Trilinear neutral gauge boson couplings in effective theories

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We list all the lowest dimension effective operators inducing off-shell trilinear neutral gauge boson couplings $ZZ\gamma$, $Z\gamma\gamma$, and ZZZ within the effective Lagrangian approach, both in the linear and nonlinear realizations of $\mathrm{SU}(2)_\mathrm{L}\times \mathrm{U}(1)_\mathrm{Y}$ gauge symmetry. In the linear scenario we find that these couplings can be generated only by dimension-8 operators necessarily including the Higgs boson field, whereas in the nonlinear case they are induced by dimension-6 operators. We consider the impact of these couplings on some precision measurements such as the magnetic and electric dipole moments of fermions, as well as the Z boson rare decay $Z\rightarrow \nu\bar{\nu}\gamma$. If the underlying new physics is of a decoupling nature, it is not expected that trilinear neutral gauge boson couplings may affect considerably any of these observables. On the contrary, it is just in the nonlinear scenario where these couplings have the more promising prospects of being perceptible through high precision experiments.

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I. INTRODUCTION

The present agreement between experimental data and the standard model (SM) suggests that the energy scale Λ associated with any new physics should be large compared with the electroweak scale $v=(\sqrt{2}G_F)^{1/2}=246$ GeV. To infer the existence of new particles as heavy as Λ through their virtual effects, effective Lagrangian (EL) techniques have been extensively used to study quantities which are forbidden or highly suppressed within the SM [1–3]. Among these quantities, self-couplings of electroweak gauge bosons constitute a sensitive probe of nonstandard interactions [4]. Experimental bounds on possible anomalous $W^+W^-Z(\gamma)$ couplings have reached an accuracy of the few percent level in both hadronic and leptonic colliders [5,6], but the situation looks less promising for anomalous ZZZ, $ZZ\gamma$, and $Z\gamma\gamma$ couplings [7].[1] Unlike $W^+W^-Z(\gamma)$ couplings, trilinear neutral gauge boson couplings (TNGBCs) vanish when the three bosons are real. Another interesting peculiarity of TNGBCs is that they must be induced by loop effects in any renormalizable theory since they cannot possess a renormalizable structure. In the SM, TNGBCs are generated at the one-loop level by fermion triangles [8], being very suppressed even in the presence of a fourth fermion family [9]. It follows that it is convenient to carry out a model independent study of TNGBCs using the EL method to parametrize any anomalous contribution. Within this approach, there are two well-motivated schemes to parametrize the virtual effects of physics beyond the Fermi scale via effective operators involving only SM fields, namely the linear and the nonlinear realizations.

In the linear realization or decoupling scenario it is assumed that the light spectrum of particles, which fill out multiplets of the electroweak $\mathrm{SU}(2)_\mathrm{L}\times \mathrm{U}(1)_\mathrm{Y}$ gauge group, includes at least the physical Higgs boson of the SM. Because of the decoupling theorem, virtual effects of heavy physics cannot affect low energy processes dramatically. Nonetheless, any new effect, in spite of its smallness, may have significant effects on the couplings which are absent or highly suppressed within the SM. Starting from the SM fields and assuming lepton and baryon number conservation, there is no way to construct any odd dimension operator respecting the linearly realized $\mathrm{SU}(2)_\mathrm{L}\times \mathrm{U}(1)_\mathrm{Y}$ symmetry. As for dimension 6, operators of this class were comprehensively studied in [10]. It was shown that there are 84 independent dimension-6 operators.

In the case of the nonlinear realization or nondecoupling scenario, the parametrization of new physics effects arises when it is assumed that the Higgs bosons are very heavy or do not exist at all. The scalar sector is comprised only by Goldstone bosons, which transform nonlinearly under the $\mathrm{SU}(2)_\mathrm{L}\times \mathrm{U}(1)_\mathrm{Y}$ group. It is also possible to introduce light scalar fields in this parametrization, but they cannot be recognized as Higgs bosons since such fields do not couple to the remaining light particles as dictated by the Higgs mechanics [11]. Since the low energy theory is nonrenormalizable under the Dyson prescription, heavy physics does not decouple from the low energy processes. We may think of this scenario as the one in which the EL parametrizes unknown physics which would not obey the Higgs mechanism. In this case, the most important operators are the ones which

[1] Throughout this work we consider the general case of off-shell bosons, unless stated otherwise, but they will be denoted by V rather than V^*.

induce the masses of the W and Z gauge bosons, prescribing also the general structure of the $W^+W^-Z(\gamma)$ couplings [12]. These operators have dimension 2 and 4.

At the lowest order, anomalous $W^+W^-Z(\gamma)$ couplings are induced by dimension-6 operators in the decoupling scenario and by dimension-4 operators in the nonlinear scheme. In contrast, TNGBCs are induced by dimension-8 operators in the linear realization and by dimension-6 operators in the nonlinear one. In the latter case there are also some dimension-4 operators which give rise to the ZZZ coupling, but they are proportional to the scalar part of the Z boson $(\partial_\mu Z^\mu)$. It can be shown that such operators may be eliminated by means of a transformation which leaves invariant the S matrix [13]. Consequently, any anomalous contribution to TNGBCs is expected to be more suppressed than those inducing nonstandard $W^+W^-Z(\gamma)$ couplings. It must be stressed, however, that any potential effect must be carefully examined as it may constitute clear evidence of new physics.

The structure of TNGBCs has already been studied in the context of effective theories, initially at the level of vertex functions [14]. However, in this approach the case was considered where two particles are real and just one is virtual. It is only recently that analysis of the off-sell vertices has been done under the $\mathrm{U}(1)_{\mathrm{em}}$ gauge invariant framework, including the study of the respective EL. By invoking Bose symmetry, Lorentz covariance, and electromagnetic gauge invariance, the most general structures inducing TNGBCs with three off-shell neutral bosons were constructed [15]. As was shown in [16], the $\mathrm{U}(1)_{\mathrm{em}}$ gauge invariant framework is equivalent to the nonlinearly realized $\mathrm{SU}(2)_{\mathrm{L}}\times \mathrm{U}(1)_{\mathrm{Y}}$ invariant case. Such an equivalence is explicit in the unitary gauge. The choice of using either framework is only a matter of convenience. In particular, the nonlinear scheme is suitable for loop calculations, as the presence of Goldstone bosons allows one to quantize the theory with the aid of a renormalizable R_ξ gauge.

It is clear that a comprehensive study of TNGBCs must include both linear and nonlinear schemes. To our knowledge the former has never been studied before. One of the aims of the present paper is to present a complete list of the effective operators which induce TNGBCs at the lowest order in both realizations of the $\mathrm{SU}(2)_{\mathrm{L}}\times \mathrm{U}(1)_{\mathrm{Y}}$ gauge symmetry. Not all the operators that can be constructed respecting the Lorentz and electroweak symmetries are independent since a certain class of general transformations allows one to rule out some of them without affecting the *S*-matrix elements [17]. In the course of our classification we found operators with terms containing higher derivatives which resemble the covariant structure of the equations of motion; there were also operators with terms which are proportional to the scalar part of the Z boson $(\partial_\mu Z^\mu)$. It has been shown in [13,18] that both types of structures can be eliminated in favor of other operators already present in the effective Lagrangian. Such a procedure is only valid at first order in the unknown effective parameters of the theory as any effective Lagrangian is assumed to describe the effects of well-behaved new physics just in this approximation. Consequently, after performing the required transformation, the equations of motions can be used to eliminate any redundant structure, expressing the respective operator in terms of other ones. This whole procedure does not affect the *S*-matrix elements. In order to present all the independent operators, we will classify them according to the following criterion: those which cannot be reduced by using the equations of motion will be referred to as irreducible; the remaining ones will be referred to as reducible.

After classifying the operators, our paper will be concerned with the sensitivity of some precision experiments to new physics effects arising from TNGBCs. Although persuasive theoretical arguments indicate that trilinear gauge boson couplings are not expected to be larger than 1% [19,20], the Large Hadron Collider (LHC) and the planned Next Linear Collider (NLC) are expected to constrain them at a level of 10^{-4}–10^{-6} [4,21]. As long as TNGBCs are concerned, the size of their effects will be suppressed by powers of $(v/\Lambda)^4$ and $(v/\Lambda)^2$ in the linear and the nonlinear scenarios, respectively. We will examine whether some high precision measurements may lead to any reasonable bound on these couplings. The anomalous $W^+W^-\gamma(Z)$ couplings have been constrained from a global analysis of the LEP and SLC observables at the Z pole [2]. To draw any inference about the size of TNGBCs we will consider the muon $g-2$ value, the known limit on the electric dipole moment (EDM) of the electron, and the current limit on the rare decay $Z\to \nu\bar{\nu}\gamma$.

Our paper is organized as follows. All the lowest dimension operators that generate TNGBCs in the linear scheme are presented in Sec. II, following the already explained classification criterion. The respective Lagrangians are shown explicitly. In Sec. III, a similar analysis within the nonlinear scenario is presented. Section. IV is devoted to examining the constraints on the couplings out of high precision experiments. Finally, the conclusions are presented in Sec. V.

II. DECOUPLING SCENARIO

This section focuses on the itemization of all the lowest dimension operators that generate at least one of the couplings ZZZ, $ZZ\gamma$, or $Z\gamma\gamma$ within the linear realization of the $\mathrm{SU}(2)_{\mathrm{L}}\times \mathrm{U}(1)_{\mathrm{Y}}$ electroweak group. To construct a basis of independent operators with a given dimension, we must consider some aspects concerning the independence of the S matrix under a wide class of transformations which leave it invariant [17]. For instance, it was shown in [18] that some operators, which consist of a piece containing higher derivatives, can be eliminated in favor of others by using a specific transformation, leaving unchanged the *S*-matrix elements at any order of perturbation theory. Another situation arises when an operator is proportional to the scalar part of the Z boson. While the latter kind of structures give vanishing contributions when the Z boson is on mass shell or is virtual but couples to light fermions, the situation is not the same in the case of the top quark. In this respect, this kind of operator can also be eliminated by transformation which does not alter the *S*-matrix elements [13]. It must be noted that both transformations are equivalent to applying the equations of motion. Beside these considerations, we have made a systematic use of integration by parts to rule out any operator related to others through a surface term. Consequently, we will catalog the operators inducing TNGBCs as reducible or irreducible.

Any $\mathrm{SU(2)_L \times U(1)_Y}$ invariant involving only bosonic fields can be constructed out of the covariant structures $B_{\mu\nu}$, $\mathbf{W}_{\mu\nu}=\frac{1}{2}\sigma^i W^i_{\mu\nu}$, Φ, and $D_\mu\Phi$, where the covariant derivative is defined as $D_\mu=\partial_\mu - ig\mathbf{W}_\mu - ig' B_\mu$, and Φ is the Higgs doublet. Using these basic structures, we can build the following $\mathrm{SU(2)_L \times U(1)_Y}$ invariant and Lorentz covariant structures of dimension 2–5:

$$B_{\mu\nu},\ \Phi^\dagger\Phi,\ \Phi^\dagger D_\mu\Phi,\ \Phi^\dagger\mathbf{W}_{\mu\nu}\Phi,\ B_{\mu\nu}B^{\lambda\rho},\ \mathrm{Tr}[\mathbf{W}_{\mu\nu}\mathbf{W}^{\lambda\rho}],$$
$$\Phi^\dagger(D_\mu D_\nu + D_\nu D_\mu)\Phi,\ \Phi^\dagger\mathbf{W}_{\mu\nu}D_\lambda\Phi. \tag{1}$$

Note that another set of $\mathrm{SU(2)_L \times U(1)_Y}$ invariant and Lorentz covariant structures can be generated by operating with the ordinary derivative on these expressions. Any nonrenormalizable bosonic operator can be built by choosing the appropriate combinations of these structures to form Lorentz scalars. The ordinary derivative can act on the last expressions in several ways, but the contractions $\partial^\mu B_{\mu\nu}$ and $\partial^\mu(\Phi^\dagger D_\mu\Phi)$, being proportional to the scalar part of the Z boson, are special because in both cases we can use the equations of motion to eliminate the resulting operator.

Let us now discuss the general Lorentz structure of TNGBCs. The lowest dimension operators which can be assembled out of the basic structures have dimension 6 [10]. It is easy to see that no dimension-6 operator induces TNGBCs, which unavoidably leads one to consider dimension-8 operators. In principle, the combination which can give rise to TNGBCs may involve the 4-vectors A_μ and Z_μ, together with the antisymmetric tensors $F_{\mu\nu}=\partial_\mu A_\nu - \partial_\nu A_\mu$ and $Z_{\mu\nu}=\partial_\mu Z_\nu - \partial_\nu Z_\mu$. Owing to $\mathrm{U(1)_{em}}$ gauge symmetry, the electromagnetic field can only appear as A_μ through the respective covariant derivative, which operates on charged fields only. Therefore, the photon must appear in any term through the tensor field $F_{\mu\nu}$. As a result of the antisymmetry of the $F_{\mu\nu}$ and $Z_{\mu\nu}$ tensors, it is not possible to generate TNGBCs using only these structures: it would be necessary to have at our disposal three antisymmetric tensors. There follows the absence of the $\gamma\gamma\gamma$ vertex in this gauge invariant scheme.

To construct the ZZZ, $ZZ\gamma$, and $Z\gamma\gamma$ vertices, we must use at least a Z boson in the Z_μ form, which is allowed because this field couples to neutral fields. The 4-vector Z_μ is contained in the covariant derivative, which in the bosonic sector operates only on the Higgs doublet. As a consequence, the Higgs mechanism plays a special role in this type of couplings. In particular, the Higgs presence increases the dimension at which the operators can be generated in comparison to the nonlinear case, where this field is absent. The Z boson may appear through the combinations $Z_{\lambda\rho}Z_{\mu\nu}$, $Z_\lambda Z_{\mu\nu}$, $Z_\mu Z_\nu$, and Z_μ. The building blocks necessary to construct these couplings are $\Phi^\dagger D_\mu\Phi$, $\Phi^\dagger(D_\mu D_\nu + D_\nu D_\mu)\Phi$, and $\Phi^\dagger\mathbf{W}_{\mu\nu}D_\lambda\Phi$, which, after spontaneous symmetry breaking (SSB), induce the structures Z_μ, $Z_\mu Z_\nu$, and $Z_{\mu\nu}(F_{\mu\nu})Z_\lambda$, respectively. The irreducible operators may contribute to any process through the specific structure of TNGBCs, while the reducible ones may contribute to it via contact diagrams in which an internal line associated with either a Z boson or a photon has been amputated, for instance when the equations of motion are used to replace the term $\partial_\mu B^{\mu\nu}$ with the respective current. Therefore, the irreducible operators deserve a more careful study than the reducible ones. We will present thus the Lagrangians and vertex functions in the irreducible case, whereas in the reducible one we will list only the respective operators and the Lagrangian prescribing the off-shell electromagnetic properties of the Z boson. In the next section we will enumerate the operators of dimension 8 that generate TNGBCs.

A. Irreducible operators

We begin by classifying the operators which cannot be eliminated using the equations of motion. We will categorize them according to CP symmetry.

1. CP-odd operators

The operators we are interested in have the form $\mathcal{O}_i\partial^\rho\mathcal{O}_j$, where $\mathcal{O}_i$ is any of the $\mathrm{SU(2)_L \times U(1)_Y}$ invariant expressions shown in Eq. (1). Given these operators it is immediate to construct the new ones $(\partial^\rho\mathcal{O}_i)\mathcal{O}_j$, which also belong to the irreducible group, but they are not independent at all since they are related to the original operators through a surface term. Bearing this in mind, we obtain the following four independent CP-odd operators of dimension 8:

$$\mathcal{O}_{WW1}=i\,2\partial^\lambda(\Phi^\dagger D_\mu\Phi)\mathrm{Tr}[\mathbf{W}^{\mu\nu}\mathbf{W}_{\lambda\nu}]+\mathrm{H.c.}, \tag{2}$$

$$\mathcal{O}_{WB1}=i\,(\Phi^\dagger\mathbf{W}_{\mu\nu}D_\lambda\Phi)\partial^\lambda B^{\mu\nu}+\mathrm{H.c.}, \tag{3}$$

$$\mathcal{O}_{WB2}=i(\Phi^\dagger\mathbf{W}_{\mu\nu}D_\lambda\Phi)\partial^\mu B^{\lambda\nu}+\mathrm{H.c.}, \tag{4}$$

$$\mathcal{O}_{BB1}=i\,(\Phi^\dagger D_\mu\Phi)B_{\lambda\nu}\partial^\lambda B^{\mu\nu}+\mathrm{H.c.} \tag{5}$$

Notice that the operator $\mathcal{O}_{BB1}$ contains three $\mathrm{SU(2)_L \times U(1)_Y}$ invariant structures which can be contracted with the ordinary derivative in three different ways, leading to the same number of operators. One of them, namely $i\partial^\lambda(\Phi^\dagger D_\mu\Phi)B_{\lambda\nu}B^{\mu\nu}$, is irreducible, but can be expressed by means of integration by parts in terms of $\mathcal{O}_{BB1}$ and the reducible operator $i(\Phi^\dagger D_\mu\Phi)(\partial^\lambda B_{\lambda\nu})B^{\mu\nu}$, which will be considered later.

2. CP-odd structure of the ZZZ, ZZγ, and Zγγ couplings

After decomposing the operators in terms of the mass eigenstate fields, we are left with several Lorentz structures corresponding to TNGBCs, though not all of them are independent: some structures are identical, which is manifest after a subtle manipulation of their Lorentz indices, whereas other ones are related through a surface term. Consequently, the ZZZ, $ZZ\gamma$, and $Z\gamma\gamma$ couplings can be described by the following independent Lorentz structures:

$$\mathcal{L}^{CP\text{-}odd}_{L\text{-}ZZZ}=f^{ZZZ}_{L1}Z_\lambda Z_{\mu\nu}\partial^\lambda Z^{\mu\nu}+f^{ZZZ}_{L2}Z_{\mu\nu}Z^{\lambda\nu}\partial_\lambda Z^\mu, \tag{6}$$

$$\mathcal{L}^{CP\text{-}odd}_{L\text{-}ZZ\gamma}=f^{ZZ\gamma}_{L1}Z^{\mu\nu}F_{\lambda\nu}\partial^\lambda Z_\mu+f^{ZZ\gamma}_{L2}Z_\lambda Z_{\mu\nu}\partial^\lambda F^{\mu\nu}+f^{ZZ\gamma}_{L3}Z_\lambda F_{\mu\nu}\partial^\lambda Z^{\mu\nu}, \tag{7}$$

$$\mathcal{L}^{CP\text{-}odd}_{L\text{-}Z\gamma\gamma}=f^{Z\gamma\gamma}_{L1}F^{\mu\nu}F_{\lambda\nu}\partial^{\lambda}Z_{\mu}+f^{Z\gamma\gamma}_{L2}Z_{\lambda}F_{\mu\nu}\partial^{\lambda}F^{\mu\nu}, \quad (8)$$

where L is a subscript standing for the linear scheme. The coefficients f^{VVV}_{Li} in turn depend on the other ones $\epsilon_j=(m_Z/\Lambda)^4\alpha_j$, with α_j the constant factor accompanying each effective operator in Eqs. (2)–(5). We thus have

$$f^{VVV}_{Li}=f(\epsilon_{WW1},\epsilon_{WB1},\ldots). \quad (9)$$

Detailed expressions of each factor f^{VVV}_{Li} as well as the respective vertex functions are available to the interested reader in [22].

3. *CP*-even operators

Operators of this kind can be obtained from the *CP*-odd ones by replacing each strength tensor with its respective dual, namely $\tilde{\mathbf{W}}_{\mu\nu}=(1/2)\epsilon_{\mu\nu\lambda\rho}\mathbf{W}^{\lambda\rho}$, and a similar expression for $\tilde{B}_{\mu\nu}$. There is a couple of independent *CP*-even operators associated with each one of the *CP*-odd operators $\mathcal{O}_{WW1}$, $\mathcal{O}_{WB2}$, and $\mathcal{O}_{BB1}$. Note that in these operators both $\mathbf{W}$ tensors are contracted via only one of their indices, leading to two independent combinations of the dual tensor. On the other hand, in $\mathcal{O}_{WB1}$ the $\mathbf{W}$ and B tensors appear contracted by both indices. Since the two possible combinations of dual tensors are equivalent, just one *CP*-even operator can be constructed from $\mathcal{O}_{WB1}$. In this way, there are seven independent *CP*-even operators:

$$\mathcal{O}_{\tilde{W}W1}=i2\,\partial^{\lambda}(\Phi^{\dagger}D_{\mu}\Phi)\mathrm{Tr}[\tilde{\mathbf{W}}^{\mu\nu}\mathbf{W}_{\lambda\nu}]+\mathrm{H.c.}, \quad (10)$$

$$\mathcal{O}_{W\widetilde{W1}}=i2\,\partial^{\lambda}(\Phi^{\dagger}D_{\mu}\Phi)\mathrm{Tr}[\mathbf{W}^{\mu\nu}\tilde{\mathbf{W}}_{\lambda\nu}]+\mathrm{H.c.}, \quad (11)$$

$$\mathcal{O}_{W\tilde{B}1}=i(\Phi^{\dagger}\mathbf{W}_{\mu\nu}D_{\lambda}\Phi)\partial^{\lambda}\tilde{B}^{\mu\nu}+\mathrm{H.c.}, \quad (12)$$

$$\mathcal{O}_{\tilde{W}B2}=i(\Phi^{\dagger}\tilde{\mathbf{W}}_{\mu\nu}D_{\lambda}\Phi)\partial^{\mu}B^{\lambda\nu}+\mathrm{H.c.}, \quad (13)$$

$$\mathcal{O}_{W\tilde{B}2}=i(\Phi^{\dagger}\mathbf{W}_{\mu\nu}D_{\lambda}\Phi)\partial^{\mu}\tilde{B}^{\lambda\nu}+\mathrm{H.c.}, \quad (14)$$

$$\mathcal{O}_{\tilde{B}B1}=i(\Phi^{\dagger}D_{\mu}\Phi)\tilde{B}_{\lambda\nu}\partial^{\lambda}B^{\mu\nu}+\mathrm{H.c.}, \quad (15)$$

$$\mathcal{O}_{B\tilde{B}1}=i(\Phi^{\dagger}D_{\mu}\Phi)B_{\lambda\nu}\partial^{\lambda}\tilde{B}^{\mu\nu}+\mathrm{H.c.} \quad (16)$$

We can make the ordinary derivative operate on the remaining $\mathrm{SU(2)_L}\times\mathrm{U(1)_Y}$ invariant terms out of which the previous operators are constructed. The resulting operators are also of the irreducible kind, but they are not independent since, as explained in the *CP*-odd case, all of them are related to the first ones through a surface term.

4. *CP*-even structure of the ZZZ, $ZZ\gamma$, and $Z\gamma\gamma$ couplings

After a careful analysis of the Lorentz structure induced by the *CP*-even operators, we find that the ZZZ, $ZZ\gamma$, and $Z\gamma\gamma$ couplings are characterized, respectively, by two, five, and three independent Lorentz structures

$$\mathcal{L}^{CP\text{-}even}_{L\text{-}ZZZ}=g^{ZZZ}_{L1}Z_{\lambda}Z_{\mu\nu}\partial^{\lambda}\tilde{Z}^{\mu\nu}+g^{ZZZ}_{L2}Z_{\lambda}Z_{\mu\nu}\partial^{\mu}\tilde{Z}^{\lambda\nu}, \quad (17)$$

$$\mathcal{L}^{CP\text{-}even}_{L\text{-}ZZ\gamma}=g^{ZZ\gamma}_{L1}\tilde{F}_{\mu\nu}Z^{\lambda\nu}\partial_{\lambda}Z^{\mu}+g^{ZZ\gamma}_{L2}\tilde{Z}_{\mu\nu}F^{\lambda\nu}\partial_{\lambda}Z^{\mu}+g^{ZZ\gamma}_{L3}\tilde{Z}_{\lambda\nu}F^{\mu\nu}\partial^{\lambda}Z_{\mu}+g^{ZZ\gamma}_{L4}Z^{\lambda}F^{\mu\nu}\partial_{\mu}\tilde{Z}_{\lambda\nu}+g^{ZZ\gamma}_{L5}Z^{\lambda}Z^{\mu\nu}\partial_{\mu}\tilde{F}_{\lambda\nu}, \quad (18)$$

$$\mathcal{L}^{CP\text{-}even}_{L\text{-}Z\gamma\gamma}=g^{Z\gamma\gamma}_{L1}\partial_{\lambda}Z^{\mu}\tilde{F}_{\mu\nu}F^{\lambda\nu}+g^{Z\gamma\gamma}_{L2}\partial_{\lambda}Z^{\mu}\tilde{F}^{\lambda\nu}F_{\mu\nu}+g^{Z\gamma\gamma}_{L3}Z_{\lambda}F_{\mu\nu}\partial^{\mu}\tilde{F}^{\lambda\nu}. \quad (19)$$

The coefficients g^{VVV}_{Li} are related to the coefficients associated with the *CP*-odd operators (10)–(16) and obey a relation similar to Eq. (9), with the appropriate substitutions. Details regarding these coefficients as well as the respective vertex functions can be found in [22].

B. Reducible operators

The operators belonging to the reducible class are proportional to the $\mathrm{SU(2)_L}\times\mathrm{U(1)_Y}$ invariants $\partial^{\mu}(\Phi^{\dagger}D_{\mu}\Phi)$ and $\partial^{\mu}B_{\mu\nu}$. While the operators with the term $\partial^{\mu}(\Phi^{\dagger}D_{\mu}\Phi)$ are proportional to the scalar part of the Z boson, those proportional to the $\partial^{\mu}B_{\mu\nu}$ have the peculiarity that they generate the Lorentz structures required to define the off-shell electromagnetic properties of the Z boson, namely the transition magnetic (electric) dipole and quadrupole moments. All of these operators can be reduced to others by using the equations of motion. To define these structures, it will be necessary to include some operators of dimension 10, but as they can always be expressed in terms of other operators we will content ourselves with listing them. We will also present the Lagrangian prescribing the off-shell electromagnetic properties of the Z boson. The operators will be classified according to these properties.

1. Operators that generate the off-shell electromagnetic properties of the *Z* boson

These operators are proportional to the $\mathrm{SU(2)_L}\times\mathrm{U(1)_Y}$ invariant $\partial^{\mu}B_{\mu\nu}$, being given by

$$\mathcal{O}_{WB3}=i(\Phi^{\dagger}\mathbf{W}^{\mu\nu}D_{\mu}\Phi)\partial^{\lambda}B_{\lambda\nu}+\mathrm{H.c.}, \quad (20)$$

$$\mathcal{O}_{BB3}=i(\Phi^{\dagger}D_{\mu}\Phi)B^{\mu\nu}\partial^{\lambda}B_{\lambda\nu}+\mathrm{H.c.}, \quad (21)$$

$$\mathcal{O}_{\tilde{W}B3}=i(\Phi^{\dagger}\tilde{\mathbf{W}}^{\mu\nu}D_{\mu}\Phi)\partial^{\lambda}B_{\lambda\nu}+\mathrm{H.c.}, \quad (22)$$

$$\mathcal{O}_{\tilde{B}B3}=i(\Phi^{\dagger}D_{\mu}\Phi)\tilde{B}^{\mu\nu}\partial^{\lambda}B_{\lambda\nu}+\mathrm{H.c.} \quad (23)$$

To define the off-shell electromagnetic properties of the Z boson, it is necessary to include the following operators of dimension 10:

$$\mathcal{O}^{10}_{WB}=i(\Phi^{\dagger}\mathbf{W}^{\mu\nu}D_{\lambda}\Phi)\partial_{\mu}\partial^{\lambda}\partial^{\rho}B_{\rho\nu}+\mathrm{H.c.}, \quad (24)$$

$$\mathcal{O}^{10}_{BB}=i(\Phi^{\dagger}D_{\lambda}\Phi)B^{\mu\nu}\partial_{\mu}\partial^{\lambda}\partial^{\rho}B_{\rho\nu}+\mathrm{H.c.}, \quad (25)$$

$$\mathcal{O}^{10}_{\tilde{W}B}=i(\Phi^{\dagger}\tilde{\mathbf{W}}^{\mu\nu}D_{\lambda}\Phi)\partial_{\mu}\partial^{\lambda}\partial^{\rho}B_{\rho\nu}+\mathrm{H.c.}, \quad (26)$$

$$\mathcal{O}^{10}_{\tilde{B}B}=i(\Phi^{\dagger}D_{\lambda}\Phi)\tilde{B}^{\mu\nu}\partial_{\mu}\partial^{\lambda}\partial^{\rho}B_{\rho\nu}+\mathrm{H.c.} \quad (27)$$

We have excluded any redundant operator, such as the ones related through a surface term. The operator $\mathcal{O}_{DB} = \Phi^\dagger (D_\mu D_\nu + D_\nu D_\mu)\Phi \partial^\mu \partial_\lambda B^{\lambda\nu}$, which does not contribute to the electromagnetic properties of the Z boson, can be eliminated by using the equations of motion. The Lorentz structures defining the off-shell electromagnetic properties of the Z boson can be conveniently parametrized by the following Lagrangian

$$\mathcal{L}_{ZZ\gamma} = -e\left[(h_1^Z F^{\mu\nu} + h_3^Z \tilde{F}^{\mu\nu}) Z_\mu \frac{\partial^\lambda Z_{\lambda\nu}}{m_Z^2} + (h_2^Z F^{\mu\nu} + h_4^Z \tilde{F}^{\mu\nu}) Z^\lambda \frac{\partial_\mu \partial_\lambda \partial^\rho Z_{\rho\nu}}{m_Z^4} \right], \tag{28}$$

where the transition moments are given by

$$\mu_Z = -\frac{e}{\sqrt{2} m_Z} \frac{E_\gamma^2}{m_Z^2} (h_1^Z - h_2^Z), \tag{29a}$$

$$Q_Z^e = -\frac{2\sqrt{10} e}{m_Z^2} h_1^Z, \tag{29b}$$

$$d_Z = -\frac{e}{\sqrt{2} m_Z} \frac{E_\gamma^2}{m_Z^2} (h_3^Z - h_4^Z), \tag{29c}$$

$$Q_Z^m = -\frac{2\sqrt{10} e}{m_Z^2} h_3^Z, \tag{29d}$$

with μ_Z (d_Z) the off-shell magnetic (electric) dipole moment and $Q_Z^m (Q_Z^e)$ the magnetic (electric) quadrupole moment of the Z boson. The coefficients h_i^Z are defined in [22].

2. Operators proportional to the scalar part of the Z boson

These operators are characterized by the $SU(2)_L \times U(1)_Y$ invariant $\partial_\mu(\Phi^\dagger D^\mu \Phi)$. There are three CP-odd operators of this type:

$$\mathcal{O}_{WW2} = i2\partial_\lambda(\Phi^\dagger D^\lambda \Phi) \mathrm{Tr}[\mathbf{W}_{\mu\nu} \mathbf{W}^{\mu\nu}] + \mathrm{H.c.}, \tag{30}$$

$$\mathcal{O}_{BB2} = i\partial_\lambda(\Phi^\dagger D^\lambda \Phi) B_{\mu\nu} B^{\mu\nu} + \mathrm{H.c.}, \tag{31}$$

$$\mathcal{O}_{D\Phi} = i\Phi^\dagger (D_\mu D_\nu + D_\nu D_\mu)\Phi \partial^\mu (\Phi^\dagger D^\nu \Phi) + \mathrm{H.c.} \tag{32}$$

The last operator generates only the ZZZ coupling, which can be expressed by integration by parts as a coupling proportional to the scalar part of the Z boson. As for CP-even operators, there are only a pair of this kind:

$$\mathcal{O}_{W\tilde{W}2} = i2\partial_\lambda(\Phi^\dagger D^\lambda \Phi) \mathrm{Tr}[\mathbf{W}_{\mu\nu} \tilde{\mathbf{W}}^{\mu\nu}] + \mathrm{H.c.}, \tag{33}$$

$$\mathcal{O}_{B\tilde{B}2} = i\partial_\lambda(\Phi^\dagger D^\lambda \Phi) B_{\mu\nu} \tilde{B}^{\mu\nu} + \mathrm{H.c.} \tag{34}$$

We disregarded any operator which can be expressed as a linear combination of those given above.

III. NONDECOUPLING SCENARIO

In the situation where the new physics effects do not decouple from low energy physics, the relevant $SU(2)_L \times U(1)_Y$ invariant structures are the same as in the linear case, with the Higgs doublet being replaced by the following unitary matrix:

$$U = \exp\left[\frac{2i\sigma^i \phi^i}{v}\right], \tag{35}$$

where the ϕ^i scalars would become Goldstone bosons. The covariant derivative in the nonlinear realization of the $SU(2)_L \times U(1)_Y$ group is defined as $\mathbf{D}_\mu U = \partial_\mu U + ig\mathbf{W}_\mu U - ig' U \mathbf{B}_\mu$, with the Abelian field defined as $\mathbf{B}_\mu = (\sigma^3/2) B_\mu$. The basic structures out of which TNGBCs can be constructed are the $SU(2)_L \times U(1)_Y$ invariants $\mathrm{Tr}[\sigma^3 U^\dagger \mathbf{D}_\mu U]$, $\mathrm{Tr}[U^\dagger (\mathbf{D}_\mu \mathbf{D}_\nu + \mathbf{D}_\nu \mathbf{D}_\mu) U]$, and $\mathrm{Tr}[U^\dagger \mathbf{W}_{\mu\nu} \mathbf{D}_\lambda U]$, which in mass units have dimension 1, 2, and 3. Like their linear counterparts, these invariants are essential to construct any TNGBCs because they induce the Lorentz structures Z_μ, $Z_\mu Z_\nu$, and $Z_\lambda Z_{\mu\nu}(F_{\mu\nu})$. Since these structures have a lower dimension than their analogous structures in the linear case, it is not only possible to construct dimension-6 operators inducing TNGBCs but a larger number of independent operators. As we will show below, there are some operators of dimension 4 which induce the ZZZ coupling, though not the $ZZ\gamma$ and $Z\gamma\gamma$ ones. Nevertheless, such operators are proportional to the scalar part of the Z boson and belong to the reducible group. We will use the same criterion as in the linear case to classify all of the independent operators. We will refrain from any technical details already explained while discussing the linear scenario if it is not relevant for the present discussion.

A. Irreducible operators

These operators are proportional to the $SU(2)_L \times U(1)_Y$ invariant structures $\mathrm{Tr}[\sigma^3 U^\dagger \mathbf{D}_\mu U]$ and $\mathrm{Tr}[U^\dagger \mathbf{W}_{\mu\nu} \mathbf{D}_\lambda U]$. We will classify them according to CP symmetry.

1. CP-odd operators

The dimension-6 operators resembling those of the linear scenario are the following:

$$\mathcal{L}_{WW1} = 2i\frac{\lambda_{WW1}}{\Lambda^2} \partial^\lambda \mathrm{Tr}[\sigma^3 U^\dagger \mathbf{D}_\mu U] \mathrm{Tr}[\mathbf{W}^{\mu\nu} \mathbf{W}_{\lambda\nu}] + \mathrm{H.c.}, \tag{36}$$

$$\mathcal{L}_{WB1} = i\frac{\lambda_{WB1}}{\Lambda^2} \mathrm{Tr}[U^\dagger \mathbf{W}_{\mu\nu} \mathbf{D}_\lambda U] \partial^\lambda B^{\mu\nu} + \mathrm{H.c.}, \tag{37}$$

$$\mathcal{L}_{WB2} = i\frac{\lambda_{WB2}}{\Lambda^2} \mathrm{Tr}[U^\dagger \mathbf{W}_{\mu\nu} \mathbf{D}_\lambda U] \partial^\mu B^{\lambda\nu} + \mathrm{H.c.}, \tag{38}$$

$$\mathcal{L}_{BB1} = i\frac{\lambda_{BB1}}{\Lambda^2} \mathrm{Tr}[\sigma^3 U^\dagger \mathbf{D}_\mu U] B_{\lambda\nu} \partial^\lambda B^{\mu\nu} + \mathrm{H.c.}, \tag{39}$$

where we are using the symbol Λ, introduced in the linear case, to denote the new physics scale. As the structure $\mathrm{Tr}[\sigma^3 U^\dagger \mathbf{D}_\mu U]$ has dimension 1, we can construct three new independent operators of dimension 6 which have no dimension-8 counterpart in the linear realization. They are given by

$$\mathcal{L}_{DD} = i\frac{\lambda_{DD}}{\Lambda^2}\mathrm{Tr}[\sigma^3 U^\dagger \mathbf{D}_\mu U]\Box \mathrm{Tr}[\sigma^3 U^\dagger \mathbf{D}_\nu U]\partial^\mu \times \mathrm{Tr}[\sigma^3 U^\dagger \mathbf{D}_\mu U] + \mathrm{H.c.}, \tag{40}$$

$$\mathcal{L}_{DB1} = \frac{\lambda_{DB1}}{\Lambda^2}\mathrm{Tr}[\sigma^3 U^\dagger \mathbf{D}_\mu U]\partial_\lambda \mathrm{Tr}[\sigma^3 U^\dagger \mathbf{D}_\nu U]\partial^\lambda B^{\mu\nu} + \mathrm{H.c.}, \tag{41}$$

$$\mathcal{L}_{DB2} = \frac{\lambda_{DB2}}{\Lambda^2}\mathrm{Tr}[\sigma^3 U^\dagger \mathbf{D}_\mu U]\partial_\nu \mathrm{Tr}[\sigma^3 U^\dagger \mathbf{D}_\lambda U]\partial^\lambda B^{\mu\nu} + \mathrm{H.c.} \tag{42}$$

Note that in the linear scheme the operators corresponding to $\mathcal{L}_{DD}$ have dimension 12, whereas those related to $\mathcal{L}_{DB1}$ and $\mathcal{L}_{DB2}$ are of dimension 10. These operators have the peculiarity that they induce TNGBCs exclusively; i.e., there are no interactions containing a charged W boson, which can be seen by noting that the structure $\mathrm{Tr}[\sigma^3 U^\dagger \mathbf{D}_\mu U]$ is proportional to the Z_μ boson in the unitary gauge. While the first one of these operators induces only the ZZZ coupling, the remaining ones generate both the ZZZ and $ZZ\gamma$ couplings. There is no $Z\gamma\gamma$ coupling arising from these kind of operators, which implies that the Lorentz structure of it is the same in both the linear and the nonlinear realizations of the electroweak group, at least at the lowest order.

2. *CP*-odd structure of the ZZZ, $ZZ\gamma$, and $Z\gamma\gamma$ couplings.

After decomposing the nonlinear CP-odd operators in terms of the physical fields, we have found that the ZZZ coupling can be described by five independent Lorentz structures, and so is the $ZZ\gamma$ vertex. On the other hand, the $Z\gamma\gamma$ coupling becomes changed, as compared to its counterpart in the linear case, in its coefficients but not in its Lorentz structure. We thus have

$$\mathcal{L}^{CP\text{-}odd}_{NL\text{-}ZZZ} = \mathcal{L}^{CP\text{-}odd}_{L\text{-}ZZZ} + f^{ZZZ}_{NL3} Z_\mu \Box Z_\nu \partial^\mu Z^\nu + f^{ZZZ}_{NL4} Z_\mu \partial_\lambda Z_\nu \partial^\lambda Z^{\mu\nu} + f^{ZZZ}_{NL5} Z_\mu \partial_\nu Z_\lambda \partial^\lambda Z^{\mu\nu}, \tag{43}$$

$$\mathcal{L}^{CP\text{-}odd}_{NL\text{-}ZZ\gamma} = \mathcal{L}^{CP\text{-}odd}_{L\text{-}ZZ\gamma} + f^{ZZ\gamma}_{NL4} Z_\mu \partial_\lambda Z_\nu \partial^\lambda F^{\mu\nu} + f^{ZZ\gamma}_{NL5} Z_\mu \partial_\nu Z_\lambda \partial^\lambda F^{\mu\nu}, \tag{44}$$

$$\mathcal{L}^{CP\text{-}odd}_{NL\text{-}Z\gamma\gamma} = \mathcal{L}^{CP\text{-}odd}_{L\text{-}Z\gamma\gamma}, \tag{45}$$

with the respective coefficients obtained from those of the linear scenario through the relation

$$f^{VVV}_{NLi} = \left(\frac{\Lambda}{m_Z}\right)^2 f^{VVV}_{Li}, \tag{46}$$

whereas the remaining ones together with the respective vertex functions can be found in [22]. Unless stated otherwise, we will denote by λ_i rather than α_i the coefficient associated with each operator in the nonlinear scenario.

3. *CP*-even operators

There are eight operators of this kind. Seven of them can be easily obtained from their linear counterparts whereas a new one is obtained from the CP-odd operator $\mathcal{L}_{DB1}$ when the tensor $B_{\mu\nu}$ is replaced by its dual. The CP-odd operator which is equivalent to $\mathcal{L}_{DB2}$ is not independent as it generates TNGBCs with a Lorentz structure already induced by the operators resembling those of the linear case. In this way, the independent CP-even operators are

$$\tilde{\mathcal{L}}_{\tilde{W}W1} = 2i\frac{\lambda_{\tilde{W}W1}}{\Lambda^2}\partial^\lambda \mathrm{Tr}[\sigma^3 U^\dagger \mathbf{D}_\mu U]\mathrm{Tr}[\tilde{\mathbf{W}}^{\mu\nu}\mathbf{W}_{\lambda\nu}] + \mathrm{H.c.}, \tag{47}$$

$$\tilde{\mathcal{L}}_{W\tilde{W}1} = 2i\frac{\lambda_{W\tilde{W}1}}{\Lambda^2}\partial^\lambda \mathrm{Tr}[\sigma^3 U^\dagger \mathbf{D}_\mu U]\mathrm{Tr}[\mathbf{W}^{\mu\nu}\tilde{\mathbf{W}}_{\lambda\nu}] + \mathrm{H.c.}, \tag{48}$$

$$\tilde{\mathcal{L}}_{W\tilde{B}1} = i\frac{\lambda_{W\tilde{B}1}}{\Lambda^2}\mathrm{Tr}[U^\dagger \mathbf{W}_{\mu\nu}\mathbf{D}_\lambda U]\partial^\lambda \tilde{B}^{\mu\nu} + \mathrm{H.c.}, \tag{49}$$

$$\tilde{\mathcal{L}}_{\tilde{W}B2} = i\frac{\lambda_{\tilde{W}B2}}{\Lambda^2}\mathrm{Tr}[U^\dagger \tilde{\mathbf{W}}_{\mu\nu}\mathbf{D}_\lambda U]\partial^\mu B^{\lambda\nu} + \mathrm{H.c.}, \tag{50}$$

$$\tilde{\mathcal{L}}_{W\tilde{B}2} = i\frac{\lambda_{W\tilde{B}2}}{\Lambda^2}\mathrm{Tr}[U^\dagger \mathbf{W}_{\mu\nu}\mathbf{D}_\lambda U]\partial^\mu \tilde{B}^{\lambda\nu} + \mathrm{H.c.}, \tag{51}$$

$$\tilde{\mathcal{L}}_{\tilde{B}B1} = i\frac{\lambda_{\tilde{B}B1}}{\Lambda^2}\mathrm{Tr}[\sigma^3 U^\dagger \mathbf{D}_\mu U]\tilde{B}_{\lambda\nu}\partial^\lambda B^{\mu\nu} + \mathrm{H.c.}, \tag{52}$$

$$\tilde{\mathcal{L}}_{B\tilde{B}1} = i\frac{\lambda_{B\tilde{B}1}}{\Lambda^2}\mathrm{Tr}[\sigma^3 U^\dagger \mathbf{D}_\mu U]B_{\lambda\nu}\partial^\lambda \tilde{B}^{\mu\nu} + \mathrm{H.c.}, \tag{53}$$

$$\tilde{\mathcal{L}}_{D\tilde{B}1} = i\frac{\lambda_{D\tilde{B}1}}{\Lambda^2}\mathrm{Tr}[\sigma^3 U^\dagger \mathbf{D}_\mu U]\partial_\lambda \mathrm{Tr}[\sigma^3 U^\dagger \mathbf{D}_\nu U]\partial^\lambda \tilde{B}^{\mu\nu} + \mathrm{H.c.} \tag{54}$$

4. *CP*-even structure of the ZZZ, $ZZ\gamma$, and $Z\gamma\gamma$ couplings

As far as their Lorentz structure is concerned, both the ZZZ and the $ZZ\gamma$ couplings differ from their analogues in the linear realization as they now receive a contribution arising from the operator $\tilde{\mathcal{L}}_{D\tilde{B}1}$, that is

$$\mathcal{L}^{CP\text{-}even}_{NL\text{-}ZZZ} = \mathcal{L}^{CP\text{-}even}_{L\text{-}ZZZ} + g^{ZZZ}_{NL3} Z_\mu \partial_\lambda Z_\nu \partial^\lambda \tilde{Z}^{\mu\nu}. \tag{55}$$

$$\mathcal{L}^{CP\text{-}even}_{NL\text{-}ZZ\gamma} = \mathcal{L}^{CP\text{-}even}_{L\text{-}ZZ\gamma} + g^{ZZ\gamma}_{NL6} Z_\mu \partial_\lambda Z_\nu \partial^\lambda \tilde{F}^{\mu\nu}. \quad (56)$$

On the other hand, the $Z\gamma\gamma$ vertex coincides with the one of its linear counterpart. As for the coefficients g^{VVV}_{NLi}, they are given in terms of the linear ones by means of a relation similar to Eq. (46). Once again, detailed expressions of these coefficients as well as the respective vertex functions are presented in [22].

B. Reducible operators

We can classify the reducible operators in those contributing to the off-shell electromagnetic properties of the Z boson and those which are proportional to the scalar part of the Z boson.

1. Operators that generate the off-shell electromagnetic properties of the Z boson

These operators are proportional to the $SU(2)_L \times U(1)_Y$ invariant $\partial_\mu B^{\mu\nu}$ and are obtained from their linear counterpart by replacing $\Phi^\dagger D_\mu \Phi$ with $\mathrm{Tr}[\sigma^3 U^\dagger \mathbf{D}_\mu U]$. This give rise to dimension-6 and dimension-8 operators. The ones of dimension 6 are given by

$$\mathcal{L}_{WB3} = i \frac{\lambda_{WB3}}{\Lambda^2} \mathrm{Tr}[U^\dagger \mathbf{W}^{\mu\nu} \mathbf{D}_\mu U] \partial^\lambda B_{\lambda\nu} + \mathrm{H.c.}, \quad (57)$$

$$\mathcal{L}_{BB3} = i \frac{\lambda_{BB3}}{\Lambda^2} \mathrm{Tr}[U^\dagger \mathbf{D}_\mu U] B^{\mu\nu} \partial^\lambda B_{\lambda\nu} + \mathrm{H.c.}, \quad (58)$$

$$\mathcal{L}_{\tilde{W}B3} = i \frac{\lambda_{\tilde{W}B3}}{\Lambda^2} \mathrm{Tr}[U^\dagger \tilde{\mathbf{W}}^{\mu\nu} \mathbf{D}_\mu U] \partial^\lambda B_{\lambda\nu} + \mathrm{H.c.}, \quad (59)$$

$$\mathcal{L}_{\tilde{B}B3} = i \frac{\lambda_{\tilde{B}B3}}{\Lambda^2} \mathrm{Tr}[\sigma^3 U^\dagger \mathbf{D}_\mu U] \tilde{B}^{\mu\nu} \partial^\lambda B_{\lambda\nu} + \mathrm{H.c.} \quad (60)$$

Just as in the linear realization, there is another CP-odd dimension-6 operator given by

$$\mathcal{L}_{DB} = \frac{\lambda_{DB}}{\Lambda^2} \mathrm{Tr}[U^\dagger (\mathbf{D}_\mu \mathbf{D}_\nu + \mathbf{D}_\nu \mathbf{D}_\mu) U] \partial^\mu \partial_\lambda B^{\lambda\nu}, \quad (61)$$

which, however, does not contribute to the electromagnetic properties of the Z boson. The operators of dimension 8, necessary to an adequate definition of the electric and magnetic transition dipole and quadrupole moments, are

$$\mathcal{L}^8_{WB} = i \frac{\lambda^8_{WB}}{\Lambda^4} \mathrm{Tr}[U^\dagger \mathbf{W}^{\mu\nu} \mathbf{D}_\lambda U] \partial_\mu \partial^\lambda \partial^\rho B_{\rho\nu} + \mathrm{H.c.}, \quad (62)$$

$$\mathcal{L}^8_{BB} = i \frac{\lambda^8_{BB}}{\Lambda^4} \mathrm{Tr}[\sigma^3 U^\dagger \mathbf{D}_\lambda U] B^{\mu\nu} \partial_\mu \partial^\lambda \partial^\rho B_{\rho\nu} + \mathrm{H.c.}, \quad (63)$$

$$\mathcal{L}^8_{\tilde{W}B} = i \frac{\lambda^8_{\tilde{W}B}}{\Lambda^4} \mathrm{Tr}[U^\dagger \tilde{\mathbf{W}}^{\mu\nu} \mathbf{D}_\lambda U] \partial_\mu \partial^\lambda \partial^\rho B_{\rho\nu} + \mathrm{H.c.}, \quad (64)$$

$$\mathcal{L}^8_{\tilde{B}B} = i \frac{\lambda^8_{\tilde{B}B}}{\Lambda^4} \mathrm{Tr}[\sigma^3 U^\dagger \mathbf{D}_\lambda U] \tilde{B}^{\mu\nu} \partial_\mu \partial^\lambda \partial^\rho B_{\rho\nu} + \mathrm{H.c.} \quad (65)$$

They induce the off-shell electromagnetic properties of the Z boson through the Lagrangian given in Sec. II. The coefficients $h^Z_{1,3}$ and $h^Z_{2,4}$ are obtained from those of the linear scenario after multiplying the latter by $(\Lambda/m_Z)^2$ and $(\Lambda/m_Z)^4$, respectively.

2. Operators that are proportional to the scalar part of the Z boson

These operators are proportional to the $SU(2)_L \times U(1)_Y$ invariant $\partial_\mu \mathrm{Tr}[\sigma^3 U^\dagger \mathbf{D}^\mu U]$. As previously mentioned, there are a pair of dimension four CP-odd operators which generate just the ZZZ vertex. They are given by

$$\mathcal{L}^4_1 = i\lambda_1 \mathrm{Tr}[\sigma^3 U^\dagger \mathbf{D}_\nu U] \mathrm{Tr}[\sigma^3 U^\dagger \mathbf{D}^\nu U] \partial_\mu \mathrm{Tr}[\sigma^3 U^\dagger \mathbf{D}^\mu U] + \mathrm{H.c.}, \quad (66)$$

$$\mathcal{L}^4_2 = i\lambda_2 \mathrm{Tr}[\sigma^3 U^\dagger \mathbf{D}^\nu U] \partial^\mu \mathrm{Tr}[U^\dagger (\mathbf{D}_\mu \mathbf{D}_\nu + \mathbf{D}_\nu \mathbf{D}_\mu) U] + \mathrm{H.c.} \quad (67)$$

The linear counterpart of the operator $\mathcal{L}^4_1$ has dimension 10, while the one associated with $\mathcal{L}^4_2$ has dimension 8, as described in Sec. II. The remaining operators have dimension 6, and are obtained from those given in the linear case by the replacement of $\Phi^\dagger D_\mu \Phi$ by $\mathrm{Tr}[\sigma^3 U^\dagger \mathbf{D}_\mu U]$. There are four operators of this type: one pair of CP-odd ones as well as one pair of CP-even ones:

$$\mathcal{L}_{WW2} = 2i \frac{\lambda_{WW2}}{\Lambda^2} \partial_\lambda \mathrm{Tr}[\sigma^3 U^\dagger \mathbf{D}^\lambda U] \mathrm{Tr}[\mathbf{W}_{\mu\nu} \mathbf{W}^{\mu\nu}] + \mathrm{H.c.}, \quad (68)$$

$$\mathcal{L}_{BB2} = i \frac{\lambda_{BB2}}{\Lambda^2} \partial_\lambda \mathrm{Tr}[U^\dagger \mathbf{D}^\lambda U] B_{\mu\nu} B^{\mu\nu} + \mathrm{H.c.}, \quad (69)$$

$$\mathcal{L}_{W\tilde{W}2} = 2i \frac{\lambda_{\tilde{W}W2}}{\Lambda^2} \partial_\lambda \mathrm{Tr}[\sigma^3 U^\dagger \mathbf{D}^\lambda U] \mathrm{Tr}[\mathbf{W}_{\mu\nu} \tilde{\mathbf{W}}^{\mu\nu}] + \mathrm{H.c.}, \quad (70)$$

$$\mathcal{L}_{B\tilde{B}2} = i \frac{\lambda_{B\tilde{B}2}}{\Lambda^2} \partial_\lambda \mathrm{Tr}[U^\dagger \mathbf{D}^\lambda U] B_{\mu\nu} \tilde{B}^{\mu\nu} + \mathrm{H.c.} \quad (71)$$

IV. CONSTRAINTS FROM PRECISION MEASUREMENTS

Once a complete treatment of the effective operators inducing TNGBCs has been presented within both the linear and the nonlinear realizations of the $SU(2)_L \times U(1)_Y$ gauge symmetry, our major concern lies in how to get bounds on the respective coefficients of these operators from current phenomenology. In this respect, considerable work exists in the literature where bounds on anomalous trilinear gauge boson couplings $W^+W^-\gamma$ have been analyzed. To this purpose, measurements on some observables have been extensively used, such as the magnetic and electric dipole moments of elementary fermions and the $Z \to \bar{b}b$ branching fraction, as well as the processes $e^+e^- \to WW$ and $W^* \to W\gamma$ [4]. As for TNGBCs, bounds on these couplings have been obtained through the processes $e^+e^- \to Z\gamma(Z)$ and $q\bar{q} \to Z\gamma(Z)$, although such studies involve only those operators in which two gauge bosons are on-shell. To obtain bounds on our operators, we will follow a similar approach as that in previous works. We will also consider the rare decay $Z \to \nu\bar{\nu}\gamma$, which is affected at the tree level by TNGBCs through the $ZZ\gamma$ vertex. Since its SM contribution is insignificant [9], this process might offer an invaluable mode to unravel any latent new physics effect.

A. Decoupling scenario

We will start by examining the situation in the decoupling scheme of the EL. Before performing any explicit calculation, it is worth estimating on a general basis the size of the TNGBCs. In this respect, it was pointed out that persuasive theoretical arguments indicate that one loop generated anomalous trilinear gauge boson couplings are unlikely expected to be above the 1% level [23]. Indeed, the fact that TNGBCs are induced at the one loop level suggests that they are of order $(g/4\pi)^2$ in a wide class of models. It has also been conjectured that even in theories with underlying strong dynamics, trilinear gauge couplings are not expected to have a sizable enhancement. In the SM, the $ZZ\gamma(Z)$ couplings are severely constrained even in the presence of a fourth fermion family and are thus out of the range of detectability [9,15]. Regarding the bounds arising from phenomenological grounds, we would like to begin by examining in a qualitative way whether the current measurements on the magnetic and electric dipole moments of elementary fermions can give any useful bounds on TNGBCs.

The effective operators presented so far not only induce TNGBCs but also anomalous $W^+W^-\gamma$ couplings. An exhaustive analysis of phenomenological constraints would require one to compute every contribution to the observable under study, including the ones coming from all of the lower dimension operators inducing vertices which also affect the process. For the sake of simplicity, a crude estimate can be obtained if just some operators are considered at a time. In the specific case of the magnetic moment of leptons, which receives contributions from CP-even operators exclusively, a profuse work has been devoted to study comprehensively the contributions from the lowest order effective operators respecting the $\mathrm{SU}(2)_\mathrm{L} \times \mathrm{U}(1)_\mathrm{Y}$ gauge invariance, linearly and nonlinearly realized, which induce nonstandard anomalous couplings. In this respect, there are one loop generated operators of dimension 6 which induce $W^+W^-\gamma$ couplings, but not TNGBCs. These operators contribute to the magnetic moment of leptons via their insertion in the loop diagram depicted in Fig. 1 [24]. Second, some dimension-6 operators directly induce the magnetic moment term at the tree level, though they are generated at the one loop level. Finally, the redefinition of the gauge fields, necessary to an adequate definition of the quadratic part of the theory, also affects the

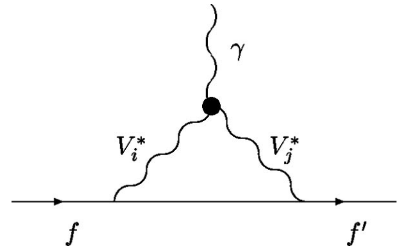

FIG. 1. Contribution from TNGBCs to the anomalous magnetic moment of fermions in the effective Lagrangian approach.

anomalous magnetic moment value. To obtain bounds on the coefficients of the CP conserving operators, the full contribution to the anomalous magnetic moment of the muon was computed [25]. To this end, the strategy which has proved to be the most suitable for estimating the size of loops involving an effective vertex is that of dimensional regularization, together with the modified minimal subtraction ($\overline{\mathrm{MS}}$) renormalization scheme. According to this approach and retaining only the leading logarithmic dependence on the new physics scale Λ, it was found that the contribution from dimension-6 operators inducing the $W^+W^-\gamma$ vertex is given by

$$\delta a_\mu = \eta_0 \left(\frac{m_\mu}{\Lambda}\right)^2 O(\log \Lambda^2/m_W^2)\, \alpha_\mathrm{L}, \tag{72}$$

where η_0 is a factor dependent on the particular graph, and α_L is directly related to the operator coefficients. Numerically, one obtains from this equation $|\delta a_\mu|/10^{-9} = \alpha_\mathrm{L}(1 + \log \Lambda)/\Lambda^2$, with Λ in TeV. If the accepted lowest value of 1 TeV for the new physics scale Λ is taken, we are left with the unpromising result that the operator coefficient should be of order $O(1)$ to have any chance of being detected. But this result is far beyond the estimate of α_L being of order $(g/4\pi)^2$. Indeed, only the direct contribution is expected to give a measurable contribution to the magnetic moment of the muon. In view of this result, it is natural to think that we should not expect a better situation for TNGBCs since they are generated by higher order operators. We note that dimension-8 operators are suppressed by the factor $(v/\Lambda)^2$, with $v = 246$ GeV the vacuum expectation value, with respect to dimension-6 operators. A rough estimate is obtained if we multiply Eq. (72) by the suppression factor and evaluate again at $\Lambda = 1$ TeV. We obtain the discouraging result that α_L should be of order $O(100)$, which is very unlikely to occur, to allow any TNGBCs to be experimentally detected. By way of illustration, we have explicitly computed the contribution to the muon anomalous magnetic moment which is obtained by introducing in the one loop diagram of Fig. 1 the effective $ZZ\gamma$ vertex associated with the factor $g_{L1}^{ZZ\gamma}$ in Eq. (18). After isolating the divergent part, the application of the $\overline{\mathrm{MS}}$ scheme gives

$$\delta a_\mu^{L1} = \frac{t_w(4s_w^2 - 1)g}{256\pi^2}\left(\frac{m_Z}{\Lambda}\right)^2\left(\frac{m_\mu}{\Lambda}\right)^2\left[\log\left(\frac{\Lambda}{m_Z}\right)^2 + \frac{3}{4}\right]\tilde{\epsilon}_{L1}, \tag{73}$$

with

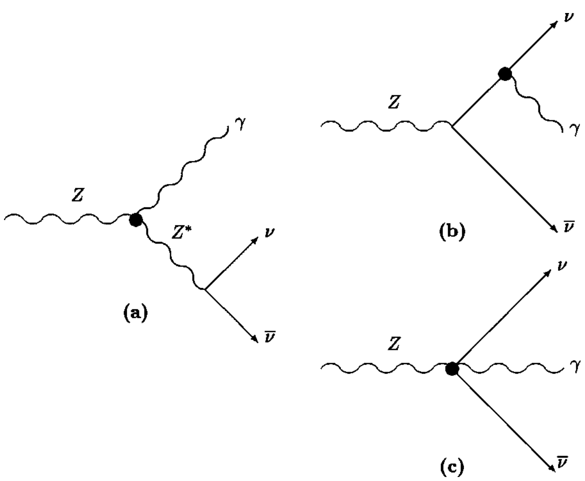

FIG. 2. Feynman diagrams contributing to the decay $Z\rightarrow\nu\bar{\nu}\gamma$ in the effective Lagrangian approach.

$$\tilde{\epsilon}_{L1}=2c_w(\alpha_{\tilde{W}W1}+\alpha_{\tilde{B}B1})+s_w(\alpha_{\tilde{W}B2}+2\alpha_{W\tilde{B}1}). \quad (74)$$

After numerical evaluation, we find that the actual bound on any $\tilde{\alpha}_i$ is looser than the rough estimate. We thus see that it seems there are few hopes that a reasonable bound on CP-even TNGBCs could be obtained from precision measurements on the magnetic moment of the muon. Although we have analyzed only one vertex, the same result is expected for the remaining ones. In fact, the Lorentz structure which parametrizes TNGBCs does not differ essentially in each case [22]. The most optimum situation is the one where all of the contributions add up coherently, though there is no compelling reason to expect that.

A similar analysis can be done for the CP-odd operators which contribute to the electric dipole moment of fermions. In this case a strong bound, from precision measurements on the electric dipole moment of the neutron, exists on dimension-6 operators inducing anomalous $W^+W^-\gamma$ couplings [26]. The respective operator coefficients are constrained to lie below the 10^{-3} level. Since our CP-odd operators, which also induce anomalous $W^+W^-\gamma$ couplings, are of dimension 8 in the decoupling scenario, we could not expect to get a better bound for their coefficients. Once more, a rough estimate would be obtained by dividing the bound on dimension-6 operators by the suppression factor $(v/\Lambda)^2$.

Now let us focus on the rare Z boson decay $Z\rightarrow\nu\bar{\nu}\gamma$, which has been studied within both the SM realm and the EL approach [3,9]. It was shown that the SM contribution turns out to be negligible small, with a branching ratio of order 10^{-10} [9]. In the EL approach, this process arises at the tree level, as depicted in Fig. 2. In addition it has also the advantage of receiving contributions from TNGBCs only through the $ZZ\gamma$ vertex. Although there are also lower dimension effective operators contributing to $Z\rightarrow\nu\bar{\nu}\gamma$ through the Feynman diagrams of Figs. 2b and 2c [3], we will not include those contributions in here since they are not associated with TNGBCs. Furthermore, we are only interested in estimating the best possible bound on TNGBCs.

The measurement of energetic single photons at LEP arising from the decay $Z\rightarrow\nu\bar{\nu}\gamma$ has been used to put a direct limit on the magnetic moment of the τ neutrino [27]. For the purpose of the present analysis, the search for energetic single-photon events in the data collected by the L3 Collaboration may be translated into bounds on TNGBC. In order to reduce backgrounds, the L3 collaboration required the photon energy to be greater than one-half the e^+e^- beam energy. A limit was obtained on the branching ratio for $Z\rightarrow\nu\bar{\nu}\gamma$ of 1 part in a 10^6 when the photon energy is above 30 GeV [27]. To calculate the decay width, we will follow closely the notation of [9]. Expressing the invariant amplitude $\mathcal{M}$ in terms of the scaled variables $x=2k_1p_1/m_Z^2$ and $y=2k_1p_2/m_Z^2$, the $Z(k_2)\rightarrow A(k_1)\nu(p_1)\overline{\nu(p_2)}$ decay width is given by

$$\Gamma(Z\rightarrow\bar{\nu}\nu\gamma)=\frac{m_Z}{256\pi^3}\int_0^1 dx\int_0^{1-x}dy|\bar{\mathcal{M}}|^2. \quad (75)$$

We have not imposed any energy cutoff since it is better to estimate the TNGBC bounds in a conservative way. From Eqs. (7) and (18) one obtains

$$|\bar{\mathcal{M}}|^2=\frac{1}{32}[(x^2+y^2)(1-x-y)-4xy](\alpha^2+\tilde{\alpha}^2), \quad (76)$$

$$\alpha\equiv\alpha_L=\alpha_{L1}+\alpha_{L2}-\alpha_{L3}, \quad (77a)$$

$$\tilde{\alpha}\equiv\tilde{\alpha}_L=\tilde{\alpha}_{L1}-\tilde{\alpha}_{L2}-\tilde{\alpha}_{L3}+\tilde{\alpha}_{L4}+\tilde{\alpha}_{L5}. \quad (77b)$$

As natural, there is no interference between CP violating and CP conserving couplings. The coefficients α_{Li} ($\tilde{\alpha}_{Li}$), are related to the factors $f_{Li}^{ZZ\gamma}$ ($g_{Li}^{ZZ\gamma}$), which in turn depend on the CP-odd (CP-even) operator coefficients, via the relation

$$\alpha_{Li}=\left(\frac{g\,m_Z^2}{c_w}\right)f_{Li}^{ZZ\gamma}, \quad (78a)$$

$$\tilde{\alpha}_{Li}=\left(\frac{g\,m_Z^2}{c_w}\right)g_{Li}^{ZZ\gamma}. \quad (78b)$$

After integration of Eq. (75) we have

$$BR(Z\rightarrow\bar{\nu}\nu\gamma)=2.912\times10^{-5}(\alpha_L^2+\tilde{\alpha}_L^2). \quad (79)$$

Taking the value $\Lambda=1$ TeV and considering the L3 bound on the respective branching fraction, we obtain again the result that the size of the $ZZ\gamma$ coupling should be beyond any reasonable expectation to become perceptible through the process $Z\rightarrow\nu\bar{\nu}\gamma$. Stated in other words, we may not expect moderate bounds from this process. The reason for such a discouraging result is the natural suppression of dimension-8 operators. Our viewpoint would be more pessimistic if we consider that in this calculation only those contributions arising from effective operators inducing the $ZZ\gamma$ coupling have been included. However, there is no compelling reason to disregard any other new physics contributions, such as the ones coming from the Feynman diagrams shown in the Figs. 2a and 2b [3]. In view of our results, it is conceivable to state that any TNGBCs associated with underlying physics respecting linearly the $\mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$ symmetry would not be measurable through the processes investigated in this work. However, we cannot discard the case in which a certain TNGBC is given by a sum of loops whose contributions add up coherently to give a large value.

B. Nondecoupling scenario

We now turn to analyze the situation in the nonlinear scenario, where TNGBCs are generated by dimension-6 operators. Therefore, we might expect a better situation than that in the decoupling scenario. We will see that the discussion for the linear scenario can be easily translated to comprise the nonlinear case. To begin with, in the nonlinear scenario the CP-even $ZZ\gamma$ vertex is parametrized by one extra Lorentz structure in addition to the respective ones appearing in the decoupling case. The results given in Eqs. (73) and (74) for the linear scenario can be directly used if we consider the substitution rules $\tilde{\epsilon}_{L1} \to (\Lambda/m_Z)^2 \tilde{\epsilon}_{NL1}$ and $\alpha_i \to \lambda_i$. The leading term obtained by including in the loop graph of Fig. 1 the $ZZ\gamma$ vertex associated with the coefficient $g_{NL1}^{ZZ\gamma}$ in Eq. (56) is thus

$$\delta a_\mu^{NL1} = \frac{t_w(4s_w^2-1)g}{256\pi^2}\left(\frac{m_\mu}{v}\right)^2\left[\log\left(\frac{v}{m_Z}\right)^2 + \frac{3}{4}\right]\tilde{\epsilon}_{NL1}, \tag{80}$$

where we have employed the conservative value $\Lambda \to v$. Numerically one obtains $\delta a_\mu^{NL1} = -0.767\times 10^{-9}\tilde{\epsilon}_{NL1}$. On the other hand, the more recent data collected through the BNL E281 experiment together with the SM predictions put a bound on any new physics contribution to a_μ of $1.12\times 10^{-9} < \delta a_\mu < 7.56\times 10^{-9}$ at 95% C.L. [28]. As a consequence, probing δa_μ^{NL} at the $\pm 10^{-9}$ level provides a sensitivity to $\tilde{\epsilon}_{NL1}$ of about $O(1)$ at most, which translates into a loose bound for the operator coefficients λ_i. This situation is not better than the result obtained in [25] for the dimension-4 operators inducing $W^+W^-\gamma$ couplings within the nonlinear scheme. Moreover, as there are other sources of new physics which can affect the anomalous magnetic moment, it is hard to think that any TNGBC could be competitive in this process, even in the nonlinear scenario.

Regarding the rare decay $Z \to \nu\bar{\nu}\gamma$, after the inclusion of all the contributions arising from the $ZZ\gamma$ vertex we have that Eq. (76) remains valid, though Eqs. (77a) and (77b) now read

$$\alpha \equiv \alpha_{NL} = \alpha_{NL1} + \alpha_{NL2} - \alpha_{NL3} + 2\,\alpha_{NL4} + \alpha_{NL5}, \tag{81a}$$

$$\tilde{\alpha} \equiv \tilde{\alpha}_{NL} = \tilde{\alpha}_{NL1} - \tilde{\alpha}_{NL2} - \tilde{\alpha}_{NL3} + \tilde{\alpha}_{NL4} + \tilde{\alpha}_{NL5} + 2\,\tilde{\alpha}_{NL6}. \tag{81b}$$

The new coefficients α_{NLi} and $\tilde{\alpha}_{NLi}$ are obtained, with the adequate subscript substitutions, via the relations (78a) and (78b), which also hold for the nonlinear scenario. We will only concentrate in the CP-conserving term, which has been widely studied in the literature. Equation (79) and the L3 limit for the respective branching ratio give the bound $|\tilde{\alpha}_{NL}| < 1.8\times 10^{-1}$ if $\alpha_{NL}=0$. This is a more promising result than that previously found in the linear scenario. In fact, there exists a direct relation between the bound just obtained within the nonlinear scenario and the ones presented elsewhere under the parametrization derived in [14]. It will be shown below that $\alpha_{NL} = 2g^2 h_{10}^Z/(c_w s_w)$ and $\tilde{\alpha}_{NL} = 2g^2 h_{30}^Z/(c_w s_w)$ correspond to the low energy limit of the form factors h_i^Z used extensively to study the $ZZ\gamma$ vertex in the case in which one Z boson and the photon are on shell [29]. Our bound translates thus into

$$|h_{30}^Z| < 0.38, \tag{82}$$

if $h_{10}^Z = 0$, which agrees with previous bounds [30]. Of course, the same result applies to h_{10}^Z when $h_{30}^Z=0$. In this analysis, we have considered that the SM contribution to the rare decay $Z\to\nu\bar{\nu}\gamma$ is negligible, which is a good approximation since it was found that the branching ratio is of order 10^{-10} [9]. We have also neglected the contributions coming from the operators which give rise to the effective vertices shown in Figs. 2(b) and 2(c). This is the most optimum scenario indeed. It is likely that any TNGBC may be screened by any other sources of new physics arising from lower dimension operators. Therefore, a more comprehensive analysis must be done to disentangle any new physics contributing to the processes e^+e^- $(q\bar{q}) \to Z\gamma$ [29].

C. Connection with results derived within the $\mathrm{U}(1)_{\mathrm{em}}$ formalism

TNGBCs were studied for the first time long ago, although only one particle was allowed to be off shell [14]. Following that approach, it became customary to parametrize any new physics effects inducing TNGBCs by certain structures derived out of $\mathrm{U}(1)_{\mathrm{em}}$ gauge invariance and Lorentz covariance, as well as Bose symmetry, which corresponds to the so-called $\mathrm{U}(1)_{\mathrm{em}}$ framework [21]. The coefficients of such Lorentz structures are taken to be form factors which actually comprise all our ignorance of the underlying dynamics inducing TNGBCs. In general, these form factors depend on the squared momenta of the participating particles, but such a dependence is unknown since it is to be prescribed by up to now unknown physics. Then it is necessary to make some assumptions to describe the form factor behavior. In particular, much work has been done to constrain the low energy values of the form factors through $Z\gamma$ production in e^+e^- and $q\bar{q}$ collisions at LEP, the Tevatron, and the future LHC [21].

The above formalism is to be contrasted with the approach followed in this work, which in turn is well suited for studying new physics effects in a model independent way, and no form factors nor extra assumptions on the unknown physics are required, but all our ignorance of the new physics lies in dimensionless (or dimensionful) coefficients associated with each effective operator, which in turn only depend on the new physics energy scale. Another peculiarity of the EL formalism is that we are allowed to know what operators the new physics comes from, in contrast to the form factor scheme where we only know that the form factors themselves are generated at a given order in the $U(1)_{\rm em}$ effective Lagrangian. To establish a direct connection between these two different formalisms is not immediate. In a previous work both approaches were studied, within the $U(1)_{\rm em}$ gauge invariant scheme [15]. The explicit relation was also shown between the form factors and the coefficients associated with the effective operators arising from the $U(1)_{\rm em}$ framework. At this point, it is worth examining the connection between our own results, when it is considered the case of only one off-shell particle, and those derived from the form factor parametrization. We will show that in the case where the form factors are given their low energy values h_{i0}^Z, there is a simple relation indeed. To this end, we will consider only the $ZZ\gamma$ coupling, since it is the only coupling involved in the rare process $Z\to\nu\bar{\nu}\gamma$.

The most general structure for the $ZZ\gamma$ vertex respecting Lorentz covariance, $U(1)_{\rm em}$ gauge invariance and Bose symmetry is given by

$$\Gamma^{ZZ\gamma}_{\alpha_1\,\alpha_2\,\alpha}(k_1,k_2,k) = \frac{ie(k_2^2-m_Z^2)}{m_Z^2}\left(h_1^Z(k^{\alpha_1}g^{\alpha\alpha_2}-k^{\alpha_2}g^{\alpha\alpha_1}) + \frac{h_2^Z}{m_Z^2}k_2^{\alpha_1}(k_2\cdot k\, g^{\alpha_2\alpha}-k^{\alpha_2}k_2^{\alpha}) + h_3^Z\epsilon^{\alpha_1\alpha_2\alpha\mu}k^{\mu} + \frac{h_4^Z}{m_Z^2}k_2^{\alpha_1}\epsilon^{\alpha_2\alpha\mu\nu}k_{1\mu}k_{2\nu}\right), \tag{83}$$

where all momenta are taken as incoming. Any term proportional to k^{α} and $k_1^{\alpha_1}$ has been omitted and the same is true for those proportional to $k_2^{\alpha_2}$ because it is also assumed that the virtual Z boson couples to light fermions, as actually happens in the decay $Z\to ll\gamma$. In this parametrization, the CP-conserving terms $h_{1,2}^Z$ as well as the CP-violating ones $h_{3,4}^Z$ are form factors which depend on the dynamics of the underlying new physics. Within the $U(1)_{\rm em}$ formalism, as far as the form factors $h_{1,3}^Z$ are concerned, they receive contributions from dimension-6 operators, whereas the ones $h_{2,4}^Z$ can be induced by dimension-8 or higher operators. Based on the unitarity requirement, some authors have found it convenient to use the approximation $h_i^Z=h_{i0}^Z/(1+s/\Lambda^2)^n$, with n an integer, h_{i0}^Z the form factor low energy value, and s the squared momentum of the virtual Z boson [29]. If the energy scale Λ associated with the new physics inducing TNGBCs is larger than the energy scale involved in the process, i.e. the squared momentum of the virtual particle, it is a good approximation to use the low energy values of the form factors. After replacing $h_i^Z\to h_{i0}^Z$ in Eq. (83), we are left with the expression for the $ZZ\gamma$ vertex which is to be compared with the one obtained from our results in the nonlinear scenario.

Considering the above assumptions, we can obtain from Eqs. (7), (18), (44), and (56), the expression for the $ZZ\gamma$ vertex arising from the lower dimension operators within either the linear scenario or the nonlinear one [22]. After a judicious manipulation and with the aid of Shouten's identity, we obtain

$$\Gamma^{ZZ\gamma}_{\alpha_1\alpha_2\alpha}(k_1,k_2,k)=(k_2^2-m_Z^2)[g^{ZZ\gamma}(k_{\alpha_2}g_{\alpha_1\alpha}-k_{\alpha_1}g_{\alpha_2\alpha}) + f^{ZZ\gamma}\epsilon_{\alpha_1\alpha_2\alpha\mu}k^{\mu}], \tag{84}$$

which has an obvious relation with Eq. (83). Instead of giving explicit expressions for $f^{ZZ\gamma}$ and $g^{ZZ\gamma}$, it is useful to establish the relation of h_{10}^Z and h_{30}^Z with the coefficients α and $\tilde{\alpha}$ appearing in Eqs. (77a),(77b) in the linear scenario and Eqs. (81a),(81b) in the nonlinear one, that is

$$h_{10}^Z=\frac{c_W\, s_W\, \alpha_L}{2g^2}, \tag{85a}$$

$$h_{30}^Z=\frac{c_W\, s_W\, \tilde{\alpha}_L}{2g^2}. \tag{85b}$$

The same relation holds for these coefficients in the nonlinear scenario.

Finally, we would like to note some interesting points. Although the $ZZ\gamma$ has the same Lorentz structure in both realizations of the $SU(2)_L\times U(1)_Y$ gauge symmetry, the main difference is that the operators inducing these structures are of dimension 6 in the nonlinear scenario, whereas in the linear case they are induced by dimension eight operators. As a result, though the bounds found for the coefficients h_{10}^Z and h_{30}^Z apply in both scenarios, if they were translated into the operator coefficients α_i and λ_i, looser bounds would be obtained in the linear scenario. Regarding the remaining TNGBCs, a similar analysis following the lines sketched above was done for the ZZZ and $Z\gamma\gamma$ couplings. It was found that our results agree with those previously presented. Another interesting point to be noted is that, since the operators which induce the most general TNGBC vertices also induce those couplings with only one off-shell particle, any bound which has been put on the latter will be immediately applicable to the former.

V. CONCLUSIONS

In this work we have presented an analysis of trilinear neutral gauge boson couplings, ZZZ, $ZZ\gamma$, and $Z\gamma\gamma$, under the context of the effective Lagrangian approach, both in the linear and the nonlinear realizations of the $SU(2)_L\times U(1)_Y$ gauge symmetry. Particular emphasis has been given to the linear scenario since the current literature lacks an analysis along these lines. The most general case with three off-shell bosons is considered. In the linear scenario these couplings receive contributions from dimension-8 operators, whereas in the nonlinear scenario they are induced by dimension six operators. Based on general considerations and actual calculations, we conclude that, if the until now unknown physics underlying the SM is of a decoupling nature, it is not expected that TNGBCs could have a considerable impact either through their virtual effects or via direct production. In contrast, if new physics effects arise from a strong coupling regime at higher energies which is responsible for the breaking of the $SU(2)_L \times U(1)_Y$ symmetry (endowing the gauge bosons with mass), the possibility of measuring their effects still remains. The EL approach indicates that, owing to the suppression of the operators inducing TNGBC, it is difficult that the effects arising from them may compete with those coming from other sources of new physics induced by lower dimension operators. However, it may happen that some fortuitous fact, such as some resonant effect, could give rise to large TNGBCs in a particular model. In this context, it would be useful a study in a model dependent way to have more evidences which could lead us to a deeper understanding of TNGBCs.

Note added in proof. After the submission of this paper we became aware of Ref. [31], where *CP*-violating TNGBCs were studied within the minimal supersymmetric standard model.

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