P and T odd asymmetries in lepton flavor violating τ decays

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We calculate the differential cross sections of the processes in which one of the pair created τ particles at an e^+e^- collider decays into lepton flavor violating final states, e.g., $\tau\to\mu\gamma$, $\tau\to3\mu$, $\tau\to\mu ee$. Using the correlations between angular distributions of both sides of τ decays, we can obtain information on parity and CP violations of lepton flavor nonconserving interactions. The formulas derived here are useful in distinguishing different models, since each model of physics beyond the standard model predicts different angular correlations. We also calculate angular distributions of the major background process to $\tau\to l\gamma$ search, namely, $\tau\to l\nu\bar{\nu}\gamma$, and discuss the usefulness of the angular correlation for background suppression.

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I. INTRODUCTION

Recent results from neutrino experiments such as the Super-Kamiokande experiment strongly suggest neutrino oscillation so that there are flavor mixings in the lepton sector [1]. This implies that charged lepton flavor violating processes such as $\mu\to e\gamma$, $\tau\to\mu\gamma$, $\tau\to e\gamma$, etc., also occur at some level. It is, therefore, important to search for lepton flavor violation (LFV) in rare decay processes of the muon and τ.

The prediction of the branching fraction of LFV processes depends on models of physics beyond the standard model. In the minimal extension of the standard model, which takes into account neutrino oscillations by the seesaw mechanism of neutrino mass generation [2], the expected branching fraction is too small to be observable in the near future [3]. On the other hand, in supersymmetric (SUSY) models, the prediction can be close to the current experimental upper bound. In this case, flavor mixing in the slepton mass matrix becomes a new source of LFV. Even in the minimal supergravity scenario [4], in which the slepton mass matrix is proportional to a unit matrix at the Planck scale, the renormalization effects due to LFV interactions can induce sizable slepton mixings [5]. For example, such LFV Yukawa interactions exist in the SUSY grand unified theory (GUT) model [6], SUSY model with right-handed neutrinos [7], and SUSY models with exotic vectorlike leptons [8]. Another interesting possibility is models with extra dimensions, where the neutrino masses and mixings are obtained from the Yukawa interaction between the ordinary left-handed leptons and the gauge-singlet neutrinos which propagate in the bulk of extra dimensions [9]. This Yukawa interaction breaks the lepton flavor conservation and the Kaluza-Klein modes of the bulk neutrinos can enhance $\mu\to e\gamma$ decay, $\tau\to\mu\gamma$ decay, etc., through the loop diagrams [10].

In the muon decay, the polarized muon experiments provide useful information on the nature of LFV interactions [11,12]. We can define a parity (P) odd asymmetry for $\mu\to e\gamma$ process and P and time reversal (T) odd asymmetries for $\mu\to3e$ processes. These asymmetries are useful to distinguish different models. For example, in the SU(5) SUSY GUT model with small and intermediate values of $\tan\beta$ (a ratio of two vacuum expectation values of Higgs fields), only $\mu^+\to e_L^+\gamma$ (or $\mu^-\to e_R^-\gamma$) occurs because LFV is induced through the right-handed slepton sector. On the other hand, SUSY models with right-handed neutrinos predict $\mu^+\to e_R^+\gamma$ (or $\mu^-\to e_L^-\gamma$), and SUSY models with vectorlike leptons can induce both $\mu^+\to e_L^+\gamma$ and $\mu^+\to e_R^+\gamma$ depending on how the interaction breaks lepton flavor conservation. In models with extra dimensions, only $\mu^+\to e_R^+\gamma$ can occur. As for the T odd asymmetry in the $\mu\to3e$ process, it was shown that asymmetry could be sizable in the SU(5) SUSY GUT [11,12].

In this paper, we discuss the LFV processes of τ decays such as $\tau\to\mu\gamma$, $\tau\to3\mu$, $\tau\to\mu ee$, taking into account P and T odd asymmetries. In the $\tau^+\tau^-$ pair production at e^+e^- collisions, we can extract information on the spin of the decaying τ particle from the angular distribution of the τ decay products in the opposite side. Using this technique, we can obtain the P and T odd asymmetry defined in the rest frame of τ. The method of the spin correlation has been developed since the days before the discovery of τ particle [13]. There have been many works on the spin correlation method in search of anomalous coupling involving τ [14]. In those references various energy and angular correlations as well as asymmetries are introduced to extract P and T odd quantities. We have applied the same formalism in order to obtain the information on LFV interactions under P and T symmetries. We also calculate angular correlation of the process where one of the τ's decays through the $\tau\to l\nu\bar{\nu}\gamma$ mode. This mode is a background process to the $\tau\to l\gamma$ search if the neutrinos carry little energy. As in the muon case [15], the angular correlation is useful to identify the background process and background suppression is effective for $\tau^-\to\mu_L^-\gamma$ ($\tau^+\to\mu_R^+\gamma$) search.

This paper is organized as follows. In Sec. II, we introduce a formalism to calculate the spin correlation. In Sec. III,

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we present a differential cross section of the production and decays of $\tau^+\tau^-$ at e^+e^- colliders where one of τ's decays in $\tau\to\mu\gamma$ or $\tau\to e\gamma$ modes, and show how to extract the P odd asymmetry of τ decay. In Sec. IV, P and T odd asymmetries in three body LFV decays ($\tau\to 3\mu$, $\tau\to\mu ee$, etc.) are considered. In Sec. V, we consider the $\tau\to l\nu\bar{\nu}\gamma$ mode and show that the analysis of the angular distribution is useful for background suppression of the $\tau\to\mu\gamma$ and $\tau\to e\gamma$ searches. A summary and discussion are given in Sec. VI. The Appendixes contain the derivation of basic formulas and a list of kinematical functions.

II. GENERAL FORMULA FOR SPIN CORRELATION

In this section, we present general formulas used in the calculation of differential cross sections and spin correlations.

We calculate differential cross sections of $e^+e^-\to\tau^+\tau^-\to f_B f_A$, where $f_B(f_A)$ represents the decay products of τ^+ (τ^-). If the intermediate states were spinless particles, the cross section is simply a product of a production cross section and decay branching ratios. However, in the case of spin 1/2 particles, we have to take into account spin correlation between two intermediate particles. If we take $\tau^+\to f_B$ to be a LFV decay mode, we can measure P and T violation of LFV interactions by using the angular correlations of decay products of τ^+ and τ^-.

The differential cross section of $e^+e^-\to\tau^+\tau^-\to f_B f_A$ is given by

$$d\sigma=d\sigma^P dB^{\tau^-\to f_A}dB^{\tau^+\to f_B}+\sum_{a,b=1}^{3} d\Sigma^P_{ab}dR_a^{\tau^-\to f_A}dR_b^{\tau^+\to f_B} \tag{1}$$

and

$$d\sigma^P=\frac{d^3p_A}{(2\pi)^3 2p_A^0}\frac{d^3p_B}{(2\pi)^3 2p_B^0}\frac{1}{2s}(2\pi)^4 \times\delta^4(p_A+p_B-p_{e^+}-p_{e^-})\alpha^P, \tag{2}$$

$$dB^{\tau^-\to f_A}=\frac{1}{\Gamma}\frac{d^3q_1}{(2\pi)^3 2q_1^0}\cdots\frac{d^3q_n}{(2\pi)^3 2q_n^0}\frac{1}{2m_\tau}(2\pi)^4 \times\delta^4\left(\sum_{i=1}^{n}q_i-p_A\right)\alpha^{D-}, \tag{3}$$

$$dB^{\tau^+\to f_B}=\frac{1}{\Gamma}\frac{d^3q_{n+1}}{(2\pi)^3 2q_{n+1}^0}\cdots\frac{d^3q_{n+m}}{(2\pi)^3 2q_{n+m}^0} \times\frac{1}{2m_\tau}(2\pi)^4\delta^4\left(\sum_{i=n+1}^{n+m}q_i-p_B\right)\alpha^{D+}, \tag{4}$$

$$d\Sigma^P_{ab}=\frac{d^3p_A}{(2\pi)^3 2p_A^0}\frac{d^3p_B}{(2\pi)^3 2p_B^0}\frac{1}{2s}(2\pi)^4 \times\delta^4(p_A+p_B-p_{e^+}-p_{e^-})\rho^P, \tag{5}$$

$$dR_a^{\tau^-\to f_A}=\frac{1}{\Gamma}\frac{d^3q_1}{(2\pi)^3 2q_1^0}\cdots\frac{d^3q_n}{(2\pi)^3 2q_n^0}\frac{1}{2m_\tau}(2\pi)^4 \times\delta^4\left(\sum_{i=1}^{n}q_i-p_A\right)\rho_a^{D-}, \tag{6}$$

$$dR_b^{\tau^+\to f_B}=\frac{1}{\Gamma}\frac{d^3q_{n+1}}{(2\pi)^3 2q_{n+1}^0}\cdots\frac{d^3q_{n+m}}{(2\pi)^3 2q_{n+m}^0} \times\frac{1}{2m_\tau}(2\pi)^4\delta^4\left(\sum_{i=n+1}^{n+m}q_i-p_B\right)\rho_b^{D+}, \tag{7}$$

where we assume that f_A is a n body system and f_B is a m body system. $p_{e^+}(p_{e^-})$ is the $e^+(e^-)$ four-momentum, p_B (p_A) is the $\tau^+(\tau^-)$ four-momentum, and q_i's are the momenta of final state particles. s is determined as $s=(p_{e^+}+p_{e^-})^2$. Γ and m_τ are the width and the mass of the τ, respectively. In order to define α^P, α^{D-}, α^{D+}, ρ^P_{ab}, ρ_a^{D-}, and ρ_b^{D+}, we first write down the invariant amplitude of $e^+e^-\to\tau^+\tau^-\to f_B f_A$ as follows:

$$M=\frac{e^2}{s}\bar{A}(\not{p}_A+m_\tau)\gamma^\mu(\not{p}_B-m_\tau)B\bar{v}_{e^+}\gamma_\mu u_{e^-} \times\frac{1}{p_A^2-(m_\tau-i\Gamma/2)^2}\frac{1}{p_B^2-(m_\tau-i\Gamma/2)^2}, \tag{8}$$

where $v_{e^+}(u_{e^-})$ is the wave function of the positron (electron) and A and B are spinors which include wave functions of final states and interaction vertices. By using the Bouchiat-Michel formulas [16] and the narrow width approximation, α^P, α^{D-}, α^{D+}, ρ^P_{ab}, ρ_a^{D-}, and ρ_b^{D+} are given by

$$\alpha^P=\frac{1}{4}\frac{e^4}{s^2}\mathrm{Tr}[(\not{p}_A+m_\tau)\gamma^\mu(\not{p}_B-m_\tau)\gamma^\nu] \times\mathrm{Tr}[\not{p}_{e^+}\gamma_\mu\not{p}_{e^-}\gamma_\nu], \tag{9}$$

$$\alpha^{D-}=\frac{1}{2}\{\bar{A}(\not{p}_A+m_\tau)A\}, \tag{10}$$

$$\alpha^{D+}=\frac{1}{2}\{\bar{B}(\not{p}_B-m_\tau)B\}, \tag{11}$$

$$\rho^P_{ab}=\frac{1}{4}\frac{e^4}{s^2}\mathrm{Tr}[\gamma_5\not{s}_A^a(\not{p}_A+m_\tau)\gamma^\mu\gamma_5\not{s}_B^b(\not{p}_B-m_\tau)\gamma^\nu] \times\mathrm{Tr}[\not{p}_{e^+}\gamma_\mu\not{p}_{e^-}\gamma_\nu], \tag{12}$$

$$\rho_a^{D-}=\frac{1}{2}\{\bar{A}\gamma_5\not{s}_A^a(\not{p}_A+m_\tau)A\}, \tag{13}$$

$$\rho_b^{D+}=\frac{1}{2}\{\bar{B}\gamma_5\not{s}_B^b(\not{p}_B-m_\tau)B\}, \tag{14}$$

where the spins of the final state fermions are summed over, and the four-vectors $(s_A^a)^\mu$ and $(s_B^b)^\nu$ $(a,b=1,2,3)$ are a set of vectors which satisfy following equations:

$$p_A\cdot s_A^a=p_B\cdot s_B^b=0, \tag{15}$$

$$s_A^a\cdot s_A^b=s_B^a\cdot s_B^b=-\delta^{ab}, \tag{16}$$

$$\sum_{a=1}^{3}(s_A^a)_\mu(s_A^a)_\nu=-g_{\mu\nu}+\frac{p_{A\mu}p_{A\nu}}{m_\tau^2},$$

$$\sum_{b=1}^{3}(s_B^b)_\mu(s_B^b)_\nu=-g_{\mu\nu}+\frac{p_{B\mu}p_{B\nu}}{m_\tau^2}. \tag{17}$$

The derivation of the above result is shown in Appendix A. Notice that $d\sigma^P$, $dB^{\tau^-\to f_A}$, and $dB^{\tau^+\to f_B}$ in Eq. (1) are the $\tau^+\tau^-$ production cross section and τ decay branching ratios, in which the spins of τ's are averaged, and $d\Sigma_{ab}^P$, $dR_a^{\tau^-\to f_A}$, and $dR_b^{\tau^+\to f_B}$ represent the spin correlation effects of this process.

In the above formulas, it is assumed that τ pair production occurs through photon exchange. It is straightforward to include the contribution from Z boson exchange and γ-Z interference. If we consider the e^+e^- center of mass energy to be in the range of the $\tau^+\tau^-$ threshold energy considered in the τ-charm factory or the $\Upsilon(4S)$ resonance energy where e^+e^- B factories are operated, these effects only contribute to the production cross section at the level of $O(10^{-4})$ of the photon exchanging diagram.

III. PARITY ASYMMETRY IN $\tau\to\mu\gamma$ DECAY

Let us calculate the cross section of $e^+e^-\to\tau^+\tau^-\to\mu^+\gamma+f_A$ processes. For f_A, we consider hadronic and leptonic modes such as ($\pi\nu$, $\rho\nu$, $a_1\nu$, and $l\overline{\nu}\nu$). Below we neglect the muon mass compared to the τ mass, and therefore all formulas can be applied also to the $\tau\to e\gamma$ process. The effective Lagrangian for $\tau^+\to\mu^+\gamma$ decay is given by

$$\mathcal{L}=-\frac{4G_F}{\sqrt{2}}\{m_\tau A_R\overline{\tau}\sigma^{\mu\nu}P_L\mu F_{\mu\nu}+m_\tau A_L\overline{\tau}\sigma^{\mu\nu}P_R\mu F_{\mu\nu}+\text{H.c.}\}, \tag{18}$$

where G_F is the Fermi coupling constant, $P_L=(1-\gamma_5)/2$, and $P_R=(1+\gamma_5)/2$. In this paper, we use the conventions $\sigma^{\mu\nu}=i/2[\gamma^\mu,\gamma^\nu]$, $F_{\mu\nu}=\partial_\mu A_\nu-\partial_\nu A_\mu$, and $D_\mu=\partial_\mu+ieA_\mu$ for electrons where e(70) is the positron charge. The operator with the coupling constant A_R (A_L) induces the $\tau^+\to\mu_R^+\gamma$ ($\tau^+\to\mu_L^+\gamma$) decay. As mentioned in the Introduction, each model of physics beyond the standard model predicts a different ratio of A_L and A_R. For example, the SU(5) SUSY GUT in the minimal supergravity scenario predicts that only A_L has a nonvanishing value for small and intermediate values of $\tan\beta$. Therefore the separate determination of A_L and A_R provides us important information on the origin of LFV. For this purpose, we need information about the τ polarization. This can be done by observing angular distributions of the final state of τ decay in the opposite side in the modes of $\tau\to\pi\nu$, $\tau\to\rho\nu$, $\tau\to a_1\nu$, and $\tau\to l\overline{\nu}\nu$, because these processes proceed due to the *V-A* interaction and therefore have a specific angular distribution with respect to polarization of τ. Using τ^+-τ^- spin correlation, we can determine $|A_L|^2$ and $|A_R|^2$, separately.

We first define three coordinate systems (Fig. 1). The first coordinate system (frame 1) is the center of mass frame of the e^+e^- collision in which the z axis is taken to be the e^+ momentum direction. The second one (frame 2) is the rest frame of the τ^+, and the third one (frame 3) is the rest frame of the τ^-. More explicitly, the relation of a four-vector in the three systems is given as follows:

$$\xi_1^\mu=\begin{pmatrix}1&0&0&0\\0&\cos\theta_\tau&0&\sin\theta_\tau\\0&0&1&0\\0&-\sin\theta_\tau&0&\cos\theta_\tau\end{pmatrix}\begin{pmatrix}\gamma&0&0&\gamma\beta_\tau\\0&1&0&0\\0&0&1&0\\\gamma\beta_\tau&0&0&\gamma\end{pmatrix}\xi_2^\mu$$

$$=\begin{pmatrix}1&0&0&0\\0&1&0&0\\0&0&-1&0\\0&0&0&-1\end{pmatrix}\begin{pmatrix}1&0&0&0\\0&\cos\theta_\tau&0&-\sin\theta_\tau\\0&0&1&0\\0&\sin\theta_\tau&0&\cos\theta_\tau\end{pmatrix}\times\begin{pmatrix}\gamma&0&0&\gamma\beta_\tau\\0&1&0&0\\0&0&1&0\\\gamma\beta_\tau&0&0&\gamma\end{pmatrix}\xi_3^\mu, \tag{19}$$

where $\gamma=\sqrt{s}/(2m_\tau)$ and $\beta_\tau=\sqrt{1-4m_\tau^2/s}$, and the four-vectors ξ_{1-3} are defined in frames 1–3, respectively. We calculate the production process in frame 1 and the $\tau^+(\tau^-)$

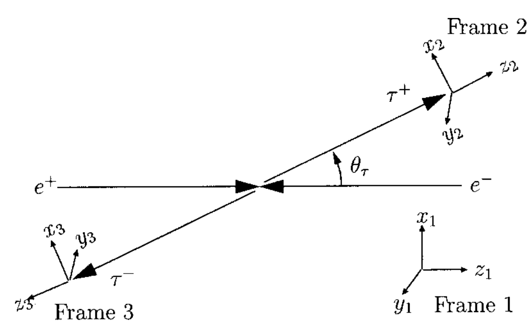

FIG. 1. The coordinate systems. The plane determined by e^+e^- and $\tau^+\tau^-$ momentum vectors corresponds to the xz planes in each of three coordinate systems.

decay in frame 2 (frame 3). In the calculations, we choose the spin vectors $(s_A^a)^\mu, (s_B^a)^\mu$ as follows:

$$(s_A^a)^\mu = \begin{pmatrix} 0 \\ \delta_{a\mu} \end{pmatrix} \quad \text{(in frame 3)}, \tag{20}$$

$$(s_B^b)^\mu = \begin{pmatrix} 0 \\ \delta_{b\mu} \end{pmatrix} \quad \text{(in frame 2)}. \tag{21}$$

The production cross section and spin dependence term are obtained from Eqs. (9) and (12) as follows:

$$d\sigma^P = \frac{d\Omega_\tau}{4\pi}\frac{\pi\alpha^2}{s}\sqrt{1-\frac{4m_\tau^2}{s}}\left\{\left(1+\frac{4m_\tau^2}{s}\right)+\left(1-\frac{4m_\tau^2}{s}\right)\cos^2\theta_\tau\right\}, \tag{22}$$

$$d\Sigma_{ab}^P = \frac{d\Omega_\tau}{4\pi}\frac{\pi\alpha^2}{s}\sqrt{1-\frac{4m_\tau^2}{s}}\begin{pmatrix} \left(1+\frac{4m_\tau^2}{s}\right)\sin^2\theta_\tau & 0 & -\frac{2m_\tau}{\sqrt{s}}\sin 2\theta_\tau \\ 0 & \left(1-\frac{4m_\tau^2}{s}\right)\sin^2\theta_\tau & 0 \\ \frac{2m_\tau}{\sqrt{s}}\sin 2\theta_\tau & 0 & -\left(1-\frac{4m_\tau^2}{s}\right)-\left(1+\frac{4m_\tau^2}{s}\right)\cos^2\theta_\tau \end{pmatrix}, \tag{23}$$

where θ_τ is the angle between the e^+ and τ^+ directions in the frame 1, and $d\Omega_\tau$ is a solid angle element of τ^+, $d\Omega_\tau = d\cos\theta_\tau\, d\phi_\tau$.

For decay processes, we take $\tau^+\to\mu^+\gamma$ for the τ^+ side and hadronic ($\tau^-\to\pi^-\nu$, $\tau^-\to\rho^-\nu$, and $\tau^-\to a_1^-\nu$) and leptonic ($\tau^-\to l^-\overline{\nu}\nu$) decays for the τ^- side. $dB^{\tau^+\to\mu^+\gamma}$ and $dR_b^{\tau^+\to\mu^+\gamma}$ [see Eq. (1)] can be calculated from Eqs. (11) and (14) in which the spinor B is given by

$$B = \frac{8i}{\sqrt{2}} G_F m_\tau \sigma^{\mu\nu}(q_\gamma)_\mu (A_R P_L + A_L P_R)\epsilon_\nu^* v(q_\mu), \tag{24}$$

where ϵ_ν is the polarization vector of the photon, and $(q_\gamma)_\mu$ is the momentum of the photon, and $v(q_\mu)$ is the wave function of the muon. These quantities are given as follows:

$$dB^{\tau^+\to\mu^+\gamma} = \frac{d\Omega_\mu}{4\pi}\frac{1}{\Gamma}\frac{2}{\pi} G_F^2 m_\tau^5 (|A_L|^2+|A_R|^2), \tag{25}$$

$$dR_b^{\tau^+\to\mu^+\gamma} = \frac{d\Omega_\mu}{4\pi}\frac{1}{\Gamma}\frac{2}{\pi} G_F^2 m_\tau^5 (|A_L|^2-|A_R|^2) \times \begin{pmatrix} \sin\theta_\mu\cos\phi_\mu \\ \sin\theta_\mu\sin\phi_\mu \\ \cos\theta_\mu \end{pmatrix}, \tag{26}$$

where (θ_μ, ϕ_μ) are angles in the polar coordinate for a unit vector of the muon momentum direction in frame 2. The three components in Eq. (26) correspond to $b=1$, 2, and 3.

Next, we list dB and dR for the τ^- decay in each mode of $\tau^-\to\pi^-\nu$, $\tau^-\to\rho^-\nu$, $\tau^-\to a_1^-\nu$, and $\tau^-\to l^-\overline{\nu}\nu$. For $\tau^-\to\pi^-\nu$ decay, the spinor A in Eqs. (10) and (13) is given by

$$A = 2iV_{ud} f_\pi G_F \not{q}_\pi P_L u(q_\nu), \tag{27}$$

where f_π is the pion decay constant, q_π is the momentum of the pion, and $u(q_\nu)$ is the neutrino wave function. Then, $dB^{\tau^-\to\pi^-\nu}$ and $dR_a^{\tau^-\to\pi^-\nu}$ are given by

$$dB^{\tau^-\to\pi^-\nu} = \frac{d\Omega_\pi}{4\pi}\frac{1}{\Gamma}\frac{1}{8\pi}|V_{ud}|^2 f_\pi^2 G_F^2 m_\tau^3, \tag{28}$$

$$dR_a^{\tau^-\to\pi^-\nu} = dB^{\tau^-\to\pi^-\nu}\begin{pmatrix} \sin\theta_\pi\cos\phi_\pi \\ \sin\theta_\pi\sin\phi_\pi \\ \cos\theta_\pi \end{pmatrix}, \tag{29}$$

where (θ_π, ϕ_π) are the polar angles of π^- momentum in frame 3 and $d\Omega_\pi = d\cos\theta_\pi\, d\phi_\pi$ (we use a similar notation in the following expressions). Here we neglect the mass of the pion. As before the three elements in Eq. (29) correspond to $a=1$, 2, and 3. Similar results can be obtained for the vector mesons. The spinor A for $\tau\to\rho\nu$, $\tau\to a_1\nu$ is given by

$$A = -2V_{ud} g_V G_F \not{\epsilon}_V P_L u(q_\nu), \tag{30}$$

where g_V and ϵ_V are the decay constant and polarization vector of the corresponding vector mesons, respectively. From this expression, we can obtain dB and dR for the longitudinally polarized vector mesons, e.g., $\tau^-\to\rho^-(L)\nu$ and $\tau^-\to a_1^-(L)\nu$ as follows:

$$dB^{\tau^- \to V(L)^- \nu} = \frac{d\Omega_V}{4\pi} \frac{1}{\Gamma} \frac{1}{8\pi} |V_{ud}|^2 \left(\frac{g_V}{m_V^2} \right)^2 \times G_F^2 m_\tau^3 m_V^2 \left(1 - \frac{m_V^2}{m_\tau^2} \right), \quad (31)$$

$$dR_a^{\tau^- \to V(L)^- \nu} = dB^{\tau^- \to V(L)^- \nu} \begin{pmatrix} \sin\theta_V \cos\phi_V \\ \sin\theta_V \sin\phi_V \\ \cos\theta_V \end{pmatrix}, \quad (32)$$

where m_V and (θ_V, ϕ_V) are the mass and polar angles of the corresponding vector meson, respectively. For the transversely polarized vector mesons, the spin dependence terms have a minus sign contrary to the case of the pion and longitudinally polarized vector mesons:

$$dB^{\tau^- \to V(T)^- \nu} = \frac{d\Omega_V}{4\pi} \frac{1}{\Gamma} \frac{1}{8\pi} |V_{ud}|^2 \left(\frac{g_V}{m_V^2} \right)^2 \times G_F^2 m_\tau^3 m_V^2 \left(1 - \frac{m_V^2}{m_\tau^2} \right) \frac{2m_V^2}{m_\tau^2}, \quad (33)$$

$$dR_a^{\tau^- \to V(T)^- \nu} = dB^{\tau^- \to V(T)^- \nu} \begin{pmatrix} -\sin\theta_V \cos\phi_V \\ -\sin\theta_V \sin\phi_V \\ -\cos\theta_V \end{pmatrix}. \quad (34)$$

For leptonic decays, after integrating over the phase space of the neutrinos, the branching ratio and the spin dependence term are given by

$$dB^{\tau^- \to l^- \bar{\nu}\nu} = \frac{d\Omega_l}{4\pi} dx \frac{1}{\Gamma} \frac{G_F^2 m_\tau^5}{192\pi^3} 2x^2(3-2x), \quad (35)$$

$$dR_a^{\tau^- \to l^- \bar{\nu}\nu} = \frac{d\Omega_l}{4\pi} dx \frac{1}{\Gamma} \frac{G_F^2 m_\tau^5}{192\pi^3} 2x^2(1-2x) \times \begin{pmatrix} \sin\theta_l \cos\phi_l \\ \sin\theta_l \sin\phi_l \\ \cos\theta_l \end{pmatrix}, \quad (36)$$

where we neglect the masses of the leptons (e and μ) and x is the lepton energy normalized by the maximum energy $m_\tau/2$, i.e., $x = 2E_l/m_\tau$, and (θ_l, ϕ_l) are the polar angles of the lepton in frame 3.

Substituting these results into the formula in Eq. (1), we obtain the differential cross sections of each process. For example, the differential cross section of the $e^+e^- \to \tau^+\tau^- \to \mu^+\gamma + \pi^-\nu$ process is given by

$$d\sigma(e^+e^- \to \tau^+\tau^- \to \mu^+\gamma + \pi^-\nu) = \frac{\sigma(e^+e^- \to \tau^+\tau^-)}{\frac{4}{3}\left(1 + \frac{2m_\tau^2}{s}\right)} B(\tau^- \to \pi^-\nu)\, B(\tau^+ \to \mu^+\gamma) \frac{d\Omega_\tau}{4\pi} \frac{d\Omega_\gamma}{4\pi} \frac{d\Omega_\pi}{4\pi}$$
$$\times \left[\left(1 + \frac{4m_\tau^2}{s}\right) + \left(1 - \frac{4m_\tau^2}{s}\right)\cos^2\theta_\tau + A_P \left(\sin\theta_\pi \cos\phi_\pi \quad \sin\theta_\pi \sin\phi_\pi \quad \cos\theta_\pi \right) \right.$$
$$\left. \times \begin{pmatrix} \left(1 + \frac{4m_\tau^2}{s}\right)\sin^2\theta_\tau & 0 & -\frac{2m_\tau}{\sqrt{s}}\sin 2\theta_\tau \\ 0 & \left(1 - \frac{4m_\tau^2}{s}\right)\sin^2\theta_\tau & 0 \\ \frac{2m_\tau}{\sqrt{s}}\sin 2\theta_\tau & 0 & -\left(1 - \frac{4m_\tau^2}{s}\right) - \left(1 + \frac{4m_\tau^2}{s}\right)\cos^2\theta_\tau \end{pmatrix} \begin{pmatrix} \sin\theta_\mu \cos\phi_\mu \\ \sin\theta_\mu \sin\phi_\mu \\ \cos\theta_\mu \end{pmatrix} \right], \quad (37)$$

where

$$\sigma(e^+e^-\to\tau^+\tau^-)=\frac{4\pi\alpha^2}{3s}\sqrt{1-\frac{4m_\tau^2}{s}}\left(1+\frac{2m_\tau^2}{s}\right) \tag{38}$$

is the $\tau^+\tau^-$ production cross section. The branching ratio of $\tau^-\to\pi^-\nu$ and $\tau^+\to\mu^+\gamma$ is given by

$$B(\tau^-\to\pi^-\nu)=\frac{1}{\Gamma}\frac{1}{8\pi}|V_{ud}|^2 f_\pi^2 G_F^2 m_\tau^3, \tag{39}$$

$$B(\tau^+\to\mu^+\gamma)=\frac{1}{\Gamma}\frac{2}{\pi}G_F^2 m_\tau^5(|A_L|^2+|A_R|^2), \tag{40}$$

and the asymmetry parameter A_P is defined as follows:

$$A_P\equiv\frac{|A_L|^2-|A_R|^2}{|A_L|^2+|A_R|^2}. \tag{41}$$

We can see that the measurement of angular correlation of the pion and muon momentum enables us to determine the parameter A_P, so that we can obtain $|A_L|^2$ and $|A_R|^2$ separately.

A simpler expression can be obtained if we integrate over the angle θ_τ, ϕ_τ, ϕ_π, and ϕ_γ in Eq. (37). The differential cross section is given by

$$\begin{aligned}d\sigma(e^+e^-\to\tau^+\tau^-\to\mu^+\gamma+\pi^-\nu)\\ =\sigma(e^+e^-\to\tau^+\tau^-)B(\tau^+\to\mu^+\gamma)B(\tau^-\to\pi^-\nu)\\ \times\frac{d\cos\theta_\mu}{2}\frac{d\cos\theta_\pi}{2}\left(1-\frac{s-2m_\tau^2}{s+2m_\tau^2}A_P\cos\theta_\mu\cos\theta_\pi\right).\end{aligned} \tag{42}$$

Notice that angular distribution in the rest frames of τ^+ and τ^- can be easily converted to the energy distribution in the center of mass frame of the e^+e^- collision. We obtain

$$\begin{aligned}d\sigma(e^+e^-\to\tau^+\tau^-\to\mu^+\gamma+\pi^-\nu)\\ =\sigma(e^+e^-\to\tau^+\tau^-)B(\tau^+\to\mu^+\gamma)B(\tau^-\to\pi^-\nu)\\ \times\frac{s}{s-4m_\tau^2}dz_\mu dz_\pi\left(1-\frac{s(s-2m_\tau^2)}{(s-4m_\tau^2)(s+2m_\tau^2)}\right.\\ \left.\times A_P(2z_\mu-1)(2z_\pi-1)\right),\end{aligned} \tag{43}$$

where $z_\mu=E_\mu/E_\tau$ ($z_\pi=E_\pi/E_\tau$), and E_μ, E_π, and $E_\tau=\sqrt{s}/2$ are the energies of the muon, pion, and τ in the center of mass frame, respectively.

The angular (or energy) distributions in Eq. (42) [Eq. (43)] can be understood as follows. Because of the helicity conservation of the $\tau^+\tau^-$ production process, the helicities of τ^+ and τ^- are correlated; namely, $\tau_L^+\tau_R^-$ or $\tau_R^+\tau_L^-$ is produced. This means that two τ spins are parallel in the limit of $\sqrt{s}\gg m_\tau$. In the decay process, the π^- tends to be emitted in the spin direction of τ^- for $\tau^-\to\pi^-\nu$, because of the V-A interaction. On the other hand, for $\tau^+\to\mu^+\gamma$ decay, the muon tends to be emitted in the same direction of the τ^+ spin if $A_P>0$. Therefore the differential branching ratio is enhanced (suppressed) if the sign of $\cos\theta_\mu\cos\theta_\pi$ is negative (positive). In other words, pion and muon energies in the center of mass frame of the e^+e^- collision have a negative correlation if $A_P>0$. If $A_P<0$, we have an opposite correlation.

We can define an asymmetry $A^{\mu^+\gamma,\pi^-\nu}$ by the following asymmetric integrations:

$$A^{\mu^+\gamma,\pi^-\nu}=\frac{\int d\cos\theta_\mu\, d\cos\theta_\pi\, w(\cos\theta_\mu,\cos\theta_\pi)\dfrac{d^2\sigma}{d\cos\theta_\mu d\cos\theta_\pi}}{\sigma(e^+e^-\to\tau^+\tau^-)B(\tau^+\to\mu^+\gamma)B(\tau^-\to\pi^-\nu)}=\frac{N^{++}+N^{--}-N^{+-}-N^{-+}}{N^{++}+N^{--}+N^{+-}+N^{-+}}, \tag{44}$$

where the weight function $w(u,v)$ is defined by

$$w(u,v)=\frac{uv}{|uv|}, \tag{45}$$

and shown in Fig. 2. In Eq. (44), $N^{\pm\pm}$ are the event numbers where the first $\pm$ represents the sign of $\cos\theta_\mu$ and the second one is that of $\cos\theta_\pi$, respectively. $A^{\mu^+\gamma,\pi^-\nu}$ is related to the parameter A_P by

$$A^{\mu^+\gamma,\pi^-\nu}=-\frac{s-2m_\tau^2}{4(s+2m_\tau^2)}A_P. \tag{46}$$

In Fig. 3, the $\sqrt{s}$ dependence of $A^{\mu^+\gamma,\pi^-\gamma}$ is shown for $A_P=-1$. We can see that the asymmetry is already close to the maximal value at the B-factory energy.

It is straightforward to extend the above formula to other cases. We only present here formulas corresponding to Eq. (42) for different decay modes of τ^-:

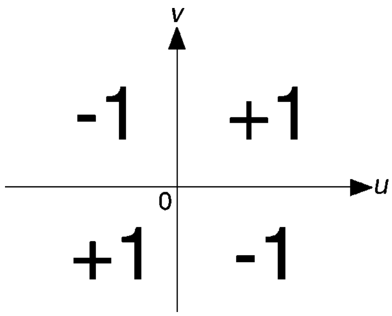

FIG. 2. The weight function $w(u,v)$.

$$d\sigma(e^+e^-\to\tau^+\tau^-\to\mu^+\gamma+V^-\nu)$$
$$=\sigma(e^+e^-\to\tau^+\tau^-)B(\tau^+\to\mu^+\gamma)B(\tau^-\to V^-\nu)$$
$$\times\frac{d\cos\theta_\mu}{2}\frac{d\cos\theta_V}{2}$$
$$\times\left(1\pm\frac{s-2m_\tau^2}{s+2m_\tau^2}A_P\cos\theta_\mu\cos\theta_V\right), \qquad (47)$$

where $+$ corresponds to the vector mesons with transverse polarization $V=\rho(T),a_1(T)$ and $-$ corresponds to those with longitudinal polarization $V=\rho(L),a_1(L)$. For leptonic decay, we obtain

$$d\sigma(e^+e^-\to\tau^+\tau^-\to\mu^+\gamma+l^-\overline{\nu}\nu)$$
$$=\sigma(e^+e^-\to\tau^+\tau^-)B(\tau^+\to\mu^+\gamma)B(\tau^-\to l^-\overline{\nu}\nu)$$
$$\times\frac{d\cos\theta_\mu}{2}\frac{d\cos\theta_l}{2}dx\;2x^2$$
$$\times\left\{3-2x-\frac{s-2m_\tau^2}{s+2m_\tau^2}(1-2x)A_P\cos\theta_\mu\cos\theta_l\right\}. \qquad (48)$$

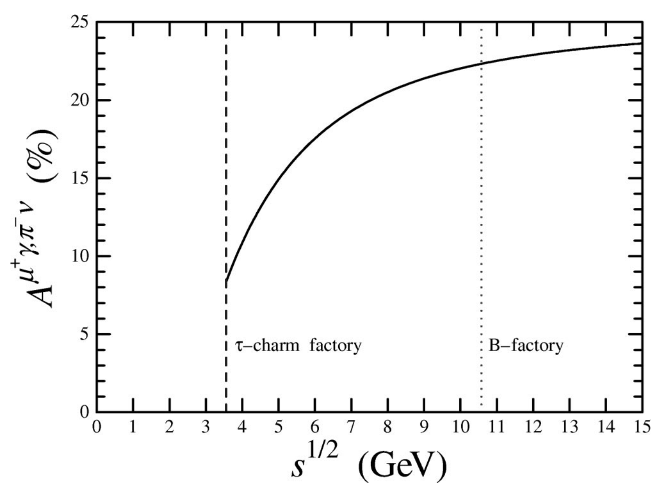

FIG. 3. The observable asymmetry $A^{\mu^+\gamma,\pi^-\nu}$ vs $\sqrt{s}$ for $A_P=-1$. The dashed line represents the $\sqrt{s}$ of τ-charm factory and the dotted line represents that of B factory.

The measurement of the polarization of the vector mesons can be done by the analysis of the distribution of the two (or three) pions from the ρ (a_1) meson decay [17].

In the case of τ^- decays into $\mu^-\gamma$ and τ^+ decays via the *V-A* interaction, the $dR_a^{\tau^-\to f_A}$ and $dR_b^{\tau^+\to f_B}$ acquire extra minus signs. For example,

$$dR_a^{\tau^-\to\mu^-\gamma}=-\frac{d\Omega'_\mu}{4\pi}\frac{1}{\Gamma}\frac{2}{\pi}G_F^2m_\tau^5(|A_L|^2-|A_R|^2)\times\begin{pmatrix}\sin\theta'_\mu\cos\phi'_\mu\\ \sin\theta'_\mu\sin\phi'_\mu\\ \cos\theta'_\mu\end{pmatrix}, \qquad (49)$$

$$dR_b^{\tau^+\to\pi^+\overline{\nu}}=-\frac{d\Omega'_\pi}{4\pi}\frac{1}{\Gamma}\frac{1}{8\pi}|V_{ud}|^2f_\pi^2G_F^2m_\tau^3\times\begin{pmatrix}\sin\theta'_\pi\cos\phi'_\pi\\ \sin\theta'_\pi\sin\phi'_\pi\\ \cos\theta'_\pi\end{pmatrix}, \qquad (50)$$

where (θ'_μ,ϕ'_μ) $[(\theta'_\pi,\phi'_\pi)]$ are the polar angle of the muon [pion] momentum in frame 3 [frame 2]. The formula in Eq. (42) can be applied to the $\tau^-\to\mu^-\gamma$ case by the replacement of (θ_μ,θ_π) by $(\theta'_\mu,\theta'_\pi)$, and therefore same angular and energy correlation holds as in the $\tau^+\to\mu^+\gamma$ case. In a similar way, we can obtain the formulas corresponding to Eqs. (47) and (48) for the $\tau^-\to\mu^-\gamma$ case by the replacement of (θ_V,θ_l) by (θ'_V,θ'_l), where $\theta'_V(\theta'_l)$ is the angle between the vector meson (lepton) momentum and τ^+ direction in frame 2.

IV. *P* AND *T* ASYMMETRIES IN LFV THREE BODY τ DECAYS

In this section, we consider LFV three body decays, i.e., $\tau\to3\mu$, $\tau\to3e$, $\tau\to\mu ee$, and $\tau\to e\mu\mu$. Within the approximation that the muon and electron masses are neglected, $\tau\to3\mu$ and $\tau\to3e$ (or $\tau\to\mu ee$ and $\tau\to e\mu\mu$) give the same formula, so that we only consider $\tau\to3\mu$ and $\tau\to\mu ee$ processes. In these processes, we can define the *P* odd as well as *T* odd asymmetries of τ decays.

For $\tau^+\to\mu^+\mu^+\mu^-$ decay, the effective Lagrangian is given by

$$\mathcal{L}=-\frac{4G_F}{\sqrt{2}}\{m_\tau A_R\overline{\tau}\sigma^{\mu\nu}P_L\mu F_{\mu\nu}+m_\tau A_L\overline{\tau}\sigma^{\mu\nu}P_R\mu F_{\mu\nu}$$
$$+g_1(\overline{\tau}P_L\mu)(\overline{\mu}P_L\mu)+g_2(\overline{\tau}P_R\mu)(\overline{\mu}P_R\mu)$$
$$+g_3(\overline{\tau}\gamma^\mu P_R\mu)(\overline{\mu}\gamma_\mu P_R\mu)+g_4(\overline{\tau}\gamma^\mu P_L\mu)(\overline{\mu}\gamma_\mu P_L\mu)$$
$$+g_5(\overline{\tau}\gamma^\mu P_R\mu)(\overline{\mu}\gamma_\mu P_L\mu)+g_6(\overline{\tau}\gamma^\mu P_L\mu)$$
$$\times(\overline{\mu}\gamma_\mu P_R\mu)+\text{H.c.}\}. \qquad (51)$$

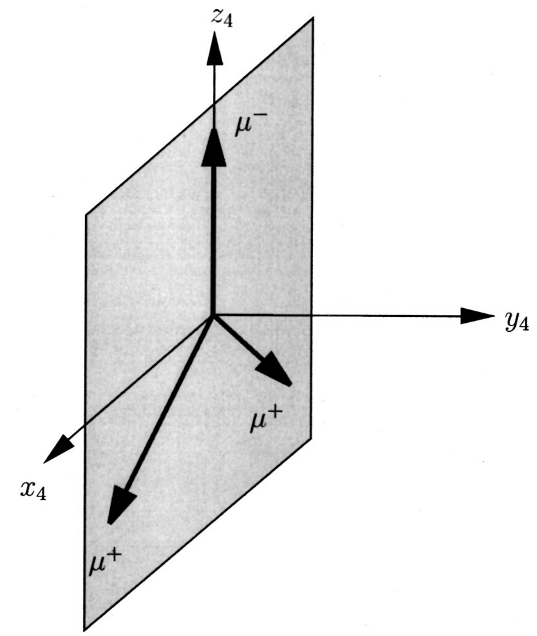

FIG. 4. The coordinate system in the $\tau \to 3\mu$ calculation.

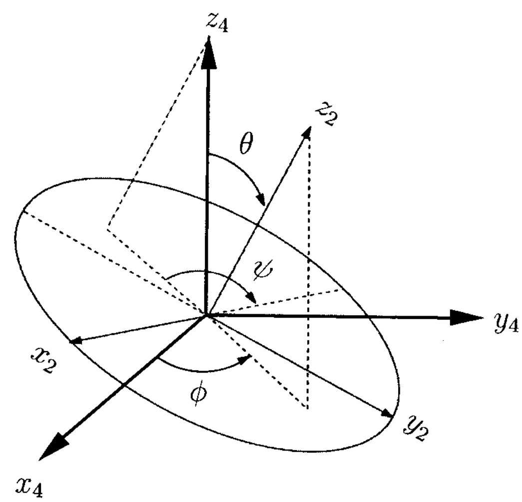

FIG. 5. The relation between frame 2 and frame 4.

With this Lagrangian in Eq. (51), we can calculate the differential branching ratio $dB^{\tau^+ \to 3\mu}$ and the spin dependence term $dR_b^{\tau^+ \to 3\mu}$ in Eq. (1). In order to calculate these quantities we first define the Lorentz frame (frame 4) for the three body decays [12]. Frame 4 is the rest frame of τ^+ and we take the z direction to be the μ^- momentum direction, and the xz plane to be the decay plane. The x direction is determined so that the x component of the momentum for the μ^+ with larger energy is positive. The coordinate system is shown in Fig. 4. Any four-vector in frame 4 is related to that in frame 2 by Euler rotation with three angles (θ, ϕ, ψ) as follows (Fig. 5):

$$\xi_4^\mu = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\phi & -\sin\phi & 0 \\ 0 & \sin\phi & \cos\phi & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\theta & 0 & \sin\theta \\ 0 & 0 & 1 & 0 \\ 0 & -\sin\theta & 0 & \cos\theta \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\psi & -\sin\psi & 0 \\ 0 & \sin\psi & \cos\psi & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \xi_2^\mu, \tag{52}$$

where

$$0 \leq \theta \leq \pi, \quad 0 \leq \phi \leq 2\pi, \quad 0 \leq \psi \leq 2\pi. \tag{53}$$

We also define the energy variables $x_1 = 2E_1/m_\tau$ and $x_2 = 2E_2/m_\tau$ where $E_1(E_2)$ is the energy of μ^+ with a larger (smaller) energy in the rest frame of τ^+.

With these angles and energy variables, the branching ratio and spin dependence term can be expressed as follows:

$$dB^{\tau^+ \to 3\mu} = \frac{1}{\Gamma} \frac{m_\tau^5 G_F^2}{256\pi^5} dx_1\, dx_2\, d\cos\theta\, d\phi\, d\psi\, X, \tag{54}$$

$$dR_b^{\tau^+ \to 3\mu} = \frac{1}{\Gamma} \frac{m_\tau^5 G_F^2}{256\pi^5} dx_1\, dx_2\, d\cos\theta\, d\phi\, d\psi \times \begin{pmatrix} -Ys_\theta c_\psi + Z(c_\theta c_\phi c_\psi - s_\phi s_\psi) + W(c_\theta s_\phi c_\psi + c_\phi s_\psi) \\ Ys_\theta s_\psi + Z(-c_\theta c_\phi s_\psi - s_\phi c_\psi) + W(-c_\theta s_\phi s_\psi + c_\phi c_\psi) \\ Yc_\theta + Zs_\theta c_\phi + Ws_\theta s_\phi \end{pmatrix}, \tag{55}$$

where $s_\theta(s_\phi, s_\psi)$ and $c_\theta(c_\phi, c_\psi)$ represent $\sin\theta(\sin\phi, \sin\psi)$ and $\cos\theta(\cos\phi, \cos\psi)$, respectively. The functions X, Y, Z, and W are defined as follows:

$$X=\left(\frac{|g_1|^2}{16}+\frac{|g_2|^2}{16}+|g_3|^2+|g_4|^2\right)\alpha_1(x_1,x_2)+(|g_5|^2+|g_6|^2)\alpha_2(x_1,x_2)+(|eA_R|^2+|eA_L|^2)\alpha_3(x_1,x_2)$$
$$-\mathrm{Re}(eA_Rg_4^*+eA_Lg_3^*)\alpha_4(x_1,x_2)-\mathrm{Re}(eA_Rg_6^*+eA_Lg_5^*)\alpha_5(x_1,x_2), \tag{56}$$

$$Y=\left(\frac{|g_1|^2}{16}-\frac{|g_2|^2}{16}+|g_3|^2-|g_4|^2\right)\alpha_1(x_1,x_2)+\mathrm{Re}(eA_Rg_4^*-eA_Lg_3^*)\alpha_4(x_1,x_2)$$
$$-\mathrm{Re}(eA_Rg_6^*-eA_Lg_5^*)\alpha_5(x_1,x_2)+(|g_5|^2-|g_6|^2)\beta_1(x_1,x_2)+(|eA_R|^2-|eA_L|^2)\beta_2(x_1,x_2), \tag{57}$$

$$Z=(|g_5|^2-|g_6|^2)\gamma_1(x_1,x_2)+(|eA_R|^2-|eA_L|^2)\gamma_2(x_1,x_2)-\mathrm{Re}(eA_Rg_4^*-eA_Lg_3^*)\gamma_3(x_1,x_2)$$
$$+\mathrm{Re}(eA_Rg_6^*-eA_Lg_5^*)\gamma_4(x_1,x_2), \tag{58}$$

$$W=-\mathrm{Im}(eA_Rg_4^*+eA_Lg_3^*)\gamma_3(x_1,x_2)+\mathrm{Im}(eA_Rg_6^*+eA_Lg_5^*)\gamma_4(x_1,x_2), \tag{59}$$

where $e(>0)$ is the positron charge and functions α_{1-5}, β_{1-2}, and γ_{1-4} are given in Appendix B. Notice that the Y and Z terms represent P odd quantities with respect to the τ^+ spin in the rest frame of τ^+ and the W term represents a T odd quantity. These are the same as the P and T odd terms considered in the differential decay width of $\mu^+\to e^+e^+e^-$ [12]. The sign differences in some terms of the above expressions from the formulas in Ref. [12] are due to the difference of the sign convention in the definition of the covariant derivative.

The differential cross section is obtained by substituting this into Eq. (1). In the case that the opposite side τ decays into $\pi^-\nu$, we obtain, after integrating over ϕ_π, ψ, θ_τ, and ϕ_τ,

$$d\sigma(e^+e^-\to\tau^+\tau^-\to\mu^+\mu^+\mu^-+\pi^-\nu)=\sigma(e^+e^-\to\tau^+\tau^-)B(\tau^-\to\pi^-\nu)\left(\frac{m_\tau^5G_F^2}{128\pi^4}\Big/\Gamma\right)\frac{d\cos\theta_\pi}{2}\,dx_1\,dx_2\,d\cos\theta\,d\phi$$
$$\times\left[X-\frac{s-2m_\tau^2}{s+2m_\tau^2}\{Y\cos\theta+Z\sin\theta\cos\phi+W\sin\theta\sin\phi\}\cos\theta_\pi\right]. \tag{60}$$

The terms X, Y, Z, and W can be extracted by the following (asymmetric) integrations:

$$\int d\cos\theta d\phi d\cos\theta_\pi\,\frac{d^5\sigma}{dx_1dx_2d\cos\theta d\phi d\cos\theta_\pi}\left\{\sigma(e^+e^-\to\tau^+\tau^-)B(\tau^-\to\pi^-\nu)\left(\frac{m_\tau^5G_F^2}{32\pi^3}\Big/\Gamma\right)\right\}^{-1}$$
$$=X, \tag{61}$$

$$\int d\cos\theta d\cos\theta_\pi w(\cos\theta,\cos\theta_\pi)\,\frac{d^4\sigma}{dx_1dx_2d\cos\theta d\cos\theta_\pi}\left\{\sigma(e^+e^-\to\tau^+\tau^-)B(\tau^-\to\pi^-\nu)\left(\frac{m_\tau^5G_F^2}{32\pi^3}\Big/\Gamma\right)\right\}^{-1}$$
$$=-\frac{(s-2m_\tau^2)}{4(s+2m_\tau^2)}Y, \tag{62}$$

$$\int d\phi d\cos\theta_\pi w(\cos\phi,\cos\theta_\pi)\,\frac{d^4\sigma}{dx_1dx_2d\phi d\cos\theta_\pi}\left\{\sigma(e^+e^-\to\tau^+\tau^-)B(\tau^-\to\pi^-\nu)\left(\frac{m_\tau^5G_F^2}{32\pi^3}\Big/\Gamma\right)\right\}^{-1}$$
$$=-\frac{(s-2m_\tau^2)}{4(s+2m_\tau^2)}Z, \tag{63}$$

$$\int d\phi d\cos\theta_\pi w(\sin\phi,\cos\theta_\pi)\,\frac{d^4\sigma}{dx_1dx_2d\phi d\cos\theta_\pi}\left\{\sigma(e^+e^-\to\tau^+\tau^-)B(\tau^-\to\pi^-\nu)\left(\frac{m_\tau^5G_F^2}{32\pi^3}\Big/\Gamma\right)\right\}^{-1}$$
$$=-\frac{(s-2m_\tau^2)}{4(s+2m_\tau^2)}W. \tag{64}$$

Notice that the function W represents a CP violating LFV interaction. We can see that this is induced by the relative phase between the photon-penguin coupling constants (A_L and A_R) and the four-fermion coupling constants (g_3–g_6).

A similar formula can be obtained for the $\tau^+\to\mu^+e^+e^-$ decay. The effective Lagrangian for the $\tau^+\to\mu^+e^+e^-$ is given by

$$\mathcal{L}=-\frac{4G_F}{\sqrt{2}}\{m_\tau A_R\overline{\tau}\sigma^{\mu\nu}P_L\mu F_{\mu\nu}+m_\tau A_L\overline{\tau}\sigma^{\mu\nu}P_R\mu F_{\mu\nu}+\lambda_1(\overline{\tau}P_L\mu)(\overline{e}P_Le)+\lambda_2(\overline{\tau}P_L\mu)(\overline{e}P_Re)+\lambda_3(\overline{\tau}P_R\mu)(\overline{e}P_Le)$$
$$+\lambda_4(\overline{\tau}P_R\mu)(\overline{e}P_Re)+\lambda_5(\overline{\tau}\gamma^\mu P_L\mu)(\overline{e}\gamma_\mu P_Le)+\lambda_6(\overline{\tau}\gamma^\mu P_L\mu)(\overline{e}\gamma_\mu P_Re)+\lambda_7(\overline{\tau}\gamma^\mu P_R\mu)(\overline{e}\gamma_\mu P_Le)$$
$$+\lambda_8(\overline{\tau}\gamma^\mu P_R\mu)(\overline{e}\gamma_\mu P_Re)+\lambda_9(\overline{\tau}\sigma^{\mu\nu}P_L\mu)(\overline{e}\sigma_{\mu\nu}e)+\lambda_{10}(\overline{\tau}\sigma^{\mu\nu}P_R\mu)(\overline{e}\sigma_{\mu\nu}e)+\text{H.c.}\}. \tag{65}$$

In this calculation, we define frame 4′ which is almost the same as frame 4 in the $\tau^+\to\mu^+\mu^+\mu^-$ case. The definition is obtained by the replacement of μ^- of $\tau^+\to\mu^+\mu^+\mu^-$ by e^- of $\tau^+\to\mu^+e^+e^-$, the μ^+ with a larger energy of $\tau^+\to\mu^+\mu^+\mu^-$ by μ^+ of $\tau^+\to\mu^+e^+e^-$, and μ^+ with a smaller energy of $\tau^+\to\mu^+\mu^+\mu^-$ by e^+ of $\tau^+\to\mu^+e^+e^-$. If we take the definition of $(\theta,\ \phi,\ \psi)$ in such a way that the same relation is satisfied as in Eq. (52), the branching ratio and the spin dependence term are given by

$$dB^{\tau^+\to\mu^+e^+e^-}=\frac{1}{\Gamma}\frac{m_\tau^5G_F^2}{256\pi^5}\,dx_1dx_2d\cos\theta d\phi d\psi X', \tag{66}$$

$$dR_b^{\tau^+\to\mu^+e^+e^-}=\frac{1}{\Gamma}\frac{m_\tau^5G_F^2}{256\pi^5}\,dx_1dx_2d\cos\theta d\phi d\psi$$
$$\times\begin{pmatrix}-Y's_\theta c_\psi+Z'(c_\theta c_\phi c_\psi-s_\phi s_\psi)+W'(c_\theta s_\phi c_\psi+c_\phi s_\psi)\\ Y's_\theta s_\psi+Z'(-c_\theta c_\phi s_\psi-s_\phi c_\psi)+W'(-c_\theta s_\phi s_\psi+c_\phi c_\psi)\\ Y'c_\theta+Z's_\theta c_\phi+W's_\theta s_\phi\end{pmatrix}, \tag{67}$$

where the functions X', Y', Z', and W' are given by

$$X'=(|eA_R|^2+|eA_L|^2)A_1(x_1,x_2)+\text{Re}(eA_R\lambda_5^*+eA_L\lambda_8^*)A_2(x_1,x_2)+\text{Re}(eA_R\lambda_6^*+eA_L\lambda_7^*)A_3(x_2)$$
$$+(|\lambda_1|^2+|\lambda_2|^2+|\lambda_3|^2+|\lambda_4|^2)A_4(x_1)+(|\lambda_5|^2+|\lambda_8|^2)A_5(x_1,x_2)+(|\lambda_6|^2+|\lambda_7|^2)A_6(x_2)$$
$$+(|\lambda_9|^2+|\lambda_{10}|^2)A_7(x_1,x_2)+\text{Re}(\lambda_1\lambda_9^*+\lambda_4\lambda_{10}^*)A_8(x_1,x_2), \tag{68}$$

$$Y'=-\text{Re}(eA_R\lambda_5^*-eA_L\lambda_8^*)A_2(x_1,x_2)+\text{Re}(eA_R\lambda_6^*-eA_L\lambda_7^*)A_3(x_1,x_2)-(|\lambda_5|^2-|\lambda_8|^2)A_5(x_1,x_2)$$
$$+(|eA_R|^2-|eA_L|^2)B_1(x_1,x_2)+(|\lambda_1|^2+|\lambda_2|^2-|\lambda_3|^2-|\lambda_4|^2)B_2(x_1,x_2)$$
$$+(|\lambda_6|^2-|\lambda_7|^2)B_3(x_1,x_2)+(|\lambda_9|^2-|\lambda_{10}|^2)B_4(x_1,x_2)+\text{Re}(\lambda_1\lambda_9^*-\lambda_4\lambda_{10}^*)B_5(x_1,x_2), \tag{69}$$

$$Z'=(|eA_R|^2-|eA_L|^2)C_1(x_1,x_2)+\text{Re}(eA_R\lambda_5^*-eA_L\lambda_8^*)C_2(x_1,x_2)+\text{Re}(eA_R\lambda_6^*-eA_L\lambda_7^*)C_3(x_1,x_2)$$
$$+(|\lambda_1|^2+|\lambda_2|^2-|\lambda_3|^2-|\lambda_4|^2)C_4(x_1,x_2)+\{|\lambda_6|^2-|\lambda_7|^2+\text{Re}(-2\lambda_1\lambda_9^*+2\lambda_4\lambda_{10}^*)\}C_5(x_1,x_2)$$
$$+(|\lambda_9|^2-|\lambda_{10}|^2)C_6(x_1,x_2), \tag{70}$$

$$W'=\text{Im}(eA_R\lambda_5^*+eA_L\lambda_8^*)C_2(x_1,x_2)+\text{Im}(eA_R\lambda_6^*+eA_L\lambda_7^*)C_3(x_1,x_2)+\text{Im}(\lambda_1\lambda_9^*+\lambda_4\lambda_{10}^*)C_7(x_1,x_2). \tag{71}$$

The functions A_{1-8},B_{1-5},C_{1-7} are given in Appendix B. The X', Y', Z', and W' can be extracted in the same way as in Eqs. (61)–(64).

Next we consider the decay mode of $\tau^+\to\mu^-e^+e^+$. This case is different from above in the point that both $\tau\to e$ and $\mu\to e$ transitions are necessary. The effective Lagrangian for this process is given by

$$\mathcal{L}=-\frac{4G_F}{\sqrt{2}}\{g_1'(\overline{\tau}P_Le)(\overline{\mu}P_Le)+g_2'(\overline{\tau}P_Re)(\overline{\mu}P_Re)+g_3'(\overline{\tau}\gamma^\mu P_Re)(\overline{\mu}\gamma_\mu P_Re)$$
$$+g_4'(\overline{\tau}\gamma^\mu P_Le)(\overline{\mu}\gamma_\mu P_Le)+g_5'(\overline{\tau}\gamma^\mu P_Re)(\overline{\mu}\gamma_\mu P_Le)+g_6'(\overline{\tau}\gamma^\mu P_Le)(\overline{\mu}\gamma_\mu P_Re)+\text{H.c.}\}. \tag{72}$$

If we take a coordinate system similar to frame 4, in which the larger (smaller) energy μ^+ is replaced by the larger (smaller) energy e^+, $dB^{\tau^+\to\mu^-e^+e^+}$ and $dR_b^{\tau^+\to\mu^-e^+e^+}$ are given by

$$dB^{\tau^+\to\mu^-e^+e^+}=\frac{1}{\Gamma}\frac{m_\tau^5G_F^2}{256\pi^5}\,dx_1dx_2d\cos\theta d\phi d\psi X'', \tag{73}$$

$$dR_b^{\tau^+\to\mu^+e^+e^-}=\frac{1}{\Gamma}\frac{m_\tau^5G_F^2}{256\pi^5}\,dx_1dx_2d\cos\theta d\phi d\psi\begin{pmatrix}-Y''s_\theta c_\psi+Z''(c_\theta c_\phi c_\psi-s_\phi s_\psi)\\ Y''s_\theta s_\psi+Z''(-c_\theta c_\phi s_\psi-s_\phi c_\psi)\\ Y''c_\theta+Z''s_\theta c_\phi\end{pmatrix}, \tag{74}$$

where functions X'', Y'', and Z'' are given by

$$X''=\left(\frac{|g_1'|^2}{16}+\frac{|g_2'|^2}{16}+|g_3'|^2+|g_4'|^2\right)\alpha_1(x_1,x_2)+(|g_5'|^2+|g_6'|^2)\alpha_2(x_1,x_2), \tag{75}$$

$$Y''=\left(\frac{|g_1'|^2}{16}-\frac{|g_2'|^2}{16}+|g_3'|^2-|g_4'|^2\right)\alpha_1(x_1,x_2)+(|g_5'|^2-|g_6'|^2)\beta_1(x_1,x_2), \tag{76}$$

$$Z''=(|g_5'|^2-|g_6'|^2)\gamma_1(x_1,x_2), \tag{77}$$

where α_{1-2}, β_1, and γ_1 are the same functions that we defined in the $\tau^+\to\mu^+\mu^+\mu^-$ calculation. X'', Y'', and Z'' can be extracted by asymmetric integrations as before, but we cannot obtain information on CP violation in this case.

Notice that the above three cases exhaust all possibilities in the three body decay of τ to e and/or μ as long as we neglect the electron and muon masses compared to the τ mass. Namely, the formula for other cases can be obtained by appropriate replacements of e and/or μ.

The formulas for LFV decays with τ^- can be obtained in a similar substitution as the $\tau\to\mu\gamma$ case. Using appropriate angles of τ^- decay in frame 3 and τ^+ decay in frame 2, dR_b gets an extra minus sign in Eqs. (55), (67), and (74).

V. $\tau\to\mu\nu\bar{\nu}\gamma$ PROCESS AND BACKGROUND SUPPRESSIONS

In this section, we consider the background processes for the $\tau\to\mu\gamma$ search, and we show that the measurement of angular distributions is useful in identifying the background process. In the muon decay, the physical background can be suppressed if we use polarized muons [15]. In the following, we show a similar suppression mechanism holds for τ decay if we use the spin correlation.

One of the main backgrounds for the $\tau\to\mu\gamma$ search comes from the kinematical end point region of the $\tau\to\mu\nu\bar{\nu}\gamma$ process where two neutrinos carry out a little energy at the rest frame of τ. In the following, we assume that τ^+ decays into $\mu^+\nu\bar{\nu}\gamma$ and τ^- decays through one of hadronic and leptonic decay processes. For the τ^- decay, the differential branching ratio and the spin dependence term are given in Eqs. (28)–(36). For $\tau^+\to\mu^+\nu\bar{\nu}\gamma$, these quantities are given by

$$dB^{\mathrm{B.G.}}=\frac{1}{\Gamma}\frac{G_F^2m_\tau^5\alpha}{3\times2^{11}\pi^5}\,dxdydzd\Omega_\mu\,\sin z\,\frac{\beta_\mu}{y}F, \tag{78}$$

$$dR_b^{\mathrm{B.G.}}=\frac{1}{\Gamma}\frac{G_F^2m_\tau^5\alpha}{3\times2^{11}\pi^5}\,dxdydzd\Omega_\mu\,\sin z\,\frac{\beta_\mu}{y}\times(-\beta_\mu G+H\cos z)\begin{pmatrix}\sin\theta_\mu\cos\phi_\mu\\ \sin\theta_\mu\sin\phi_\mu\\ \cos\theta_\mu\end{pmatrix}, \tag{79}$$

where x and y are the muon and photon energies normalized by $m_\tau/2$, respectively, and (θ_μ,ϕ_μ) is the polar coordinate of the unit vector of the muon momentum direction, all defined in the rest frame of τ^+ (frame 2). Here $\beta_\mu=\sqrt{1-4r/x^2}$ with $r\equiv m_\mu^2/m_\tau^2$. The angle z is defined by $z\equiv\pi-\theta_{\mu\gamma}$, where $\theta_{\mu\gamma}$ is the angle between the muon and photon momentum in the same frame. These quantities can be obtained by a simple replacement from the formula of the differential decay width for the radiative muon decay presented in Ref. [18]. For completeness, the functions F, G, and H are given in Appendix B.

The background comes from the kinematical region near $x=1+r$ and $y=1-r$, at which the branching fraction vanishes. However, with finite detector resolutions, this kinematical region gives physical backgrounds. If we take the signal region as $1+r-\delta x\leqslant x\leqslant 1+r$ and $1-r-\delta y\leqslant y\leqslant 1-r$, the leading terms of the branching ratio and spin dependence term expanded in terms of r, δx, and δy, after integrating over z, are given by

$$dB^{\mathrm{B.G.}} \simeq \frac{1}{\Gamma}\frac{G_F^2 m_\tau^5 \alpha}{3\times 2^{11}\pi^5}\, d\Omega_\mu\left(\delta x^4 \delta y^2 + \frac{8}{3}\,\delta x^3 \delta y^3\right), \tag{80}$$

$$dR_b^{\mathrm{B.G.}} \simeq \frac{1}{\Gamma}\frac{G_F^2 m_\tau^5 \alpha}{3\times 2^{11}\pi^5}\, d\Omega_\mu\left(-\delta x^4 \delta y^2 + \frac{8}{3}\,\delta x^3 \delta y^3\right) \times \begin{pmatrix} \sin\theta_\mu \cos\phi_\mu \\ \sin\theta_\mu \sin\phi_\mu \\ \cos\theta_\mu \end{pmatrix}. \tag{81}$$

Then after integrating over ϕ_μ, ϕ_π, ϕ_τ, and θ_τ, the differential cross section for $e^+e^- \to \tau^+\tau^- \to \mu^+\nu\bar{\nu}\gamma + \pi^-\nu$ is given by

$$\begin{aligned} d\sigma(e^+e^- &\to \tau^+\tau^- \to \mu^+\nu\bar{\nu}\gamma + \pi^-\nu) \\ &= \sigma(e^+e^- \to \tau^+\tau^-) B(\tau^- \to \pi^-\nu) \\ &\quad \times \left(\frac{G_F^2 m_\tau^5 \alpha}{3\times 2^9 \pi^4}\Big/\Gamma\right)\frac{d\cos\theta_\mu}{2}\,\frac{d\cos\theta_\pi}{2} \\ &\quad \times\left\{\left(\delta x^4\delta y^2 + \frac{8}{3}\delta x^3 \delta y^3\right) - \frac{s-2m_\tau^2}{s+2m_\tau^2}\right. \\ &\quad \times\left.\left(-\delta x^4 \delta y^2 + \frac{8}{3}\delta x^3\delta y^3\right)\cos\theta_\mu \cos\theta_\pi\right\}. \end{aligned} \tag{82}$$

If the photon energy resolution is worse than the muon energy resolution, the term $\delta x^4 \delta y^2$ is small compared to $(8/3)\delta x^3 \delta y^3$. In such a case, the angular distribution is similar to the $A_R = 0$, $A_L \neq 0$ case of the $\tau \to \mu\gamma$ angular distribution. See Eqs. (25), (26), and (42). This feature is useful for the background suppressions for the $\tau^+ \to \mu_R^+ \gamma$ search because signal and background processes have different angular correlations. For the $\tau^+ \to \mu_L^+\gamma$ search, the signal to background ratio is almost the same even if we take into account angular correlations.

A similar background suppression works for the $\tau \to e\gamma$ case because Eqs. (80) and (81) do not include the mass of the muon explicitly.

VI. SUMMARY AND DISCUSSION

In this paper, we have calculated the differential cross sections of $e^+e^- \to \tau^+\tau^- \to f_B f_A$ processes, where one of the τ's decays through LFV processes. Using spin correlations of $\tau^+\tau^-$, we show that the P odd asymmetry of $\tau \to \mu\gamma$ and $\tau \to e\gamma$ and P and T asymmetries of three body LFV decays of τ can be obtained by angular correlations. These P and T odd quantities are important to identify a model of new physics responsible for LFV processes.

We have also considered the background suppression of the $\tau \to \mu\gamma$ and $\tau \to e\gamma$ searches by the angular distributions. We see that the analysis of the angular distributions is useful for the $\tau^+ \to \mu_R^+ \gamma(\tau^- \to \mu_L^- \gamma)$ and $\tau^+ \to e_R^+ \gamma(\tau^- \to e_L^-\gamma)$ searches.

We would like to give a rough estimate of the number of $\tau^+\tau^-$ pairs needed for the asymmetry measurement at the B-factory energy. As an example, we take the $\tau \to \mu\gamma$ process for LFV decay and $\tau \to \pi\nu$, $\rho\nu$, and $a_1(\to \pi^\pm\pi^\mp\pi^\mp)\nu$ for the opposite side τ decay. For $\tau \to \pi\nu$, we use the angular distribution in Eq. (42). In $\tau \to \rho\nu$ and $\tau \to a_1\nu$, we have to look at the angular distribution of two or three pions in addition to the $\cos\theta_\mu$ and $\cos\theta_V$ distributions in order to use the information on the ρ and a_1 polarizations. With help of optimized observable quantities defined in Ref. [19], the statistical errors for the determination of the parameter A_P with N signal events are $3.4/\sqrt{N}$, $4.6/\sqrt{N}$, and $9.5/\sqrt{N}$ for $\tau \to \pi\nu$, $\tau \to \rho\nu$, and $\tau \to a_1\nu$, respectively. The combined error is then given by

$$\sigma_{A_P} = \frac{1}{\sqrt{2N_\tau B_{\tau\to\mu\gamma}}}\left(\frac{\epsilon_\pi B_\pi}{3.4^2} + \frac{\epsilon_\rho B_\rho}{4.6^2} + \frac{\epsilon_{a_1} B_{a_1}}{9.5^2}\right)^{-1/2}, \tag{83}$$

where ϵ_π, ϵ_ρ, and ϵ_{a_1} are the signal selection efficiencies for these modes, and $B_{\tau\to\mu\gamma}$, B_π, B_ρ, and B_{a_1} are the τ decay branching ratios, namely, $B_{\tau\to\mu\gamma} = B(\tau \to \mu\gamma)$, $B_\pi = B(\tau \to \pi\nu) = 0.11$, $B_\rho = B(\tau \to \rho\nu) = 0.25$, and $B_{a_1} = B(\tau \to a_1\nu \to \pi^\pm\pi^\mp\pi^\mp\nu) = 0.09$. Here N_τ is the total number of τ pair. If we assume that the $B(\tau \to \mu\gamma)$ is 1×10^{-6}, which is just below the current experimental bound [20], and the signal selection efficiency is 10%–20%, $(2.5\text{–}5)\times 10^8$ $\tau^+\tau^-$ pairs are required in order to distinguish $A_P = +1$ and -1 at 3σ level. This number means that the ongoing B-factory experiments could provide useful information on the LFV interaction if the $B(\tau \to \mu\gamma)$ is close to the current experimental bound.

In this paper, we only consider τ decay. We can obtain similar information in muon decay experiments if initial muons are polarized. Although highly polarized muons are available experimentally, a special setup for the production and transportation of a muon beam is necessary for an actual experiment. The advantage of the τ case is that we can extract the information on τ spins by looking at the decay distribution of the other side of τ decay so that we do not need a special requirement for the experimental setup.

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APPENDIX A: THE DERIVATION OF THE GENERAL FORMULAS

In this section, we derive Eq. (1) from the amplitude in Eq. (8).

By using the completeness relation of the fermion spinors, the amplitude squared is deformed to

$$|\overline{A}(\not{p}_A+m_\tau)\gamma^\mu(\not{p}_B-m_\tau)B|^2=\left|\sum_{\lambda_1=\pm}\sum_{\lambda_2=\pm}\overline{A}u(\mathbf{p}_A,\lambda_1)\overline{u}(\mathbf{p}_A,\lambda_1)\gamma^\mu v(\mathbf{p}_B,\lambda_2)\overline{v}(\mathbf{p}_B,\lambda_2)B\right|^2$$
$$=\sum_{\lambda_1=\pm}\sum_{\lambda_2=\pm}\sum_{\lambda_1'=\pm}\sum_{\lambda_2'=\pm}[\overline{A}u(\mathbf{p}_A,\lambda_1)\overline{u}(\mathbf{p}_A,\lambda_1')A]$$
$$\times[\overline{u}(\mathbf{p}_A,\lambda_1)\gamma^\mu v(\mathbf{p}_B,\lambda_2)\overline{v}(\mathbf{p}_B,\lambda_2')\gamma^\nu u(\mathbf{p}_A,\lambda_1')][\overline{B}v(\mathbf{p}_B,\lambda_2')\overline{v}(\mathbf{p}_B,\lambda_2)B], \tag{A1}$$

where λ's are the spin eigenvalues. The spin summation can be performed by using the Bouchiat-Michel formulas as follows [16,21]:

$$|\overline{A}(\not{p}_A+m_\tau)\gamma^\mu(\not{p}_B-m_\tau)B|^2=\alpha^{D_-}\,\mathrm{Tr}[(\not{p}_A+m_\tau)\gamma^\mu(\not{p}_B-m_\tau)\gamma^\nu]\alpha^{D_+}$$
$$+\alpha^{D_-}\mathrm{Tr}[(\not{p}_A+m_\tau)\gamma^\mu\gamma_5\not{s}_B^b(\not{p}_B-m_\tau)\gamma^\nu]\rho_b^{D_+}$$
$$+\rho_a^{D_-}\,\mathrm{Tr}[\gamma_5\not{s}_A^a(\not{p}_A+m_\tau)\gamma^\mu(\not{p}_B-m_\tau)\gamma^\nu]\alpha^{D_+}$$
$$+\rho_a^{D_-}\,\mathrm{Tr}[\gamma_5\not{s}_A^a(\not{p}_A+m_\tau)\gamma^\mu\gamma_5\not{s}_B^b(\not{p}_B-m_\tau)\gamma^\nu]\rho_b^{D_+}, \tag{A2}$$

where

$$\alpha^{D_-}=\frac{1}{2}\{\overline{A}(\not{p}_A+m_\tau)A\},\quad \alpha^{D_+}=\frac{1}{2}\{\overline{B}(\not{p}_B-m_\tau)B\}, \tag{A3}$$

$$\rho_a^{D_-}=\frac{1}{2}\{\overline{A}\gamma_5\not{s}_A^a(\not{p}_A+m_\tau)A\},$$
$$\rho_b^{D_+}=\frac{1}{2}\{\overline{B}\gamma_5\not{s}_B^b(\not{p}_B-m_\tau)B\}, \tag{A4}$$

where $(s_A^a)^\mu$ and $(s_B^b)^\nu$ are four vectors which satisfy the following equations:

$$p_A\cdot s_A^a=p_B\cdot s_B^b=0, \tag{A5}$$

$$s_A^a\cdot s_A^b=s_B^a\cdot s_B^b=-\delta^{ab}, \tag{A6}$$

$$\sum_{a=1}^{3}(s_A^a)_\mu(s_A^a)_\nu=-g_{\mu\nu}+\frac{p_{A\mu}p_{A\nu}}{m_\tau^2},$$
$$\sum_{b=1}^{3}(s_B^b)_\mu(s_B^b)_\nu=-g_{\mu\nu}+\frac{p_{B\mu}p_{B\nu}}{m_\tau^2}. \tag{A7}$$

The second and third terms in Eq. (A2) vanish because the production parts are antisymmetric on μ and ν indices while the square of the electromagnetic current from e^+e^- collision is symmetric on μ and ν indices. Explicit calculation gives

$$\mathrm{Tr}[(\not{p}_A+m_\tau)\gamma^\mu\gamma_5\not{s}_B^b(\not{p}_B-m_\tau)\gamma^\nu]$$
$$=4im_\tau\epsilon^{\mu\nu\rho\sigma}p_{B\rho}(s_B^b)_\sigma+4im_\tau\epsilon^{\mu\nu\rho\sigma}p_{A\rho}(s_B^b)_\sigma, \tag{A8}$$

$$\mathrm{Tr}[\gamma_5\not{s}_A^a(\not{p}_A+m_\tau)\gamma^\mu(\not{p}_B-m_\tau)\gamma^\nu]$$
$$=4im_\tau\epsilon^{\mu\nu\rho\sigma}p_{B\rho}(s_A^a)_\sigma+4im_\tau\epsilon^{\mu\nu\rho\sigma}p_{A\rho}(s_A^a)_\sigma, \tag{A9}$$

$$\sum_{\mathrm{spin}}|\overline{v}_{e^+}\gamma_\mu u_{e^-}|^2=\mathrm{Tr}[\not{p}_{e^+}\gamma_\mu\not{p}_{e^-}\gamma_\nu]$$
$$=4p_{e^+\mu}p_{e^-\nu}+4p_{e^+\nu}p_{e^-\mu}-4g_{\mu\nu}p_{e^+}\cdot p_{e^-}. \tag{A10}$$

Using the narrow width approximation

$$\left|\frac{1}{q^2-(m-i\Gamma/2)^2}\right|^2\simeq\frac{\pi}{m\Gamma}\delta(q^2-m^2), \tag{A11}$$

the first and last terms in Eq. (A2) give formula (1) after the phase space integral.

APPENDIX B: THE KINEMATICAL FUNCTIONS

In this section, we list the kinematical functions used in the formulas of branching ratios.

The functions α_{1-5}, β_{1-2}, and γ_{1-4} in the $\tau^+\to\mu^+\mu^+\mu^-$ and $\tau^+\to\mu^-e^+e^+$ decay calculations are given as follows. These functions are the same as those used in $\mu^+\to e^+e^+e^-$ decay [12]. Here x_1 and x_2 are given by

$x_1=2E_1/m_\tau$ and $x_2=2E_2/m_\tau$:

$$\alpha_1(x_1,x_2)=8(2-x_1-x_2)(x_1+x_2-1), \tag{B1}$$

$$\alpha_2(x_1,x_2)=2\{x_1(1-x_1)+x_2(1-x_2)\}, \tag{B2}$$

$$\alpha_3(x_1,x_2)=8\left\{\frac{2x_2^2-2x_2+1}{1-x_1}+\frac{2x_1^2-2x_1+1}{1-x_2}\right\}, \tag{B3}$$

$$\alpha_4(x_1,x_2)=32(x_1+x_2-1), \tag{B4}$$

$$\alpha_5(x_1,x_2)=8(2-x_1-x_2), \tag{B5}$$

$$\beta_1(x_1,x_2)=\frac{2(x_1+x_2)(x_1^2+x_2^2)-6(x_1+x_2)^2+12(x_1+x_2)-8}{2-x_1-x_2}, \tag{B6}$$

$$\beta_2(x_1,x_2)=\frac{8}{(1-x_1)(1-x_2)(2-x_1-x_2)}\times\{2(x_1+x_2)(x_1^3+x_2^3)-4(x_1+x_2)\times(2x_1^2+x_1x_2+2x_2^2)+(19x_1^2+30x_1x_2+19x_2^2)-12(2x_1+2x_2-1)\}, \tag{B7}$$

$$\gamma_1(x_1,x_2)=\frac{4\sqrt{(1-x_1)(1-x_2)(x_1+x_2-1)}(x_2-x_1)}{2-x_1-x_2}, \tag{B8}$$

$$\gamma_2(x_1,x_2)=32\sqrt{\frac{x_1+x_2-1}{(1-x_1)(1-x_2)}}\,\frac{(x_1+x_2-1)(x_2-x_1)}{2-x_1-x_2}, \tag{B9}$$

$$\gamma_3(x_1,x_2)=16\sqrt{\frac{x_1+x_2-1}{(1-x_1)(1-x_2)}}\,(x_1+x_2-1)(x_2-x_1), \tag{B10}$$

$$\gamma_4(x_1,x_2)=8\sqrt{\frac{x_1+x_2-1}{(1-x_1)(1-x_2)}}\,(2-x_1-x_2)(x_2-x_1). \tag{B11}$$

The functions A_{1-8}, B_{1-5}, and C_{1-7} in the $\tau^+\to\mu^+e^+e^-$ decay calculation are given by

$$A_1(x_1,x_2)=\frac{8\,(2-x_1-4\,x_2+2\,x_1\,x_2+2\,x_2^2)}{1-x_1}, \tag{B12}$$

$$A_2(x_1,x_2)=-8\,(x_1+x_2-1), \tag{B13}$$

$$A_3(x_2)=-8\,(1-x_2), \tag{B14}$$

$$A_4(x_1)=\frac{x_1(1-x_1)}{2}, \tag{B15}$$

$$A_5(x_1,x_2)=2(2-x_1-x_2)\,(x_1+x_2-1), \tag{B16}$$

$$A_6(x_2)=2\,x_2(1-x_2), \tag{B17}$$

$$A_7(x_1,x_2)=-8(4-5\,x_1+x_1^2-8\,x_2+4\,x_1\,x_2+4\,x_2^2), \tag{B18}$$

$$A_8(x_1,x_2)=-4(1-x_1)\,(x_1+2\,x_2-2), \tag{B19}$$

$$B_1(x_1,x_2)=\frac{-8}{(1-x_1)(2-x_1-x_2)}\times(-6+8x_1-3x_1^2+12x_2-11x_1x_2+2x_1^2x_2-8x_2^2+4x_1x_2^2+2x_2^3), \tag{B20}$$

$$B_2(x_1,x_2)=\frac{-(1-x_1)(2-2\,x_1+x_1^2-2\,x_2+x_1\,x_2)}{2(2-x_1-x_2)}, \tag{B21}$$

$$B_3(x_1,x_2)=\frac{2(1-x_2)(2-2\,x_1-2\,x_2+x_1\,x_2+x_2^2)}{2-x_1-x_2}, \tag{B22}$$

$$B_4(x_1,x_2)=\frac{8}{2-x_1-x_2}\times(-10+16x_1-7x_1^2+x_1^3+22x_2-23x_1x_2+5x_1^2x_2-16x_2^2+8x_1x_2^2+4x_2^3), \tag{B23}$$

$$B_5(x_1,x_2)=\frac{4(1-x_1)(2-4\,x_1+x_1^2-4\,x_2+3\,x_1\,x_2+2\,x_2^2)}{2-x_1-x_2}, \tag{B24}$$

$$C_1(x_1,x_2)=\frac{-16(x_1+x_2-1)\sqrt{(1-x_1)(1-x_2)(x_1+x_2-1)}}{(1-x_1)(2-x_1-x_2)}, \tag{B25}$$

$$C_2(x_1,x_2)=\frac{8(x_1+x_2-1)\sqrt{(1-x_1)(1-x_2)(x_1+x_2-1)}}{1-x_1}, \tag{B26}$$

$$C_3(x_1,x_2)=\frac{-8(1-x_2)\sqrt{(1-x_1)(1-x_2)(x_1+x_2-1)}}{1-x_1}, \tag{B27}$$

$$C_4(x_1,x_2)=\frac{(1-x_1)\sqrt{(1-x_1)(1-x_2)(x_1+x_2-1)}}{2-x_1-x_2}, \tag{B28}$$

$$C_5(x_1,x_2)=\frac{4(1-x_2)\sqrt{(1-x_1)(1-x_2)(x_1+x_2-1)}}{2-x_1-x_2}, \tag{B29}$$

$$C_6(x_1,x_2) = \frac{-16\sqrt{(1-x_1)(1-x_2)(x_1+x_2-1)}(3-x_1-2x_2)}{2-x_1-x_2}, \tag{B30}$$

$$C_7(x_1,x_2)=8\sqrt{(1-x_1)(1-x_2)(x_1+x_2-1)}. \tag{B31}$$

Finally, the functions F, G, and H in the $\tau\to\mu\nu\bar{\nu}\gamma$ decay calculation are given by

$$F=F^{(0)}+rF^{(1)}+r^2F^{(2)}, \tag{B32}$$

$$G=G^{(0)}+rG^{(1)}+r^2G^{(2)}, \tag{B33}$$

$$H=H^{(0)}+rH^{(1)}+r^2H^{(2)}, \tag{B34}$$

where $F^{(0)-(2)}$, $G^{(0)-(2)}$, and $H^{(0)-(2)}$ are the functions of $x(\equiv 2E_\mu/m_\tau)$, $y(\equiv 2E_\gamma/m_\tau)$, $d(\equiv 1+\beta_\mu\cos z)$ with $\beta_\mu=\sqrt{1-4r/x^2}$ $(r\equiv m_\mu^2/m_\tau^2)$ and $z=\pi-\theta_{\mu\gamma}$. These functions are given by

$$F^{(0)}(x,y,d)=\frac{-8(-3+2x+2y)(2x^2+2xy+y^2)}{d}+8x\{x^2(2+4y)+y(-3+y+y^2)+x(-3+y+4y^2)\}-2x^2y\{-6+y(5+2y)+2x(4+3y)\}d+2x^3y^2(2+y)d^2, \tag{B35}$$

$$F^{(1)}(x,y,d)=\frac{32(x+y)(-3+2x+2y)}{xd^2}+\frac{8\{6x^2+(6-5y)y-2x(4+y)\}}{d}-8x\{-4-(-3+y)y+3x(1+y)\}+6x^2y(2+y)d, \tag{B36}$$

$$F^{(2)}(x,y,d)=\frac{-32(-4+3x+3y)}{x\,d^2}+\frac{48y}{d}, \tag{B37}$$

$$G^{(0)}(x,y,d)=\frac{-8x\{4x^2+y(-1+2y)+x(-2+6y)\}}{d}+4x^2\{-2+3y+4y^2+x(4+6y)\}-4x^3y(2+y)d, \tag{B38}$$

$$G^{(1)}(x,y,d)=\frac{32(-1+2x+2y)}{d^2}+\frac{8x(6x-y)}{d}-12x^2(2+y), \tag{B39}$$

$$G^{(2)}(x,y,d)=\frac{-96}{d^2}, \tag{B40}$$

$$H^{(0)}(x,y,d)=\frac{-8y(x+y)(-1+2x+2y)}{d}+4xy\{2x^2+2y(1+y)+x(-1+4y)\}-2x^2y^2(-1+4x+2y)d+2x^3y^3d^2, \tag{B41}$$

$$H^{(1)}(x,y,d)=\frac{32y(-1+2x+2y)}{xd^2}-\frac{8y(-2+x+5y)}{d}-4x(3x-2y)y+6x^2y^2d, \tag{B42}$$

$$H^{(2)}(x,y,d)=\frac{-96y}{xd^2}+\frac{48y}{d}. \tag{B43}$$

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