Large neutrino mixing from renormalization group evolution

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The renormalization group evolution equation for two neutrino mixing is known to exhibit a nontrivial ''fixed point'' structure corresponding to maximal mixing at the weak scale. Their presence provides a natural explanation of the observed maximal mixing of $\nu_\mu-\nu_\tau$ if the ν_μ and ν_τ are assumed to be quasidegenerate at the seesaw scale without constraining the mixing angles at that scale. In particular, it allows them to be similar to the quark mixings as in generic grand unified theories. We discuss implementation of this program in the case of the minimal supersymmetric standard model and find that the predicted mixing remains stable and close to its maximal value, for all energies below the O (TeV) SUSY scale. We also discuss how a particular realization of this idea can be tested in neutrinoless double beta decay experiments.

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I. INTRODUCTION

Theoretical understanding of experimentally measured neutrino anomalies poses a major challenge to unified gauge theories, especially since $\nu_\mu-\nu_\tau$ mixing has been observed to be close to maximal through atmospheric neutrino flux measurements whereas the mixing in the corresponding quark sector is small. The problem is so severe that, only over the limited span of the last two years, nearly a hundred models have been proposed where considerable effort has been devoted to accommodate large neutrino mixing [1]. There are also interesting suggestions to understand this large mixing in the context of various grand unified theories including SO(10) [2,3], which unify both quarks and leptons. It is however fair to say that no convincing and widely accepted natural model has yet emerged.

With a view to simplifying model building, we recently suggested criteria for radiative magnification of neutrino mixing [4,5] which allow a small mixing at high scale to be amplified to large mixing at the weak scale after renormalization group evolution. The only condition that needs to be satisfied is that the ν_μ and ν_τ be quasidegenerate in mass, for example, as would be independently required if Liquid Scintillation Neutrino Detector (LSND) results are confirmed. In such models, there is no need to impose special constraints on the theory at high scale beyond those needed to guarantee quasidegeneracy. They would, therefore, require less theoretical input compared to the case where one tries to obtain both degeneracy (should it be phenomenologically warranted) and maximal mixing at the high (seesaw) scale within the framework of quark lepton unification.

A key role in the above scenario is played by the renormalization group equations for neutrino masses and mixings [6,7]. In this paper, we exploit one of the most interesting and highly appealing aspect of renormalization group (RG) running of gauge and Yukawa couplings, i.e., the emergence of a behavior which has similarity to the fixed point structure in many renormalizable field theories. For simplicity we will call this fixed point (FP), even though it is different from the case of the behavior of the top Yukawa coupling [8] discussed in grand unified theories. As we will see later, our ''FP'' behavior works only for a certain range of parameters of neutrino masses in the theory.

It was noted in [6] that neutrino mixings can have fixed points corresponding to maximal mixing and several examples were given to illustrate this point in the standard model and two Higgs model. The desirable value of $\sin^2(2\theta)\sim1$ was shown to arise in these models both at the electroweak and at intermediate scales of order 10^8 GeV or so depending on the model parameters at the high scale. Our goal in the present work is to extend the discussion of [6] to supersymmetric theories [minimal supersymmetric standard model (MSSM)] and delineate the constraints on the high scale theory under which the fixed point (or maximal mixing) occurs around the weak scale. We discuss the conditions under which the value of the mixing remains stable as the energy is varied from TeV to M_Z scale. A crucial requirement for the fixed point to occur is that the muon and the tau neutrinos must be quasidegenerate. Our analysis further

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clarifies the idea of radiative magnification discussed in Refs. [4,5]. We point out that in a special class of models which extend this idea to the case of three degenerate neutrinos, searches for neutrinoless double beta decay can provide a test of these models.

The paper is organized as follows. In Sec. II, we discuss radiative correction and derivation of RG equation (RGEs) for mixing angle in the standard model (SM) and MSSM. In Sec. III, we obtain an analytic solution for the RGE and demonstrate explicitly the renormalization group fixed point (RGFP) structure. In Sec. IV, we show how the FP occurs naturally at the weak scale for quasidegenerate neutrinos leading to the condition of radiative magnification. We also derive a new stability criterion and show how the FP and magnification occur in MSSM starting from small mixings as in the quark sector. In Sec. V, we comment on tests of the radiative magnification scheme in neutrinoless double beta decay searches.

II. RADIATIVE CORRECTIONS AND RGE FOR NEUTRINO MIXING

In both SM and MSSM, we consider radiative corrections in the flavor basis to the light Majorana neutrino mass matrix, $m_{\alpha\beta}$, which is a 5-dim operator scaled by the high mass, M_N (e.g., seesaw or Majorana neutrino mass scale), where the mass matrix is generated, with

$$\mathcal{L}_{\nu\nu}=-\nu_\alpha^T C^{-1} m_{\alpha_\beta}\nu_\beta+\text{H.c.} \tag{2.1}$$

As a result of one-loop radiative corrections the RGEs below $\mu=M_N$ are [6,7]

$$\text{SM:}\quad 16\pi^2\frac{dm}{dt}=[-3g_2^2+2\lambda+\mathrm{Tr}(6Y_U^\dagger Y_U+6Y_D^\dagger Y_D+2Y_E^\dagger Y_E)]m-\frac{1}{2}[m(Y_EY_E^\dagger)+(Y_EY_E^\dagger)^T m]. \tag{2.2}$$

$$\text{MSSM:}\quad 16\pi^2\frac{dm}{dt}=\left[-\frac{6}{5}g_1^2-6g_2^2+\mathrm{Tr}(6Y_U^\dagger Y_U)\right]m+[m(Y_EY_E^\dagger)+(Y_EY_E^\dagger)^T m]. \tag{2.3}$$

In Eqs. (2.2) and (2.3) $g_1(g_2)$ are the $\mathrm{U}(1)_Y[\mathrm{SU}(2)_L]$ gauge couplings and λ is the Higgs quartic coupling (in SM) whereas $Y_U(Y_D)$ and Y_E are the Yukawa matrices for the up (down) quarks and charged leptons. We work in the charged lepton diagonal basis where the unitary matrix $U_{\alpha i}$ that transforms the mass basis to flavor basis is identified as the standard Maki-Nakagawa-Sakata (MNS) matrix [9]. Using Eq. (2.2) and (2.3), the mass matrix is evolved from the high scale down to $t(=\ln\mu)<t_0(=\ln M_N)$ in the SM or MSSM;

$$\text{SM:}\quad m(\mu)=\left(\frac{\mu}{M_N}\right)^{(\lambda/8\pi^2)} I_{g_2}^{-3}I_{\mathrm{top}}^6 I_b^3 I_\tau \times\begin{bmatrix} m_{ee}^0 I_e' & m_{e\mu}^0\sqrt{I_e'I_\mu'} & m_{e\tau}^0\sqrt{I_e'I_\tau'}\\ m_{\mu e}^0\sqrt{I_e'I_\mu'} & m_{\mu\mu}^0 I_\mu' & m_{\mu\tau}^0\sqrt{I_\mu'I_\tau'}\\ m_{\tau e}^0\sqrt{I_e'I_\tau'} & m_{\tau\mu}^0\sqrt{I_\mu'I_\tau'} & m_{\tau\tau}^0 I_\tau'\end{bmatrix}. \tag{2.4}$$

$$\text{MSSM:}\quad m(\mu)=I_{g_1}^{-6/5}I_{g_2}^{-6}I_{\mathrm{top}}^6 \times\begin{bmatrix} m_{ee}^0 I_e^2 & m_{e\mu}^0 I_eI_\mu & m_{e\tau}^0 I_eI_\tau\\ m_{\mu e}^0 I_eI_\mu & m_{\mu\mu}^0 I_\mu^2 & m_{\mu\tau}^0 I_\mu I_\tau\\ m_{\tau e}^0 I_eI_\tau & m_{\tau\mu}^0 I_\mu I_\tau & m_{\tau\tau}^0 I_\tau^2\end{bmatrix}. \tag{2.5}$$

Here,

$$I_h(\equiv I_h'^{-1})=e^{\delta_h}=\exp\left(\frac{1}{16\pi^2}\int_{t_0}^{t}h^2(t')dt'\right), \tag{2.6}$$

and h denotes the gauge coupling (g_1,g_2) or the Yukawa-coupling-eigenvalue for quarks and charged leptons $(y_{\mathrm{top}},y_b,y_\tau,y_\mu,y_e)$. When the running vacuum expectation value (VEV) of the up type Higgs doublet in MSSM is taken into account the common factor in Eq. (2.5) is changed with the replacement, $I_{g_1}^{-6/5}I_{g_2}^{-6}I_{\mathrm{top}}^6\to I_{g_1}^{-9/10}I_{g_2}^{-9/2}$ and similarly in SM. In subsequent discussions for the mixing angle we ignore common renormalization factors in Eqs. (2.4) and Eq. (2.5) as they cancel out in the relevant expressions. At any value of $t<t_0$,

$$\tan 2\theta(t)=\frac{2m_{\mu\tau}(t)}{m_{\tau\tau}(t)-m_{\mu\mu}(t)}, \tag{2.7}$$

and Eqs. (2.2), (2.3), and (2.7) give the RGEs for $\sin^2(2\theta)$,

$$\text{SM:}\quad 16\pi^2\frac{d\sin^2 2\theta}{dt}=\sin^2 2\theta\cos^2 2\theta(y_\tau^2-y_\mu^2)\frac{m_{\tau\tau}+m_{\mu\mu}}{m_{\tau\tau}-m_{\mu\mu}}=\sin^2 2\theta\frac{(m_{\tau\tau}^2-m_{\mu\mu}^2)(y_\tau^2-y_\mu^2)}{(m_{\tau\tau}-m_{\mu\mu})^2+4m_{\mu\tau}^2}. \tag{2.8}$$

MSSM:

$$16\pi^2\frac{d\sin^2 2\theta}{dt}=-2\sin^2 2\theta\cos^2 2\theta(y_\tau^2-y_\mu^2)\times\frac{m_{\tau\tau}+m_{\mu\mu}}{m_{\tau\tau}-m_{\mu\mu}}=-2\sin^2 2\theta\frac{(m_{\tau\tau}^2-m_{\mu\mu}^2)(y_\tau^2-y_\mu^2)}{(m_{\tau\tau}-m_{\mu\mu})^2+4m_{\mu\tau}^2}. \tag{2.9}$$

All quantities in the right-hand side (RHS) of Eqs. (2.8) and (2.9) are t-dependent. As was noted in [6], both RGEs have one trivial fixed point at $\sin^2 2\theta=0$ and the other nontrivial fixed point at $\sin^2 2\theta=1$. Recently the FP structure of the MNS matrix has been investigated in [10]. Assuming that the initial high-scale texture of the mass matrix is such that the nontrivial fixed point occurs at a scale $\mu_c(M_Z \leqslant \mu_c < M_N, t_c = \ln \mu_c)$, we have the FP condition

$$\sin^2 2\theta(t_c)=1, \tag{2.10}$$

or, equivalently,

$$m_{\tau\tau}(t_c)=m_{\mu\mu}(t_c). \tag{2.11}$$

III. ANALYTIC FORMULA AND FIXED POINT

Before obtaining analytic solutions to Eqs. (2.8) and (2.9), it is worthwhile to explain why resonance structures in the numerical solutions [6,10–12] in the $\sin^2 2\theta(t)$ vs t plots are expected for specific textures of $m^0_{\alpha\beta}$.

Noting that

$$|y_\tau(t)|^2 \gg |y_\mu(t)|^2, \tag{3.1}$$

Eq. (2.5) states that in MSSM, as t decreases below t_0, the ratio, $R_\tau(t)=m_{\tau\tau}(t)/m^0_{\tau\tau}$ decreases faster from its high scale value $R_\tau(t_0)=1$, as compared to the rate of decrease of the ratio $R_\mu(t)=m_{\mu\mu}(t)/m^0_{\mu\mu}$. In particular, the relations in Eqs. (2.10) and (2.11) are satisfied at $t=t_c$ if

$$m^0_{\tau\tau} e^{2\delta_\tau(t_c)}=m^0_{\mu\mu}. \tag{3.2}$$

For the FP to occur at $t_c<t_0$, the high scale texture must be such that $m^0_{\tau\tau}$ and $m^0_{\mu\mu}$ are comparable but unequal with $m^0_{\tau\tau}>m^0_{\mu\mu}$. Lower values of t_c correspond to larger differences between $m^0_{\tau\tau}$ and $m^0_{\mu\mu}$. Also note that since $\delta_\tau(t_c)$ depends on $\tan\beta$ parameter of MSSM, the initial values $m^0_{\tau\tau,\mu\mu}$ will depend on it in a crucial manner.

From Eq. (2.9) it is clear that when $m_{\tau\tau}(t_c)=m_{\mu\mu}(t_c)$, the slope of the curve in the $\sin^2 2\theta(t)$ vs t plot vanishes at $t=t_c$. For $t>t_c$, $m_{\tau\tau}(t)>m_{\mu\mu}(t)$, the slope is negative, but for $t<t_c$, $m_{\tau\tau}(t)<m_{\mu\mu}(t)$, the slope is positive as given by the RGE. Negative (positive) slope to the right (left) with vanishing slope at $t=t_c$ is the characteristic feature of a resonance curve as predicted by Eq. (2.9) for MSSM. A similar result emerges for SM from Eq. (2.8) with a somewhat different high scale condition with $m^0_{\mu\mu}>m^0_{\tau\tau}$ and the ratios $R_\tau(t)$ and $R_\mu(t)$ increase as t decreases below t_0. Thus it is clear that for certain given textures at high scale ($m^0_{\mu\mu}, m^0_{\tau\tau}$ and $m^0_{\mu\tau}$) resonance occurs at $t=t_c$ converting small mixing at high scale to large mixing at lower scales.

In spite of the terse nature of the RHS of Eq. (2.9), using the almost exact approximation, $|\delta_\tau(t)| \gg |\delta_\mu(t)|$, we have integrated it to obtain an analytic solution for the RG evolution of $\sin^2 2\theta$ in the MSSM for all values of $\mu<M_N$,

$$\sin^2 2\theta(t)=\sin^2 2\theta_0 \frac{[(m^0_{\tau\tau}-m^0_{\mu\mu})^2+4m^{0\,2}_{\mu\tau}]e^{2\delta_\tau(t)}}{(m^0_{\tau\tau}e^{2\delta_\tau(t)}-m^0_{\mu\mu})^2+4m^{0\,2}_{\mu\tau}e^{2\delta_\tau(t)}}, \tag{3.3}$$

where θ_0 is the high scale mixing angle with

$$\tan 2\theta_0=\frac{2m^0_{\mu\tau}}{(m^0_{\tau\tau}-m^0_{\mu\mu})}. \tag{3.4}$$

Given

$$m_{\mu\mu}(t)/m_{\tau\tau}(t)=(m^0_{\mu\mu}/m^0_{\tau\tau})e^{2\delta_\mu(t)}e^{-2\delta_\tau(t)},$$

$$m_{\mu\tau}(t)/m_{\tau\tau}(t)=(m^0_{\mu\tau}/m^0_{\tau\tau})e^{\delta_\mu(t)}e^{-\delta_\tau(t)},$$

and using Eqs. (2.5)–(2.7), Eq. (3.3) may be recognized as the approximation $|\delta_\mu(t)| \ll |\delta_\tau(t)|$ to the following exact analytic solution of Eq. (2.9),

$$\sin^2 2\theta(t)=\sin^2 2\theta_0 \frac{[(m^0_{\tau\tau}-m^0_{\mu\mu})^2+4m^{0\,2}_{\mu\tau}]e^{2\delta_\tau(t)}e^{2\delta_\mu(t)}}{(m_{\tau\tau}(t)-m_{\mu\mu}(t))^2+4m_{\mu\tau}(t)^2}. \tag{3.5}$$

Replacing $\theta(t)\to\theta(\mu)$, $m_{ij}(t)\to m_{ij}(\mu)$, $\theta_0\to\theta(M)$, and $m^0_{ij}\to m_{ij}(M)$, formulas (3.3) or (3.5) can be used to derive $\theta(\mu)$ from $\theta(M)$ or vice versa for all values of $\mu<M \leqslant M_N$. It is interesting to note that these analytic solutions exhibit both the resonance as well as the nontrivial FP structure explicitly. While detailed features of resonance such as the t-dependent width, maximal mixing at the peak, and smaller mixings for $t>t_c$ or $t<t_c$ are clearly exhibited, the FP structure is proved as follows. At $t=t_c$, when Eqs. (2.11) or (3.2) are satisfied, the quantity inside the parenthesis in the denominator of Eqs. (3.5) or (3.3) vanishes. Then using Eqs. (3.4), (3.3), and (3.5) give

$$\sin^2 2\theta(t_c)=\sin^2 2\theta_0\left[\frac{(m^0_{\tau\tau}-m^0_{\mu\mu})^2}{4m^{0\,2}_{\mu\tau}}+1\right]=\sin^2 2\theta_0 + \cos^2 2\theta_0=1. \tag{3.6}$$

It is to be noted that Eq. (3.6) holds for all initial values of $\theta_0<\pi/4$, thus demonstrating the fixed point behavior corresponding to maximal mixing. Although the relation (3.6) appears to be true also for $\theta_0=\pi/4$ showing that maximal mixing remains maximal at $t=t_c$, the RG evolution equations never satisfy $m_{\tau\tau}(t_c)=m_{\mu\mu}(t_c)$ for $t_c \ll t_0$ if we start with the initial condition $m^0_{\tau\tau}=m^0_{\mu\mu}$ which is necessary for $\theta_0=\pi/4$. In fact nearly maximal mixings at the high scale are damped out to small mixings at lower scales ($\mu \ll M_N$) due to nonvanishing contributions of the quantity $[m_{\tau\tau}(t_c)-m_{\mu\mu}(t_c)]^2$ in the RHS of Eqs. (3.3) or (3.5). Thus the analytic formula, apart from demonstrating the FP structure and resonance behavior, also explains why large mixing at high scales is damped out to small mixings near the weak scale. Also, the zero mixing angle does not run and continues to be zero down to $\mu=M_Z$. Similar analytic solutions are also obtained for SM exhibiting the FP structure with the replacement $2\delta_i(t)\to-\delta_i(t)$, $i=\mu,\tau$ in Eqs. (3.3)–(3.5).

In almost all cases of RG fixed point discussed so far in the literature, the FP structure is revealed through the differential RGEs and demonstrated through numerical solutions only. But in the present case, apart from the differential RGE and numerical solutions (see Sec. IV), the analytic solutions also exhibit the FP structure explicitly as demonstrated through Eqs. (3.3)–(3.6).

IV. RADIATIVE MAGNIFICATION THROUGH THE FIXED POINT AND STABILITY

When the condition in Eqs. (2.10) or (2.11) is satisfied for $t_c = t_S = \ln M_S$ (M_S = SUSY scale), the FP may manifest as a large neutrino mixing observed at low energies, for example, in the $\nu_\mu - \nu_\tau$ oscillation scenario necessary to solve the atmospheric neutrino anomaly. In terms of the high scale mass eigenvalues (m_2^0, m_3^0), mixing angle (θ_0), and radiative correction parameters, the condition for FP manifestation at $\mu = \mu_c = M_S$ then reduces to

$$(m_2^0 - m_3^0) c_{2\theta_0} = 2\delta_\tau(t_S)(m_2^0 s_{\theta_0}^2 + m_3^0 c_{\theta_0}^2) - 2\delta_\mu(t_S)(m_2^0 c_{\theta_0}^2 + m_3^0 s_{\theta_0}^2), \qquad (4.1)$$

where $s_{\theta_0} = \sin\theta_0$, $c_{\theta_0} = \cos\theta_0$, $c_{2\theta_0} = \cos 2\theta_0$ and $s_{2\theta_0} = \sin 2\theta_0$. Taking $M_S = M_Z$, this is recognized exactly as the condition that was derived in [4,5] for magnifying small mixing at high scale to large mixing at low energies through radiative corrections. But, as noted here, the condition is exact, needs no fine tuning, and emerges as a natural consequence of the manifestation of the FP at the weak scale. For small mixing angles at $\mu = M_N$, similar to those existing in the quark sector (e.g., $\theta_0 \approx V_{cb} \approx 0.04$), $c_{\theta_0} \approx c_{2\theta_0} \approx 1$ and $s_{\theta_0}^2 \sim 0$, it is clear that the condition (4.1) cannot be satisfied if the masses m_2^0 and m_3^0 are hierarchial, or exactly degenerate having the same ($m_2^0 = m_3^0$) or opposite CP parity ($m_2^0 = -m_3^0$). Also it cannot be satisfied if the masses are quasidegenerate with opposite CP parity ($m_2^0 = -m_3^0$). It can be satisfied only if the masses are quasidegenerate at the high scale having the same CP parity ($m_2^0 = m_3^0$). Since δ_τ is negative, a necessary prediction of MSSM is that $m_3^0 > m_2^0$. In the SM, $2\delta_\tau(t_Z)$ and $2\delta_\mu(t_Z)$ in Eq. (4.1) are replaced by $-\delta_\tau(t_Z)$ and $-\delta_\mu(t_Z)$, respectively, and Eq. (4.1) predicts $m_2^0 > m_3^0$. These requirements in MSSM or SM are analogous to the occurrence of quasi fixed points in top-quark Yukawa coupling where right order of the top quark mass is obtained only for certain strong interaction couplings. We emphasize that the observed large neutrino mixing in the $\nu_\mu - \nu_\tau$ sector predicts the corresponding ν_2 (ν_3) masses to be quasidegenerate with the same CP parity as a necessary requirement in order that the FP manifests at the lower scale. Under the condition (2.11), with $t_c = t_S$, the mass eigenvalues at $\mu = M_S$ are

$$m_2(t_S) = (m_2^0 c_{\theta_0}^2 + m_3^0 s_{\theta_0}^2)[1 + 2\delta_\mu(t_S)] - (m_3^0 - m_2^0) c_{\theta_0} s_{\theta_0} \times [1 + \delta_\tau(t_S) + \delta_\mu(t_S)], \qquad (4.2)$$

$$m_3(t_S) = (m_2^0 c_{\theta_0}^2 + m_3^0 s_{\theta_0}^2)[1 + 2\delta_\mu(t_S)] + (m_3^0 - m_2^0) c_{\theta_0} s_{\theta_0} \times [1 + \delta_\tau(t_S) + \delta_\mu(t_S)]. \qquad (4.3)$$

Taking the high scale mixings to be small, we obtain the mass squared difference at $\mu = M_S$,

$$\Delta m^2 \equiv m_3^2 - m_2^2 \approx \Delta m^{02} s_{2\theta_0}[1 + \delta_\tau(t_S)], \qquad (4.4)$$

where

$$\Delta m^{02} \approx 2 m_2^{02}(e^{-2\delta_\tau(t_S)} - 1) \approx -4 m_2^{02} \delta_\tau(t_S). \qquad (4.5)$$

Before proceeding further, we show analytically how the stability of radiative magnification is controlled by the high scale mixing angle. To generate nearly maximal mixing at a lower scale ($\mu = \mu_c = M_S = M_Z$) starting from small mixing as in the quark sector at the high scale (e.g., $\theta_0 \approx V_{cb} \approx 0.04$), the FP position is desired to be stable near $M_S = M_Z$. As the FP is a consequence of radiative corrections, the stability must be guaranteed against smaller changes in the neutrino mass matrix due to higher order corrections. To maintain such stability this requires the mixing to be nearly maximal within at least $\mu \sim$ few (M_Z). In fact we show that radiative stability is ensured over a larger range. We define the range, $t = t_S$ to t_Γ ($\mu = M_S$ to μ_Γ), within which the mixing remains nearly maximal. Noting that

$$\delta_\tau(t_\Gamma) = \delta_\tau(t_S) + \epsilon_\tau(t_\Gamma), \qquad (4.6)$$

with

$$\epsilon_\tau(t_\Gamma) \approx \frac{y_\tau^2}{16\pi^2} \ln\frac{\mu_\Gamma}{M_S} = \frac{m_\tau^2(1 + \tan^2\beta)}{16\pi^2 v^2} \ln\frac{\mu_\Gamma}{M_S}, \qquad (4.7)$$

which remains small ($|\epsilon_\tau| \ll 1$) over a wide range of y_τ, we use the FP condition (2.11) and (3.2), in Eqs. (3.3) and (3.5) to obtain

$$\sin^2 2\theta(t_\Gamma) \approx \frac{1 + 2\delta_\tau(t_\Gamma)}{[1 + 2\delta_\tau(t_\Gamma) + (m_{\mu\mu}^{0\ 2}/m_{\mu\tau}^{0\ 2})\epsilon^2(t_\Gamma)]}. \qquad (4.8)$$

The stability criterion for the FP position and radiative magnification at $\mu \approx M_S$ may be stated as

$$\frac{y_\tau^4 m_{\mu\mu}^{0\ 2}}{256\pi^4 m_{\mu\tau}^{0\ 2}} \left(\ln\frac{\mu_\Gamma}{M_S}\right)^2 \ll 1. \qquad (4.9)$$

This clearly has the implication that arbitrarily small values of high scale mixing cannot maintain a stable FP whereas zero initial mixing continues to remain zero at all lower values of t and is never magnified. For smaller values of θ_0 or $m_{\mu\tau}^0$, the contribution of the third term in the denominator in Eq. (4.8) becomes larger leading to sharper decrease of the predicted low scale mixing angle from its maximal fixed point value. This results in the smaller width of the resonance for smaller values of high scale mixing (θ_0 or $m_{\mu\tau}^0$). This feature is clearly exhibited through Figs. 1 and 2, where we have presented $\sin 2\theta(\mu)$ for $\mu = 100$ GeV–1 TeV taking $M_S = M_Z$, $M_N = 10^{13}$ GeV, $\tan\beta = 50$ and $y_\tau = 0.49$ with $e^{2\delta_\tau(M_Z)} = 0.929$. The high scale parameters for Fig. 1 are $m_{\tau\tau}^0 \approx m_3^0 \approx 0.28$ eV, $m_{\mu\mu}^0 \approx m_2^0 \approx 0.26$ eV, and $m_{\mu\tau}^0 \approx 0.0044$

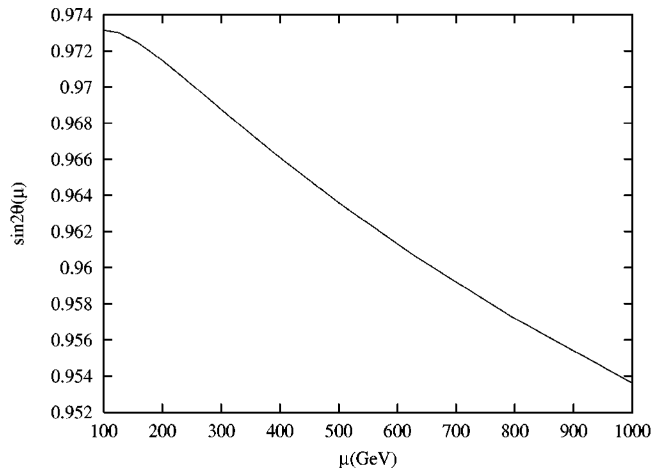

FIG. 1. Manifestation of the fixed point at the weak scale and stability of radiative magnification of small high scale mixing $\sin\theta_0 \approx 0.22$ to nearly maximal mixing at low energies for $y_\tau \approx 0.48$, $\tan\beta \approx 50$, $m^0_{\mu\mu} \approx m^0_2 \approx 0.26$ eV, $m^0_{\tau\tau} \approx m^0_3 \approx 0.28$ eV, and $m^0_{\mu\tau} = 0.0044$ eV consistent with atmospheric neutrino data ($\Delta m^2 \approx 4\times 10^{-3}$ eV^2).

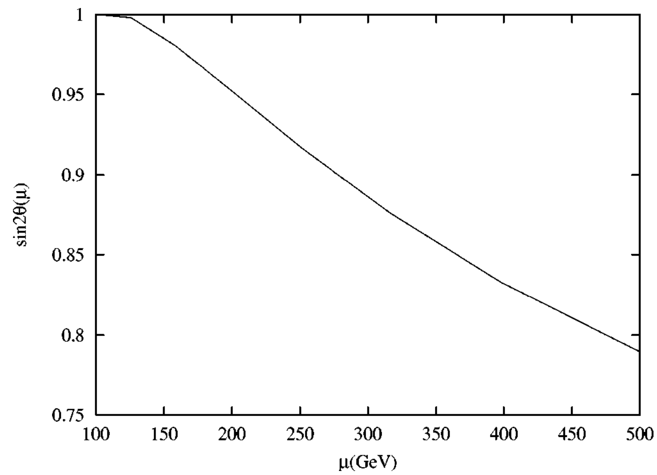

FIG. 2. Same as Fig. 1 but for $\sin\theta_0 \approx V_{cb} \approx 0.04$, $y_\tau \approx 0.49$, $m^0_{\mu\mu} \approx m^0_2 \approx 0.25$ eV, $m^0_{\tau\tau} \approx m^0_3 \approx 0.27$ eV, $m^0_{\mu\tau} \approx 0.0008$ eV, and $\Delta m^2 \approx 7\times 10^{-4}$ eV^2 consistent with atmospheric neutrino data.

eV corresponding to $\theta_0 = 0.22$ consistent with $\Delta m^2 \approx 4 \times 10^{-3}$ eV^2 needed for atmospheric neutrino data. For Fig. 2 these parameters are $m^0_{\tau\tau} \approx m^0_3 \approx 0.27$ eV, $m^0_{\mu\mu} \approx m^0_2 \approx 0.25$ eV, and $m^0_{\mu\tau} \approx 0.0008$ eV corresponding to $\theta_0 = V_{cb} = 0.04$ consistent with $\Delta m^2 \approx 7\times 10^{-3}$ eV^2. It is clear that in Fig. 2 the width is substantially narrower than Fig. 1 and $\sin 2\theta(\mu)$ reduces by nearly 20% from its maximal value over the range of 100–500 GeV. Such energy dependent mixing between the two neutrinos, as a prediction of MSSM when both the FP and the SUSY scale are at $M_S = M_Z$ might be possible to testify or falsify in the future by high energy neutrino experiments.

In contrast to the energy dependent mixing discussed above, for the first time we find here a very attractive new feature of the other class of MSSM with higher SUSY scale $M_S = O$ (TeV) where stable and almost energy independent mixing, close to its maximal value, is predicted over a wider range of energy scale $\mu = M_Z$–few TeV starting from the high scale mixing similar to the quark sector, $\theta_0 = V_{cb} \approx 0.04$. Using the technique explained above, the high scale parameters are chosen to have the RG fixed point at the SUSY scale $M_S = 1$ or few TeV. Then the origin of negligible energy dependence in the predicted mixing at all lower energy scales is explained by noting the non-SUSY SM prediction for which $y_\tau \approx 0.01$ below M_S,

SM: $M_Z \leqslant \mu \leqslant M_S$,

$$\sin^2 2\theta(\mu) = \sin^2 2\theta(M_S) \frac{\{[m_{\tau\tau}(M_S) - m_{\mu\mu}(M_S)]^2 + 4m^2_{\mu\tau}(M_S)\}(M_S/\mu)^{y^2_\tau/16\pi^2}}{[m_{\tau\tau}(M_S)(M_S/\mu)^{y^2_\tau/16\pi^2} - m_{\mu\mu}(M_S)]^2 + 4m^2_{\mu\tau}(M_S)(M_S/\mu)^{y^2_\tau/16\pi^2}} \tag{4.10}$$

$$= \sin^2 2\theta(M_S) \frac{4m^2_{\mu\tau}(M_S)(M_S/\mu)^{y^2_\tau/16\pi^2}}{m^2_{\mu\mu}(M_S)[(M_S/\mu)^{y^2_\tau/16\pi^2} - 1]^2 + 4m^2_{\mu\tau}(M_S)(M_S/\mu)^{y^2_\tau/16\pi^2}}, \tag{4.11}$$

where Eq. (4.11) has been obtained from Eq. (4.10) by using the FP condition $m_{\tau\tau}(M_S) = m_{\mu\mu}(M_S)$. Then, because of smallness of the τ-Yukawa coupling in the SM with $y_\tau \simeq 0.01$ in Eq. (4.11) there is negligible μ dependence and the predicted mixing remains stable, close to its maximal value, for all values of μ below $M_S = O$ (TeV). This behavior is shown in Fig. 3 for initial values of $\theta_0 = V_{cb} = 0.04$, $m^0_3 \approx m^0_{\tau\tau} = 0.1543$ eV, $m^0_2 \approx m^0_{\mu\mu} = 0.1434$ eV, $m^0_{\mu\tau} = 0.00044$ eV, $\Delta m^2 = 4\times 10^{-3}$ eV^2 and other values of parameters same as in Figs. 1 and 2, but now having the FP at $M_S \approx 1$ TeV. The dashed line of Fig. 3 shows the continuation of the resonance structure at $\mu_c \approx 1$ TeV when the SUSY scale is M_Z and the evolution of mixing throughout is as in MSSM given by Eqs. (3.3) or (3.5). The part of the solid line below $\mu \approx 1$ TeV exhibiting almost flat behavior of the predicted mixing angle, with $\sin 2\theta(\mu) \approx 0.99$, has been obtained using Eq. (4.11) with $y_\tau = 0.01$ and corresponds to the FP and the SUSY scale both at $M_S \approx 1$ TeV. In this case the formula in Eq. (4.8) applies to the part of the curve above $M_S \approx 1$ TeV. Thus, we have shown for the first time that after radiative magnification through manifestation of fixed point at $\mu = M_S = O$ (TeV) the predicted mixing remains stable and

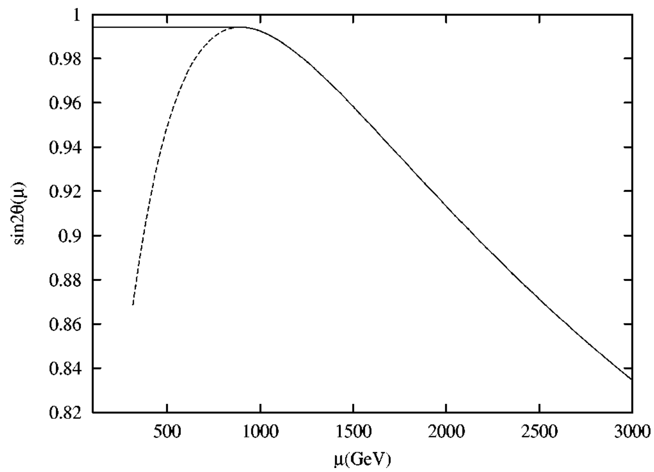

FIG. 3. Radiative magnification and an explicit demonstration of stability of large neutrino mixing at lower scales for $\mu \leqslant M_S$. The high scale parameters are $M_N = 10^{13}$ GeV, $\sin\theta_0 = V_{cb} = 0.04$, $m^0_{\tau\tau} \approx m^0_3 = 0.1543$ eV, $m^0_{\mu\mu} \approx m^0_2 = 0.1434$ eV, $m^0_{\mu\tau} \approx 0.00044$ eV with $\Delta m^2 \approx 4\times 10^{-3}$ eV^2. The dashed line is the continuation of the resonance curve with the FP at ≈ 1 TeV and SUSY scale $M_S = M_Z$. The part of the solid line almost flat below 1 TeV has been obtained with both the FP and SUSY scale at ≈ 1 TeV. The value of y_τ is 0.49(0.01) for MSSM(SM) corresponding to $\tan\beta = 50$.

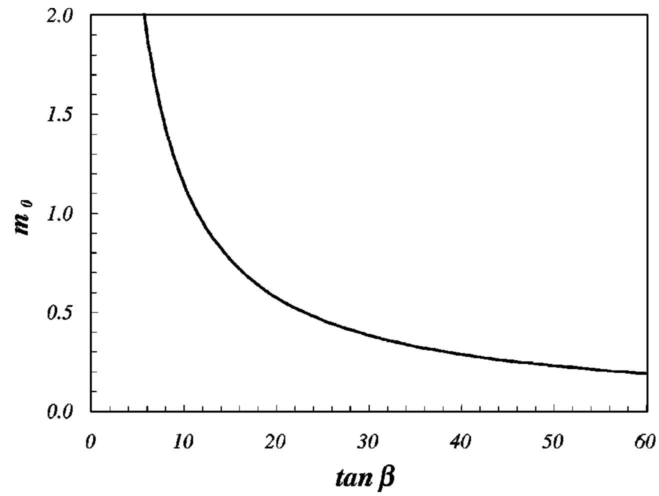

FIG. 4. Allowed lower bounds for the common mass m_0 (in eV), for varying $\tan\beta$, with $\Delta m^2_{atm} \approx 4\times 10^{-3}$ eV^2, $M_N = 10^{13}$ GeV, and $M_S = 1$ TeV. For any other value of M_N/M_S, the corresponding lower value for m_0 scales accordingly.

close to its maximal value at all lower energy scales. In this regard our analysis favors the class of MSSM with O (TeV) SUSY scale.

As explained in [4,5] while keeping the quasidegenerate eigenstates ν_2 and ν_3 to have the same CP parity for radiative magnification, it is necessary to have CP parity of ν_1 to be opposite to prevent radiative magnification in the $\nu_e - \nu_\tau$ sector from small values of θ_{13} which are consistent with CHOOZ-PALOVERDE [13] bound. In this case the solar neutrino anomaly is explained by $\nu_e \to \nu_\mu$ oscillation through small angle MSW effect.

V. TESTING RADIATIVE MAGNIFICATION BY NEUTRINOLESS DOUBLE BETA DECAY

In this section, we briefly remark on the implications of our magnification scheme for neutrinoless double beta decay experiments.

So far we have considered only two generation mixing. In complete models, one will have to embed this mechanism into scenarios with three generations or three generations plus a sterile neutrino. In the former case, if ν_μ and ν_τ are degenerate, then we have all three neutrinos nearly degenerate in mass in order to fit solar and atmospheric neutrino data. In particular, we could have all three neutrinos to have the same CP. An example of such an extension is given in [5]. We see below that in this particular embedding of our scenario, neutrinoless double beta decay can provide a test of the idea of radiative magnification of the atmospheric neutrino mixing.

Neutrinoless double beta decay experiment measures

$$m_{ee} = \sum_k U^2_{ek} m_k, \tag{5.1}$$

where k denotes the mass eigenstate label. For our case with $U_{ek} \ll 1$, $m_{ee} \simeq m_0$ where m_0 is the common mass of all the neutrinos. We will show now that for the radiative magnification scheme to work, one must have a lower limit on the common mass of all neutrinos m_0 which depends on the value of $\tan\beta$ of the MSSM. From Eq. (4.5), we have the lower bound

$$\frac{4\pi^2 v^2 \Delta m^2_{atm}}{m^2_\tau (1+\tan^2\beta)\ln(M_N/M_S)} \leqslant m^2_0. \tag{5.2}$$

In Fig. 4, for a fixed M_N/M_S, we show the variation of the lower bound on m_0 with $\tan\beta$. We have chosen $M_N = 10^{13}$ GeV, $M_S = 1$ TeV and we see that for lower $\tan\beta$ values, the lower bound on the common mass increases. In fact, for small initial mixings, a lower common mass implies a larger $\tan\beta$. For large $\tan\beta \approx 50$–60, with $M_N = 10^{13}$ GeV and $M_S = 1$ TeV, the lower bound on the common mass varies in the range $m_0 \approx 0.18$–0.20 eV. Thus, once supersymmetry is discovered and the value of $\tan\beta$ is determined, combining this with the improved searches for neutrinoless double beta decay [14], one can test the idea of radiative magnification for the three generation model. In particular, note that the lower limit on m_0 predicted above is very near the present upper limits. This should provide strong motivation to improve the limits on the lifetime of neutrinoless double beta decay.

VI. CONCLUSION

We presented the analytic formula for RG evolution of neutrino mixing which demonstrates explicitly the FP structure corresponding to maximal neutrino mixing at the weak scale leading to the condition of radiative magnification. We-have derived stability criterion for radiative magnification and show that the radiatively magnified two-neutrino mixing, predicted by the RG fixed point structure, remains stable and close to its maximal value, for all energy scales below the O (TeV) SUSY scale in MSSM. This result is specific to the MSSM and cannot be realized in non-SUSY SM. When this mechanism is applied to the standard model, one gets only a resonance structure with maximal mixing at M_Z and smaller mixings at all higher scales which are energy dependent. Our numerical computations with the help of the analytic formulas clearly show that radiative magnification of high scale neutrino mixing takes place for quasidegenerate neutrinos having the same CP parity and it remains stable only for the MSSM. We point out a very interesting test of the three generation embedding of this model by improving limits on the common mass m_0 from $0\nu\beta\beta$ searches.[1]

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