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Shapiro conjecture: Prompt or delayed collapse in the head-on collision of neutron stars?

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We study the question of prompt versus delayed collapse in the head-on collision of two neutron stars. We show that the prompt formation of a black hole is possible, contrary to a conjecture of Shapiro which claims that collapse is delayed until after neutrino cooling. An understanding of the limitation of the conjecture is provided in terms of the many time scales involved in the problem. General relativistic simulations with the full set of Einstein equations coupled to the general relativistic hydrodynamic equations are carried out.

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INTRODUCTION

The study of the coalescence of neutron stars (NSs) is important for gravitational wave astronomy and high energy astronomy. However, at present we lack even a qualitative understanding of the process. One issue is the prompt versus delayed collapse problem. While we expect that two $1.4M_{\odot}$ NSs when merged will eventually collapse to form a black hole, the collapse could be delayed by fragmentation or mass shredding, angular momentum hang-up, and/or shock heating. The time scale of the collapse has important implications for the gravitational wave signals to be detected by the Laser Interferometric Gravitional Wave Observatory (LIGO) [1] and for models of gamma-ray bursts. We focus on the issue of prompt versus delayed collapse in this paper.

Recently, Shapiro [2] put up an argument suggesting that one may be able to answer this question without numerical simulations, at least for the case of head-on collisions. The ''Shapiro conjecture'' goes as follows: Given the conditions (i) that the two NSs are colliding head-on after falling in from infinity, and (ii) the NSs are described by a polytropic equation of state (EOS) $P=K\rho^{\Gamma}$ (with K a function of the entropy and the polytropic index Γ remaining constant throughout the collision process), it is conjectured that no prompt collapse can occur for an arbitrary Γ and an arbitrary initial K. The basic argument is that the potential energy when converted to thermal energy by shock heating is always enough to support the merged object, until neutrino cooling sets in.

The argument based on conservation is appealing, and provides useful understanding for a range of the NS coalescence problems. However, there is a major assumption for the argument to go through, namely, the collision process can be approximated by a quasiequilibrium process, in two senses: (a) The coalescing matter can be described by one *single* EOS everywhere (K is a function of time but not space), and (b) whether it collapses or not is determined by hydrostatic equilibrium conditions, i.e., whether a stable equilibrium configuration *exists* or not. This quasiequilibrium assumption is not self-evident for the head-on collision of heavier NSs. It could happen that the coalesced object collapses before it can thermalize in the sense of (a) above, *or* the collision process is so dynamic that even though a stable equilibrium state exists, it is not attained in the collapse process. The outcome depends on the various time scales in the problem.

TIME SCALE CONSIDERATIONS

We examine this assumption of ''quasiequilibrium'' and see if it can be justified under the conditions of (i) and (ii) above. We note that the collision process involves many time scales, and at least six are relevant for our present consideration: (1) The time scale associated with the infall velocity: $t_i=R/V_i$; $R=$ the radius of the NS, $V_i=$ infall velocity at the point of contact. (2) The time scale associated with the local sound velocity: $t_s=R/V_s$; $V_s=$ sound velocity. (3) The time scale associated with the velocity of the shock (in the rest frame of fluid): $t_{sh}=R/V_{sh}$; $V_{sh}=$ shock velocity. (4) The time scale for the merged object to thermalize, in the sense of being describable by one single EOS (same K everywhere): t_e. (5) The time scale of neutrino cooling t_n. (6) The time scale of the gravitational collapse t_c.

Some comments on these time scales are in order. We focus on the case of two $1.4M_{\odot}$ NSs. We model them with a polytropic EOS with a polytropic index of $\Gamma=2$. The initial K of the two stars is 1.16×10^5 cm^5/g s^2. (Maximum stable mass of these values of K and Γ is $1.46M_{\odot}$.) We note that the argument in [2] is applicable to all polytropic models.

For this model, V_i is (somewhat larger than) the Newtonian value $\sim 0.28c$, as can be estimated by $\sqrt{GM/(2R)}$; the diameter of the NSs is about 26 km (the isotropic coordinate radius of this NS is 9.3 km; the proper radius is 13 km). Hence the time scale associated with the infall velocity t_i is about (smaller than) 0.16 ms. To estimate the second time scale t_s, note that the sound velocity V_s depends strongly on the dynamical process and the region under consideration.

For the model mentioned above, the initial central rest mass density of the NSs is about 1.5×10^{15} g/cm^3; V_s there is about $0.5c$. With the density elsewhere initially lower than this value, but higher in some period in the central region of the collision, V_s varies but is roughly $0.5c$. Thus, t_s is roughly 0.1 ms. To estimate the third time scale t_{sh} requires an estimation of the velocity of the shock V_{sh} produced in the collision. The locally measured proper velocity of the shock V_{sh} is higher than, but of the same order of magnitude as, the sound speed V_s at a fraction of c in the head-on collision case. Hence t_{sh} is also of order 0.1 ms. These three time scales determine the time scale 4 which is central to our discussion. In near static situation, or when the bulk velocity of matter is small ($V_i \ll V_s$ and $V_i \ll V_{sh}$), t_e can be taken to be a few times t_s or t_{sh}. (Note that t_e is much shorter than the heat conduction time scale, with the shock being the main dissipation mechanism.) However, the value of t_e in a highly dynamic situation with V_i comparable to V_s and V_{sh} is an important issue to be discussed below. The 5th time scale t_n governs the final settling down of the merged object after t_e. t_n is of the order of seconds, orders of magnitude longer than the first four time scales. The gravitational collapse time scale t_c is controlled by these time scales 1–5. It can be as short as t_i, or as long as t_n. For the collision of two $1.4M_\odot$ NSs, the merged object would have to collapse after t_c, if not before, for most reasonable EOS. We call collapse that occurs on the first four time scales prompt collapse, and collapse that occurs on a longer time scale, such as t_n, delayed collapse. For more general coalescence processes, there can be other time scales involved, e.g., the time scale of angular momentum transfer t_a, and the time scale of gravitational wave emission t_g. However, for the case of head-on collision with the stars falling in from infinity, we expect strong shock heating causing t_n to be shorter than t_g. We do not have to consider t_a and t_g in our present consideration.

In Shapiro's argument, the time scale 4, t_e, is implicitly taken to be the shortest time scale in the problem, so that the system can be described by a single EOS at any instant in the collision process. The above discussion suggests that this may not be true for the two $1.4M_\odot$ NS collision case. Indeed, the relations between the time scales 1, 2, and 3 strongly affect t_e. With t_i comparable to t_s and t_{sh}, dynamic effects are important, and t_e can be longer than t_i. In particular, with matter falling in at high speed along the axis of the collision, the speed of the shock wave in that direction would be significantly reduced, until *after* t_i, delaying ''thermalization'' of the coalescing objects. For situations like this, arguments based on a uniform EOS throughout the coalescing object cannot be justified. Indeed, when the infalling time scale t_i is comparable to the other time scales in the process, it could happen that even if a hydrostatic stable equilibrium configuration exists, the dynamics of the system might not lead to that configuration and the time scale of collapse could be as short as t_i.

Another way of looking at the problem is to imagine we tie the two stars on strings and lower them towards one another in a quasistationary fashion while depositing the potential energy extracted back to the two stars. For this case Shapiro's argument would be applicable. However, for a NS collision with the time scales discussed above, one would have to examine the dynamics of the infall to determine whether a prompt or delayed collapse would occur. In short, as both a thermally supported merged object and a black hole can have the same rest mass and total energy, arguments based solely on conservation of mass and energy without taking dynamics into consideration cannot rule out one outcome from the other.

We note that the above time scale considerations suggest that whether it is a delayed or prompt collapse in head-on collision can depend on the initial NS's configuration. It does *not* imply prompt collapse by itself. To demonstrate that a prompt collapse results, one has to perform a fully relativistic simulation.

Our NASA Neutron Star Grand Challenge collaboration is developing a multipurpose 3D numerical code for relativistic astrophysics and gravitational wave astronomy (see http://wugrav.wustl.edu/Relativ/nsgc.html). This code contains the Einstein equations coupled to the general relativistic hydrodynamic equations. For a description of various aspects of the code see [3]. Testbeds and methods for evolving neutron stars have been given in [4], and will not be repeated in this paper. While this multipurpose code is still under development for various capabilities in treating a broad class of astrophysical scenarios, in this paper we focus on the results obtained by applying this code to the head-on collision problem.

SIMULATION RESULTS

We show the $1.4M_\odot$ head-on collision case. The stars are modeled as given above. We put the two TOV solutions at a proper distance of $d=44$ km apart (slightly more than $3R$ separation) along the z axis, and boost them towards one another at the speed (as measured at infinity) of $\sqrt{GM/d}$ (the Newtonian infall velocity). The metric and extrinsic curvature of the two boosted TOV solutions are superimposed by (i) adding the off-diagonal components of the metric, (ii) adding the diagonal components of the metric and subtracting 1, and (iii) adding the components of the extrinsic curvature. The resulting matter distribution, momentum distributions, conformal part of the metric, and transverse traceless part of the extrinsic curvature are then used as input to York's procedure [5] for determining the initial data. The initial data then satisfy the complete set of Hamiltonian and momentum constraints to high accuracy (terms in the constraints cancel to 10^{-6}), *and* physically represent two NSs in head-on collision falling in from infinity, at least up to the Newtonian order.

The initial data are evolved with the numerical methods described in [4]. Various singularity avoiding slicings have been used (maximal and $1+\log$ slicings most extensively), yielding basically the same results. The simulations have been carried out with resolutions ranging from $\Delta x=1.48$ km to 0.246 km (13 to 76 grid points across each NS, and 32^3 to 192^3 for the entire grid) for convergence and accuracy analysis.

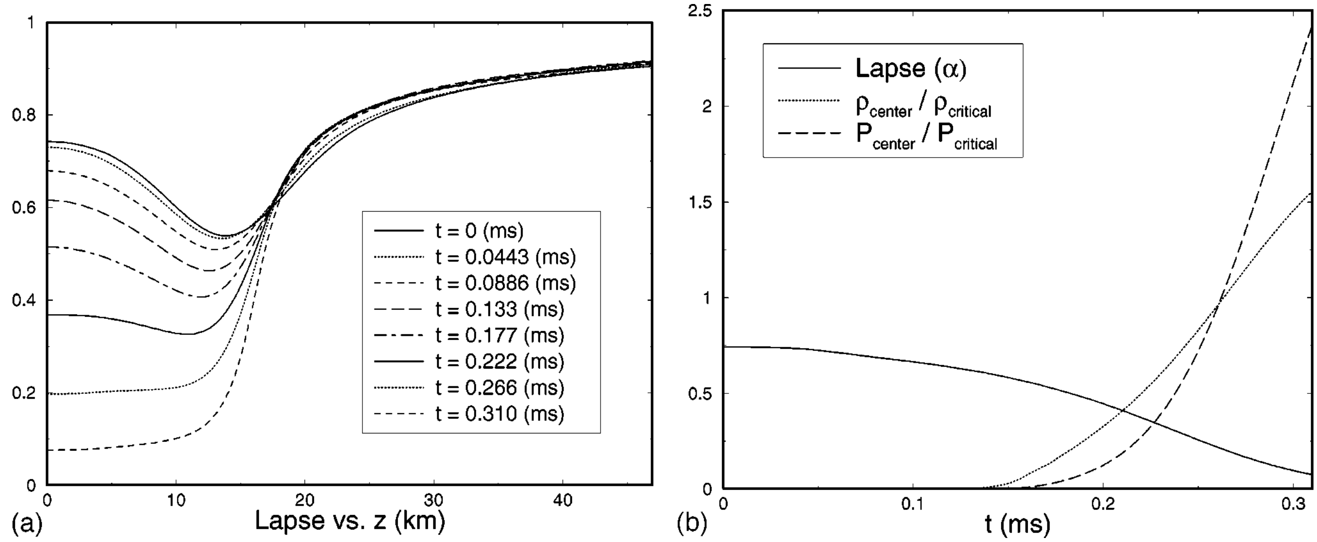

FIG. 1. (a) The lapse (α) along the z axis is displayed at various times. This simulation used 192^3 grid points, with $\Delta x = 0.246$ km. (b) The evolution of the lapse (α), the rest mass density (ρ), and the pressure (P) in the region centered at the point $x = y = z = 0$ to the time $t = 0.31$ ms.

In Fig. 1(a) we show the collapse of the lapse along the $x = y = 0$ line from $t = 0$ ms to $t = 0.31$ ms at intervals of 0.044 ms. (With the reflection symmetry across the $z = 0$ plane and the axisymmetry of the head-on collision, we only need to evolve the first octant.) At $t = 0.31$ ms the lapse has collapsed significantly.

Figure 1(b) shows the time development of the lapse, the (proper) rest mass density ρ, and the pressure P at the origin, scaled by the critical secular stability values $\rho_{critical}$ and $P_{critical}$, the values beyond which a static TOV solution is unstable to collapse for the given polytropic coefficient K and index Γ. We note that the effective $K(=P/\rho^2)$ is time dependent due to shock heating. At coordinate time $t = 0.26$ ms we see that both ρ and P surpass $\rho_{critical}$ and $P_{critical}$, indicating a collapse.

In Fig. 2 we show the position of the apparent horizon (AH). To confirm the location of the AH, convergence tests both in terms of resolution and in terms of location of the computational boundary have been carried out. (For a discussion of the AH finder, see [6]). We have also explicitly determined trapped surfaces bounded by the AH for the confirmation of a collapsed region. The solid and long-dashed lines correspond to the AH locations at resolutions of $\Delta x = 0.492$ km and $\Delta x = 0.246$ km, while the dotted line corresponds to $\Delta x = 0.492$ km but with the outer boundary two times further out. Although the coordinate position of the AH is substantially elongated in the z direction, the AH is actually quite spherical. The proper circumference on the x-y plane (equatorial) is close to the circumference on the x-z plane (polar), with the latter being 52.9 ± 1.9 km. For comparison, $4\pi M_{AH}$ is 52.9 ± 2.1 km, where M_{AH} is the mass of the AH (we note that a substantial part of the matter in the system is enclosed within the AH).

Figure 3 shows contour lines in the $y = 0$ plane of the log of the gradient of the rest mass density $\log(\sqrt{\nabla^i(\rho)\nabla_i(\rho)})$ at time $t = 0.31$ ms. We see a sharp peak at a coordinate radius of ~ 6.0 km. The sharp change in rest mass density indicates a shock, stronger in the infalling direction (z), while weaker near the equatorial plane. The shock is moderately relativistic with a Lorentz factor of about 1.2. The shock is well captured in this 192^3 run with high resolution shock capturing (HRSC) general relativistic hydro (GR-hydro) treatment. Comparing to Fig. 2, we see that the shock front is inside the AH in all directions at this time, although it is still moving outward in coordinate location.

In Fig. 4 we show the convergence of the Hamiltonian

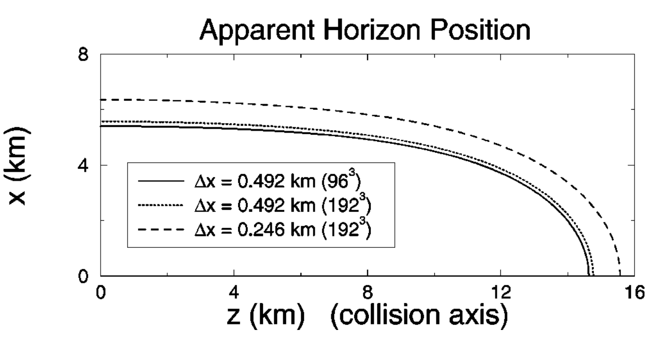

FIG. 2. The position of the AH at different resolutions and outer boundary locations, all at $t = 0.31$ ms.

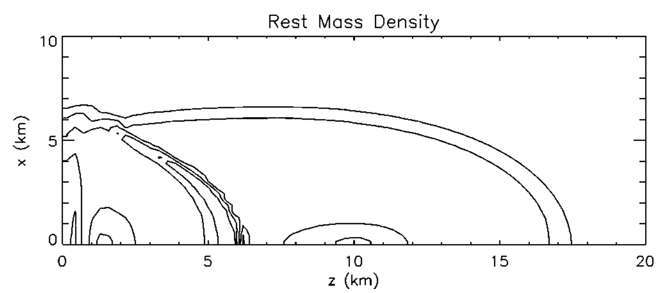

FIG. 3. Equally spaced contour lines of the log of the gradient of the rest mass density $\log(\sqrt{\nabla^i(\rho)\nabla_i(\rho)})$, showing a shock front at coordinate radius ~ 6.0 km.

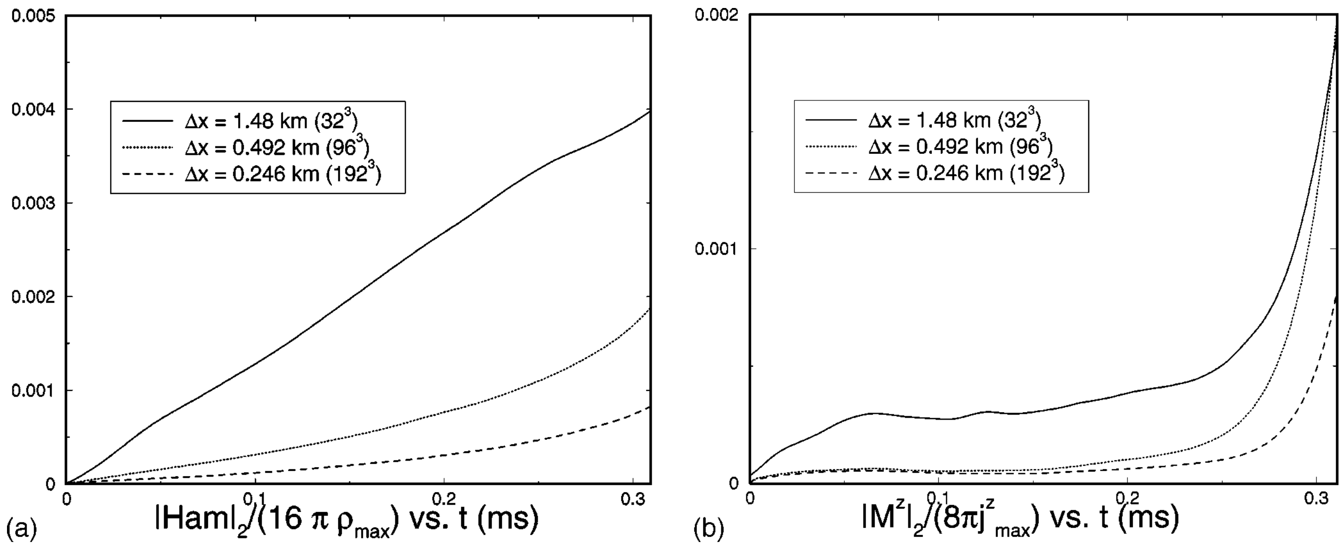

FIG. 4. The evolutions of the $L2$ norms of (a) the Hamiltonian constraint, and (b) the z component of the momentum constraint.

and the z-momentum constraints for a measure of the accuracy of the simulation. The evolution of the $L2$ norms (integrated squared) of the constraints are scaled by the maximum of the matter terms in the constraints ($16\pi\rho_{ADM}$ and $8\pi j^z_{ADM}$, respectively). The solid, dotted, and dashed lines represent the constraints at resolutions $\Delta x = 1.48$ km, 0.492 km, and 0.246 km, respectively. These *long time scale* convergence tests indicate that our numerical evolution is *stable and convergent* for the time scale of our present problem. Towards the end we see that the error is increasing rapidly; an examination of the spatial distribution of the constraints shows that the error is due to the familiar problem of resolving the ''grid stretching'' peaks of a black hole metric. Extensive convergence analysis of many of the variables involved in the simulation has been carried out and will be presented in a followup paper. We have carefully examined the code's ability to correctly capture and propagate shocks in dynamical spacetimes. We have also performed simulations with the initial boost velocity increased by 10% (generating more shock heating) and confirmed that our results are not sensitive to the initial velocity.

To further substantiate the above results, we have carried out simulations of head-on collisions of lower mass NSs. In Fig. 5 we verify that the numerical results have the same scaling as the GR-hydro equations on the analytic level under the coordinate transformation $(t,x,y,z) \rightarrow (\sigma t, \sigma x, \sigma y, \sigma z)$. For $\sigma = 0.067$, the $1.4 M_\odot$ NS studied above is scaled to a $0.1 M_\odot$ NS with $K = 5.20 \times 10^2$ cm^5/g s^2. We again show $\log(\sqrt{\nabla^i(\rho)\nabla_i(\rho)})$ at the scaled time $t = 0.021$ ms when the AH is found. The contour plot is exactly the same as that of Fig. 3 when plotted with the corresponding scale length. In Fig. 6, we again show the $0.1 M_\odot$ NS but now with the original K value $K = 1.6 \times 10^5$ cm^5/g s^2 (the same value as the $1.4 M_\odot$ case). At $t = 1.11$ ms, the shock wave has travelled through more than half of the star, and the central density has begun to decrease (first bounce), the lapse starts increasing from its minimum value of 0.96. No AH is formed. To illustrate that this configuration does not collapse to a black hole promptly, we plot the maximum rest mass density of the entire configuration as a function of time in Fig. 7. Notice that by $t = 1.5$ ms, the maximum rest mass density of the entire configuration is less than that of the initial central rest mass density of the initial stars; the first bounce is well under way by this time.

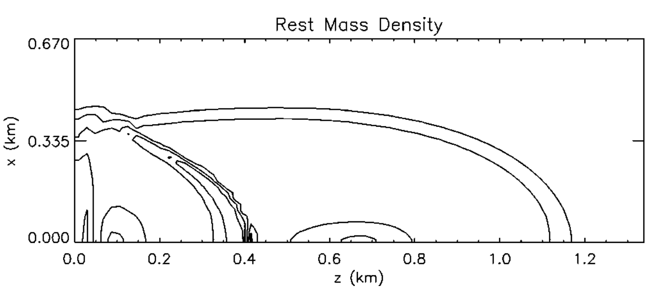

FIG. 5. Contour lines of the log of the gradient of the rest mass density $\log(\sqrt{\nabla^i(\rho)\nabla_i(\rho)})$, for the head-on collision of two $0.1 M_\odot$ stars, with $K = 5.20 \times 10^2$ cm^5/g s^2. The coordinate time is 0.021 ms.

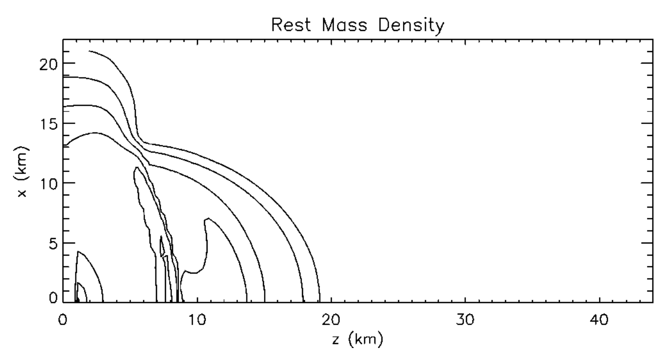

FIG. 6. Contour lines of the log of the gradient of the rest mass density $\log(\sqrt{\nabla^i(\rho)\nabla_i(\rho)})$, for the head-on collision of two $0.1 M_\odot$ stars, with $K = 1.16 \times 10^5$ cm^5/g s^2, at the time of first bounce $t = 1.11$ ms.

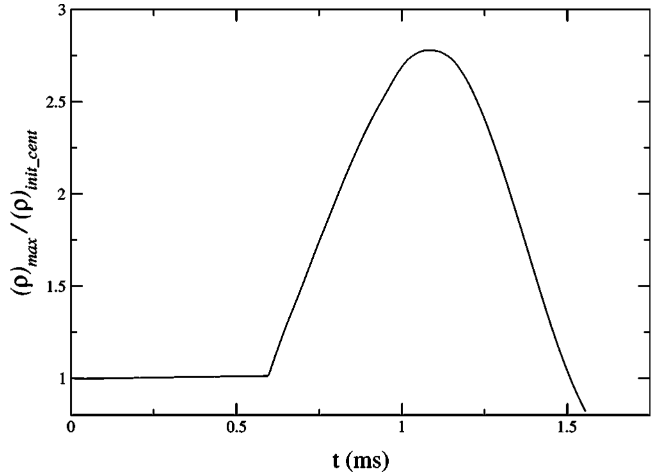

FIG. 7. Maximum rest mass density $(\rho)_{max}$ as a function of time for the head-on collision of two $0.1M_\odot$ stars, with $K=1.16 \times 10^5$ cm^5/g s^2. The rest mass density ρ is normalized by the central density of each star at the initial time. The bounce occurs at roughly $t=1.1$ ms (see also Fig. 6). By $t=1.5$ ms, the first bounce is well underway with the maximum density dropping below 1.

CONCLUSIONS

We pointed out that there is an assumption in Shapiro's conjecture, namely, the head-on collision process is in quasiequilibrium [in the sense of (a) and (b) above]. We showed that this may not be true for the collision of two $1.4M_\odot$ NSs. We substantiated our argument with a simulation solving the full set of coupled Einstein and general relativistic hydrodynamic equations. We confirmed the prompt formation of a black hole in the infalling time scale t_i with an apparent horizon found 0.16 ms after the point of contact.

In this paper we concentrate on the head-on collision process under the same conditions as in Shapiro's conjecture. As the time scale argument given above is rather general, and in particular does not depend on the polytropic EOS, we expect the same argument to be applicable to more general situations. An investigation of the prompt versus delayed collapse problem of head-on collisions with realistic EOSs, more realistic initial conditions (initial data setup with post-Newtonian formulation), and with a determination of the critical point between delayed versus prompt collapse will be given in followup papers.

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