe with Λ, having $\Omega_\Lambda \sim 0.72$ in Einstein gravity best fits the data sets. But the candidate Λ as the constituent of the major energy density is troubled by the fact that it has an energy scale which is $\sim 10^{-123}$ lower than the normal energy scale predicted by most particle physics models. So to find some alternative candidate for this acceleration a dynamical Λ [7] in the form of a scalar field with some self-interacting potential [8] is considered whose slowly varying energy density mimics an effective cosmological constant. The idea of this candidate, called *quintessence* [7], is borrowed from the inflationary phase of the early universe, with the difference that it evolves at a much lower energy scale. The energy density of this field, though dominant at present epoch, must remain subdominant at a very early stage and has to evolve in such a way that it becomes comparable with the matter density Ω_m now. This type of specific evolution, better known as the ''*cosmic coincidence*'' [9] problem, needs several constraints and fine-tuning of parameters for the potential used to model quintessence with minimally coupled scalar field. A new form of quintessence field called ''*tracker field*'' [10] has been proposed to solve the cosmic coincidence problem. It has an equation of motion with an attractorlike solution in a sense that for a wide range of initial conditions the equation of motion converges to the same solution.

There are a number of quintessence models which have been put forward and most of which involve a minimally coupled scalar field with different potentials dominating over the kinetic energy of the field. A purely exponential potential is one of the widely studied cases [11]. In spite of the other advantages the energy density is not enough to make up for the missing part. The inverse power law is the other potential [8–10] that has been studied extensively for quintessence models, particularly for solving the cosmic coincidence problem. Though the problems are resolved successfully with this potential, the predicted value for the equation of state for the quintessence field γ_Q is not in good agreement with the observed results. In search of proper potentials that would eliminate the problems, new types of potentials, like $V_0[\cos h\lambda\phi - 1]^p$ [12] and $V_0 \sin h(\alpha\sqrt{k_0}\Delta\phi)^\beta$ [6,13] have been considered, which have asymptotic forms like the inverse power-law or exponential ones. Different physical considerations have led to the study of other types of potentials also [14]. Recently Saini *et al.* [15] have reconstructed the potential in context of general relativity and a minimally coupled quintessence field from the expression of the luminosity distance $d_L(z)$ as a function of redshift obtained from the observational data. However, none of these potentials are entirely free of problems. Hence there is still a need to identify appropriate potentials to explain current observations [11].

Most of the studies regarding accelerated expansion have been done with a minimally coupled scalar field representing the quintessence. It has been recently shown by Pietro and Demaret [16] that for the constant scalar field equation of state, which is a good approximation for a tracker field solution, the field equations and the conservation equations strongly constrain the scalar field potential, and most of the widely used potential for quintessence, such as the inverse power-law one, the exponential or the cosine form, are incompatible with these constraints. The minimally coupled self-interacting models will also be ruled out if the observations predict that the missing component of the energy density obeys an equation of state $p = \gamma\rho$ with $\gamma < -1 (\rho \geqslant 0)$, and this sort of equation of state is in reasonable agreement with different observations [17]. Also the inequality $dH^2(z)/dz \geqslant 3\Omega_{m0}H_0(1+z)^2$ should satisfy for the minimally coupled scalar field and its violation will certainly point towards a theory of non-Einstein gravity such as scalar

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tensor theories where the scalar field is non minimally coupled to gravity.

There have been quite a few attempts at treating this problem with the nonminimally coupled scalar fields. Scaling attractor solutions are available in the literature with the exponential [14] and power-law [14,18] potentials in nonminimally coupled theories. Faraoni [19] has studied different potentials with a nonminimal coupling term $\psi R(\phi^2/2)$ for the present acceleration. There have been different approaches also for solving the problem in general scalar tensor theory, sometimes called *extended* or *generalized* quintessence, not only because this theory is considered to be the most natural alternative to general relativity; there are other strong motivations [20] also. People such as Bertolo *et al.* [21], Bertolami *et al.* [22], and Ritis *et al.* [23] have found tracking solutions in scalar tensor theories with different types of power-law potential. In another work, Sen *et al.* [24] have found the potential relevant to power-law expansion in Brans-Dicke cosmology. Like Saini *et al.* [15], Boisseau *et al.* [25] have reconstructed the potential from the luminosity-redshift relation available from the observations in the context of scalar tensor theory.

Very recently, McDonald [26] has investigated the possibility of modeling a dynamical cosmological constant with a scalar field which has undergone a very recent phase transition. For this he has considered a standard ϕ^4 potential for the scalar field with an additional time dependent mass squared term in the potential which becomes negative very recently. For this kind of model, phase transition occurs very recently at redshift $z \leqslant 1.2$.

In this paper we have investigated whether nonminimally coupled self-interacting scalar fields such as a Brans-Dicke-(BD-) type scalar field with this kind of potential can successfully drive the late time acceleration for the flat universe. In the context of the Brans-Dicke (BD) theory [27] with a self-interacting potential and a matter field, the action is given by

$$\mathcal{S}=\int d^4x\sqrt{-g}\left(\phi R-\frac{\omega}{\phi}\phi^{\alpha}\phi_{\alpha}-V(\phi)+\mathcal{L}_m\right) \tag{1}$$

(we have chosen the unit $8\pi G_0=c=1$). We have chosen the ϕ^4 potential with a time-dependent mass squared term which has already become negative after a phase transition in recent time [26]:

$$V(\phi)=\lambda\phi^4-\mu^2(t)\phi^2, \tag{2}$$

where

$$\mu^2(t)=\overline{\mu}_0^2\left(\frac{R_c}{R}\right)^n=\frac{\mu_0^2}{R^n}, \tag{3}$$

$\mu_0^2=\overline{\mu}_0^2R_c^n$. λ is a constant and n is an integer. The time-dependent mass squared term with integer n can arise naturally in plausible models and one can find a detailed discussion in Ref. [26].

As a matter field we would consider first a perfect fluid and then a fluid having negative pressure. An effective negative pressure and hence an acceleration can be achieved by a dissipative mechanism modeled commonly by fluid viscosities. It has been proposed recently that the cold dark matter (CDM) must self-interact in order to explain the detailed structure of the galactic halos [28]. This self-interaction will naturally create a viscous pressure whose magnitude will depend on the mean free path of the CDM particles. An effective negative pressure in CDM can also be created from cosmic antifriction which is closely related to particle production out of the gravitational field [29]. Since the negative pressure can be modeled in different ways, we are not *a priori* assuming any specific model for this negative pressure.

In this work we find that it is not possible to have a late time power-law accelerated expansion when the CDM is a perfect fluid, but a dissipative CDM fluid in BD cosmology with such a potential like Eq. (2) can successfully drive a late time accelerated expansion. In the next section we treat the field equations and find the solutions in both cases. We also calculate some cosmological parameters to match the accelerated model with observation. The third section is the concluding section where we have discussed different features of this model.

II. FIELD EQUATIONS AND SOLUTIONS

The gravitational field equations derived from the action (1) by varying the action with respect to the metric is

$$G_{\mu\nu}=\frac{T_{\mu\nu}}{\phi}+\frac{\omega}{\phi^2}\left(\phi_\mu\phi_\nu-\frac{1}{2}g_{\mu\nu}\phi_\alpha\phi^\alpha\right)+\frac{1}{\phi}[\phi_{\mu;\nu}-g_{\mu\nu}\Box\phi]-g_{\mu\nu}\frac{V(\phi)}{2\phi}, \tag{4}$$

where $T_{\mu\nu}$ represents the energy-momentum tensor of the matter field. We have assumed the matter content of the universe to be composed of a fluid represented by the energy-momentum tensor

$$T_{\mu\nu}=(\rho+P)v_\mu v_\nu+Pg_{\mu\nu}, \tag{5}$$

where ρ and P are the energy density and effective pressure of the fluid, respectively, and v_μ is the four velocity of the fluid, i.e., $v_\mu v^\mu=-1$. The effective pressure of the fluid includes the thermodynamic pressure p and a negative pressure π, which could arise either because of the viscous effect or due to particle production, i.e.,

$$P=p+\pi. \tag{6}$$

The wave equation that follows from Eq. (1), by varying the action with respect to the scalar field ϕ, is

$$\Box\phi=\frac{T}{2\omega+3}+\frac{1}{2\omega+3}\left(\phi\frac{dV(\phi)}{d\phi}-2V(\phi)\right). \tag{7}$$

For our choice of potential (2), the field Eqs. (4) and the wave Eq. (7) becomes, respectively,

$$3\frac{\dot{R}^2}{R^2}+3\frac{\dot{R}}{R}\frac{\dot{\phi}}{\phi}-\frac{\omega}{2}\frac{\dot{\phi}^2}{\phi^2}-\frac{\lambda}{2}\phi^3+\frac{\mu_0^2}{2R^n}\phi=\frac{\rho}{\phi}, \tag{8}$$

$$2\frac{\ddot{R}}{R}+\frac{\dot{R}^2}{R^2}+\frac{\ddot{\phi}}{\phi}+2\frac{\dot{R}}{R}\frac{\dot{\phi}}{\phi}+\frac{\omega}{2}\frac{\dot{\phi}^2}{\phi^2}-\frac{\lambda}{2}\phi^3+\frac{\mu_0^2}{2R^n}\phi=-\frac{(p+\pi)}{\phi}, \tag{9}$$

and

$$\ddot{\phi}+3\frac{\dot{R}}{R}\dot{\phi}=\frac{\rho-3(p+\pi)}{2\omega+3}-\frac{1}{2\omega+3}\left[2\lambda\phi^4+\frac{n\mu_0^2\phi^2}{R^n}\frac{\frac{\dot{R}}{R}}{\frac{\dot{\phi}}{\phi}}\right]. \tag{10}$$

We have assumed the standard Friedman-Robertson-Walker metric with the signature convention $(-,+,+,+)$ and R is the scale factor. We restrict ourselves for the spatially flat metric only. We work in the Jordan frame. One interesting thing about BD theory in the Jordan frame is that the conservation equation holds for the matter and scalar field separately. Or in a slightly different way, the Bianchi identity along with the wave Eq. (7) gives the matter conservation equation

$$\dot{\rho}+3\frac{\dot{R}}{R}(\rho+p+\pi)=0. \tag{11}$$

We assume that both the scale factor and the scalar field evolve as the power function of time

$$R=R_0\left(\frac{t}{t_0}\right)^{\alpha} \quad \text{and} \quad \phi=\phi_0\left(\frac{t}{t_0}\right)^{\beta}, \tag{12}$$

where the subscript 0 refers to the values of the parameters at the present epoch. In order to get an accelerated expansion for such evolution of the universe the deceleration parameter has to be negative, which immediately restricts the parameter α to be greater than 1. For such an expansion the solution for the matter density is

$$\rho=\rho_c t^{\beta-2}, \tag{13}$$

where

$$\rho_c=\frac{3\alpha\phi_0}{t_o^{\beta}}\left[\frac{2\alpha+\beta(1+\alpha)-\beta^2(1+\omega)}{2-\beta}\right]. \tag{14}$$

First we consider a normal perfect fluid with no negative pressure i.e., $\pi=0$ in Eq. (6). Then, for the power-law evolution, the thermodynamic pressure of the fluid becomes

$$p=p_c t^{\beta-2}, \tag{15}$$

where

$$p_c=\frac{(2-\beta-3\alpha)\phi_0}{t_o^{\beta}}\left[\frac{2\alpha+\beta(1+\alpha)-\beta^2(1+\omega)}{2-\beta}\right]. \tag{16}$$

The power-law solution is consistent with the field equations (8), (9), and (10) only if

$$\beta=-\frac{2}{3} \quad \text{and} \quad \alpha n-\beta=2, \quad \text{i.e.,} \quad \alpha n=\frac{4}{3}. \tag{17}$$

So the acceleration demands $n<\frac{4}{3}$, and as n is a positive integer, it restricts $n=1$ in our model. Again, from Eq. (13) the weak energy condition $(\rho>0)$ demands

$$\omega<3\alpha-\frac{5}{2}. \tag{18}$$

From Eqs. (15) and (13) it is clear that the perfect fluid follows an equation of state of the form $p=\gamma_m\rho$. The index γ_m is given by

$$\gamma_m=\frac{2-\beta}{3\alpha}-1, \tag{19}$$

where γ_m lies within the interval $0<\gamma_m<1$. This restricts α within the range $\frac{4}{9}<\alpha<\frac{8}{9}$. In fact, for the present matter dominated universe ($\gamma_m=0$), $\alpha=\frac{8}{9}$. But this does not satisfy the criteria for acceleration $(\alpha>1)$ and hence for a simple power-law-type expansion the universe decelerates with a perfect fluid CDM $(0\leqslant\gamma_m<1)$ with a potential (2) in BD theory.

Now we consider a CDM which has a dissipative effect and we are particularly interested in a present day universe, i.e, $p=0$. Under such a condition Eq. (11) takes the form

$$\dot{\rho}+3\frac{\dot{R}}{R}(\rho+\pi)=0. \tag{20}$$

As is mentioned earlier, this type of dissipative effect in Friedmann-Robertson-Walker (FRW) cosmology can be modeled in two ways. Generally, the dissipative effect is accounted by conventional bulk viscous effect. In the FRW universe the bulk viscosity can be modeled within the framework of nonequilibrium thermodynamics proposed by Israel and Stewart [30]. According to this theory the bulk viscous pressure π follows the transport equation

$$\pi+\tau\dot{\pi}=-3\eta H-\frac{\tau\pi}{2}\left(3H+\frac{\dot{\tau}}{\tau}-\frac{\dot{T}}{T}-\frac{\dot{\eta}}{\eta}\right), \tag{21}$$

where the positive definite quantity η stands for the coefficient of bulk viscosity, T is the temperature of the fluid, and τ is the relaxation time associated with the dissipative effect, i.e., the time taken by the system to reach the equilibrium state if the dissipative effect is suddenly switched off. Considering the divergence term in the square bracket to be small, i.e., $(R^3\tau/\eta T)$ to be constant, the equation can be approximated to a simpler form,

$$\pi+\tau\dot{\pi}=-3\eta H. \tag{22}$$

In literature this is commonly described as a truncated version of the full nonequilibrium thermodynamics. The viscous effects are assumed to be not so large as observation seems to rule out huge entropy production on large scales [31]. Usually τ is expressed as η/ρ so as to ensure that the viscous signal does not exceed the speed of light [32] and also $(\tau H)^{-1}=\nu$, where $\nu>1$ for a consistent hydrodynamical description of the fluid [33]. With these two assumptions Eq. (22) becomes

$$\nu H+\frac{\dot{\pi}}{\pi}=-\frac{3\rho H}{\pi}. \tag{23}$$

In a very recent work Chimento *et al.* [34] have shown that a mixture of a minimally coupled self interacting scalar field and a perfect fluid is unable to drive the accelerated expansion and solve the cosmic coincidence problem at the same time, while the mixture of a dissipative CDM with bulk viscosity along with a minimally coupled self-interacting scalar field can successfully drive the accelerated expansion and solve the cosmic coincidence problem simultaneously.

An effective negative pressure can also be created from cosmic antifriction which is closely related to particle production out of gravitational field. In a recent paper Zimdahl *et al.* [29] have shown that one can have a negative π if there exists a particle number nonconserving interaction inside matter. This may happen due to particle production out of the gravitational field. In this case, the matter is of course not a dissipative fluid, but a perfect fluid with varying particle number. Though substantial particle production is an event that occurs in the early universe, Zimdahl *et al.* have shown that an extremely small particle production rate can also cause sufficiently negative π to violate strong energy conditions.

We do not *a priori* assume any specific model for this dissipative effect, rather we only assume the existence of a negative π. For a similar kind of evolution of the scale factor and the scalar field given by Eq. (12), the energy density for the fluid with negative pressure is also given by Eqs. (13) and (14). From Eqs. (20) and (13), one can easily find that

$$\pi=\frac{(2-\beta-3\alpha)\phi_0}{t_o^{\beta}}\left[\frac{2\alpha+\beta(1+\alpha)-\beta^2(1+\omega)}{2-\beta}\right]t^{\beta-2}. \tag{24}$$

From Eq. (24) one can easily check that to have a negative π, one should have $3\alpha>2-\beta$ which essentially means $\alpha>\frac{8}{9}$. This suits the condition for acceleration as an α is needed to be greater than 1 for that. One can also check that for this set of solutions given by Eqs. (12), (13), and (24) and from Eqs. (23), the condition $\nu>1$ holds provided $2-\beta>0$, which is very much true in our case. This is important for the hydrodynamical description if the CDM is assumed to be a conventional viscous fluid.

To have a clear picture of the expansion of the universe and the missing energy, we further study the energy density and pressure of the geometric scalar field. The expressions for the energy density and the pressure of the scalar field can be derived from the field Eqs. (8) and (9) to be

$$\rho_{\phi}=\left[\frac{\omega}{2}\frac{\dot{\phi}^2}{\phi}+\frac{V}{2}-3\frac{\dot{R}}{R}\dot{\phi}\right] \tag{25}$$

and

$$p_{\phi}=\left[\frac{\omega}{2}\frac{\dot{\phi}^2}{\phi}-\frac{V}{2}+\ddot{\phi}+2\frac{\dot{R}}{R}\dot{\phi}\right]. \tag{26}$$

In the case of power-law expansion (12) and potential-like (2) the energy density of the BD field becomes

$$\rho_{\phi}=\frac{\alpha\phi_0}{2t_0^{\beta}}\left\{3\alpha+\omega+\frac{5}{2}\right\}t^{\beta-2} \tag{27}$$

and pressure of the BD field is

$$p_{\phi}=\left[-\frac{\alpha}{2}\left(3\alpha+\omega+\frac{5}{2}\right)+\frac{2}{3}\left(\alpha+\frac{2}{3}\omega+\frac{5}{3}\right)\right]\frac{\phi_0}{t_0^{\beta}}t^{\beta-2}. \tag{28}$$

The positivity condition for the scalar energy density demands

$$\omega>-\left(3\alpha+\frac{5}{2}\right) \tag{29}$$

which eventually restricts ω beyond some lower value. So essentially the two positivity energy conditions (18) and (29) limit the range of ω within $-(3\alpha+\frac{5}{2})<\omega<3\alpha-\frac{5}{2}$. Clearly a barotropic relation ($p_{\phi}=\gamma_{\phi}\rho_{\phi}$) is followed by the scalar field, where the adiabatic index γ_{ϕ} is given by

$$\gamma_{\phi}=-1+\frac{\frac{2}{3}\left(\alpha+\frac{2}{3}\omega+\frac{5}{3}\right)}{\frac{\alpha}{2}\left(3\alpha+\omega+\frac{5}{2}\right)}. \tag{30}$$

The range of γ_{ϕ} that agrees with the observational data and describes the current acceleration for the universe well is $-0.6>\gamma_{\phi}>-1$. One can adjust the value of α and ω so as to get the required value of γ_{ϕ}. We now recast Eq. (8) in the form

$$\Omega_m+\Omega_{\phi}=1, \tag{31}$$

where the density parameters for matter Ω_m and scalar field Ω_{ϕ} are defined to be (see Ref. [35])

$$\Omega_m=\frac{\rho}{3H^2\phi}\quad\text{and}\quad\Omega_{\phi}=\frac{\rho_{\phi}}{3H^2\phi}. \tag{32}$$

The expression for density parameters at present epoch are

$$\Omega_{m0}=\frac{\rho_0}{3H_0^2\phi_0}=\frac{1}{2}-\frac{1}{6\alpha}\left(\omega+\frac{5}{2}\right) \tag{33}$$

and

$$\Omega_{\phi 0}=\frac{\rho_{\phi 0}}{3H_0^2\phi_0}=\frac{1}{2}+\frac{1}{6\alpha}\left(\omega+\frac{5}{2}\right). \tag{34}$$

Like γ_ϕ, the value of Ω_{m0} that suits best the luminosity distance-redshift data for type Ia supernovas is $\Omega_{m0}=0.28$ and in a similar fashion like γ_ϕ, one can adjust the α and ω value to get the required value of Ω_{m0} that tallies with the observation.

So far α is restricted only by the deceleration parameter that it should be greater than 1 for the universe to accelerate. And the positivity energy conditions limits ω within the range $-(3\alpha+\frac{5}{2})<\omega<3\alpha-\frac{5}{2}$. So if a small value is chosen for $\alpha(\sim 1)$, ω is also restricted accordingly and a suitable choice of both parameters can be made to find the allowable range of γ_ϕ and Ω_{m0} that matches observation. Kaplinghat *et al.* [36] and others [37] have pointed out that for power-law cosmologies, high redshift data and present age of the universe restricts α to a value ≈ 1. In a very recent investigation we have found that the best fit value of α with the supernova Ia data for power-law cosmology is approximately 1.25 [38]. Hence this small value for α restricts ω also to be small. But this squarely contradicts the solar system bound on $\omega(>600)$. To accommodate such large value of ω, α should be large. But for the large values of α the universe accelerates faster (almost like de Sitter expansion) and γ_ϕ asymptotically approaches -1. It is quite unlikely that the universe presently accelerates in such a high power-law fashion and such α values do not match with present observation. We will discuss more about this point in the discussion section.

We wish to find the range of the parameters α and ω allowable in our model, that suits the permissible range of γ_ϕ and Ω_{m0} of the quintessence proposals. In Fig. 1, we have shown the allowed region in the (α,ω) parameter space (shaded portion in the figure) for the specified range of γ_ϕ $(-0.6>\gamma_\phi>-0.8)$ and Ω_{m0} $(0.5>\Omega_{m0}>0.3)$, where we have assumed α is small. It can be noticed that the allowable range of both the parameters α and ω obeys the constraints (18) and (29) imposed on them by physical conditions.

A point to note here is that in BD theory the gravitational coupling G varies inversely with the scalar field ϕ. At present time ϕ approaches a constant value ϕ_0, the inverse of which gives the present Newtonian constant G_N. In the weak-field limit the present Newtonian coupling and the asymptotic value of ϕ is related by

$$G_N=\frac{2\omega+4}{2\omega+3}\,\frac{1}{\phi_0}. \tag{35}$$

The present day variation of the gravitational coupling G, is $\dot{G}/G|_0=(2/3\alpha)H_0$, where $H_0(=\alpha/t_0)$ is the Hubble parameter at present. For any value of α that allows acceleration this rate is $<10^{-10}$ per year [39].

Another important point to mention here is that time variation of G does not directly affect the nuclear process of the early universe. But the expansion rate of the universe in this type of theory does influence the primeval nucleosynthesis [40]. A fixed value for the parameter $\alpha>1$ for all epochs implies that universe is always accelerating which seriously contradicts the nucleosynthesis scenario. One way to avoid such a problem is to consider ω as a function of the scalar field ϕ. In a recent work Banerjee and Pavon [41] have shown that with $\omega(\phi)$ one can have a decelerating radiation dominated era in the early time and accelerated matter dominated era in the late time. But in their case also, ω has to be small asymptotically to have a late time acceleration for the universe.

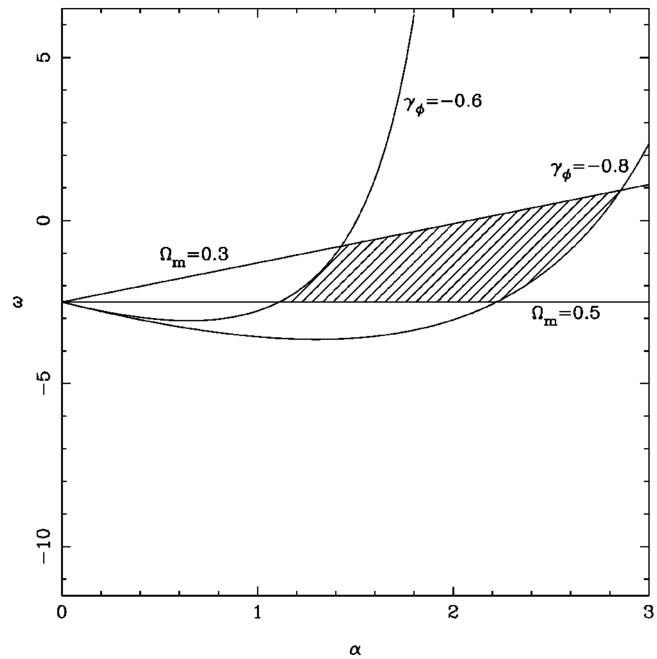

FIG. 1. ω vs α for $-0.6>\gamma_\phi>-0.8$ and $0.5>\Omega_{m0}>0.3$.

To analyze the nature of acceleration and our ansatz more critically, it is interesting to match different cosmological parameters with observations. We intend to find the age of the universe and the luminosity distance-redshift relation compatible with our model, probing the background dynamics, that could differentiate between different types of universe.

Since one of the main incentives for reconsidering the introduction of the cosmological constant was the age of the universe, we first consider the age of the universe suggested in our model and the constraints imposed on it by observations. Equation (8) can also be presented as

$$H^2=H_0^2[\Omega_{m0}+\Omega_{\phi 0}](1+z)^{2/\alpha}, \tag{36}$$

where z is the redshift defined by

$$1+z=\frac{R_{observed}}{R_{emitted}}. \tag{37}$$

From Eq. (36) we find the age of the observable universe for a given redshift z is

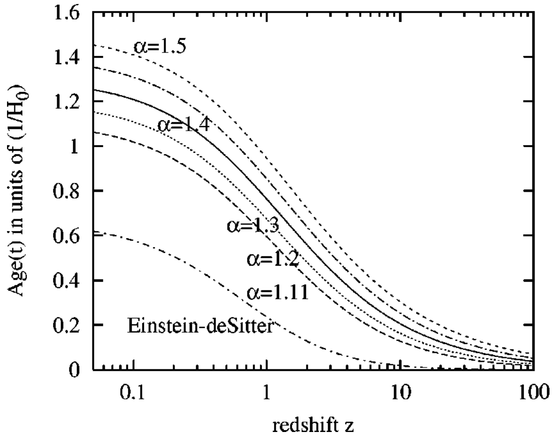

FIG. 2. Age (t) vs redshift z for different α values.

$$t_0 - t = \frac{\alpha}{H_0(\Omega_{m0}+\Omega_{\phi 0})^{1/2}}\left[1-\frac{1}{(1+z)^{1/\alpha}}\right]. \quad (38)$$

Of course for $t=0$, i.e., for infinite redshift, the age of the universe is $t_0=\alpha/H_0$.

An old object observed at a certain redshift selects all models with at least that age at that given redshift. In that respect, several age constraints have recently appeared in the literature [42]. For example, the age of the radio galaxy 53W091 observed at a redshift $z=1.55$ puts a lower bound of 3.5 Gyr at that redshift. The quasar observed at $z=3.62$ sets a lower bound of 1.3 Gyr. In Fig. 2 we present a plot of the age of the universe as a function of redshift for various values of α. Taking into account the range of α, prescribed by Fig. 1, our universe has an age limit of $t_0 \geqslant 15.5$ Gyr. From the figure it can be seen that all the universes with that minimum age limit, except that of Einstein–de Sitter one, always accommodate these constraints. Recently Pont *et al.* [43] estimated the age of the universe to be 14 ± 2 Gyr, which is in excellent agreement with our result.

Now we would like to trace the change of luminosity distance with respect to the redshift in our model so as to compare it with the present data available. The result that reveals the so-called acceleration of the universe was the observation of the luminosity distance as a function of redshift for type Ia supernovas, which is believed to be a standard candle. From almost 60 redshifts, 42 high redshift data obtained by Supernova Cosmology Project and 18 low redshifts observed by Calan Tololo Supernova Survey favor a universe with a positive cosmological constant. Assuming flatness in the context of general relativity, the best fit for these data occurs for $\Omega_{m0}=0.28$ and $\Omega_{\lambda 0}=0.72$. Optical astronomers measure luminosities in logarithmic units, called magnitudes, given by

$$m(z)=\mathcal{M}+5\log d_L+25, \quad (39)$$

where $\mathcal{M}$ is the absolute magnitude and d_L is luminosity distance defined by

$$d_L = R(t_0)(1+z)r_1 \quad (40)$$

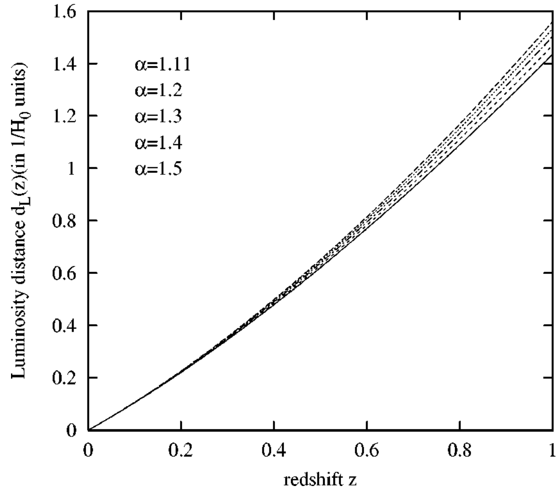

FIG. 3. Luminosity distance (d_L) vs redshift z for different α values.

for an event at $r=r_1$ at time $t=t_1$. According to our ansatz the expression for d_L is

$$d_L(z)=\frac{(1+z)}{H_0(\Omega_{m0}+\Omega_{\phi 0})^{1/2}}\int_0^z F(z')dz', \quad (41)$$

where $F(z)=1/(1+z)^{1/\alpha}$.

In Fig. 3 we have plotted this luminosity distance versus redshift for different values of α. We see that for different α values, the d_L is practically same for lower redshifts up to $z\sim 0.4$. At redshifts $z>0.4$ the curves are separated, but is not distinctly separate to discriminate and rule out different types of the models. Therefore high accuracy measurements with uncertainties at percentage level are needed in order to cleanly distinguish the models and the need to go to redshifts sensibly higher than 1 is evident. In this respect it is very much relevant to mention that Supernova Acceleration Probe (SNAP) is planned to make measurements with an accuracy at percentage level up to redshifts $z\sim 1.7$.

III. DISCUSSION

This work investigates the possibility of getting an accelerated universe in the context of the BD theory with a ϕ^4 potential having a time-dependent mass squared term and a matter field. In this work we have not used the quintessence field to trace the missing energy. The BD scalar field, which is a geometric scalar field, plays the role of dynamical Λ and provides that missing energy. It is found that for a simplistic approach of power-law expansion ($\sim t^\alpha$) a perfect fluid kind of matter (both pressureless and with pressure) cannot support a late time acceleration of the universe, if the scalar field has the potential given by Eq. (2). But a matter with a dissipative effect can provide the acceleration that agrees with the observational data sets. The dissipative effect accounted for by the negative pressure can be modeled in two ways according to recent investigations [29,34]. Particle production out of the gravitational field can give rise to negative pressure while energy can also be dissipated by the bulk viscous effect between the CDM particles. In this work though no particular model is considered for the origin of the negative pressure, it is found that $\nu > 1$. This is important for a hydrodynamical description if the CDM is assumed to be a conventional dissipative fluid. We have also calculated different parameters like the time variation of gravitational coupling, age of the universe, and the luminosity-distance redshift relation. All of these cosmological parameters agree quite well with the recent observations.

The accelerated solution depends crucially upon two parameters: α and BD parameter ω, both of which are constrained by different physical conditions. Different combinations of α and ω can produce the required values for γ_ϕ $(-0.6 > \gamma_\phi > -1)$ and Ω_m (~ 0.3) that tallies with present observation suggesting acceleration. Small α values restrict ω to small negative values and support the late time acceleration scenario quite successfully. The cosmological parameters calculated with the small value of α agrees with observations quite well. Many references [36,35,37] are available in the literature where it has been shown that this value should be very close to 1 to be consistent with observation. In one of our recent works [38] it is shown that for a simple power-law expansion $(\sim t^\alpha)$ of the universe, the best fit value of α with supernova type Ia (SNIa) data is approximately 1.25, and due to Eqs. (18) and (29), this will restrict ω to a small value. But this clearly contradicts the solar system limit $\omega > 600$. One should note that in our model, a large value of ω consistent with the solar system limit is not restricted either by physical conditions such as positivity of energy density or by the requirements of specific ranges for γ_ϕ or Ω_m consistent with the observations. Only the fact that α is not large is the prediction of the data obtained so far and this constrains ω to small value. If future observations predict large values for α then that can also be accommodated in our model with a large value for ω.

One should also note that, for $\alpha < 1.33$ which is consistent with the present data, the dissipative pressure is not sufficient to drive the acceleration alone, and the BD scalar field along with the dissipative pressure in the CDM drives the acceleration whereas the BD scalar field plays the role of the missing components of the universe.

It is also expected that after the phase transition, one expects ϕ to roll down from $\phi = 0$ taking different values in different directions causing large scale inhomogeneities. But it was argued by McDonald [26] that after this recent phase transition the universe will be filled by nontopological objects like ''*axions*'' whose radius r_ϕ is much smaller than 10 Mpc causing these axions to behave like smooth dark energy components.

The range of ω we obtained in our calculations in order to fulfil different physical conditions and also to have correct range of values for γ_ϕ, α, and Ω_m is consistent with the range obtained by other authors [24,38,41]. Also, it can be seen in Fig. 3 that for different values of ω and α it is difficult to distinguish between models up to $z \sim 1$. Hence our model is not very much fine tuned as far as the parameters α and ω are concerned.

It is also important to note that α remains constant throughout the age of the universe. This essentially means that the universe always accelerates for $\alpha > 1$, which seriously contradicts the primeval nucleosynthesis scenario. As we have mentioned earlier, one way to overcome this problem is to consider ω to be function of the scalar field ϕ [41]. A choice of ω (polynomial function of ϕ) can give a decelerating radiation era as well as accelerating matter dominated era. But then also ω asymptotically acquires a small negative value for an accelerating universe at late time. In most of the investigations done in scalar tensor theory [24,38,41] such conclusions have been arrived. The only exception so far in our knowledge is that done by Bertolami and Martins [22], where in Brans-Dicke cosmology with a ϕ^2 potential, the solution of an accelerated universe is obtained with large $|\omega|$. But there the positive energy condition on both matter and the scalar field have not been considered. There are also other evidences in literature where small $|\omega|$ has been supported. In the extended inflationary model by La and Steinhardt [44], the required value for ω is 20. The structure formation in scalar tensor theory also contradicts the solar system bound on ω [45]. Thus the problem seems to appear in different scales (astronomical and cosmological). The theory has been tested by experiments so far only in the astronomical scales and to our knowledge, no experiment has been done in cosmological scale as yet. And so the problem occurs in finding the compatibility between astronomical observation and cosmological requirements. Considering ω to be a variable to have both decelerating and accelerating phases at different epochs, while large ω values occur due to local inhomogeneities in astronomical scale to satisfy the solar system bound, can be a complete investigation and may give a satisfactory answer to this question.

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