

## Specific heat of normal, degenerate quark matter: Non-Fermi liquid corrections

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(Received 28 November 2000; published 10 May 2001)

In normal degenerate quark matter, the exchange of dynamically screened transverse gluons introduces infrared divergences in the quark self-energies that lead to the breakdown of the Fermi liquid description. If the cores of neutron stars are composed of quark matter with a normal component, cooling by direct quark URCA processes may be modified by non-Fermi liquid corrections. We find that while the quasiparticle density of states is finite and nonzero at the Fermi surface, its frequency derivative diverges and results in non-Fermi liquid corrections to the specific heat of the normal, degenerate component of quark matter. We study these non-perturbative non-Fermi liquid corrections to the specific heat and the temperature dependence of the chemical potential and show that these lead to a reduction of the specific heat.

DOI: 10.1103/PhysRevD.63.114028

PACS number(s): 12.38.Mh, 11.15.Bt

### I. INTRODUCTION AND MOTIVATION

A detailed description of the phase diagram of QCD in hot and dense environments can lead to a deeper understanding of early Universe cosmology as well as the physics of compact stellar objects. Current experimental programs at the CERN Super Proton Synchrotron (SPS) and BNL Relativistic Heavy Ion Collider (RHIC) probe QCD at high temperatures but relatively small baryon (and quark) density [1] and the future ALICE program at the CERN Large Hadron Collider (LHC) will probe at even higher temperatures and smaller densities, in conditions that prevailed a few microseconds after the big bang. The goal of these ultrarelativistic heavy ion experiments is to search for the quark gluon plasma, a novel phase of QCD in which quarks and gluons are deconfined. The region of the phase diagram of QCD for low temperatures  $T \leq 10$  MeV but densities a few times larger than that of nuclear matter, cannot be studied with terrestrial accelerators but are the realm of compact stellar objects: neutron stars and pulsars. There is the tantalizing possibility that at the core of neutron stars where the density is up to 5 times that of nuclear matter, there could be a component of cold, degenerate quark matter with temperatures  $T \leq 1$  MeV and chemical potentials  $\mu \sim 300 - 500$  MeV [2,3].

An important aspect of dense QCD is the possibility of color superconductivity [4–13]. One gluon exchange leads to a pairing instability of the quark Fermi surface in the color antitriplet channel [4], and while originally superconducting gaps were estimated to be of order  $\Delta \sim 1$  MeV more recent estimates including dynamical screening of the exchanged gluon [9] conclude that the gaps could be  $\Delta \sim 50 - 100$  MeV [5–13].

The existence of quark matter phases in the cores of neutron stars and pulsars could have distinct observational consequences. The equation of state of quark matter may lead to

pronounced delays in the spin-up history of neutron stars in low mass x-ray binaries and may explain recently observed anomalous frequency distributions [14], a deconfinement transition can lead to observational consequences in the rotational properties of pulsars [15], superconducting quark matter can influence the strength and distribution of magnetic fields of pulsars [16,17] and could influence the cooling history of young neutron stars [18] and protoneutron stars [19,20].

The properties of neutron stars and pulsars are studied by a satellite program that includes the Einstein Observatory, ROSAT, AXAF, RXTE and more recently CHANDRA and XMM that measure the (soft) x-ray emission from neutron stars, thus studying their cooling history as well as rotational properties (spin-up and spin-down).

A few seconds after the collapse of a Type II supernovae the newly born neutron star cools via the emission of neutrinos and antineutrinos and within few minutes the temperature falls to about  $10^9$  K ( $\sim 0.1$  MeV) cooling proceeds via neutrino emission for another  $10^5 - 10^6$  years before photon emission becomes the most important cooling mechanism [21–26].

The most efficient neutrino emission mechanisms are the direct URCA processes  $f_1 + l \rightarrow f_2 + \nu_l$ ;  $f_2 \rightarrow f_1 + l + \bar{\nu}_l$  with  $f_i$  being either baryons or quarks and  $l$  either electrons or muons with  $\nu_l$ ;  $\bar{\nu}_l$  their respective neutrinos and antineutrinos [18]. In the region between the crust and the core, where nuclear matter is the dominant component this process occurs with baryons and corresponds to the beta decay of the neutron and electron capture in a proton. If the core is composed of quark matter the corresponding processes are direct quark Urca,  $d \rightarrow u + e^- + \bar{\nu}_e$ ;  $u + e^- \rightarrow d + \nu_e$  which have been studied in detail in Ref. [28]. The cooling rate is determined by the equation [21–26]

$$\frac{dU}{dt} = C_v \frac{dT}{dt} = -(L_\nu + L_\gamma) \quad (1.1)$$

with  $C_v$  the specific heat (at constant volume and baryon number) and  $L_{\nu,\gamma}$  are the neutrino and photon luminosities, any other potential sources of energy loss such as magnetic field decay, differential rotation etc. had been neglected in the above equation.

The superconducting (and superfluid) component gives a contribution to the specific heat and the neutrino emissivity that is exponentially suppressed by the gap  $\propto e^{-\Delta/T}$  if  $\Delta \gg T$  [22,24,18]. Recently, however, it was noted that novel aspects of cooling during the *protonneutron* star stage, when  $T \sim 30\text{--}50$  MeV could be a result of the color superconducting transition [19] since in this regime  $\Delta \leq T$ .

For two light flavors (u,d) of quarks one gluon exchange leads to an instability in the antitriplet channel leading to a color superconducting state known as (2SC) [13,20]. This state leaves quarks of one color ungapped while the others pair up with a gap  $\Delta \sim 50\text{--}100$  MeV. The temperature of young neutron stars is  $T \sim 0.1\text{--}1$  MeV, hence in a color superconducting state the paired quarks will not contribute to cooling. Including the strange quark the color superconducting state for three flavors is a color-flavor-locked phase with all quarks paired.

If the strange quark mass is too large to allow for *S*-wave pairing in the color-flavor-locked phase it may still form a *P*-wave condensate but with a very small gap [27], in which case *S*-wave color superconductivity will be in the form of a (2SC) condensate. In this case the only color degree of freedom remaining ‘‘normal’’ and the strange quark contribute to the cooling via neutrino emissivity but the contribution from the strange quark will be suppressed by the (small) gap [27] as well as by the small Cabibbo angle [28].

There are two important quantities that determine the cooling rate: the specific heat  $C_v$  and the neutrino emissivity. Both are completely determined by the physics near the Fermi surface of the degenerate quarks and leptons (the only important ones are electrons).

Iwamoto [28] calculated the neutrino emissivity of normal quarks via the direct quark URCA processes, assuming the validity of Fermi liquid theory, and found it to be  $\epsilon \propto T_9^6$  ( $T_9 = T/10^9 K$ ). The power of temperature is found on physical grounds: each degenerate fermion (quarks and electron) has associated a phase space factor of  $d^3p/|p| \sim p_F T$  since only states within a region of width  $T$  near the Fermi surface contribute, this gives three powers of  $T$ , energy-momentum conservation restricts the neutrino to have energy-momentum  $\sim T$  and since the neutrinos are not Pauli blocked their available phase space is  $\sim E_\nu^2 dE_\nu \propto T^3$ . The energy conservation delta function leads to a factor  $1/T$  which is compensated by a factor  $E_\nu \sim T$  in the numerator for the energy loss. Thus the energy loss by neutrino emission is  $\propto \prod_{f=u,d,e} (p_F(f) T) T^3$  [28]. The linear power of  $T$  arising from phase space for each degenerate fermion is determined by the single particle density of states near the Fermi surface which also determines the linear temperature dependence of the specific heat, as can be seen from the following argument. The contribution from massless degenerate quarks to the temperature change in the internal energy is determined by the states in a region of width  $\approx T$  from the Fermi surface,

within which quarks have thermal energy  $\approx T$ . The density of single particle states at the Fermi surface  $p_F = \mu$  for relativistic fermions is  $\propto p_F^2 = \mu^2$ , thus within a region of width  $T$  around the Fermi surface there are  $\sim \mu^2 T$  states of energy  $\sim T$  leading to their contribution to the temperature change in internal energy

$$\delta U_q(\mu, T) \propto \mu^2 T^2 \rightarrow C_v \propto \mu^2 T. \quad (1.2)$$

The main reason for repeating these arguments is to highlight that both the neutrino emissivity and the specific heat of the normal component are determined by the (quasi) particle states within a region of width  $T$  near the Fermi surface.

In Refs. [9,29] it was realized that the exchange of dynamically screened gluons that lead to the pairing instability also leads to a breakdown of the Fermi liquid picture for quarks near the Fermi surface. In particular these non-Fermi liquid corrections are responsible for a decrease in the gap [29]. A detailed study of non-Fermi liquid aspects of the normal state was recently presented in [30]. There the spectral density, dispersion relation and width of quasiparticles with momenta near the Fermi surface were derived at  $T=0$  by implementing a renormalization group resummation of the leading logarithmic infrared divergences associated with the emission of soft dynamically screened transverse gluons. These non-Fermi liquid corrections modify the properties of quasiparticle states near the Fermi surface. Since cooling of degenerate quark matter is sensitive to the properties of the (quasi) particle states near the Fermi surface, it is natural to study the possible non-Fermi liquid corrections to the specific heat as well as the neutrino emissivity.

Our goal in this article is to study in detail the non-Fermi liquid corrections to the specific heat to establish if these result in an enhancement or suppression of the cooling rate via neutrino emission through direct quark Urca processes, postponing the study of non-Fermi liquid effects upon the neutrino emissivity to a forthcoming article.

## II. QUASIPARTICLE DENSITY OF STATES, CHEMICAL POTENTIAL AND SPECIFIC HEAT

Our strategy to obtain the specific heat (at constant volume and baryon density) is to first obtain the internal energy and take its temperature derivative. The internal energy in turn is the expectation value of the full Hamiltonian  $H = H_q + H_g + H_{q-g}$ , the sum of the quark, gluon and interaction parts. The regime of interest for neutron star phenomena is

$$T \approx 0.1\text{--}1 \text{ MeV}; \quad \mu_q \approx 0.3\text{--}0.5 \text{ GeV} \quad (2.1)$$

i.e.,  $T \ll \mu$ . Furthermore the scale that enters in the resummed quark propagator [30] is  $M \approx g\mu/2\pi$  at energy scales  $\sim 1$  GeV the strong coupling constant  $\alpha_s \approx 0.6$  hence  $M \sim \mu_q$  implying the hierarchy  $T \ll M \sim \mu$ . The contribution from massless gluons or Goldstone bosons remaining from the superconducting transition, to the internal energy is

$$U_g(T) \propto T^4. \quad (2.2)$$

Therefore the gluon contribution to the internal energy is subleading by a factor  $T^2/\mu^2$  as compared to the quark contribution which, as argued above is  $\delta U \sim \mu^2 T^2$ . Hence we will neglect the contribution from  $H_g$  to the internal energy. The expectation value of  $\tilde{H} = H_q + H_{q-g}$  can be obtained straightforwardly from the spectral density of the quark field by the following observation: the quark Hamiltonian including the interaction with the gluon field is bilinear in the quark field, and the equation of motion is

$$i \frac{\partial \Psi}{\partial t} = \frac{\delta \tilde{H}}{\delta \Psi^\dagger} \quad (2.3)$$

therefore it follows that

$$\int d^3x \Psi(\vec{x}, t) i \gamma^0 \partial_t \Psi(\vec{x}, t) = \tilde{H}. \quad (2.4)$$

Hence we write

$$\begin{aligned} U(T, \mu) &= \langle \tilde{H} \rangle \\ &= \int d^3x i \partial_t [\langle \Psi_b(\vec{x}', t') \Psi_a(\vec{x}, t) \rangle \gamma_{b,a}^0]_{\vec{x}'=\vec{x}; t'=t}. \end{aligned} \quad (2.5)$$

The fermion correlator can be written in terms of the spectral representation of its spatial Fourier transform as follows [30]:

$$\langle \Psi_b(\vec{x}', t') \Psi_a(\vec{x}, t) \rangle = -i S_{a,b}^<(\vec{x} - \vec{x}'; t - t') \quad (2.6)$$

$$-i S^<(\vec{k}; t - t') = \int dq_0 \rho(q_0, \vec{k}) N(q_0) e^{-iq_0(t-t')} \quad (2.7)$$

$$N(q_0) = \frac{1}{e^{\beta(q_0 - \mu)} + 1}. \quad (2.8)$$

Therefore, we find

$$\begin{aligned} \frac{U(T, \mu)}{V} &= \mathcal{U}(T, \mu) \\ &= \int \frac{d^3k}{(2\pi)^3} \int dq_0 q_0 \text{Tr}[\gamma^0 \rho(q_0, \vec{k})] N(q_0). \end{aligned} \quad (2.9)$$

In the grand canonical ensemble, the average number of particles is given by

$$\begin{aligned} N &= \int d^3x \langle \Psi(\vec{x}, t) \gamma^0 \Psi(\vec{x}, t) \rangle \\ &= \int d^3x \text{Tr}[-i S^<(\vec{x} - \vec{x}'; t - t') \gamma^0]_{\vec{x}'=\vec{x}; t'=t} \end{aligned} \quad (2.10)$$

which is independent of temperature and given in terms of the spectral density by

$$N = \int \frac{d^3k}{(2\pi)^3} \int dq_0 \text{Tr}[\rho(q_0, \vec{k}) \gamma^0] N(q_0). \quad (2.11)$$

The temperature dependence of the chemical potential  $\mu(T)$  is found by requiring that  $N$  be independent of temperature.

In the degenerate case temperature only affects states that are a distance  $\approx T$  from the Fermi surface, for positive baryochemical potential only quasiparticle states are important, since the contribution of the antiparticles will be suppressed by a factor  $\exp(-\mu/T) \ll 1$  [10,11]. For massless quarks the spectral function  $\rho(q_0, \vec{k})$  is of the form [10,11]

$$\rho(q_0, \vec{k}) = \frac{1}{2} [\mathcal{P}_-(\hat{\mathbf{k}}) \rho_-(q_0, \vec{k}) + \mathcal{P}_+(\hat{\mathbf{k}}) \rho_+(q_0, \vec{k})], \quad (2.12)$$

with  $\mathcal{P}_\pm = \gamma^0 \pm \vec{\gamma} \cdot \hat{\mathbf{k}}$  and  $\rho_\pm$  are the spectral density for quasi-antiparticles (+) and quasiparticles (-) respectively. As argued above temperature affects only states a distance  $\approx T$  from the Fermi surface, hence only quasiparticle states are important, and antiparticle states are exponentially suppressed. Therefore the contribution to the internal energy, specific heat, and (quasi) particle density for states near the Fermi surface are given by

$$\mathcal{U}(T, \mu) = 2 \int dq_0 q_0 \eta(q_0) N(q_0) \quad (2.13)$$

$$C_v = \frac{d\mathcal{U}}{dT} \quad (2.14)$$

$$\mathcal{N} = \frac{N}{V} = 2 \int dq_0 \eta(q_0) N(q_0) \quad (2.15)$$

where we have introduced the *single quasiparticle density of states*

$$\eta(q_0) = \int \frac{d^3k}{(2\pi)^3} \rho_-(q_0, \vec{k}). \quad (2.16)$$

In perturbation theory, the quark self-energy features an infrared divergence for states near the Fermi energy [9,29] which lead to the breakdown of the Fermi liquid picture. These divergences are a consequence of the exchange of dynamically screened soft transverse gluons, while longitudinal gluons are Debye screened. In Ref. [30] the leading logarithmic infrared divergences were resummed via the renormalization group at  $T=0$ , leading to a scaling form of the spectral density for frequency and momenta near the Fermi surface. The calculation for  $T \neq 0$  but in the degenerate case  $T \ll \mu$  is performed in a straightforward manner following the steps in this reference [30]. At finite temperature and chemical potential the self-energy for transverse gluons receives contributions from both the gluon and fermion thermal and dense loops [31–36]. In the degenerate case derivatives of the Fermi-Dirac distribution function are sharply peaked at the Fermi surface. For  $N_c$  colors and  $N_f$  quarks in the fundamental representation and  $q_0, k \sim \mu$  and after a

renormalization group resummation of the leading logarithmic divergences along the same lines as that in [30] we find that the improved quasiparticle spectral density for states near the Fermi surface is given by

$$\rho_{-}(q_0, \vec{k}) = \frac{\sin[\pi\lambda]}{\pi} \frac{|\tilde{q}_0| \left| \frac{\tilde{q}_0}{M} \right|^{-2\lambda}}{\left[ \left( \left| \frac{\tilde{q}_0}{M} \right|^{-2\lambda} - \tilde{k} \cos[\pi\lambda] \right)^2 + (\tilde{k} \sin[\pi\lambda])^2 \right]} \quad (2.17)$$

$$\tilde{q}_0 = q_0 - \mu; \quad \tilde{k} = k - \mu \quad (2.18)$$

$$\lambda = \frac{\alpha_s}{6\pi} \frac{N_c^2 - 1}{2N_c} \quad (2.19)$$

$$M^2(T) = \frac{\alpha_s}{2\pi} N_f \left[ \mu^2 + \frac{\pi^2}{3} T^2 \right] + \frac{\pi\alpha_s}{3} \left( \frac{N_c^2 - 1}{2N_c} \right) T^2 \quad (2.20)$$

where the Landau damping scale  $M^2(T)$  displays the contribution from quark loops (first term) and gluon loops (second term) [32]. This form of the spectral density is valid for  $|\tilde{q}_0|; |\tilde{k}| \ll M$  since the renormalization group resummation only includes the leading infrared logarithmic divergences and neglects perturbative analytic contributions. Since  $T \ll M$  this form of the quasiparticle spectral density determines the finite temperature properties in the degenerate case.

As discussed in detail in Ref. [30], this quasiparticle spectral density features a narrow resonance for  $\tilde{k} \neq 0$ , at [30]

$$\tilde{q}_0(\tilde{k}) = \text{sgn}(\tilde{k}) [|\tilde{k}| M^{-2\lambda} \cos[\pi\lambda]]^{1/(1-2\lambda)}. \quad (2.21)$$

Near the position of the resonance the spectral density can be approximated by a Breit-Wigner form [30]

$$\rho_{-}(q_0, k) \Big|_{\tilde{q}_0, k \approx \mu} = Z_p[\tilde{k}] \frac{\cos[\pi\lambda]}{\pi} \frac{\Gamma(\tilde{k})}{(\tilde{q}_0 - \tilde{q}_0(\tilde{k}))^2 + \Gamma^2(\tilde{k})} \quad (2.22)$$

$$Z_p[\tilde{k}] = \frac{\left| \frac{\tilde{q}_0(\tilde{k})}{M} \right|^{2\lambda}}{(1-2\lambda)} \quad (2.23)$$

$$\Gamma(\tilde{k}) = Z_p[\tilde{k}] |\tilde{k}| \sin[\pi\lambda]. \quad (2.24)$$

Furthermore the group velocity near the Fermi surface is given by

$$v_g(\tilde{k}) = \frac{d\tilde{q}_0}{d\tilde{k}} = Z_p[\tilde{k}] \cos[\pi\lambda]. \quad (2.25)$$

Therefore the residue of the ‘‘quasiparticle pole,’’ the ‘‘quasiparticle width’’ and the ‘‘quasiparticle group velocity’’ vanish near the Fermi surface as [30]

$$Z_p[\tilde{k}] \propto |k - k_F|^{2\lambda/(1-2\lambda)} \quad (2.26)$$

$$\Gamma(\tilde{k}) \propto |k - k_F|^{1/(1-2\lambda)} \quad (2.27)$$

$$v_g(\tilde{k}) = Z_p[\tilde{k}] \cos[\pi\lambda] \propto |k - k_F|^{2\lambda/(1-2\lambda)}. \quad (2.28)$$

The vanishing of the residue at the quasiparticle pole at the Fermi surface is the hallmark of the breakdown of Fermi liquid theory. In a degenerate Fermi gas, all of the thermodynamic response functions are determined by the quasiparticle properties near the Fermi surface. The vanishing of the quasiparticle residue at the Fermi surface, and the consequent breakdown of Fermi liquid theory has the potential for modifying the thermodynamic response functions. The important aspect that we seek to understand is how these non-Fermi liquid features affect the specific heat of quark matter which is relevant for the cooling of neutron stars with a normal component of quark matter in the core.

As discussed in [30] the non-Fermi liquid corrections to the quasiparticle spectral density are *non-perturbative* as explicitly displayed by the anomalous powers in the dispersion relation and the wave function renormalization. The renormalization group improved quasiparticle spectral density (2.17) has been obtained by a resummation of the leading infrared divergences near the Fermi surface and neglects perturbative corrections which are analytic in the frequency and momentum near the Fermi surface [30]. The validity of Eq. (2.17) is restricted to a region  $-M < \tilde{q}_0; \tilde{k} < M$  where the contribution from the dressed transverse gluons is dominated by Landau damping [30]. Away from the Fermi surface the spectral density must match smoothly to that of the fermionic collective modes (see [30,33–35]) which for frequency and momenta  $\gg M$  lead to perturbative corrections to the quasiparticle spectral density for states with frequency and momenta away from the Fermi surface.

Thus we will restrict our study of the specific heat to the *non-perturbative* aspects of the non-Fermi liquid corrections by focusing on the contribution to the specific heat of states within a width of order  $M$  near the Fermi surface and on the terms with anomalous exponents which are the hallmark of

non-Fermi liquid corrections. Therefore, we will neglect the contributions from perturbative terms since these will arise from: (i) perturbative corrections to the quasiparticle spectral density not included in Eq. (2.17) and (ii) the contribution from states a distance  $>M$  below the Fermi surface, none of which are associated with the breakdown of the Fermi liquid description.

### A. Single quasiparticle density of states

The expressions for the internal energy, specific heat and density given by Eqs. (2.13)–(2.15) require the computation of the single quasiparticle density of states (2.16). The non-Fermi liquid features of the single quasiparticle density of states can be extracted by restricting the momentum integral to states within a width of order  $M$  around the Fermi surface; hence non-Fermi liquid contributions to  $\eta(q_0)$  are given by

$$\eta(q_0) = \frac{1}{2\pi^2} \int_{-M}^M d\tilde{k} [\mu + \tilde{k}]^2 \rho_-(q_0, \tilde{k}). \quad (2.29)$$

Straightforward integration leads to the following explicit form:

$$\begin{aligned} \eta(\tilde{q}_0) &= \left(\frac{M}{\pi}\right)^2 \frac{\sin \pi\lambda}{\pi} \left\{ z - z \right. \\ &\quad \times \left. \left( \frac{\mu}{M} \sigma + z \cos \pi\lambda \right) \text{ArgTh} \left( \frac{2z \cos \pi\lambda}{1+z^2} \right) \right. \\ &\quad \left. + \frac{1}{2 \sin \pi\lambda} \left[ \left( \frac{\mu}{M} \right)^2 + 2z \frac{\mu}{M} \sigma \cos \pi\lambda + z^2 \cos 2\pi\lambda \right] \right. \\ &\quad \left. \times \left[ \pi - \arctan \left( \frac{2z \sin \pi\lambda}{1-z^2} \right) \right] \right\} \quad (2.30) \end{aligned}$$

with

$$z \equiv \left( \frac{|\tilde{q}_0|}{M} \right)^{1-2\lambda}; \quad \sigma \equiv \text{sgn}(\tilde{q}_0). \quad (2.31)$$

Since the specific heat is determined by a small region of width  $\tilde{q}_0 \approx T \ll M$  we approximate the above form of  $\eta(\tilde{q}_0)$  for  $z \ll 1$ :

$$\begin{aligned} \eta(\tilde{q}_0) &= \frac{1}{2} \left( \frac{\mu}{\pi} \right)^2 + \frac{M}{\pi^2} z \left[ \left( 1 - \frac{\mu^2}{M^2} \right) \frac{M}{\pi} \sin \pi\lambda + \mu \sigma \cos \pi\lambda \right] \\ &\quad - z^2 \frac{M}{\pi^2} \left[ \frac{2\mu\sigma}{\pi} \sin 2\pi\lambda - \frac{M}{2} \cos 2\pi\lambda \right] + \mathcal{O}(z^3). \quad (2.32) \end{aligned}$$

This expression has the correct free-field limit, which corresponds to  $\rho_-(q_0, \tilde{k}) = \delta(q_0 - k)$ . A noteworthy point to emphasize is that while the quasiparticle residue vanishes at the Fermi surface, the single quasiparticle density of states is non-vanishing and *finite* but with a divergent frequency derivative at the Fermi surface ( $\tilde{q}_0 = 0$ ). This divergence in the

frequency derivative introduces non analytic finite temperature corrections to the chemical potential which are explored below.

### B. Chemical potential

In the grand canonical ensemble, the temperature dependence of the chemical potential is obtained by requiring that the mean number of particles is *fixed* and is independent of temperature. The chemical potential is the Lagrange multiplier that enforces the constraint of a given particle number and its temperature dependence is obtained from the condition

$$\frac{d\mathcal{N}}{dT} = 0 = \int dq_0 \left[ \eta(q_0) \frac{dN(q_0)}{dT} + \frac{d\eta(q_0)}{dT} N(q_0) \right] \quad (2.33)$$

with  $\mathcal{N}$  is the particle density given by Eq. (2.15).

In the degenerate case  $dN(q_0)/dT$  is localized in a region of width  $\sim T$  around  $q_0 = \mu$  hence the first term in Eq. (2.33) above will only receive contributions from quasiparticles near the Fermi surface for which the single quasiparticle density of states is given by Eq. (2.30).

While in the free theory the single particle density of states does not depend on the temperature or chemical potential, in the interacting theory it depends on  $T$  both through  $M(T)$  given by Eq. (2.20) as well as through the chemical potential, thus the second term in Eq. (2.33) is non-vanishing. However, while the second term in Eq. (2.33) receives contributions from states well below the Fermi surface, the non-perturbative non-Fermi liquid corrections are only associated with states within a region of width  $M$  near the Fermi surface. Quasiparticle states or collective modes well below the Fermi surface give perturbative corrections which are analytic in  $\tilde{q}_0$  in the region  $\tilde{q}_0/M \ll 1$ . Such contributions will be neglected in this article.

The contribution to  $d\eta/dT$  from the quasiparticle states near the Fermi surface given by Eq. (2.30) can be obtained as follows. Writing

$$\frac{d\eta(\tilde{q}_0)}{dT} = \frac{\partial \eta(\tilde{q}_0)}{\partial \mu} \frac{d\mu}{dT} + \frac{\partial \eta(\tilde{q}_0)}{\partial M} \frac{dM}{dT} \quad (2.34)$$

it is readily apparent from Eq. (2.30) or its small  $z$  approximation Eq. (2.32) that

$$\frac{\partial \eta(\tilde{q}_0)}{\partial M} \propto \lambda; \quad \frac{dM}{dT} \propto \lambda \frac{T}{M} \quad (2.35)$$

therefore the second contribution in Eq. (2.34) is perturbatively small  $\mathcal{O}(\lambda^2 T/M)$ , does not lead to anomalous powers of the ratio  $T/M \ll 1$  and will be neglected, consistently with neglecting perturbative corrections in the spectral density and the quasiparticle density of states. Upon integrating in  $\tilde{q}_0$  the term  $[\partial \eta(\tilde{q}_0)/\partial \mu] N(\tilde{q}_0)$  up to a cutoff of order  $\sim M$ , one finds that the contribution from the first term of Eq. (2.34) is again perturbative, and of the same order as terms that have been neglected in the spectral and quasiparticle

densities. Therefore, we conclude that the contribution of the second term in Eq. (2.33) is of the same order as terms that were neglected in the perturbative expansion and hence it will be consistently neglected.

Hence, we are led to the following temperature derivative of the chemical potential to leading order

$$\frac{d\mu(T)}{dT} = - \frac{\int_{-M}^M d\tilde{q}_0 \eta(\tilde{q}_0) D(\tilde{q}_0) (\beta\tilde{q}_0)}{\int_{-M}^M d\tilde{q}_0 \eta(\tilde{q}_0) D(\tilde{q}_0)};$$

$$D(x) = \frac{e^x}{[e^x + 1]^2}. \quad (2.36)$$

The leading contribution in the weak coupling and degenerate limit can be extracted by changing variables  $\beta\tilde{q}_0 = x$  and taking  $M/T \rightarrow \infty$ . We find

$$\frac{d}{dT} \mu(T) = -4 \cos \pi\lambda F(\lambda) \left( \frac{T}{\mu(T)} \right) \left( \frac{T}{M} \right)^{-2\lambda} + \mathcal{O} \left( \left( \frac{T}{\mu(T)} \right)^2 \left( \frac{T}{M} \right)^{-4\lambda} \right) \quad (2.37)$$

leading to the following form of the temperature dependence of the chemical potential

$$\mu^2(T) = \mu^2(0) \left[ 1 - \kappa(\lambda) \left( \frac{T}{\mu(0)} \right)^2 \left( \frac{T}{M(0)} \right)^{-2\lambda} + \mathcal{O} \left( \left( \frac{T}{\mu(0)} \right)^3 \left( \frac{T}{M(0)} \right)^{-4\lambda} \right) \right] \quad (2.38)$$

where we have introduced the functions

$$F(\lambda) = \Gamma(3-2\lambda) \zeta(2-2\lambda) (1-2^{2\lambda-1}) \quad (2.39)$$

$$\begin{aligned} \kappa(\lambda) &= 8 \cos(\pi\lambda) \Gamma(2-2\lambda) \zeta(2-2\lambda) \\ &\quad \times (1-2^{2\lambda-1}) \\ &= \frac{4 \cos \pi\lambda}{1-\lambda} F(\lambda) \end{aligned} \quad (2.40)$$

in terms of the Gamma and Riemann zeta functions.

The non-analyticity of the temperature derivative of the chemical potential reflects the divergence of the frequency derivative of the single quasiparticle density of states near the Fermi surface.

### C. Specific heat

The specific heat (per unit volume) given by Eq. (2.14) can be written as

$$C_v = 2 \int dq_0 q_0 \left[ \eta(q_0) \frac{dN(q_0)}{dT} + \frac{d\eta(q_0)}{dT} N(q_0) \right]. \quad (2.41)$$

To extract the leading order in non-Fermi liquid corrections we follow the analysis invoked in obtaining the temperature derivative of the chemical potential. In the degenerate case the first term in Eq. (2.41) above is determined by the quasiparticle density of states Eq. (2.30) for  $\tilde{q}_0 \sim T \ll M$ . An estimate of the second term is obtained by following the arguments presented for the case of the chemical potential. A similar analysis reveals that the second term is perturbative in the weak coupling limit and of the same order as terms that were neglected in obtaining the spectral density and the single quasiparticle density of states near the Fermi surface. Therefore the second term in Eq. (2.41) will be consistently neglected.

Using the condition (2.33) and the fact that near the Fermi surface the quasiparticle density of states is a function of  $\tilde{q}_0$ , the leading order (in coupling) contribution to the specific heat is obtained from

$$C_v = 2 \int_{-M}^M d\tilde{q}_0 \tilde{q}_0 \eta(\tilde{q}_0) \beta D(\tilde{q}_0) \left[ \beta\tilde{q}_0 + \frac{d\mu(T)}{dT} \right] \quad (2.42)$$

with  $D(x)$  given in Eq. (2.36).

After a rescaling  $\tilde{q}_0 = xT$ , using the result (2.37) for  $d\mu(T)/dT$  and neglecting terms perturbative in the coupling  $\lambda$  we find

$$\begin{aligned} C_v &= \frac{\mu^2(0)T}{3} \left[ 1 + \frac{6}{\pi^2} [F(2\lambda-1) \cos 2\pi\lambda + 8F(\lambda)^2 \cos^2 \pi\lambda] \right. \\ &\quad \times \left( \frac{T}{\mu(0)} \right)^2 \left( \frac{T}{M(0)} \right)^{-4\lambda} - \kappa(\lambda) \left( \frac{T}{\mu(0)} \right)^2 \left( \frac{T}{M(0)} \right)^{-2\lambda} \\ &\quad \left. + \mathcal{O} \left( \left( \frac{T}{\mu(0)} \right)^3 \left( \frac{T}{M(0)} \right)^{-6\lambda} \right) \right] \end{aligned} \quad (2.43)$$

with the functions  $F(\lambda)$ ;  $\kappa(\lambda)$  given by Eqs. (2.39),(2.40) above.

The non-Fermi liquid contributions are explicit in the anomalous power laws with the temperature, which are a result of the divergent frequency derivative of the single quasiparticle density of states near the Fermi surface. We emphasize that for  $T \ll M$  the non-Fermi liquid corrections to the specific heat are much larger than those of the massless gluons or any other Goldstone bosons to the specific heat, which is of order  $T^3$ .

To illustrate the magnitude of the non-Fermi liquid corrections to the specific heat we show in Fig. 1 the function  $G(z) = 3C_v/\mu^2(0)T$  vs  $z = T/\mu(0)$  for three colors and two flavors of massless quarks. We have used the one loop QCD running coupling at a scale  $\mu(0)$  without further justification.<sup>1</sup> The solid line corresponds to  $\mu(0) = 0.5$  GeV and the dashed line to  $\mu(0) = 1$  GeV, for these values of the

<sup>1</sup>Reference [37] gives a thorough discussion of the dependence of the coupling on the chemical potential and the inherent ambiguities in the choice of scale.

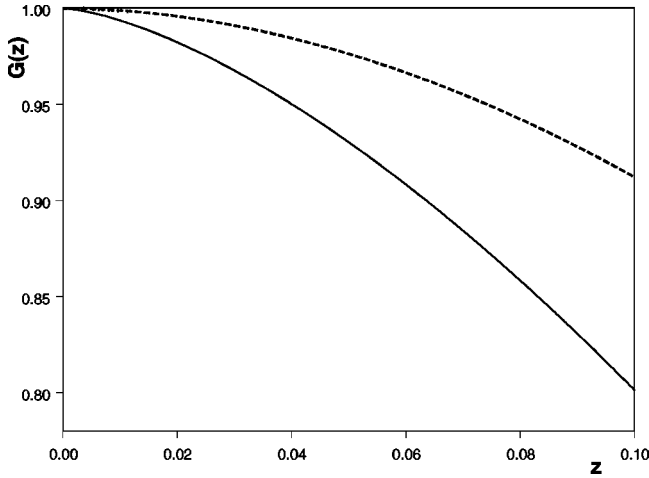


FIG. 1.  $G(z) = 3C_v/\mu^2(0)T$  vs  $z = T/\mu(0)$  for  $N_c = 3; N_f = 2$ . Solid line:  $\mu(0) = 0.5$  GeV; dashed line:  $\mu(0) = 1$  GeV.

strong coupling constant, the value of the effective coupling at these scales are  $\lambda = 0.152$  for  $\mu = 0.5$  GeV and  $\lambda = 0.046$  for  $\mu = 1$  GeV therefore the reliability of the perturbative calculation for the values of the chemical potentials expected at the core of neutron stars is at best questionable. This issue notwithstanding this qualitative estimate points out that non-Fermi liquid corrections to the specific heat result in a change of a few percent. The physical reason for this result is that whereas the quasiparticle residue (wave function renormalization) *vanishes* at the Fermi surface, the single quasiparticle density of states is non-vanishing and finite, but its frequency derivative at the Fermi surface is divergent. The finite, non-zero value of the density of states implies that the non-Fermi liquid features will only result in corrections to the Fermi liquid specific heat. The numerical estimate, with the caveat that its regime of validity is restricted to small coupling, indicates that these corrections are rather small.

Keeping higher order terms in the perturbative expansion, will not dramatically modify this conclusion since our analysis captures the most relevant aspects of the non-Fermi liquid corrections. For the values of temperature and chemical potential corresponding to young neutron stars we see that the non-Fermi liquid corrections amount to a few percent, unlikely to make a dramatic change in the cooling history of young neutron stars. For protoneutron stars with higher temperatures  $T \sim 30\text{--}50$  MeV and smaller chemical potentials, the corrections could be of order 10–15% but in this regime the reliability of the calculation is questionable. We emphasize that this numerical analysis is only intended to provide a qualitative estimate of the magnitude of the non-Fermi liquid corrections but should not be taken as a quantitative indicator since higher order terms, which are subleading and have been neglected in the perturbative expansion can become comparable in this range of couplings.

#### D. Discussion

The fact that the wave function renormalization, i.e, the residue at the “quasiparticle pole” vanishes at the Fermi surface as given by Eq. (2.26) would seem to indicate that

quasiparticle density of states should vanish near the Fermi surface with an anomalous power law. This in turn would suggest that the linear power of temperature, which is the leading contribution in the degenerate limit, would be multiplied by an anomalous power of  $T/\mu$ . However, the reason that the quasiparticle density of states remains *finite* at the Fermi surface despite the vanishing of the quasiparticle pole is because the group velocity vanishes proportionally to the wave function renormalization as displayed by Eq. (2.28). To see this feature clearly, we can approximate the spectral density (2.22) in the limit of vanishing width by

$$\rho_-(q_0, k)_{\tilde{k} \approx 0} \sim Z_p[\tilde{k}] \cos[\pi\lambda] \delta(\tilde{q}_0 - \tilde{q}_0(\tilde{k})) \quad (2.44)$$

with  $\tilde{q}_0(\tilde{k})$  given by Eq. (2.21). The integral over  $\tilde{k}$  in Eq. (2.29) can be performed by changing variables to  $\tilde{q}_0(\tilde{k})$  where the Jacobian of the transformation is precisely  $1/v_g$  with  $v_g$  the group velocity and by Eq. (2.28) this Jacobian *exactly cancels* the wave-function renormalization. This analysis clearly reveals that the reason that the quasiparticle density of states at the Fermi surface remains finite is the cancellation between the residue at the quasiparticle pole and the group velocity, which both vanish proportionally near the Fermi surface.

At this stage it is also important to compare our results to those obtained in Ref. [38] in which the infrared divergences associated with magnetic photon exchange were studied for the first time within the context of the non-relativistic electron gas. There are several important differences between our treatment and results and those of Ref. [38]. (i) In Ref. [38] the temperature derivative of the chemical potential was *neglected* in the derivative of the grand potential with respect to temperature [see Eqs. (5.6) and (5.7) of Ref. [38]]. This is an important source of discrepancy because the temperature derivative of the chemical potential has a linear term in the temperature that cancels against a similar one arising from derivatives of the density of states and the distribution function. We have included the temperature derivative of the chemical potential in the computation of the specific heat explicitly through the condition (2.33). (ii) The calculation in [38] missed the fact that the vanishing of the group velocity is compensated by the vanishing of the residue at the quasiparticle pole as explicitly discussed above. (iii) While in Ref. [38] only the perturbative logarithmic terms were accounted for, the renormalization group resummation makes clear that these logarithms sum up to anomalous power laws, a naive expansion in terms of the coupling constant reveals those logarithms, but the resummation makes explicit the *vanishing of both the residue and the group velocity*.

A consistent resummation and analysis of the quasiparticle pole and group velocity as explicitly discussed here reveals that the density of states remains finite at the Fermi surface and the non-Fermi liquid corrections are subleading.

#### III. CONCLUSIONS

The exchange of dynamically screened gluons leads to a pairing instability and at the same time to a breakdown of the Fermi liquid description of the normal state of QCD. The

superconducting component does not contribute to the cooling of young neutron stars because of the exponential suppression of the specific heat and the neutrino emissivity. If color superconductivity is in the form of a 2SC condensate, or some quarks do not pair, the normal quarks contribute to cooling of the neutron star via neutrino and antineutrino emission.

The search for novel observational aspects of quark matter at the core of neutron stars motivated us to study the effects of the breakdown of the Fermi liquid description of quasiparticle states of the normal component near the Fermi surface in the specific heat. This aspect is relevant to understand the cooling rate through direct quark URCA processes. Our analysis is based on a renormalization group improvement of the single quasiparticle spectral density in the degenerate case and reveals that non-Fermi liquid corrections tend to decrease the specific heat and therefore increase the cooling rate. However these corrections are rather small, about 10–15 % even in the most favorable scenarios, and probably unobservable in the cooling history of neutron stars or even

proton-neutron stars. A complete assessment of potential non-Fermi liquid corrections to the cooling rate still requires an understanding of these corrections on the neutrino emissivity. Both quarks and leptons (electrons) that enter in the quark direct URCA process will receive non-Fermi liquid corrections through their spectral density although the corrections for quarks, in terms of the strong coupling constant are more important than those for the electrons. We are currently studying these corrections in the weak matrix elements for the quark direct URCA processes and expect to report on our results in the near future.

#### ACKNOWLEDGMENTS

D.B. thanks NSF for support through grants PHY-9605186, PHY-9988720 and NSF-INT-9815064. H.J.d.V. thanks the CNRS-NSF collaboration for support. The authors thank K. Rajagopal and M. Alford for correspondence and comments.

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