Testing the Landau gauge operator product expansion on the lattice with a $\langle A^2\rangle$ condensate

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(Received 27 January 2001; published 16 April 2001)

Using the operator product expansion we show that the $O(1/p^2)$ correction to the perturbative expressions for the gluon propagator and the strong coupling constant resulting from lattice simulations in the Landau gauge are due to a nonvanishing vacuum expectation value of the operator $A^\mu A_\mu$. This is done using the recently published Wilson coefficients of the identity operator computed to third order, and the subdominant Wilson coefficient computed in this paper to the leading logarithm. As a test of the applicability of OPE we compare the $\langle A^\mu A_\mu\rangle$ estimated from the gluon propagator and the one from the coupling constant in the flavorless case. Both agree within the statistical uncertainty $\sqrt{\langle A^\mu A_\mu\rangle}\simeq 1.64(15)$ GeV. Simultaneously we fit $\Lambda_{\overline{\rm MS}}=233(28)$ MeV, in perfect agreement with previous lattice estimates. When the leading coefficients are only expanded to two loops, the two estimates of the condensate differ drastically. As a consequence we insist that the OPE can be applied in predicting physical quantities only if the Wilson coefficients are computed to a high enough perturbative order.

DOI: 10.1103/PhysRevD.63.114003 PACS number(s): 12.38.Aw, 11.15.Ha, 12.38.Cy, 12.38.Gc

I. INTRODUCTION

When computing an operator product in a fixed gauge, the operator product expansion (OPE) contains in general contributions from local gauge-dependent operators, even though they should not emerge in the gauge-invariant sector. For example, in Ref. [1], a detailed analysis clearly shows that operators such as $A^2=A_\mu A^\mu$ contribute to the QCD propagator OPE through a nonzero expectation value in a non-gauge-invariant ''vacuum.'' A^2 is the unique dimension-2 operator allowed to have a vacuum expectation value (VEV) and is thus the dominant nonperturbative contributor, leading to $\sim 1/p^2$ corrections to the perturbative result.

These expected $\sim 1/p^2$ have at first sight nothing to do with the possible presence of $1/p^2$ terms in *gauge-invariant* quantities such as Wilson loops [2]: since no local gauge-invariant gluonic operator of dimension less than 4 exists, it is expected from the OPE that the dominant power correction should be $\propto 1/p^4$, originating from the local and gauge-invariant $G^{\mu\nu}G_{\mu\nu}$. Of course the operator A^2 in the Landau gauge can be viewed, by simply averaging it over the gauge orbit, as a gauge-invariant nonlocal operator. But then, dealing with nonlocal operators, we lose the standard OPE power counting rule relating the power behavior of a Wilson coefficient to the dimension of the corresponding operator: there is no reason for this nonlocal operator to yield $1/p^2$ contributions in a gauge-invariant observable. It has been strongly stressed in Ref. [3] that, working in the Landau gauge, the A^2 operator plays a special role since imposing the Landau gauge condition is equivalent to asserting that A^2 is at an extremum or a saddle point on its gauge orbit. Practically, on a lattice, one fixes the Landau gauge by searching for a minimum of A^2 on the orbit. We are not able to elaborate further on the issue of what relation might exist between the *expected* $\langle A^2\rangle$ condensate in the Landau gauge and the possible *unexpected* $1/p^2$ terms in gauge-invariant quantities [2]. But we are in a position to put the first step of this possible route on a firm ground: to provide strong evidence that there is indeed an $\langle A^2\rangle$ condensate in the Landau gauge and that it is not small.

To that aim we will use heavily the OPE. We need to be sure that the OPE really works in this situation and have to invent some way of verifying this point. A success of this check would achieve several goals. First it would give strong support to the conjecture that the OPE is really working in this situation, i.e., that we do not encounter a strange situation where the OPE would have failed like the one discussed in the preceding paragraph about $\langle G^{\mu\nu}G_{\mu\nu}\rangle$. Second it would confirm that we go far enough in the perturbative expansion [the expansion in $1/\ln(p^2)$] to be able to say something sensible about the power expansion (in $1/p^2$). Third it would confirm that we really are measuring $\langle A^2\rangle$. Such checks have of course many consequences which will be further discussed in the Conclusions.

From a practical (numerical) point of view, $1/p^2$ terms provide a specially convenient way to test the OPE since they remain visible at much larger energies than the $1/p^4$ ones which would result from the gauge invariant $G^{\mu\nu}G_{\mu\nu}$, and as already mentioned, their OPE analysis is rather simple and unambiguous because A^2 is the only dimension-two operator to contribute.

A recent study of $\alpha_s^{\widetilde{\rm MOM}}(p)$, the Landau gauge coupling constant,[1] regularized on a lattice showed unequivocally the presence of $1/p^2$ power corrections still visible at energies ~ 10 GeV for which OPE contributions of the gluon condensate $\langle A^2\rangle$, were natural candidates [4]. In this term all the nonperturbative input is contained in $\langle A^2\rangle$ while the OPE Wilson coefficients can be computed in perturbation. In view of this, we proposed in a previous work [5] a procedure to test the OPE based on the determination, and further com-

[1]$\alpha_s^{\widetilde{\rm MOM}}(p)$ stands for the QCD running coupling constant nonperturbatively renormalized in a kinematically asymmetric point by following the *momentum subtraction* prescriptions $(\widetilde{\rm MOM})$.

parison, of the two estimates of the gluon condensate $\langle A^2 \rangle$ obtained from both gluon two- and three-point Green functions by means of a simultaneous matching of the lattice data to the OPE formulas derived by following standard Shifman-Vainshtein-Zakharov (SVZ) techniques [6]. Thus, our OPE matchings of lattice data provide two independent estimates of the renormalized A^2 condensate. The adequate definition of renormalized condensates and their ''universality'' when studying different Green functions was discussed in Ref. [5] in connection with the choice of truncation orders for perturbative and OPE series (see also [7,8]). In this preliminary work we described the theoretical framework for this testing procedure and we performed a first analysis of previous lattice data [9,10] but the perturbative β function was known at that time only up to two loops and our use of the OPE was limited to a sole computation of the Wilson coefficients of A^2 at the *tree level*.

After this work was completed a computation of the third coefficient of the MOM beta function β_2 was published in Ref. [11]. The authors of this last work conclude that their computed β_2 and our ''prediction'' of this coefficient based on OPE consistency [5] reasonably agree with each other. Thanks to the new information concerning the β function and to the high accuracy of our lattice results we are now in a particularly favorable situation to address further the questions we have mentioned above. This is the task we shall attack in the present paper, presenting a consistent calculation in the MOM scheme (a symmetric kinematics chosen for the vertex) with the Wilson coefficients of the identity operator computed at three loops [9–12] and the ones of A^2 computed to the leading logarithm in Sec. II B. In particular we will compare the check of the ''universality'' of the condensates when expanding the leading perturbative coefficients to three loops and when one uses only the two-loop order.

The theoretical setting of our use of OPE is described in Sec. II: the tree-level computation, presented previously in Ref. [5], is only sketched and most attention is paid to the obtention of the one-loop anomalous dimension of the Wilson coefficients. The fitting strategy is explained and the matching test performed in Sec. III. Finally, we discuss and conclude in Sec. IV.

II. OPE FOR THE GLUON PROPAGATOR AND $\alpha_s(p)$

In the present section we shall expand the three-point Green function, and hence $\alpha_S(p)$, as well as the gluon propagator, in the OPE approach up to the $1/p^2$ order. Both gluonic two- and three-point Green functions are renormalized according to the MOM scheme. Let us start with a reminder of the computation of the tree-level Wilson coefficients [5].

A. Tree-level Wilson coefficients

In the pure Yangs-Mills QCD, without quarks, the OPE yields

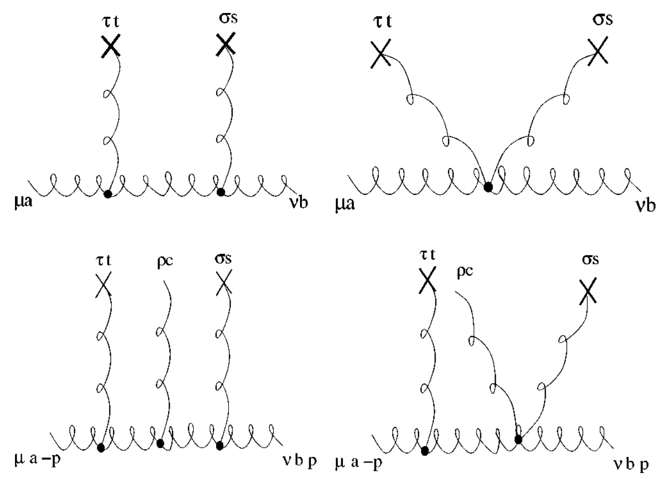

FIG. 1. Four- and five-gluon tree-level diagrams contributing (with all their possible permutations) to the Wilson coefficients of the gluon propagator and the three-gluon vertex. Crosses mark the gluon legs due to the external soft gluons.

$$T(\tilde{A}^a_\mu(-p)\tilde{A}^b_\nu(p)) = (c_0)^{ab}_{\mu\nu}(p)1 + (c_1)^{ab\mu'}_{\mu\nu a'}(p){:}A^{a'}_{\mu'}(0){:} + (c_2)^{ab\mu'\nu'}_{\mu\nu a'b'}(p){:}A^{a'}_{\mu'}(0)A^{b'}_{\nu'}(0){:} + \cdots, \tag{1}$$

$$T(\tilde{A}^a_\mu(p_1)\tilde{A}^b_\nu(p_2)\tilde{A}^c_\rho(p_3)) = (d_0)^{abc}_{\mu\nu\rho}(p_1,p_2,p_3)1 + (d_1)^{abc\mu'}_{\mu\nu\rho a'}(p_1,p_2,p_3){:}A^{a'}_{\mu'}(0){:} + (d_2)^{abc\mu'\nu'}_{\mu\nu\rho a'b'}(p_1,p_2,p_3){:}A^{a'}_{\mu'}(0)A^{b'}_{\nu'}(0){:} + \cdots, \tag{2}$$

where only normal products of local gluon field operators occur and $A(\tilde{A})$ stands for the gluon field in configuration (momentum) space, a,b being color indices and μ,ν Lorentz ones. The notation $T()$ simply refers to the standard T^* product in momentum space. The normal product of Eqs. (1),(2) should be defined in reference to the perturbative vacuum [5].

Only terms in Eqs. (1),(2) containing an even number of local gluon fields give a non-null VEV because of Lorentz invariance and of the gauge condition [5]. The coefficients c_0 and d_0 are the purely perturbative Green functions. Assuming the Wilson factorization of soft and hard gluon contributions, the relevant Wilson coefficients c_2,d_2 can be obtained, in perturbation, by computing the diagrams in Fig. 1 which represent the matrix elements of operators on the left-hand side (LHS) of Eqs. (1),(2) between soft gluons, indicated by crosses. Using also the tree-level expression for the matrix element, between the same two soft gluons, of the local operators on the RHS of Eqs. (1),(2) we obtain c_2 and d_2 by matching both sides.

Thus, in the appropriate Euclidean metrics for matching to lattice nonperturbative results, we can write

$$k^2 G^{(2)}(k^2) = Z^{MOM}(k^2) = Z^{MOM}_{n\ \text{loops}}(k^2) + \frac{3g^2\langle A^2\rangle}{4(N_c^2-1)}\frac{1}{k^2},$$

$$k^6 G^{(3)}(k^2,k^2,k^2) = k^6 G^{(3)}_{\text{pert}}(k^2,k^2,k^2) + \frac{9 g^3 \langle A^2 \rangle}{4(N_c^2-1)} \frac{1}{k^2}, \quad (3)$$

where the scalar form factors $G^{(2)}, G^{(3)}$ are defined as follows from the Green functions:

$$G^{(2)}(p^2) = \frac{\delta_{ab}}{N_c^2-1} \frac{1}{3} \left(\delta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2} \right) \langle \tilde{A}^a_\mu(-p) \tilde{A}^b_\nu(p) \rangle,$$

$$G^{(3)}(k^2,k^2,k^2) = \frac{1}{18k^2} \frac{f^{abc}}{N_c(N_c^2-1)} \langle \tilde{A}^a_\mu(p_1) \tilde{A}^b_\nu(p_2) \tilde{A}^c_\rho(p_3) \rangle \times \Bigg[(T^{\text{tree}})^{\mu_1\mu_2\mu_3} + \frac{(p_1-p_2)^\rho (p_2-p_3)^\mu (p_3-p_1)^\nu}{2k^2} \Bigg]. \quad (4)$$

For the kinematic configuration $p_1^2 = p_2^2 = p_3^2 = k^2$ the three-gluon tree-level tensor is defined as

$$(T^{\text{tree}})_{\mu_1'\mu_2'\mu_3} = [\delta_{\mu_1'\mu_2'}(p_1-p_2)_{\mu_3'} + \text{cycl. perm.}] \times \prod_{i=1,3} \left(\delta_{\mu_i'\mu_i} - \frac{p_{i\mu_i'} p_{i\mu_i}}{p_i^2} \right). \quad (5)$$

In Eqs. (3)–(5) we have dealt with bare quantities, depending only on the cutoff a^{-1} and on the momentum k. We have omitted to explicitate the dependence on the cutoff in order to simplify the notation. Using Eqs. (3) these Green functions can be conveniently renormalized by MOM prescriptions: the renormalized two-point Green function is taken equal to $1/k^2$ for $k^2 = \mu^2$:

$$k^2 G^{(2)}_R(k^2,\mu^2) \equiv \frac{k^2 G^{(2)}(k^2)}{\mu^2 G^{(2)}(\mu^2)} = c_0\left(\frac{k^2}{\mu^2}, \alpha(\mu) \right) + c_2\left(\frac{k^2}{\mu^2}, \alpha(\mu) \right) \times \frac{|A^2|_{R,\mu}}{4(N_c^2-1)} \frac{1}{k^2}. \quad (6)$$

The c_0 Wilson coefficient can be written as

$$c_0\left(\frac{k^2}{\mu^2}, \alpha(\mu) \right) = \frac{Z^{\text{MOM}}_{n\ \text{loops}}(k^2)}{Z^{\text{MOM}}(\mu^2)} = c_0(1,\alpha(\mu)) \frac{Z^{\text{MOM}}_{n\ \text{loops}}(k^2)}{Z^{\text{MOM}}_{n\ \text{loops}}(\mu^2)}, \quad (7)$$

and verifies consequently the perturbative evolution equations of Z^{MOM},

$$\frac{d \ln c_0\left(\frac{k^2}{\mu^2}, \alpha(\mu) \right)}{d \ln k^2} = -\left(\gamma_0 \frac{\alpha(k)}{4\pi} + \gamma_1 \left(\frac{\alpha(k)}{4\pi} \right)^2 + \gamma_2 \left(\frac{\alpha(k)}{4\pi} \right)^3 + \cdots \right), \quad (8)$$

where γ_1 and γ_2 depend on the perturbative scheme in which the strong coupling constant $\alpha(k)$ is defined. The boundary condition to solve Eq. (8) comes from the nonperturbative normalization of $k^2 G^{(2)}_R(k^2,\mu^2)$ to 1 at $k^2 = \mu^2$, and it results that $c_0(1,\alpha(\mu)) = 1 + O(1/\mu^2)$.

Let us recall that in the MOM prescription, the three-point Green function is renormalized by $G^{(3)}_R(k^2,\mu^2) \equiv G^{(3)}(k^2,k^2,k^2)[Z^{\text{MOM}}(\mu)]^{-3/2}$, and the MOM coupling constant follows from

$$g_R(k^2) = \frac{G^{(3)}(k^2,k^2,k^2)}{[G^{(2)}(k^2)]^3} [Z^{\text{MOM}}(k^2)]^{3/2} = k^6 G^{(3)}_R(k^2,\mu^2) \times [k^2 G^{(2)}_R(k^2,\mu^2)]^{-3/2}. \quad (9)$$

Analogously to Eq. (6) we define the renormalized three-point Green function

$$k^6 G^{(3)}_R(k^2,\mu^2) = d_0\left(\frac{k^2}{\mu^2}, \alpha(\mu) \right) + d_2\left(\frac{k^2}{\mu^2}, \alpha(\mu) \right) \frac{\langle A^2 \rangle_{R,\mu}}{4(N_c^2-1)} \frac{1}{k^2}, \quad (10)$$

where the d_0 Wilson coefficient verifies the perturbative evolution equations of $k^6 G^{(3)}_R(k^2,\mu^2)$ and the boundary condition $d_0(1,\alpha(\mu)) = g_R(\mu^2) + O(1/\mu^2)$ is immediate from Eqs. (9),(10).

In the MOM scheme, the gluon condensate $\langle A^2 \rangle_{R,\mu}$ is renormalized at μ^2 by a standard condition through division by a renormalization constant Z_{A^2}.

The c_2 and d_2 Wilson coefficients at the tree level are [5]

$$c_2(1,\alpha(\mu)) = 3\, g^2,$$

$$d_2(1,\alpha(\mu)) = 9\, g^3. \quad (11)$$

Since the three-point Green function naturally defines the MOM scheme coupling constant [see below Eq. (9)], we will perform all the coming calculations in the symmetric MOM scheme where

$$\gamma_0 = 13/2, \quad \gamma_1 = -16.9, \gamma_2 \simeq 1332.3. \quad (12)$$

In Eq. (8), $\alpha(k)$ is of course taken to be the purely perturbative running coupling constant, $g^2_{R,\text{pert}}(k^2)/(4\pi)$, obtained by integrating the beta function in the MOM scheme:

$$\frac{d}{d\ln k}\alpha(k)=\beta(\alpha(k))=-\left(\frac{\beta_0}{2\pi}\alpha^2(k)+\frac{\beta_1}{4\pi^2}\alpha^3(k)+\frac{\beta_2}{(4\pi)^3}\alpha^4(k)+\cdots\right), \quad (13)$$

where [11]

$$\beta_0=11, \quad \beta_1=51, \quad \beta_2\simeq 3088. \quad (14)$$

B. Wilson coefficient at leading logarithms

The purpose is now to compute to leading logarithms the subleading Wilson coefficients in Eqs. (6),(10). To this goal, following [13] it will be useful to consider the following matrix element:

$$\langle g_\tau^a|\tilde{A}_\rho^r(k)\tilde{A}_\sigma^s(-k)|g_\nu^b\rangle_{R,\mu}=\delta^{rs}\left(\delta_{\rho\sigma}-\frac{k_\rho k_\sigma}{k^2}\right)\times\left[\frac{c_2\left(\frac{k^2}{\mu^2},\alpha(\mu)\right)}{k^4}\frac{\langle g_\tau^a|A_R^2|g_\nu^b\rangle_\mu}{4(N_C^2-1)}+\cdots\right], \quad (15)$$

where the external gluons carry soft momenta. This ellipses refers to terms with powers of $1/k$ different from 4 (i.e., corresponding to higher dimension operators or to identity operator[2]). From Eq. (15) we get

$$\frac{4}{3}k^4\frac{\langle g_\tau^a|\tilde{A}_\rho^r(k)\tilde{A}_\sigma^s(-k)|g_\nu^b\rangle}{|g_\tau^a|A^2|g_\nu^b|}\delta^{rs}\left(\delta_{\rho\sigma}-\frac{k_\rho k_\sigma}{k^2}\right)=Z_3(\mu^2)Z_{A^2}^{-1}(\mu^2)c_2\left(\frac{k^2}{\mu^2},\alpha(\mu)\right)+\cdots\equiv\hat{Z}^{-1}(\mu^2)c_2\left(\frac{k^2}{\mu^2},\alpha(\mu)\right)+\cdots, \quad (16)$$

where $\tilde{A}_R=Z_3^{-1/2}\tilde{A}$ and $A_R^2=Z_{A^2}^{-1}A^2$, while $\hat{Z}\equiv Z_3^{-1}Z_{A^2}$ is a useful notation denoting the divergent factor of the matrix element coming from proper vertex corrections. If one takes the logarithmic derivatives with respect to μ on both sides of Eq. (16), the following differential equation is obtained:

$$\left\{-2\gamma(\alpha(\mu))_{A^2}+2\gamma(\alpha(\mu))+\frac{\partial}{\partial\ln\mu}+\beta(\alpha(\mu))\frac{\partial}{\partial\alpha}\right\}c_2\left(\frac{k^2}{\mu^2},\alpha(\mu)\right)=0. \quad (17)$$

[2]It should be remembered that other terms, such as $\partial_\mu A_\mu$, with the same dimension of A^2, do not survive.

An analogous differential equation describing the behavior of the three-point Wilson coefficient on the renormalization momentum μ can be obtained similarly:

$$\left\{-2\gamma(\alpha(\mu))_{A^2}+3\gamma(\alpha(\mu))+\frac{\partial}{\partial\ln\mu}+\beta(\alpha(\mu))\frac{\partial}{\partial\alpha}\right\}d_2\left(\frac{k^2}{\mu^2},\alpha(\mu)\right)=0. \quad (18)$$

Here we have defined

$$\gamma_{A^2}=\frac{d}{d\ln\mu^2}\ln Z_{A^2} \quad (19)$$

and $\gamma(\alpha(\mu))$ is the gluon propagator anomalous dimension. Reexpressing these evolution equations in terms of

$$\hat{\gamma}(\alpha(\mu))=\frac{d}{d\ln\mu^2}\ln\hat{Z}(\mu^2), \quad (20)$$

we obtain

$$\left\{-2\hat{\gamma}(\alpha(\mu))+\frac{\partial}{\partial\ln\mu}+\beta(\alpha(\mu))\frac{\partial}{\partial\alpha}\right\}c_2\left(\frac{k^2}{\mu^2},\alpha(\mu)\right)=0 \quad (21)$$

and

$$\left\{-2\hat{\gamma}(\alpha(\mu))+\gamma(\alpha(\mu))+\frac{\partial}{\partial\ln\mu}+\beta(\alpha(\mu))\frac{\partial}{\partial\alpha}\right\}d_2\left(\frac{k^2}{\mu^2},\alpha(\mu)\right)=0. \quad (22)$$

The leading logarithmic solution for both Eqs. (21),(22) can be written as

$$c_2\left(\frac{k^2}{\mu^2},\alpha(\mu)\right)=c_2(1,\alpha(k))\left(\frac{\alpha(k)}{\alpha(\mu)}\right)^{-\hat{\gamma}_0/\beta_0},$$
$$d_2\left(\frac{k^2}{\mu^2},\alpha(\mu)\right)=d_2(1,\alpha(k))\left(\frac{\alpha(k)}{\alpha(\mu)}\right)^{(\gamma_0-2\hat{\gamma}_0)/2\beta_0}, \quad (23)$$

where $\hat{\gamma}_0$ is defined in analogy with γ_0:

$$\hat{\gamma}(\alpha(\mu))=-\hat{\gamma}_0\frac{\alpha(k)}{4\pi}+\cdots. \quad (24)$$

The prefactors $c_2(1,\alpha(k))$ and $d_2(1,\alpha(k))$ have to be matched at the tree level to Eq. (11). The only solutions are of the form

$$c_2(1,\alpha(k))=3g_R^2(k^2)\left[1+\mathcal{O}\left(\frac{1}{\log(k/\Lambda_{\rm QCD})}\right)\right],$$

$$d_2(1,\alpha(k)) = 9\, g_R^3(k^2)\left[1+\mathcal{O}\left(\frac{1}{\log(k/\Lambda_{\rm QCD})}\right)\right]. \tag{25}$$

The $\mathcal{O}(1/\log(k/\Lambda_{\rm QCD}))$ terms are clearly of the same order as the next-to-leading contributions to the anomalous dimension which are systematically omitted in this paper.

Of course, these solutions of Eqs. (21),(22) define the dependence of the Wilson coefficients not only on the renormalization momentum, μ, but simultaneously on the momentum scale k^2. This is a straightforward consequence of standard dimensional arguments: the only dimensionless quantities are the ratio k^2/μ^2 and α. Then, as soon as one knows perturbatively $\hat{\gamma}(\alpha(\mu))$, $\gamma(\alpha(\mu))$, and $\beta(\alpha(\mu))$, the leading logarithmic behavior on k is available.

As already mentioned, the gluon propagator anomalous dimension and the beta function are known up to three loops in the MOM scheme and up to three and four loops, respectively, in the $\widetilde{\rm MOM}$ scheme. The anomalous dimension of the A^2 operator is obviously less stimulating for perturbative QCD community. We have done this calculation to one loop (see the Appendix), obtaining

$$\hat{\gamma}(\alpha(\mu)) = -\hat{\gamma}_0 \frac{\alpha(\mu)}{4\pi} + \cdots = -\frac{3N_C}{4}\frac{\alpha(\mu)}{4\pi} + \cdots \tag{26}$$

and

$$\gamma_{A^2}(\alpha(\mu)) = \frac{d}{d\ln\mu^2}\ln Z_{A^2} = -\frac{35N_C}{12}\frac{\alpha(\mu)}{4\pi} + \cdots. \tag{27}$$

C. Gluon propagator with leading logarithms for the condensate coefficient

Let us now specify our approach to lattice results. Using the definitions in Eqs. (3) and (6) we will match our lattice results to

$$\frac{Z_{\rm Latt}^{\rm MOM}(k^2,a)}{Z_{\rm Latt}^{\rm MOM}(\mu^2,a)} = k^2 G_R^{(2)}(k^2,\mu^2) + O(a^2), \tag{28}$$

where the adequate control of lattice artifacts[3] reduces the UV discretization errors to an acceptable level. From Eq. (6),

$$k^2 G_R^{(2)}(k^2,\mu^2) = c_0\left(\frac{k^2}{\mu^2},\alpha(\mu)\right) \times\left(1+\frac{c_2\left(\frac{k^2}{\mu^2},\alpha(\mu)\right)}{c_0\left(\frac{k^2}{\mu^2},\alpha(\mu)\right)}\frac{\langle A^2\rangle_{R,\mu}}{4(N_c^2-1)}\frac{1}{k^2}\right), \tag{29}$$

where we explicitly factorize the Wilson coefficient of the identity operator which, as was previously indicated, is known to three loops. Nevertheless, for consistency, all the terms *inside the parentheses* on the RHS of Eq. (29) will be developed only to leading order, including $c_0(k^2/\mu^2,\alpha(\mu))$:

$$c_{0,\rm LO}\left(\frac{k^2}{\mu^2},\alpha(\mu)\right) = \left(\frac{\alpha(k)}{\alpha(\mu)}\right)^{\gamma_0/\beta_0}. \tag{30}$$

Terms of the order of $O(1/(k^2\mu^2))$ have been neglected, as well as, of course, those of $O(1/k^4)$ coming from higher dimension operators. One free parameter, i.e., a boundary condition, has to be fitted from lattice data. It can be either $\alpha(\mu)$ or the Λ parameter, i.e., the position of the perturbative Landau pole. We choose the latter. We write $c_{0,1\ \rm loop}$ in terms of the MOM coupling constant[4] and the Λ parameter in Eq. (30) in the MOM scheme,[5]

$$\Lambda \equiv \Lambda^{\rm MOM} \simeq 3.334\,\Lambda^{\overline{\rm MS}}. \tag{31}$$

We finally obtain

$$Z_{\rm Latt}^{\rm MOM}(k^2,a) = Z_{\rm Latt}^{\rm MOM}(\mu^2,a)\, c_0\left(\frac{k^2}{\mu^2},\alpha(\mu)\right) \times\left[1+R^{(2)}\left(\ln\frac{k}{\Lambda}\right)^{(\gamma_0+\hat{\gamma}_0)/\beta_0-1}\frac{1}{k^2}\right], \tag{32}$$

where

$$R^{(2)} = \frac{6\pi^2}{\beta_0(N_c^2-1)}\left(\ln\frac{\mu}{\Lambda}\right)^{-(\gamma_0+\hat{\gamma}_0)/\beta_0}\langle A^2\rangle_{{\rm R},\mu}. \tag{33}$$

D. Running coupling constant

By taking the OPE expansions in Eqs. (10) and (6), Eq. (9) can be written as

$$g_R(k^2) = g_{R,\rm pert}(k^2)\left\{1+\frac{\langle A^2\rangle_{R,\mu}}{4(N_c^2-1)}\frac{1}{k^2} \times\left(\frac{d_2\left(\frac{k^2}{\mu^2},\alpha(\mu)\right)}{\left[d_0\left(\frac{k^2}{\mu^2},\alpha(\mu)\right)\right]} - \frac{3}{2}\frac{c_2\left(\frac{k^2}{\mu^2},\alpha(\mu)\right)}{c_0\left(\frac{k^2}{\mu^2},\alpha(\mu)\right)}\right)\right\}, \tag{34}$$

[3]See Ref. [10], where we discuss at length the artifacts of the lattice gluon propagator evaluation.

[4]$\widetilde{\rm MOM}$, for instance, or whatever renormalization scheme could be used alternatively. Our preference for the MOM scheme has been explained above.

[5]See, for instance, Ref. [14].

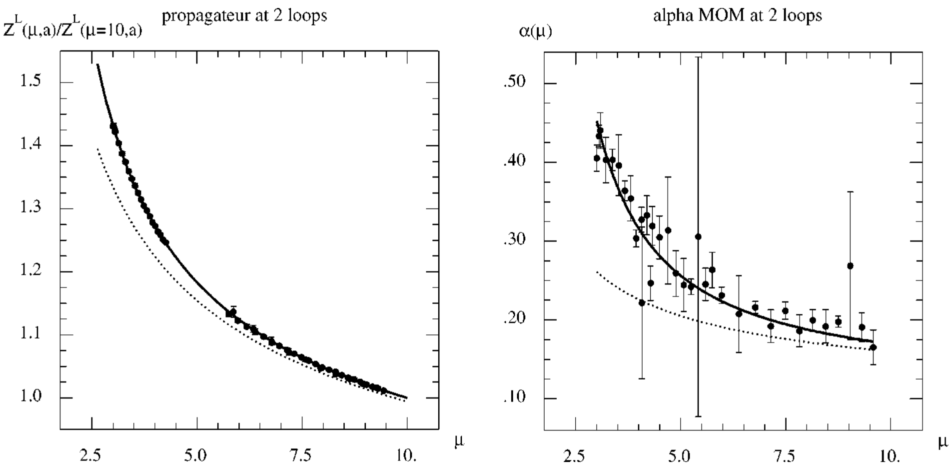

FIG. 2. Comparison of the two-loop fit to the ratio of the renormalization constants at k and at 10 GeV and to $\alpha_s(k)$ with the lattice data for $2.5<k<10$ GeV. The dotted line shows the perturbative part.

with the identification

$$g_{R,\mathrm{pert}}(k^2)\equiv d_0\left(\frac{k^2}{\mu^2},\alpha(\mu)\right)\left[c_0\left(\frac{k^2}{\mu^2},\alpha(\mu)\right)\right]^{-3/2}. \quad (35)$$

Applying the results given by Eqs. (23),(32)–(35), $\alpha_{\mathrm{MOM}}=g_{\mathrm{R}}^2/(4\pi)$ verifies

$$\alpha_{\mathrm{MOM}}(k)=\alpha_{\mathrm{pert}}(k)\left(1+R^{(3)}\left(\ln\frac{k}{\Lambda}\right)^{(\gamma_0+\hat{\gamma}_0)/\beta_0-1}\frac{1}{k^2}\right), \quad (36)$$

where

$$R^{(3)}=\frac{18\pi^2}{\beta_0(N_c^2-1)}\left(\ln\frac{\mu}{\Lambda}\right)^{-(\gamma_0+\hat{\gamma}_0)/\beta_0}\langle A^2\rangle_{\mathrm{R},\mu}. \quad (37)$$

Again, we do not retain $O(1/(k^2\mu^2),1/k^4)$ terms.

III. FITTING THE DATA TO OUR ANSÄTZE

We shall follow in this section the OPE testing approach proposed in Ref. [5]: trying a consistent description of lattice data for two- and three-gluon Green functions from Refs. [4,9,10]. We are, however, in a much better position than in [5]. In the latter work, only two-loop information was available for the beta function and d_0 while the subdominant Wilson coefficients c_2 and d_2 were computed only at the tree level. This had the practical inconvenience of preventing a simultaneous fit of both $\Lambda^{\overline{\mathrm{MS}}}$ and $\langle A^2\rangle$: the $\Lambda^{\overline{\mathrm{MS}}}$ parameter had to be taken from outside our matching procedure. Now the new input for the three-loop MOM beta function and c_0 coefficient [11] enables us to perform a self-consistent test by combining the matching of the gluon propagator and of α_s^{MOM} to formulas in Eqs. (32),(33),(36),(37), where the three quantities, $\Lambda_{\overline{\mathrm{MS}}}$ and gluon condensates from both Green functions, are taken to be fitted on the same footing. Of course, the test consists in checking the equality of the two gluon condensates obtained from those two different Green functions.

In the case of the gluon propagator, the factor $c_0(1,\alpha(\mu))Z_{\mathrm{Latt}}^{\mathrm{MOM}}(\mu^2,a)$, which carries all the logarithmic dependence on the lattice spacing, appears as an additional parameter to be fitted. As we explained in Ref. [5], a large fitting window is an important ''*ace*'' to restrict the potentially dangerous confusion between Wilson coefficients for different powers. To combine data over such a large energy window we need to match the lattice results obtained with different lattice spacings and the last factor carrying lattice spacing dependence should be independently fitted for each one. On the contrary, the running coupling constant should be regularization independent and the matching of data sets corresponding to different lattice spacing can be imposed without tuning any additional parameter (this is by itself a positive test of the goodness of the procedure used to build our data set). As a matter of fact, this is why the matching of the latter to perturbative formulas is much more constraining than that of the former in order to estimate $\Lambda_{\overline{\mathrm{MS}}}$, as discussed in Refs. [4,9,10]. The details of the lattice simulations, of the procedures used to obtain an artifact-safe data set, or of the definition of regularization-independent objects permitting lattice regularized data to be matched to continuum quantities in any scheme can be found in those references. We will now present the results of the fitting strategy just described.

A. Two-loop fit

We first perform the combined fit for the two- and three-gluon Green functions at the two-loop level for the leading Wilson coefficients. In Fig. 2, we plot lattice data and the curves given by Eqs. (32),(33),(36),(37) with the following best-fit parameter:

$$\text{propagator:}\quad \sqrt{\langle A^2\rangle_{R,\mu}}=1.64(17)\ \ \mathrm{GeV},\quad \alpha^{\mathrm{MOM}}:\quad \sqrt{\langle A^2\rangle_{R,\mu}}=3.1(3)\ \ \mathrm{GeV},$$

$$\frac{\{\sqrt{\langle A^2\rangle_{R,\mu}}\}_{\mathrm{alpha}}}{\{\sqrt{\langle A^2\rangle_{R,\mu}}\}_{\mathrm{prop}}}=1.86(4),\qquad \Lambda_{\overline{\mathrm{MS}}}=172(15)\ \ \mathrm{MeV}, \quad (38)$$

with a $\chi^2/N_{\mathrm{DF}}\simeq 1.1$ for the combined fit.

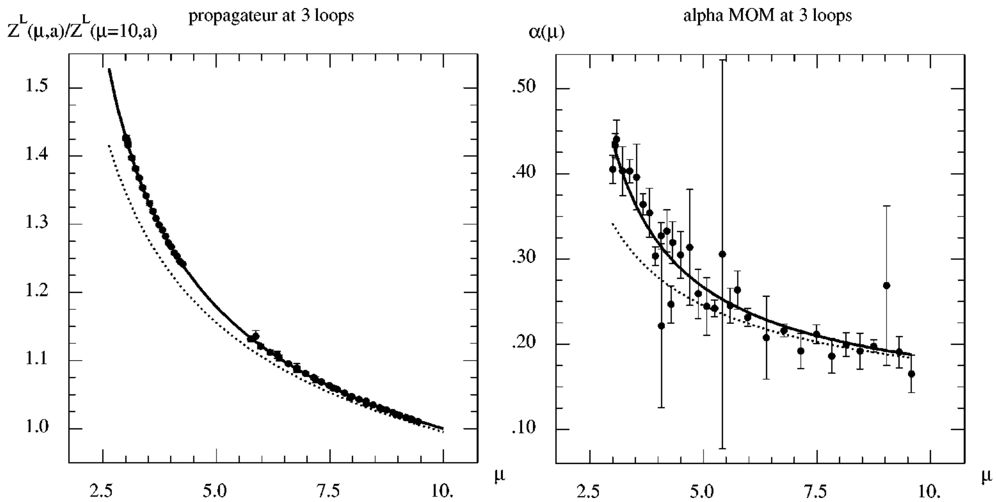

FIG. 3. Same as Fig. 2 at the three loop level.

B. Three-loops fit

The present perturbative knowledge allows a three-loop level fit for leading Wilson coefficients. Analogously to the previous paragraph, we plot in Fig. 3 the lattice data and curves given by Eqs. (32),(33),(36),(37), $Z_{R,\mathrm{pert}}^{\mathrm{MOM}}$ and α_{pert} taken at three loops, with the following best-fit parameters:

$$\text{propagator:}\quad \sqrt{\langle A^2\rangle_{R,\mu}} = 1.55(17)\ \text{GeV},\quad \alpha^{\mathrm{MOM}}:\quad \sqrt{\langle A^2\rangle_{R,\mu}} = 1.9(3)\ \text{GeV},$$

$$\frac{\{\sqrt{\langle A^2\rangle_{R,\mu}}\}_{\mathrm{alpha}}}{\{\sqrt{\langle A^2\rangle_{R,\mu}}\}_{\mathrm{prop}}}=1.21(18),\quad \Lambda_{\overline{\mathrm{MS}}}=233(28)\ \text{MeV},\qquad (39)$$

with $\chi^2/N_{\mathrm{DF}}\simeq 1.2$. Combining the results obtained from α^{MOM} and from the propagator in the standard way gives our final result $\sqrt{\langle A^2\rangle_{R,\mu}}=1.64(15)$ GeV. The renormalization scale μ is taken to be 10 GeV in both combined fits at the two- and three-loop levels. However, we have checked that, varying μ over the fitting window where we can legitimately neglect terms of $O(1/(\mu^2 k^2))$ in Eq. (37), the ratios of condensates in Eqs. (38),(39) remain essentially unmodified. In fact, that $R^{(2)}$ in Eq. (32) does not depend on μ has been explicitly tested over the fitting window [the same is obvious for $R^{(3)}$ in Eq. (36) where nothing depends on μ].

IV. DISCUSSION AND CONCLUSIONS

The gluon condensate $\langle A^2\rangle$ has been computed from the deviations of both the lattice nonperturbative evaluation of the MOM α_S and the gluon propagator from their known perturbative behavior. We have described these nonperturbative deviations using the OPE and fitting the condensate to match both sides. The use of the self-consistent fitting strategy described in the preceding section leads simultaneously to a prediction $\Lambda_{\overline{\mathrm{MS}}}$ and to two independent estimates of $\langle A^2\rangle$.

The fit using the two-loop perturbative expressions for both the MOM α_S and the gluon propagator clearly fails: *a clear disagreement between the two independent estimates of* $\sqrt{\langle A^2\rangle}$ *is found*. The ratio of both estimates is 1.86(4) from Eqs. (38). This confirms the preliminary analysis in Ref. [5], where only tree-level Wilson coefficients were computed. In this preliminary work, a self-consistent three-loop analysis was not possible because the MOM beta function was not known up to three loops. Nevertheless, we tried to fit the third coefficient of the beta function, β_2 to reach good agreement between the two estimates of $|A^2|$, the $\Lambda_{\overline{\mathrm{MS}}}$ parameter being taken from previous works to be the same for both two- and tree-point Green function matchings. Our estimate $\beta_2=7400(1500)$ was about twice larger than the result $\beta_2=3088$ in [11]. Still this fit went in the right direction, whence the authors of [11] expected their result to lead to a fair fit to lattice data.

This expectation turns out to be correct.

First, the ratio of the two estimates of $\sqrt{\langle A^2\rangle}$ *is equal to* 1.21(18), *i.e., compatible with 1, provided the leading Wilson coefficients are consistently expanded at the three-loop level and the subleading coefficients of* $\langle A^2\rangle$ *are computed to the leading logarithms. Second, in the same joint fit,* $\Lambda_{\overline{\mathrm{MS}}}$ *is estimated to be* 233(28) MeV, in amazing agreement with previous estimates of $\Lambda_{\overline{\mathrm{MS}}}$ appearing in the literature (see, for instance, [10,15]). Thus, the present analysis ends up with a twofold success and we can conclude that the OPE leads to a good description of the deviations of the running coupling constant and of the gluon propagator from their perturbative behavior in terms of perturbatively available coefficients multiplying one phenomenological condensate: the sole non vanishing nonperturbative contribution up to the order $1/p^2$, namely, the gluon condensate $\langle A^2\rangle$.

However, for this OPE description to be consistent, it is unambiguously demonstrated that the leading coefficient must be taken to three loops; on the contrary there is a clear failure at two loops, even though our analysis has been performed at an unusually large energy scale, up to 10 GeV. As was discussed in the preliminary study, such a disagreement indicates that we are, in this case, in the situation described in [8], i.e., that the perturbative order is too low to give an acceptable precision in the estimate of the power corrections. Taking into account power corrections does not make sense unless the leading contribution is computed perturbatively to a sufficient accuracy. In simple mathematical language, it makes no sense to consider the $1/p^2$ corrections when one does not consider the $1/\ln(p^2)$ corrections to a sufficient order. The OPE is often used with the leading coefficients only known to two loops (sometimes to one loop). We believe in view of our results that these attempts should be reconsidered with care.

As for us, we were in a particularly favorable situation to analyze the problem thoroughly. We have rather accurate results. The dimension-2 power correction clearly shows up clearly and can be fully consistently attributed to an $A_\mu A^\mu$ condensate in the Landau gauge in full agreement with the theoretical expectations. Since we are working in the Landau gauge, we produce (and use later on) *bare* gauge field configurations which minimize $A^\mu A_\mu$ with respect to the gauge group. This has the interesting consequence that the quantities we measure are invariant under infinitesimal gauge transformations in the vicinity of the Landau gauge. Still the link between what we call the $\langle A^2\rangle$ and the $\langle A^2_{\rm min}\rangle$ defined in [3] should be better clarified, which implies a better understanding of the renormalization procedure.[6] Taking such a direct link for granted, we can estimate from Eqs. (39) the tachyonic gluon mass defined in Ref. [3] to be[7] ~ 0.8 GeV. Using instead the notion of critical mass $M^2_{\rm crit}$, introduced in Ref. [16], which is the scale at which the nonperturbative condensate contributes 10% of the total, we estimate it for the gluon propagator to be ~ 2.6 GeV. Both these scales express a rather large contribution from the A^2 condensate.

ACKNOWLEDGMENTS

We especially thank D. Becirevic for thorough discussions at earlier stages of the work. We are grateful to V. I. Zakharov for his illuminating remarks. We are also indebted to Y. Dokshitzer and G. Korchemsky for several inspiring comments and to F. Di Renzo for clarifying the present status of their work concerning power corrections in Wilson loops. J.R-Q. thanks J. Rodríguez García and is indebted to Spanish Fundación Ramón Areces for financial support. These calculations were performed on the QUADRICS QH1 located in the Center de Ressources Informatiques (Paris-sud, Orsay) and purchased thanks to a funding from the Ministère de l'Education Nationale and the CNRS. Laboratoire de Physique Théorique is Unté Mixte de Recherche du CNRS-OMR 8627.

APPENDIX: ONE-LOOP ANOMALOUS DIMENSION OF A^2

The task of computing the one-loop anomalous dimension of the matrix element $\langle A^2\rangle$ (that of the local operator itself is directly obtained from the former as explained in Sec. II) requires only to isolate the UV divergent part of the diagrams in Fig. 4. We follow dimensional regularization prescriptions to write

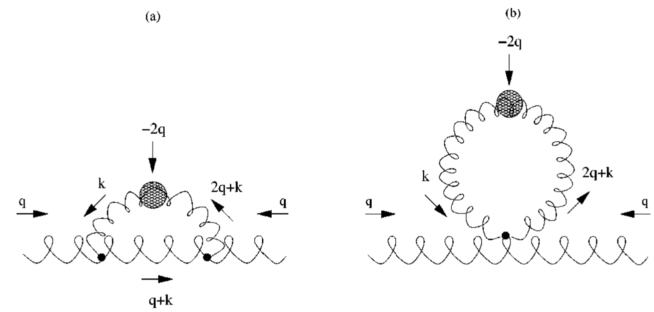

FIG. 4. The graphs involved in the computation of the anomalous dimension of A^2. The cross hatched blobs indicate the insertion of the A^2 operator; the dots are ordinary QCD vertices.

$$[\Gamma_a]^{\mu\nu}_{ab}(q,-q) = \left(g^{\mu\nu} - \frac{q^\mu q^\nu}{q^2}\right)\delta_{ab}\left(\frac{1}{\varepsilon}\left\{3N_c\frac{\alpha_b}{4\pi} + O(\alpha_b^2)\right\} + \cdots\right),$$

$$[\Gamma_b]^{\mu\nu}_{ab}(q,-q) = \left(g^{\mu\nu} - \frac{q^\mu q^\nu}{q^2}\right)\delta_{ab}\left(\frac{1}{\varepsilon}\left\{-\frac{9}{2}N_C\frac{\alpha_b}{4\pi} + O(\alpha_b^2)\right\} + \cdots\right), \tag{A1}$$

where an $a(b)$-like diagram in Fig. 2 with amputated external gluon legs is denoted by $\Gamma_{a(b)}$ and where $\epsilon \equiv 2 - d/2$. The particular kinematics we choose (see Fig. 4), where the incoming momentum flow is nonvanishing, eliminates automatically the IR divergences and makes the UV analysis easier. We require the final result for amputated diagrams to be transverse to the external momenta, but this is merely a convention to be also applied to the tree-level term. Furthermore, had we considered two different incoming momenta, the tensors in Eq. (A1) would have acquired a more complicated form.

This kind of IR regularization, by imposing a non-null incoming momentum flow to the local operator, leads of course to UV poles results equivalent to those obtained from any other one. We have tested this by considering a null incoming momentum flow and both introducing a certain cutoff to regularize IR divergences and separating IR and UV poles obtained by dimensional regularization.

If we collect the tree-level results and those from Eqs. (A1), we can write

$$\langle g^a_\mu | A^2 | g^b_\nu\rangle = 2\left(g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\right)\delta^{ab} \times \left(1 + \frac{1}{\varepsilon}\left\{\frac{3N_C}{4}\frac{\alpha_b}{4\pi} + O(\alpha_b^2)\right\} + \cdots\right), \tag{A2}$$

where the matrix element on the LHS of Eq. (A2) is defined for explicitly cut external gluons. Combinatorics gives a

[6] It is a pleasure to acknowledge discussions with V. I. Zakharov on this topic.

[7] We recall that all scale-dependent quantities are evaluated at $\mu = 10$ GeV.

multiplicity factor of 2 for *a*-like diagram, 1 for *b*-like, which have been taken into account in the last result.

We should now renormalize the matrix element defined in Eq. (A2). Our aim being to determine its anomalous dimension computation to only one-loop, the simple minimal subtraction sum (MS) prescription of simply dropping away from bare quantities the poles for $\varepsilon \to 0$ can be applied. Discrepancies between such a prescription and MOM or any other one appear only beyond one loop. Then we will have

$$\hat{Z}^{MS} = 1 + \frac{1}{\varepsilon}\left(\frac{3N_C}{4}\frac{\alpha_b}{4\pi} + O(\alpha_b^2)\right) = 1 + \frac{1}{\varepsilon}\left(\frac{3N_C}{4}\frac{\alpha^{MS}(\mu)}{4\pi} + O[(\alpha^{MS}(\mu))^2]\right). \quad (A3)$$

From Eq. (A3), the anomalous dimension can be written as (see, for instance, [12])

$$\hat{\gamma}^{\rm MS}(\alpha^{MS}(\mu)) = \frac{d}{d\ln\mu^2}\ln\hat{Z}^{\rm MS} = -\frac{3N_C}{4}\frac{\alpha^{MS}(\mu)}{4\pi} + O([\alpha^{MS}(\mu)]^2). \quad (A4)$$

Thus we obtain, in the MOM scheme,

$$\hat{\gamma}(\alpha(\mu)) = -\frac{3N_C}{4}\frac{\alpha(\mu)}{4\pi} + O([\alpha(\mu)]^2). \quad (A5)$$

From this we deduce finally, including the gluon anomalous dimension,

$$\gamma_{A^2}(\alpha(\mu)) = -\frac{35N_C}{12}\frac{\alpha(\mu)}{4\pi} + O([\alpha(\mu)]^2). \quad (A6)$$

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