Deeply virtual Compton scattering on the nucleon: Study of the twist-3 effects

N. Kivel
*Institut für Theoretische Physik, Universität Regensburg, D-93040 Regensburg, Germany
and Petersburg Nuclear Physics Institute, 188350, Gatchina, Russia*

M. V. Polyakov
*Petersburg Nuclear Physics Institute, 188350, Gatchina, Russia
and Institut für Theoretische Physik II, Ruhr-Universität Bochum, D-44780 Bochum, Germany*

M. Vanderhaeghen
Institut für Kernphysik, Johannes Gutenberg-Universität, D-55099 Mainz, Germany
(Received 18 December 2000; published 1 May 2001)

We estimate the size of the twist-3 effects on deeply virtual Compton scattering (DVCS) observables, in the Wandzura-Wilczek approximation. We present results in the valence region for the DVCS cross sections, charge asymmetries, and single spin asymmetries, to twist-3 accuracy.

DOI: 10.1103/PhysRevD.63.114014 PACS number(s): 13.60.Fz, 12.38.Bx

I. INTRODUCTION

Deeply virtual Compton scattering (DVCS) [1–3] has been studied intensively in recent years as a new hard probe to access generalized parton distributions of the nucleon, the so-called skewed parton distributions (SPDs).

After the first experimental evidence for DVCS on the proton [4–7], it is the right time to address the problem of the extraction of the twist-2 SPDs from DVCS observables. In order to be able to extract the twist-2 SPDs from DVCS observables at accessible values of the ''hard'' scale Q, one first needs to estimate the effects of higher twist (power suppressed) contributions to those observables. The first power correction to the DVCS amplitude is of order $O(1/Q)$; hence, it is called twist-3. The twist-3 corrections to the DVCS amplitude have been derived recently by several groups [8–11] using different approaches. It was shown that the twist-3 part of the amplitude depends on a new type of SPD which can be related to the twist-2 SPDs with the help of Wandzura-Wilczek (WW) relations. These relations are based on an assumption that the nucleon matrix elements of mixed quark-gluon operators are small relative to matrix elements of symmetric quark operators; for details see [10–13]. Originally, the WW relations were derived for the polarized structure functions $g_1(x)$ and $g_T(x)$ [14]. Recent experimental measurements [15] of these polarized structure functions indicate that the WW relations are satisfied to good accuracy. In the theory of the instanton vacuum, the smallness of the corrections to the WW relations for $g_T(x)$ and $h_L(x)$ can be related to the smallness of the packing fraction of the instantons in the QCD vacuum; for details see [16,17].

Our aim here is to study the twist-3 effects on DVCS observables for a nucleon target, in the Wandzura-Wilczek approximation.[1] Although on the one hand, the twist-3 effects are formally a correction when one aims to extract twist-2 SPDs from DVCS observables, they provide on the other hand an additional handle in the extraction of those same twist-2 SPDs. Indeed, in the WW approximation the twist-3 SPDs are fixed completely in terms of twist-2 SPDs and their derivatives.

In this work, we restrict our analysis to the leading order in $\alpha_s(Q^2)$, because the next leading order (NLO) corrections [19–21] were computed only for the twist-2 amplitude. Since it was found in [22] that the NLO correction to the twist-2 DVCS amplitude can be sizable, the NLO analysis for the twist-3 amplitude remains to be investigated.

In the present work, we first give in Sec. II the expressions for the DVCS amplitude on the nucleon to twist-3 accuracy, as calculated in the WW approximation.

In Sec. III, we describe the modelization of the SPDs that will be used to provide estimates for DVCS observables. We propose a two-component parametrization for the SPDs, so as to satisfy the polynomiality conditions.

In Sec. IV, we show our results for DVCS observables, and discuss the importance of the twist-3 effects on the DVCS cross sections, charge asymmetries and single spin asymmetries in the valence region.

Finally, we give our conclusions in Sec. V.

II. DVCS AMPLITUDE ON THE NUCLEON AT THE TWIST-3 ACCURACY IN WANDZURA-WILCZEK APPROXIMATION

The amplitude of the virtual Compton scattering process

$$\gamma^*(q)+N(p)\rightarrow\gamma(q')+N(p') \tag{1}$$

is defined in terms of the nucleon matrix element of the T-product of two electromagnetic currents

$$T^{\mu\nu}=-i\int d^4x\;\; e^{-i(q\cdot x)}\langle p'|T[J^{\mu}_{\rm e.m.}(x)J^{\nu}_{\rm e.m.}(0)]|p\rangle, \tag{2}$$

[1] Recently the twist-3 corrections to the DVCS amplitude on the pion target were studied in Ref. [18].

where the four-vector index $\mu(\nu)$ refers to the virtual (real) photon.

In the Bjorken limit, where $-q^2=Q^2\to\infty$, $2(p\cdot q)\to\infty$, with $x_B=Q^2/2(p\cdot q)$ constant, and $t\equiv(p-p')^2\ll Q^2$, the DVCS amplitude on the nucleon to the order $O(1/Q)$ has the form [9,10]

$$T^{\mu\nu}=\frac{1}{2}\int_{-1}^{1}dx \quad \Bigg\{\left[(-g^{\mu\nu})_\perp-\frac{P^\nu\Delta_\perp^\mu}{(P\cdot q')}\right]n^\beta F_\beta(x,\xi)C^+(x,\xi)$$
$$-\left[(-g^{\nu k})_\perp-\frac{P^\nu\Delta_\perp^k}{(P\cdot q')}\right]i\epsilon_k^{\perp\,\mu}n^\beta\tilde{F}_\beta(x,\xi)C^-(x,\xi)$$
$$-\frac{(q+4\xi P)^\mu}{(P\cdot q)}\left[(-g^{\nu k})_\perp-\frac{P^\nu\Delta_\perp^k}{(P\cdot q')}\right][F_k(x,\xi)C^+(x,\xi)$$
$$-i\epsilon_{k\rho}^\perp\tilde{F}^\rho(x,\xi)C^-(x,\xi)]\Bigg\}, \tag{3}$$

where to the twist-3 accuracy

$$P=\frac{1}{2}(p+p')=n^*,$$
$$\Delta=p'-p=-2\xi P+\Delta_\perp,$$
$$q=-2\xi P+\frac{Q^2}{4\xi}n,$$
$$q'=q-\Delta=\frac{Q^2}{4\xi}n-\Delta_\perp, \tag{4}$$

where p,p' are the momenta of the initial and final nucleon and q,q' are the momenta of the initial and final photon, respectively. The lightlike vectors n and n^* are normalized as $(n\cdot n^*)=1$. We also introduce the metric and totally antisymmetric tensors in the two dimensional transverse plane ($\varepsilon_{0123}=+1$),

$$(-g^{\mu\nu})_\perp=-g^{\mu\nu}+n^\mu n^{*\nu}+n^\nu n^{*\mu},$$
$$\epsilon_{\mu\nu}^\perp=\epsilon_{\mu\nu\alpha\beta}n^\alpha n^{*\beta}. \tag{5}$$

The leading order coefficient functions are

$$C^\pm(x,\xi)=\frac{1}{x-\xi+i\varepsilon}\pm\frac{1}{x+\xi-i\varepsilon}.$$

In the expression for the DVCS amplitude to the twist-3 accuracy the first two terms correspond to the scattering of transversely polarized virtual photons. This part of the amplitude depends only on twist-2 SPDs H,E, and $\tilde{H},\tilde{E}$ and was anticipated in Refs. [23,24]. The third term in Eq. (3) corresponds to the contribution of the longitudinal polarization of the virtual photon. This term depends only on new ‘‘transverse’’ SPDs $F_\perp^\mu$ and $\tilde{F}_\perp^\mu$. Defining the polarization vector of the virtual photon as

$$\varepsilon_L^\mu(q)=\frac{1}{Q}\left(2\xi P^\mu+\frac{Q^2}{4\xi}n^\mu\right), \tag{6}$$

we can easily calculate the DVCS amplitude for longitudinal polarization of the virtual photon ($L\to T$ transition), which is purely of twist-3,

$$\varepsilon_\mu^L T^{\mu\nu}=\frac{2\xi}{Q}\int_{-1}^{1}dx[F_\perp^\nu C^+(x,\xi)-i\varepsilon_\perp^{\nu k}\tilde{F}_{\perp k}C^-(x,\xi)]. \tag{7}$$

The skewed parton distributions F_μ and $\tilde{F}_\mu$ can be related to the twist-2 SPDs $H,E,\tilde{H}$, and $\tilde{E}$ with help of Wandzura-Wilczek relations. To derive these relations one assumes that the nonforward nucleon matrix elements of gauge invariant operators of the type $\bar{\psi}G\psi$ are small. The WW relations for the case of nucleon SPDs have the form [10,13]

$$F_\mu^{WW}(x,\xi)=\frac{\Delta_\mu}{2\xi}\left\langle\left\langle\frac{1}{M}\right\rangle\right\rangle E(x,\xi)-\frac{\Delta_\mu}{2\xi}\langle\langle\gamma_+\rangle\rangle(H+E)(x,\xi)+\int_{-1}^{1}du\, G_\mu(u,\xi)W_+(x,u,\xi)$$
$$+i\epsilon_{\perp\mu k}\int_{-1}^{1}du\,\tilde{G}^k(u,\xi)W_-(x,u,\xi), \tag{8}$$

$$\tilde{F}_\mu^{WW}(x,\xi)=\Delta_\mu\frac{1}{2}\left\langle\left\langle\frac{\gamma_5}{M}\right\rangle\right\rangle\tilde{E}(x,\xi)-\frac{\Delta_\mu}{2\xi}\langle\langle\gamma_+\gamma_5\rangle\rangle\tilde{H}(x,\xi)+\int_{-1}^{1}du\,\tilde{G}_\mu(u,\xi)W_+(x,u,\xi)$$
$$+i\epsilon_{\perp\mu k}\int_{-1}^{1}du\, G^k(u,\xi)W_-(x,u,\xi). \tag{9}$$

In Eqs. (8), (9), the following notations are used:

$$G^{\mu}(u,\xi)=\langle\langle\gamma_{\perp}^{\mu}\rangle\rangle(H+E)(u,\xi)+\frac{\Delta_{\perp}^{\mu}}{2\xi}\left\langle\left\langle\frac{1}{M}\right\rangle\right\rangle\left[u\frac{\partial}{\partial u}+\xi\frac{\partial}{\partial\xi}\right]E(u,\xi)-\frac{\Delta_{\perp}^{\mu}}{2\xi}\langle\langle\gamma_{+}\rangle\rangle\left[u\frac{\partial}{\partial u}+\xi\frac{\partial}{\partial\xi}\right](H+E)(u,\xi), \quad (10)$$

$$\widetilde{G}^{\mu}(u,\xi)=\langle\langle\gamma_{\perp}^{\mu}\gamma_5\rangle\rangle\widetilde{H}(u,\xi)+\frac{1}{2}\Delta_{\perp}^{\mu}\left\langle\left\langle\frac{\gamma_5}{M}\right\rangle\right\rangle\left[1+u\frac{\partial}{\partial u}+\xi\frac{\partial}{\partial\xi}\right]\widetilde{E}(u,\xi)-\frac{\Delta_{\perp}^{\mu}}{2\xi}\langle\langle\gamma_{+}\gamma_5\rangle\rangle\left[u\frac{\partial}{\partial u}+\xi\frac{\partial}{\partial\xi}\right]\widetilde{H}(u,\xi). \quad (11)$$

The functions $W_{\pm}(x,u,\xi)$ are the so-called Wandzura-Wilczek kernels introduced in Refs. [10,13]. They are defined as

$$\begin{aligned}W_{\pm}(x,u,\xi)=&\frac{1}{2}\left\{\theta(x>\xi)\frac{\theta(u>x)}{u-\xi}\right.\\&\left.-\theta(x<\xi)\frac{\theta(u<x)}{u-\xi}\right\}\pm\frac{1}{2}\left\{\theta(x>-\xi)\frac{\theta(u>x)}{u+\xi}\right.\\&\left.-\theta(x<-\xi)\frac{\theta(u<x)}{u+\xi}\right\}.\end{aligned} \quad (12)$$

The sandwiching between nucleon Dirac spinors is denoted by $\langle\langle\ldots\rangle\rangle=\overline{U}(p')\ldots U(p)$ and M denotes the nucleon mass. The quarks flavor dependence in the amplitude can be easily restored by the substitution

$$F_{\mu}(\widetilde{F}_{\mu})\to\sum_{q=u,d,s,\ldots}e_q^2\;F_{\mu}^{q}(\widetilde{F}_{\mu}^{q}). \quad (13)$$

The DVCS amplitude of Eq. (3) is electromagnetically gauge invariant, i.e.,

$$q_{\mu}T^{\mu\nu}=(q-\Delta)_{\nu}T^{\mu\nu}=0, \quad (14)$$

formally to the accuracy $1/Q^2$. In order to have ''absolute'' transversality of the amplitude we keep in the expression (3) terms of the order Δ^2/Q^2, by applying the prescription of [23,24], i.e.,

$$-g_{\perp}^{\mu\nu}\to-g_{\perp}^{\mu\nu}-\frac{P^{\nu}\Delta_{\perp}^{\mu}}{(P\cdot q')}, \quad (15)$$

for the twist-3 terms in the amplitude. Formally such terms are beyond our accuracy and they do not form a complete set of $1/Q^2$ contributions, but we prefer to work with the DVCS amplitude, satisfying Eq. (15) exactly.

The convolution of the leading order Wilson coefficients (6) with the WW kernels can be performed analytically (see also [18]). In particular, the part of the amplitude which contains WW kernels can be simplified using the following result for the integration over x:

$$\begin{aligned}&\int_{-1}^{1}dx\left\{\frac{1}{x-\xi+i\varepsilon}\int_{-1}^{1}du(W_{+}-W_{-})[G^{\mu}-i\varepsilon_{\perp}^{\mu k}\widetilde{G}_k]\right.\\&\left.\quad+\frac{1}{x+\xi-i\varepsilon}\int_{-1}^{1}du(W_{+}+W_{-})[G^{\mu}+i\varepsilon_{\perp}^{\mu k}\widetilde{G}_k]\right\}\\&=-i\pi\int_{\xi}^{1}du\frac{G^{\mu}-i\varepsilon_{\perp}^{\mu k}\widetilde{G}_k}{u+\xi}-i\pi\int_{-1}^{-\xi}du\frac{G^{\mu}+i\varepsilon_{\perp}^{\mu k}\widetilde{G}_k}{u-\xi}\\&\quad+\int_{-1}^{1}du\left\{\frac{G^{\mu}-i\varepsilon_{\perp}^{\mu k}\widetilde{G}_k}{u+\xi}\ln\left|\frac{u-\xi}{2\xi}\right|\right.\\&\quad\left.+\frac{G^{\mu}+i\varepsilon_{\perp}^{\mu k}\widetilde{G}_k}{u-\xi}\ln\left|\frac{u+\xi}{2\xi}\right|\right\}.\end{aligned} \quad (16)$$

Note that all integrals over u are convergent and the strongest (integrable) singularity of the integrand is logarithmic only.

III. MODELIZATION OF SPDs

In order to interpret DVCS observables, we next discuss the parametrization of the SPDs. As we address here hard exclusive reactions in the valence region at relatively small values of $-t$, we factorize the t dependence of the SPDs by the corresponding form factor (FF), so as to satisfy the first sum rule. For the SPD H, the t dependence is given by the nucleon Dirac FF. The t dependence of the SPD $\widetilde{H}$ is given by the nucleon axial FF, whereas the t dependence of the SPD $\widetilde{E}$ is given by the nucleon pseudoscalar FF, as is detailed in [25]. We stress that the factorization of the t dependence of SPDs is not expected to hold when increasing the value of $-t$. For example, the calculations of the SPD $\widetilde{H}$ in the chiral quark soliton model [26] showed that at $t\sim-0.8$ GeV^2 the deviations from the factorization ansatz for the t dependence of SPDs is as large as about 25% in some x range. In the calculations presented here, we limit ourselves to relatively small values of $-t=0.25$ GeV^2 and assume the factorization ansatz. A more realistic parametrization of SPDs at larger values of $-t$ remains to be done in a future work. We next discuss the resulting t-independent part.

For the function H^q (for each flavor q), the t-independent part is parametrized by a two-component form

$$H^{q}(x,\xi)=H_{DD}^{q}(x,\xi)+\frac{1}{N_f}D\left(\frac{x}{\xi}\right), \quad (17)$$

where H^q_{DD} is the part of the SPD which is obtained as a one-dimensional section of a two-variable double distribution F^q [27], imposing a particular skewedness ξ, as

$$H^q_{DD}(x,\xi)=\int_{-1}^{1}d\beta\int_{-1+|\beta|}^{1-|\beta|}d\alpha\,\delta(x-\beta-\alpha\xi)F^q(\beta,\alpha), \tag{18}$$

and where the D-term contribution[2] D completes the parametrization of SPDs, restoring the correct polynomiality properties [28] of SPDs [29]. The D-term contribution to SPDs has a support only for $|x|\leqslant|\xi|$, so that it is ''invisible'' in the forward limit. The D term is an isoscalar contribution, and adds therefore the same function for each flavor [in Eq. (17), $N_f=3$ is the number of active flavors].

For the double distributions, entering Eq. (18), we use the following model suggested in [27]:

$$F^q(\beta,\alpha)=\frac{\Gamma(2b+2)}{2^{2b+1}\Gamma^2(b+1)}\frac{[(1-|\beta|)^2-\alpha^2]^b}{(1-|\beta|)^{2b+1}}q(\beta), \tag{19}$$

where $q(\beta)$ is the corresponding forward parton distribution ensuring the correct forward limit for the SPD $H(x,0)=q(x)$. We use the phenomenological forward parton distributions (including their evolution) as input. In Eq. (19), the parameter b characterizes the strength of the ξ dependence of the SPD $H(x,\xi)$; the limiting case $b\to\infty$ corresponds to the ξ independent SPD $H(x,\xi)=q(x)$. The power b in Eq. (19) is a free parameter for the valence contribution (b_{val}) and for the sea/antiquark contribution (b_{sea}) to the SPD, which can be used as a fit parameter in this approach to extract SPDs from DVCS observables. The twist-2 DVCS predictions of Ref. [25] correspond to the choice $b_{val}=b_{sea}=1.0$. In the following, we also show all predictions for the value $b_{val}=b_{sea}=1.0$, in order to have a point of comparison, and refer to a future work for a more systematic study of the dependence of DVCS observables on the shape of the profile function.

The D-term contribution to the singlet SPDs $H(x,\xi)$ and $E(x,\xi)$ has the following form:

$$H^{D\ \text{term}}(x,\xi)=\theta(\xi-|x|)\ \ D\left(\frac{x}{\xi}\right), \tag{20}$$

$$E^{D\ \text{term}}(x,\xi)=-\theta(\xi-|x|)D\left(\frac{x}{\xi}\right). \tag{21}$$

Note that the D-term contribution is cancelled in the combination $H+E$ to ensure the polynomiality condition (for odd N)

$$\int_{-1}^{1}dx\,x^N[H(x,\xi)+E(x,\xi)]$$
$$=\text{polynomial of the order}\ \ (N-1)\ \ \text{in}\ \ \xi. \tag{22}$$

Note that the Nth Mellin moments of H and E separately are polynomials of order $N+1$ in ξ; only in the combination (22) are the highest powers cancelled. As a model for the SPD $E(x,\xi)$, we shall only use the contribution of the D term (21). More detailed studies of DVCS observables with the function $E(x,\xi)$ computed in the chiral quark-soliton model [31,32] will be published elsewhere.

The function $D(z)$ can be expanded in odd Gegenbauer polynomials

$$D(z,t=0)$$
$$=(1-z^2)[d_1C_1^{3/2}(z)+d_3C_3^{3/2}(z)+d_5C_5^{3/2}(z)+\cdots]. \tag{23}$$

For the moments d_1,d_3,d_5, we use the estimate which is based on the calculation of SPDs in the chiral quark-soliton model [31] at a low normalization point $\mu\approx0.6$ GeV, which gives

$$d_1\approx-4.0,\quad d_3\approx-1.2,\quad d_5\approx-0.4, \tag{24}$$

and higher moments [denoted by the ellipses in Eq. (23)] are small and are neglected in the following.[3] Notice the negative sign of the Gegenbauer coefficients for the D term in Eq. (24), as obtained in the chiral quark-soliton model.

It is interesting to note that for the case of SPDs in the pion, the value of the coefficient d_1 in the parametrization of the D term (23) can be computed in a model independent way. To do this, we can use the soft-pion theorem for the singlet SPD in the pion derived in [30]. This soft pion theorem states that the singlet SPD in the pion vanishes for $\xi=\pm1$:

$$\sum_q H_q^{(\pi)}[x,\xi=\pm1,t=0]=0. \tag{25}$$

From Eq. (25), one obtains (see also [29])

$$\int_{-1}^{1}dx\,x\left(\sum_q H_q^{(\pi)}(x,\xi,t=0)\right)=(1-\xi^2)M_2^Q. \tag{26}$$

Evaluating Eq. (26) at $\xi=0$ determines

$$M_2^Q=\int_0^1 dx\,x\sum_q\ [q^{(\pi)}(x)+\overline{q}^{(\pi)}(x)], \tag{27}$$

being the fraction of the momentum carried by the quarks and antiquarks in the pion. As the highest power in ξ in Eq.

[2]The name ''D term'' is derived from (more or less arbitrary) notation used in Ref. [29], and to some extent is confusing because of its similarity to the terminology used in supersymmetric theories, especially when the first term in Eq. (17) is called an ''F term.''

[3]In what follows we neglect the scale dependence of the result (24), as the uncertainties in the modeling of the D term are larger than the logarithmic scale dependence, for the scales considered in our calculations.

(26) (i.e., the term in ξ^2) originates solely from the D term, one easily obtains the following expression for the pion D term:

$$D^{(\pi)}(z) = -\frac{5M_2^Q}{4}(1-z^2)[C_1^{3/2}(z)+\cdots]. \tag{28}$$

We see therefore that the first Gegenbauer coefficient of the pion D term is negative and strictly nonzero. The coincidence of the signs in the nucleon and pion D terms hints that the D term in the nucleon is intimately related to the spontaneous breaking of the chiral symmetry.

When using the parametrization of Eq. (17) for the pion, the soft pion theorem (25) imposes a condition between both terms, and fixes the pion D term completely in terms of the isosinglet double distribution as follows:

$$D^{(\pi)}(z) = -\int_{-1}^{1} d\beta \int_{-1+|\beta|}^{1-|\beta|} d\alpha\, \delta(z-\beta-\alpha)\sum_q F^q(\beta,\alpha). \tag{29}$$

Therefore, when studying DVCS off the pion it is enough to model the double distribution; the corresponding D term is restored by using the soft pion theorems. For the nucleon, both terms in Eq. (17) are parametrized independently.

For the t-dependence of the D term we use, in the absence of more detailed knowledge, the same factorized form factor ansatz as for the double distribution part H_{DD}^q.

We next turn to the calculation of the twist-3 terms. For their evaluation, we need in particular derivatives of the SPD $H(x,\xi)$, which can be computed in terms of derivatives of double distributions, using the obvious result:

$$\begin{aligned}\left(x\frac{\partial}{\partial x}+\xi\frac{\partial}{\partial \xi}\right)H_{DD}(x,\xi) &= -H_{DD}(x,\xi)+\int_{-1}^{1} d\beta\frac{\beta}{\xi}\\ &\quad\times\int_{-1+|\beta|}^{1-|\beta|} d\alpha\,\delta(x-\beta-\alpha\xi)\frac{\partial F(\beta,\alpha)}{\partial\alpha}.\end{aligned} \tag{30}$$

For the model of Eq. (19), the derivative with respect to α in Eq. (30) is easily performed,

$$\frac{\partial F(\beta,\alpha)}{\partial\alpha} = -\frac{b\alpha}{2^{2b}}\frac{\Gamma(2b+2)}{\Gamma^2(b+1)}\frac{[(1-|\beta|)^2-\alpha^2]^{b-1}}{(1-|\beta|)^{2b+1}}q(\beta). \tag{31}$$

Note that although generically the derivatives of a SPD $H(x,\xi)$ with respect to x and ξ are discontinuous at the points $x=\pm\xi$, the combination $(x\partial_x+\xi\partial_\xi)H_{DD}(x,\xi)$ is continuous in these points. The contribution of the D term to the ''transverse'' SPDs $F_\perp^\mu$ and $\tilde{F}_\perp^\mu$ has been computed in Ref. [13] with the result

$$F_\mu^{\mathrm{WW}D\ \mathrm{term}}(x,\xi) = -\frac{\Delta_\mu}{2\xi M}\langle\langle 1\rangle\rangle D\left(\frac{x}{\xi}\right)\theta(|x|\leq\xi),$$

$$\tilde{F}_\mu^{\mathrm{WW}D\ \mathrm{term}}(x,\xi) = 0. \tag{32}$$

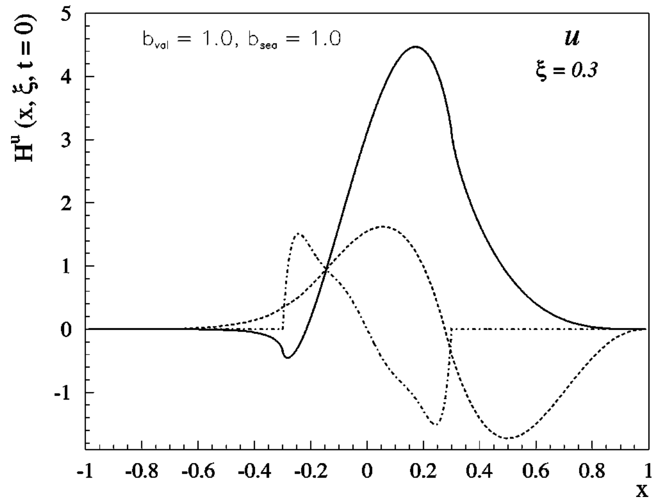

FIG. 1. SPD $H^u(x,\xi)$ for the u quark obtained from the double distribution model (19) [solid curve]; $(1+x\partial_x+\xi\partial_\xi)H^u(x,\xi)$ [dotted curve], and D term (23) [dashed-dotted curve] at a fixed value of $\xi=0.3$.

Therefore the contribution of the D term to the DVCS amplitude can be easily added separately from the contribution of the double distribution part H_{DD} of Eq. (19). Obviously the D term contributes to the real part of the amplitude only.

In Fig. 1, we plot for illustrative purposes $H^u(x,\xi)$ obtained from the model (19) with $b_{\mathrm{val}}=b_{\mathrm{sea}}=1$. We use as input for the forward quark distribution the MRST98 (more precisely the so-called ''central gluon'') parametrization [33]. In Fig. 1, we also show separately the contribution of the D term (23), as well as the combination $(1+x\partial_x+\xi\partial_\xi)H^u(x,\xi)$ which enters the twist-3 SPDs in WW approximation.

To model the SPD $\tilde{H}$, we make use of its representation in terms of double distributions in the same way as for the function H; see Eqs. (17)–(19). For the forward polarized parton distributions, we use (as in [25]) the parametrization of Ref. [34]. Note that calculations in the chiral quark-soliton model of the functions $\tilde{H}$ [26] show that the corresponding D term is very small and can be neglected.

As a model for $\tilde{E}$ we take the contribution of the pion pole [35–37,26] of the form

$$\tilde{E}^{\mathrm{pion\ pole}}(x,\xi) = \frac{4g_A^2M^2}{-t+m_\pi^2}\frac{1}{\xi}\varphi_\pi\left(\frac{x}{\xi}\right)\theta(|x|\leq\xi), \tag{33}$$

where g_A is the axial charge of the nucleon and $\varphi_\pi(z)$ is the pion distribution amplitude (DA). The calculations are performed with an asymptotic DA for the pion. Again the contribution of the pion pole to the ''transverse'' functions $F_\perp^\mu$ and $\tilde{F}_\perp^\mu$ has been computed in [13] with the result

$$F_\mu^{WW,\mathrm{pion\ pole}}(x,\xi) = 0,$$

$$\tilde{F}_{\mu}^{WW,\text{pion pole}}(x,\xi)=\frac{\Delta_{\mu}}{2M}\langle\langle\gamma_5\rangle\rangle\frac{4g_A^2M^2}{-t+m_{\pi}^2}\frac{1}{\xi}\varphi_{\pi}\left(\frac{x}{\xi}\right)\times\theta(|x|\leqslant\xi). \tag{34}$$

IV. RESULTS AND DISCUSSION

In this section, we present our results for the DVCS observables. We give all results for the invariant cross section of the $ep\to ep\gamma$ reaction, which is differential with respect to Q^2, x_B, t, and out-of-plane angle Φ ($\Phi=0°$ corresponds to the situation where the real photon is emitted in the same half plane as the leptons). The invariant $ep\to ep\gamma$ cross section is given by

$$\frac{d\sigma}{dQ^2dx_Bdtd\Phi}=\frac{1}{(2\pi)^4 32}\frac{x_By^2}{Q^4}\left(1+\frac{4M^2x_B^2}{Q^2}\right)^{-1/2}\times|T_{BH}+T_{FVCS}|^2, \tag{35}$$

where M is the nucleon mass, $y\equiv(p\cdot q)/(p\cdot k)$, and k is the initial lepton four momentum. In the $ep\to ep\gamma$ reaction, the final photon can be emitted either by the proton or by the lepton. The former process is referred to as the fully VCS process [amplitude T_{FVCS} in Eq. (35)], which includes the leptonic current. The process where the photon is emitted from the initial or final lepton is referred to as the Bethe-Heitler (BH) process [amplitude T_{BH} in Eq. (35)], and can be calculated exactly. For further technical details of the $ep\to ep\gamma$ reaction and observables, we refer to Refs. [23,38].

When calculating twist-3 effects in DVCS observables, we show all results with the exact expression for the BH amplitude, i.e., we do *not* expand the BH amplitude in powers 1/Q. Also in the actual calculations, we use the exact kinematics for the four momenta of the participating particles [the kinematics in Eq. (4) were shown to twist-3 accuracy only for simplicity of the presentation]. In this way we take (partially) kinematical higher twists into account. Furthermore, when referring to the twist-2 DVCS results, we include those higher twist effects which restore exact transversality of the amplitude as expressed through Eq. (15). Similarly, when referring to the twist-3 DVCS results (providing from a longitudinally polarized virtual photon), we include the gauge restoring higher twist terms.

In Fig. 2, we show the Φ dependence of the $ep\to ep\gamma$ cross section for a lepton (either electron or positron) of 27 GeV (accessible at HERMES). The kinematics corresponds to the valence region ($x_B=0.3$) and to a ratio $t/Q^2=0.1$ (remark that increasing the ratio t/Q^2 increases the higher twist effects).

It is first seen from Fig. 2 that in these kinematics, the pure twist-2 DVCS process without the D-term contribution (dashed curves) dominates the $ep\to ep\gamma$ cross section compared to the BH process (which is sizable only around $\Phi=0°$, where it reaches its maximal value). In absence of the BH, the twist-2 DVCS cross section would give a Φ-independent cross section which practically saturates the $ep\to ep\gamma$ cross section at $\Phi=180°$, where the BH is vanishingly small. The Φ dependence of the dashed curve in

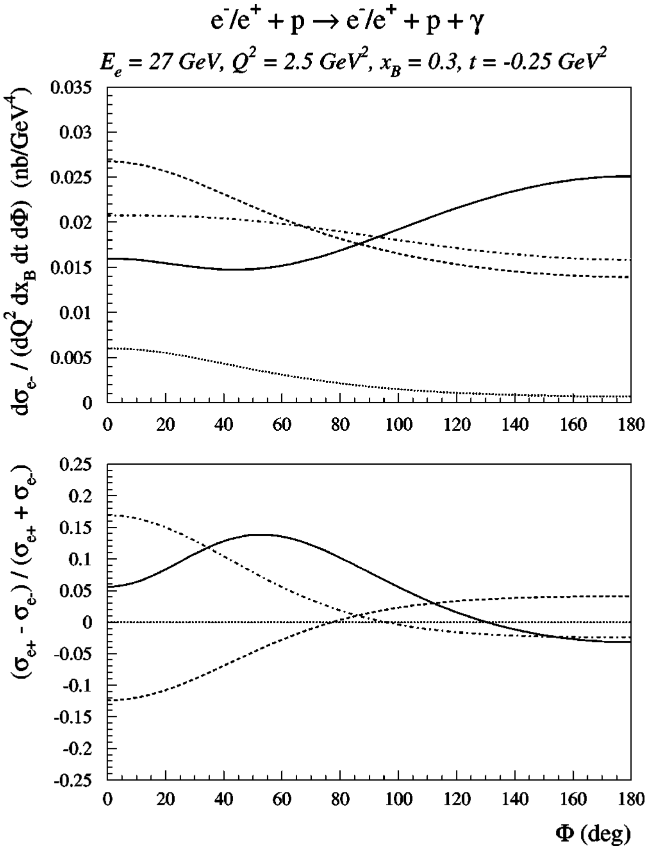

FIG. 2. Invariant cross section for the $e^-p\to e^-p\gamma$ reaction (upper panel) and DVCS charge asymmetry (lower panel) at $E_e=27$ GeV for the DVCS kinematics as indicated in the figure. Dotted curves: BH contribution; dashed curves: BH + twist-2 DVCS (without D term); dashed-dotted curves: BH + twist-2 DVCS (with D term); full curves: BH + twist-3 DVCS.

Fig. 2 can be understood as the sum of this constant twist-2 DVCS cross section, the cross section for the BH process (whose amplitude is purely real), and the interference of the BH with the relatively small real part of the DVCS amplitude (in the valence kinematics, $x_B\simeq 0.3$, shown in Fig. 2; the ratio of real to imaginary parts of the DVCS amplitude without the D term is around 15%).

When adding to the twist-2 DVCS amplitude the purely real D-term contribution, the resulting cross section for the full twist-2 DVCS process is given by the dashed-dotted curves in Fig. 2. At $\Phi=180°$ (where the BH ''contamination'' is very small), the predominantly imaginary DVCS amplitude (in the absence of the D-term contribution), and the purely real D term amplitude have only a small interference. In this region, the DVCS cross section is enhanced by about 10% when using the chiral quark-soliton model estimate of Eqs. (23), (24) for the D term. Going to $\Phi=0°$, the purely real D term amplitude interferes maximally with the BH. This interference is destructive for the electron reaction and constructive for the positron reaction. Therefore, the effect of the D term can be very clearly seen in the Φ dependence of the charge asymmetry as shown in the lower panel of Fig. 2. Including the D-term contribution, the full twist-2 DVCS charge asymmetry changes sign and obtains a rather

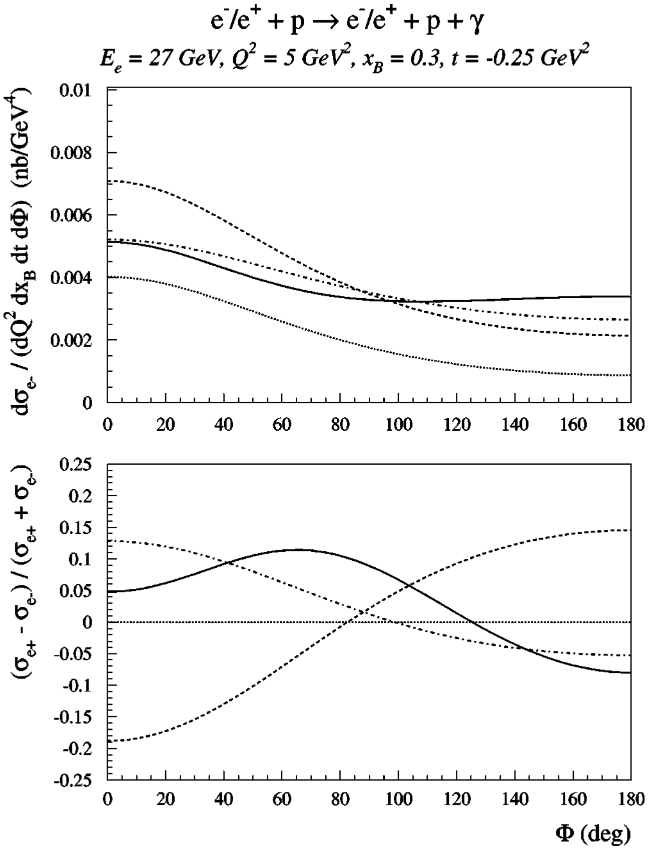

FIG. 3. Same as Fig. 2, but for $Q^2=5$ GeV^2.

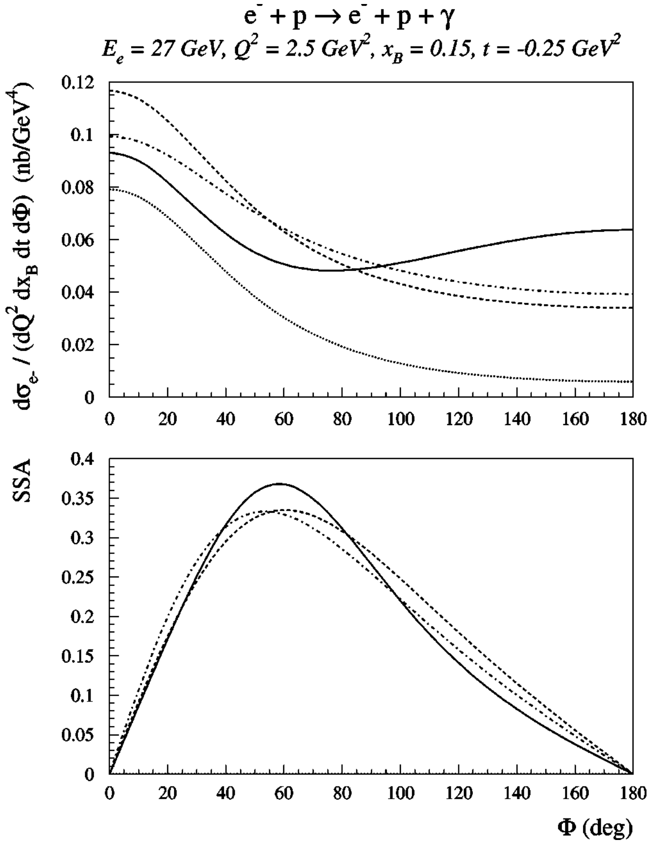

FIG. 4. Invariant cross section for the $e^- p \rightarrow e^- p \gamma$ reaction (upper panel) and DVCS SSA (lower panel) at $E_e=27$ GeV for the DVCS kinematics as indicated in the figure. Curve conventions as in Fig. 2.

large value (≈ 0.15) at $\Phi=0°$. On the other hand, at $\Phi=180°$, where the interference with the BH (and hence the difference between e^- and e^+) is small, the charge asymmetry is correspondingly small. The pronounced Φ dependence of the charge asymmetry and its value at $\Phi=0°$, provides therefore a nice observable to study the D-term contribution to the SPDs and to check the chiral quark-soliton model estimate of Eqs. (23), (24).

Adding next the twist-3 effects in the WW approximation (full curves), it is seen from Fig. 2 that they induce an additional (approximate) $\cos\Phi$ structure in the cross section. The twist-3 effects would induce an exact $\cos\Phi$ structure only when the BH amplitude is approximated by its leading term in an expansion in $1/Q$, neglecting its additional Φ dependence. In all calculations, we keep however the full Φ dependence of the BH, due to the lepton propagators, which complicates the interference at the lower Q. One sees from Fig. 2 that the interference of the twist-3 amplitude with the BH + twist-2 DVCS amplitude is destructive for $\Phi \lesssim 90°$ and constructive for $\Phi \gtrsim 90°$. At $\Phi=0°$, the twist-3 effects reduce the full twist-2 DVCS cross section (including the D term) by about 25%, whereas at $\Phi=180°$, they largely enhance the twist-2 DVCS cross section (by about 55%).

To further study the twist-3 effects on the $ep \rightarrow ep\gamma$ cross section, we show in Fig. 3 the cross section and charge asymmetry at the same E_e, x_B, and t as in Fig. 2, but at a value $Q^2=5$ GeV^2. One first sees that going to higher Q^2, at the same E_e, x_B, and t enhances the relative contribution of the BH process compared to the DVCS process. Consequently, the cross section follows much more the Φ-behavior of the BH process. For the twist-2 cross section, one sees again clearly the effect of the D term which leads, through its interference with the BH amplitude, to a charge asymmetry of opposite sign as compared to the one for the twist-2 DVCS process without the D term.

The twist-3 effects in the kinematics of Fig. 3, where the ratio t/Q^2 is only half the value of Fig. 2, are correspondingly smaller. It is furthermore seen that the twist-3 effects induce an (approximate) structure $\sim A\cos(2\Phi)$ in the charge asymmetry ($A \approx -0.08$ in the kinematics of Fig. 3). Only for out-of-plane angles $\Phi \gtrsim 130°$ one sees a deviation from the simple $\cos(2\Phi)$ structure induced by the twist-3 amplitude, due to the more complicated Φ dependence when calculating the BH amplitude exactly.

In Figs. 4 and 5, we compare the Φ dependence of the DVCS cross section and of the single spin asymmetry (SSA) corresponding to a polarized electron beam, for the same values of Q^2 and t as in Figs. 2 and 3, but for a value of $x_B=0.15$. Note that, due to parity invariance, the SSA is odd in Φ. We therefore display only half of the Φ range, i.e., Φ between $0°$ and $180°$.

When comparing Figs. 2, 3, and Figs. 4, 5, it is first seen that the cross sections for the $ep \rightarrow ep\gamma$ process increase strongly when decreasing x_B at fixed Q^2 and fixed t, mainly due to the growth of the BH amplitude. Due to this large BH

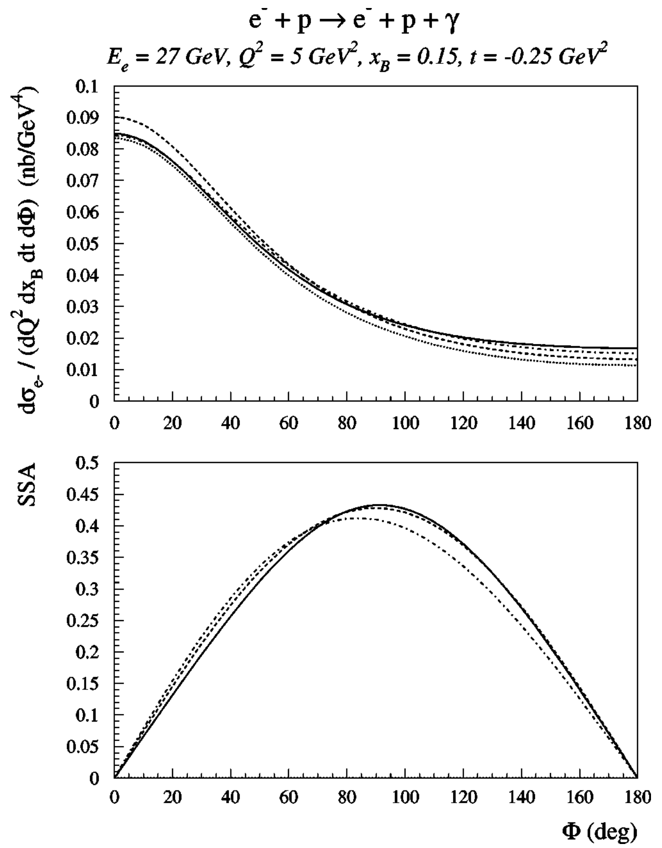

FIG. 5. Same as Fig. 4, but for $Q^2 = 5$ GeV^2.

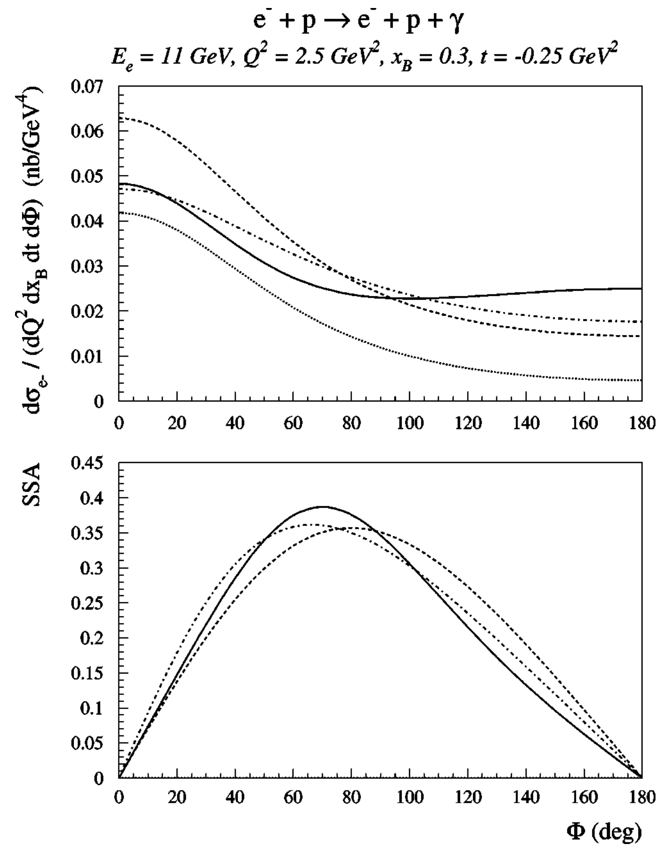

FIG. 6. Invariant cross section for the $e^- p \to e^- p \gamma$ reaction (upper panel) and DVCS SSA (lower panel) at $E_e = 11$ GeV for the DVCS kinematics as indicated in the figure. Curve conventions as in Fig. 2.

amplitude, the relative twist-3 effects in Figs. 4, 5 are smaller than the ones in Figs. 2, 3. However, the large BH amplitude leads to a large value for the SSA through its interference with the DVCS process. At (pure) twist-2 level, the SSA originates from the interference of the imaginary part of the DVCS amplitude and the (real) BH amplitude. In case the BH is approximated by its leading term in an expansion in $1/Q$, the twist-2 SSA displays a pure $\sin\Phi$ structure. Due to the more complicated Φ dependence of the BH at the lower values of Q^2, this form gets distorted and its maximum displaced, as is seen by a comparison of the twist-2 SSA in Fig. 4 ($Q^2 = 2.5$ GeV^2) and Fig. 5 ($Q^2 = 5$ GeV^2). Note that, for practical considerations, our ''twist-2'' DVCS calculations include kinematical higher twist terms as well as the gauge-restoring terms according to Eq. (15). Their effect can be seen in the slight change in the SSA (of the percent level), due to the DVCS process by itself (i.e., when increasing the real part of the DVCS amplitude by adding the D-term contribution, the curves for the SSAs are slightly displaced). A more systematic treatment of target mass corrections of order M^2/Q^2, and corrections of order t/Q^2 for DVCS, still remains to be done.

We next discuss the twist-3 effects, calculated in WW approximation, on the SSA for the DVCS process. One sees from Fig. 4 that the twist-3 corrections induce an (approximate) $\sin(2\Phi)$ structure in the SSA. The amplitude of the $\sin(2\Phi)$ term is, however, rather small and the twist-3 effects change the SSA by less than 5% in the kinematics corresponding to $t/Q^2 = 0.1$. It was checked that at $x_B = 0.3$ and for a value $t/Q^2 = 0.1$, the twist-3 effects on the SSA are of similar size. One therefore observes that although the twist-3 effects in WW approximation can provide a sizable contribution to the real part of the amplitude (Fig. 2), they modify the imaginary part, and hence the SSA, to a much lesser extent.

Finally, we show in Fig. 6 the corresponding cross sections at 11 GeV (JLab), for the same Q^2, x_B, and t as in Fig. 2. From the cross sections, one first observes again the rather large effect of the D term, which changes the twist-2 cross section by around 25%. The relative twist-3 effects on the cross section are smaller in this case than in Fig. 2, as they are on top of a larger BH amplitude. For the SSA, it is again observed that the twist-3 effects induce an approximate $\sin(2\Phi)$ structure, with amplitude of less than 5% in the valence kinematics considered here.

V. CONCLUSIONS

We have estimated the size of twist-3 effects on DVCS observables in the Wandzura-Wilczek approximation, which allows us to express the new twist-3 SPDs in terms of twist-2 SPDs and their derivatives. The twist-3 effects in WW approximation display therefore a new sensitivity to the shape of the twist-2 SPDs and provide an additional handle in the extraction of twist-2 SPDs from DVCS observables. The

WW approximation relies on the assumption that nucleon matrix elements of quark-gluon operators are small and can be neglected compared to the matrix elements of quark operators. Recent data on polarized structure functions indicate that this approximation holds to good accuracy. Therefore, the WW approximation was considered in this paper as a benchmark to study the size of the twist-3 effects on DVCS observables. It remains for future study to quantify the remaining twist-3 effects due to matrix elements of mixed quark-gluon operators.

To provide estimates and to interpret DVCS observables, we proposed a two-component parametrization for the SPDs of the nucleon as a sum of a double distribution part and a D term, so as to satisfy the polynomiality conditions. The D-term contribution is intimately related to the spontaneous breaking of the chiral symmetry and was estimated using the chiral quark-soliton model. It was shown that the effect of the D term, which gives a purely real contribution to the DVCS amplitude, can be very clearly seen in the dependence of the $ep \to ep\gamma$ cross section on the out-of-plane angle Φ. Through its interference with the BH process, the D-term contribution was seen to change the sign of the DVCS charge asymmetry and to lead to a large value for the charge asymmetry at $\Phi = 0°$. The pronounced charge asymmetry provides therefore a clear signature to study the D term and to check the chiral estimate given here. The importance of a quantitative understanding of the D term is especially emphasized by the fact that it contributes considerably to the DVCS cross section but drops out in the combination $H+E$ in Ji's angular momentum sum rule [2].

The twist-3 effects were seen to give a sizable change of the real part of the DVCS amplitude in the valence region, and they introduce an (approximate) $\cos\Phi$ structure in the cross section. In kinematics where the DVCS amplitude is comparable to the BH amplitude, one gets sizable effects. In particular, it was shown for the $e^- p \to e^- p \gamma$ reaction at $E_e = 27$ GeV, in valence kinematics ($x_B \simeq 0.3$), and for a value $t/Q^2 = 0.1$ (the value t/Q^2 governs kinematically the size of higher twist effects), that the twist-3 effects reduce the full cross section around $\Phi = 0°$ by around 25%, and largely enhance the cross section at $\Phi = 180°$ (where the BH is very small). On the charge asymmetry, the twist-3 effects introduce an (approximate) cos(2Φ) structure, with amplitude around 10% in those same kinematics.

For the single spin asymmetry, which is proportional to the imaginary part of the DVCS amplitude, the twist-3 effects introduce an (approximate) sin(2Φ) structure whose amplitude is, however, rather small, so that the SSA gets changed by less than 5% in typical valence kinematics for a value $t/Q^2 = 0.1$.

In summary, we found that the $ep \to ep\gamma$ cross section and charge asymmetry in the valence region display a sensitivity on twist-3 effects, in particular, through the real part of the amplitude. In the WW approximation, this sensitivity to the shape of the twist-2 SPDs may be exploited in fact in the extraction of those same twist-2 SPDs from DVCS observables. Such a systematic study of the extraction of twist-2 SPDs from DVCS observables to twist-3 accuracy in the WW approximation seems to be a promising subject for future work.

ACKNOWLEDGMENTS

The authors like to thank K. Goeke, P. A. M. Guichon, D. Müller, and O. V. Teryaev for useful discussions. The work of N. K. was supported by the DFG, project No. 920585; the work of M. V. P. was supported by DFG, BMFB, and COSY; the work of M. Vdh was supported by the DFG (SFB443). M. Vdh also likes to thank in particular K. Goeke for his support and warm hospitality in Bochum.

[1] D. Müller, D. Robaschik, B. Geyer, F.M. Dittes, and J. Horejsi, Fortschr. Phys. **42**, 101 (1994).
[2] X. Ji, Phys. Rev. Lett. **78**, 610 (1997); Phys. Rev. D **55**, 7114 (1997).
[3] A.V. Radyushkin, Phys. Lett. B **380**, 417 (1996).
[4] ZEUS Collaboration, P. R. Saull, ''Prompt Photon Production and Observation of Deeply Virtual Compton Scattering,'' hep-ex/0003030.
[5] H1 Collaboration, Rainer Stamen, ''Measurement of the Deeply Virtual Compton Scattering at Hera,'' H1 prelim-00-17, DIS 2000 and IHEP 2000.
[6] HERMES Collaboration, M. Amarian, ''DVCS and Exclusive Meson Production Measured by HERMES,'' talk at workshop ''Skewed Parton Distributions and Lepton-Nucleon Scattering,'' DESY 2000, http://hermes.desy.de/workshop/TALKS/talks.html
[7] V. Burkert, M. Guidal, and S. Stepanyan *et al.*, White Paper CEBAF@12 GeV, 2000, http://www.jlab.org/div_dept/physics_division/GeV.html
[8] I.V. Anikin, B. Pire, and O.V. Teryaev, Phys. Rev. D **62**, 071501 (2000).
[9] M. Penttinen, M.V. Polyakov, A.G. Shuvaev, and M. Strikman, Phys. Lett. B **491**, 96 (2000).
[10] A.V. Belitsky and D. Müller, Nucl. Phys. **B589**, 611 (2000).
[11] A.V. Radyushkin and C. Weiss, Phys. Lett. B **493**, 332 (2000); Phys. Rev. D (to be published), hep-ph/0010296.
[12] N. Kivel, M.V. Polyakov, A. Schäfer, and O.V. Teryaev, Phys. Lett. B **497**, 73 (2001).
[13] N. Kivel and M.V. Polyakov, Nucl. Phys. B (to be published) hep-ph/0010150.
[14] S. Wandzura and F. Wilczek, Phys. Lett. **72B**, 195 (1977).
[15] E155 Collaboration, G.S. Mitchell, hep-ex/9903055; E155x Collaboration, P. Bosted, Nucl. Phys. **A663**, 297 (2000).
[16] J. Balla, M.V. Polyakov, and C. Weiss, Nucl. Phys. **B510**, 327 (1998).
[17] B. Dressler and M.V. Polyakov, Phys. Rev. D **61**, 097501 (2000).
[18] A.V. Belitsky, D. Müller, A. Kirchner, and A. Schäfer, hep-ph/0011314.
[19] A.V. Belitsky and D. Müller, Phys. Lett. B **417**, 129 (1998).
[20] X. Ji and J. Osborne, Phys. Rev. D **58**, 094018 (1998).
[21] L. Mankiewicz, G. Piller, E. Stein, M. Vänttinen, and T.

Weigl, Phys. Lett. B **425**, 186 (1998).

[22] A.V. Belitsky, D. Müller, L. Niedermeier, and A. Schäfer, Phys. Lett. B **437**, 160 (1998); Nucl. Phys. **B546**, 279 (1999); Phys. Lett. B **474**, 163 (2000).

[23] P.A.M. Guichon and M. Vanderhaeghen, Prog. Part. Nucl. Phys. **41**, 125 (1998).

[24] M. Vanderhaeghen, P.A.M. Guichon, and M. Guidal, Phys. Rev. Lett. **80**, 5064 (1998).

[25] M. Vanderhaeghen, P.A.M. Guichon, and M. Guidal, Phys. Rev. D **60**, 094017 (1999).

[26] M. Penttinen, M.V. Polyakov, and K. Goeke, Phys. Rev. D **62**, 014024 (2000).

[27] A.V. Radyushkin, Phys. Rev. D **59**, 014030 (1999); Phys. Lett. B **449**, 81 (1999).

[28] X. Ji, J. Phys. G **24**, 1181 (1998).

[29] M.V. Polyakov and C. Weiss, Phys. Rev. D **60**, 114017 (1999).

[30] M.V. Polyakov, Nucl. Phys. **B555**, 231 (1999).

[31] V.Y. Petrov, P.V. Pobylitsa, M.V. Polyakov, I. Börnig, K. Goeke, and C. Weiss, Phys. Rev. D **57**, 4325 (1998).

[32] V. Petrov, P. Pobylitsa, M. Polyakov, I. Börnig, C. Weiss, and K. Goeke, in *Proceedings of the 8th International Conference on the Structure of Baryons (Baryons 98)*, Bonn, Germany, 1998, edited by D. W. Menze and B. Metsch (World Scientific, Singapore, 1999).

[33] A.D. Martin, R.G. Roberts, W.J. Stirling, and R.S. Thorne, Eur. Phys. J. C **4**, 463 (1998).

[34] E. Leader, A.V. Sidorov, and D.B. Stamenov, Phys. Rev. D **58**, 114028 (1998).

[35] L.L. Frankfurt, M.V. Polyakov, and M. Strikman, hep-ph/9808449.

[36] L. Mankiewicz, G. Piller, and A. Radyushkin, Eur. Phys. J. C **10**, 307 (1999).

[37] L.L. Frankfurt, P.V. Pobylitsa, M.V. Polyakov, and M. Strikman, Phys. Rev. D **60**, 014010 (1999).

[38] A.V. Belitsky, D. Müller, L. Niedermeier, and A. Schäfer, Nucl. Phys. **B593**, 289 (2001).